

Énumération de cartes décorées et marches dans le quart de plan

Éric Fusy - Gilles Schaeffer
Erkan Narmanli

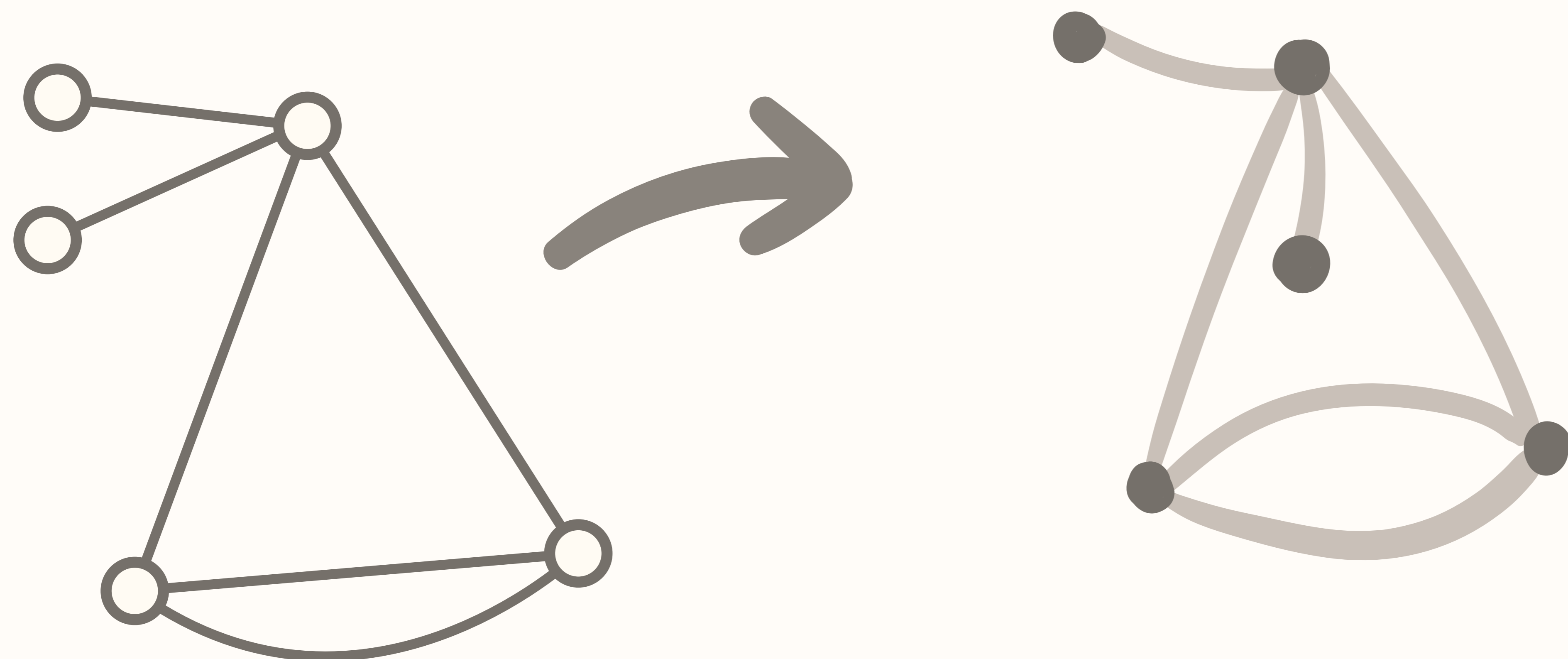


ÉCOLE
POLYTECHNIQUE

Introduction

Maps

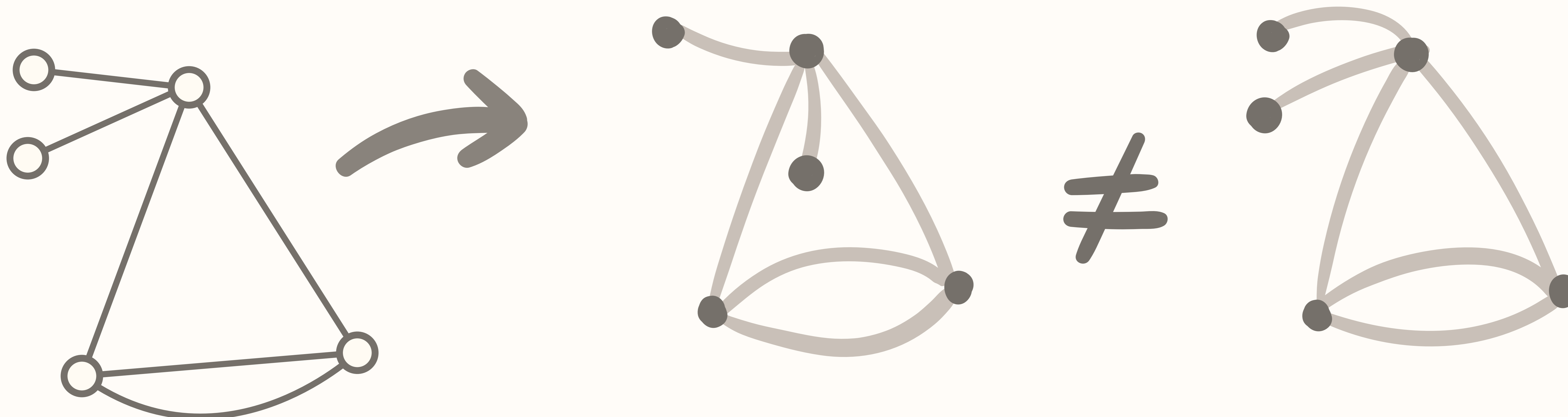
graph embedding on the plane



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Maps

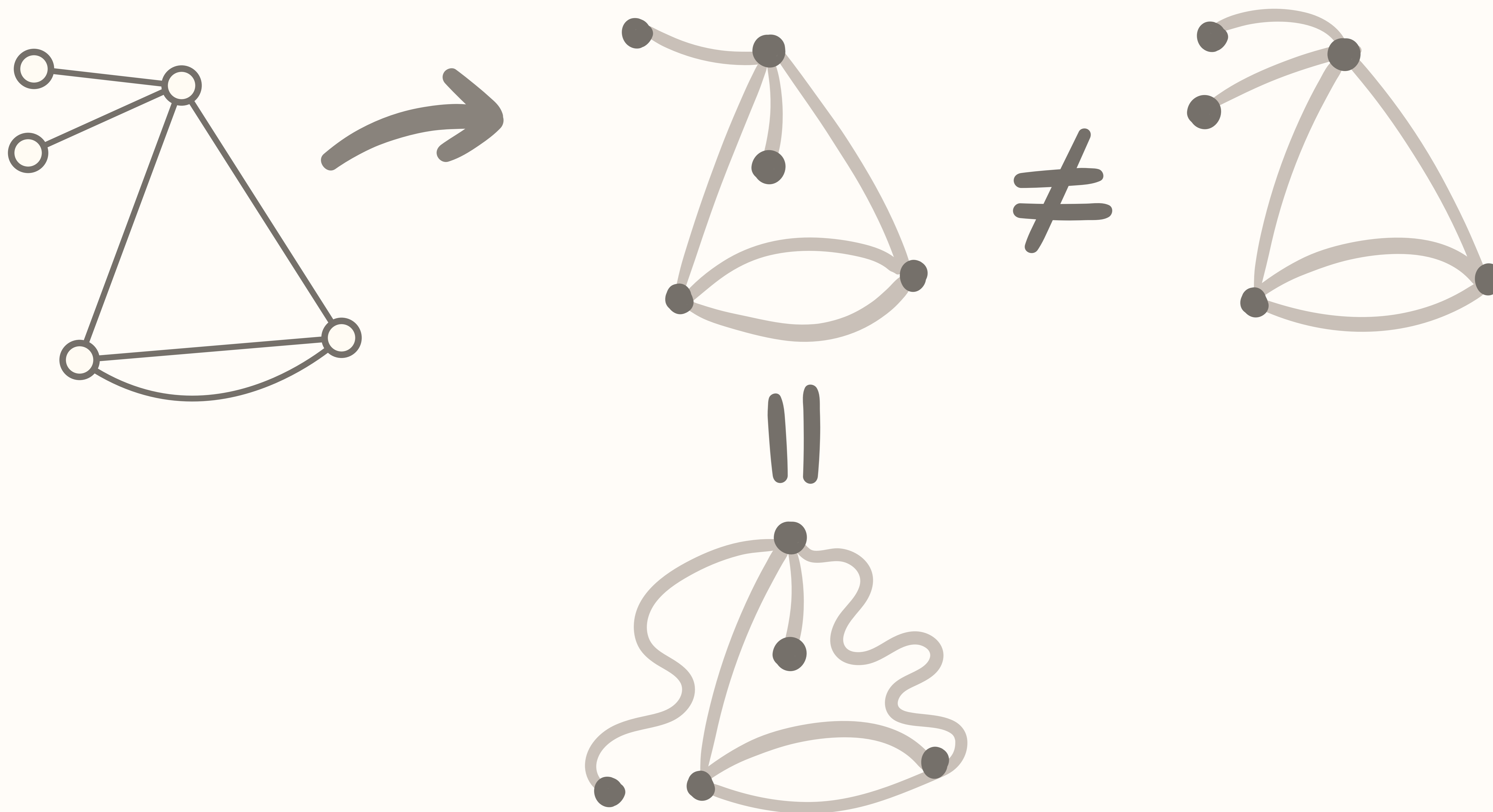
graph embedding on the plane



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Maps

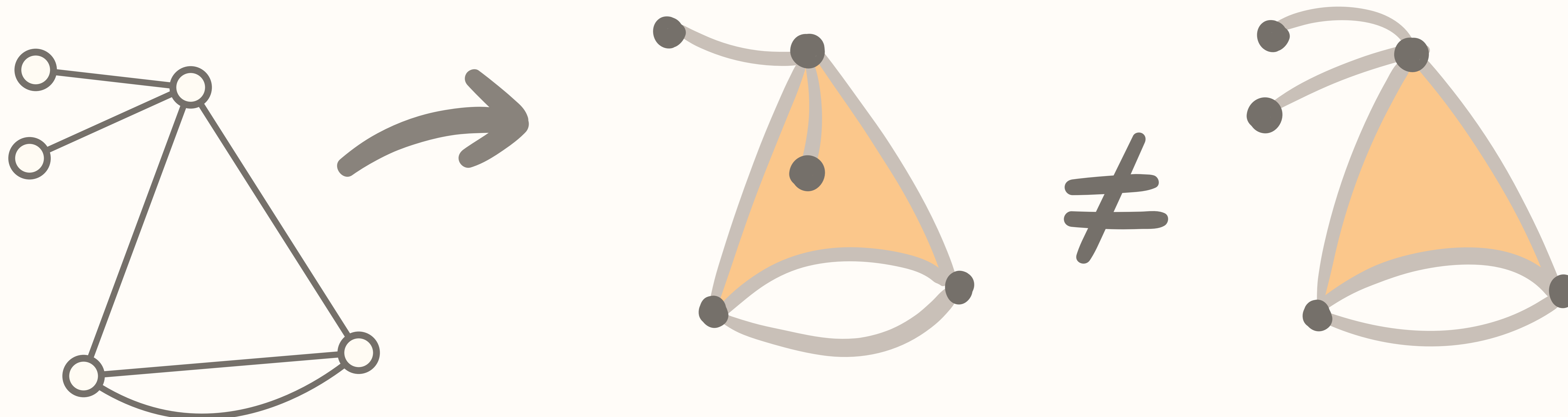
graph embedding on the plane



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Maps

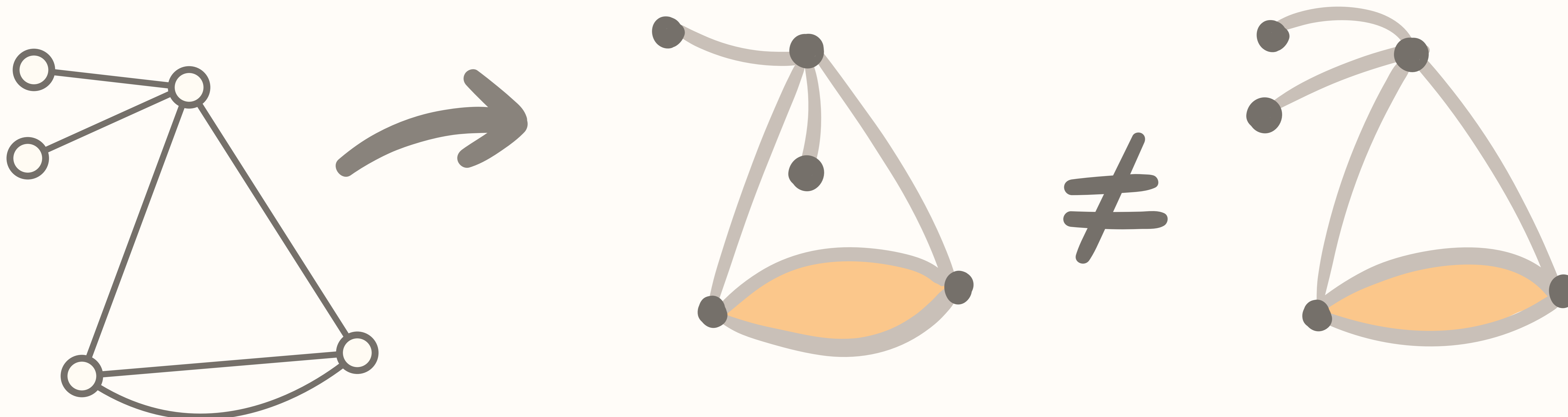
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Maps

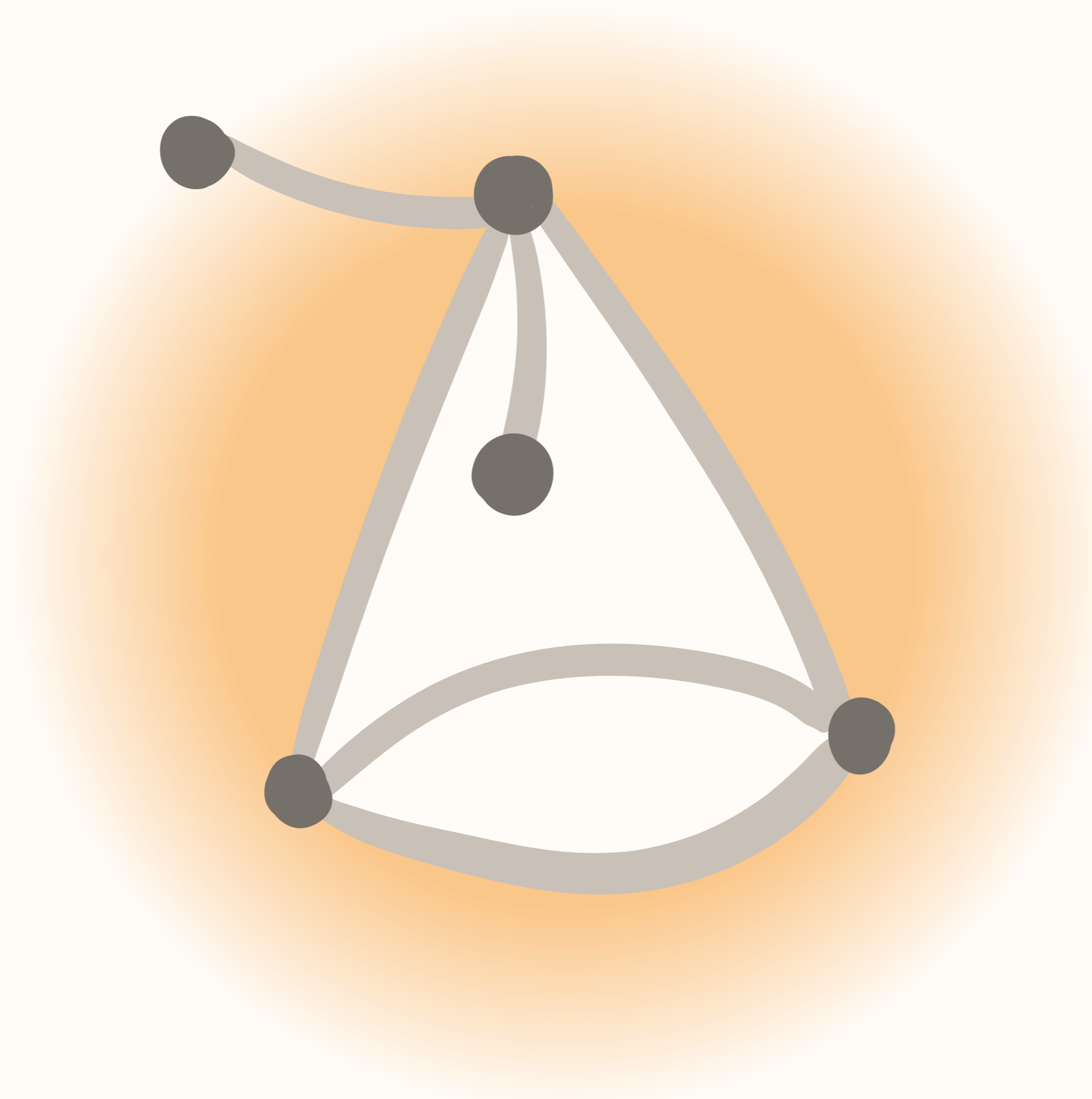
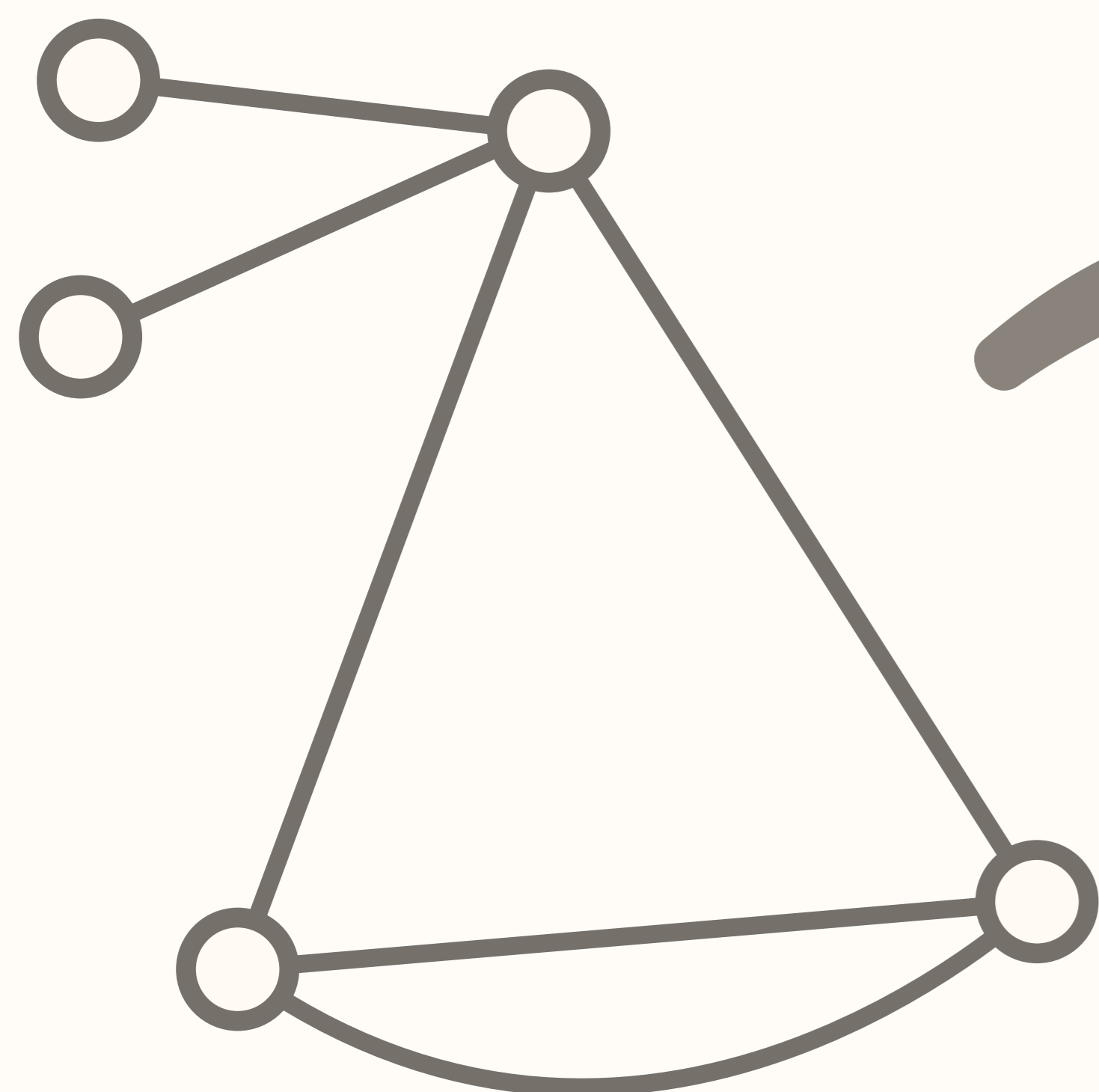
graph embedding on the plane



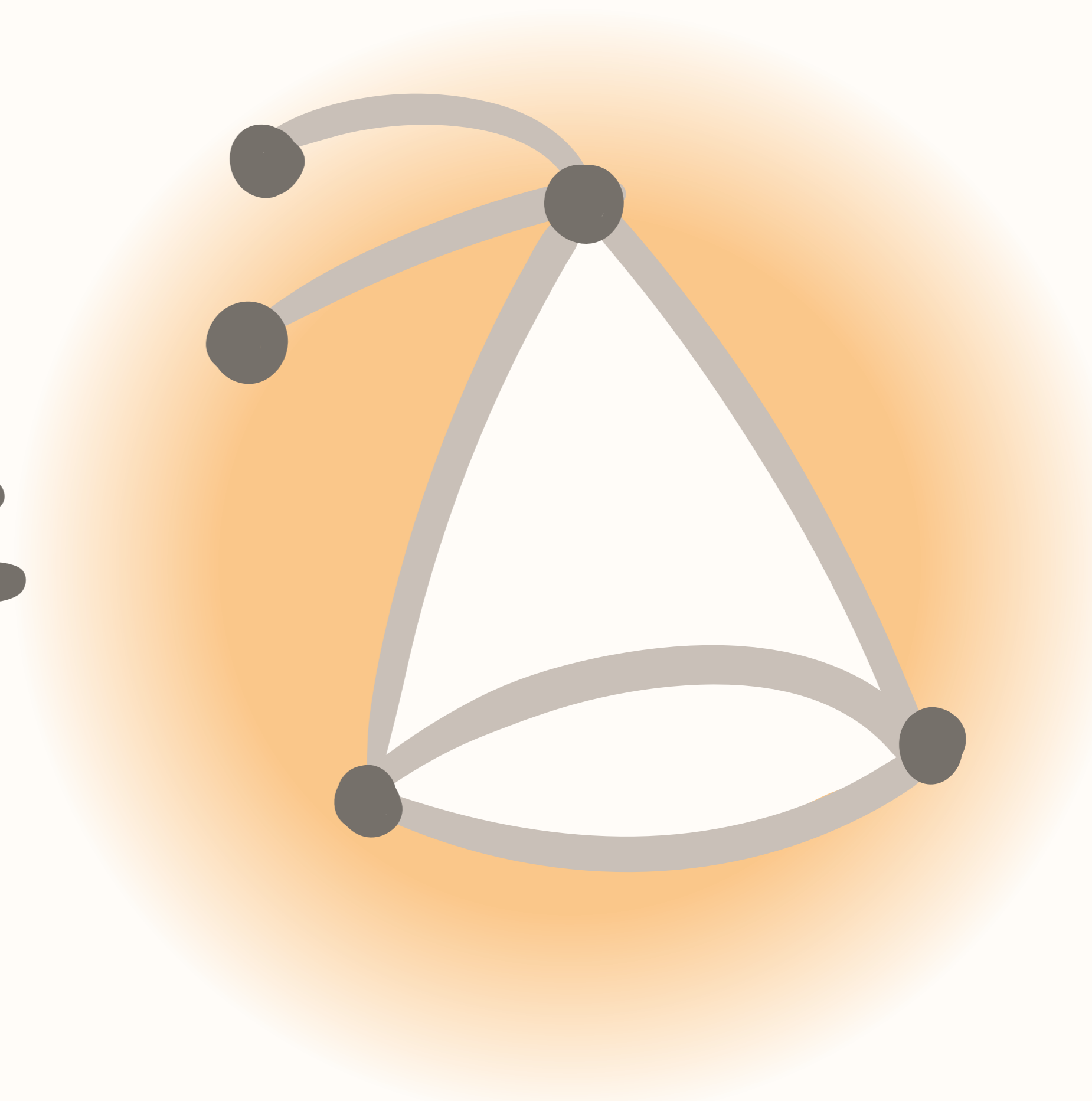
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graph embedding on the plane



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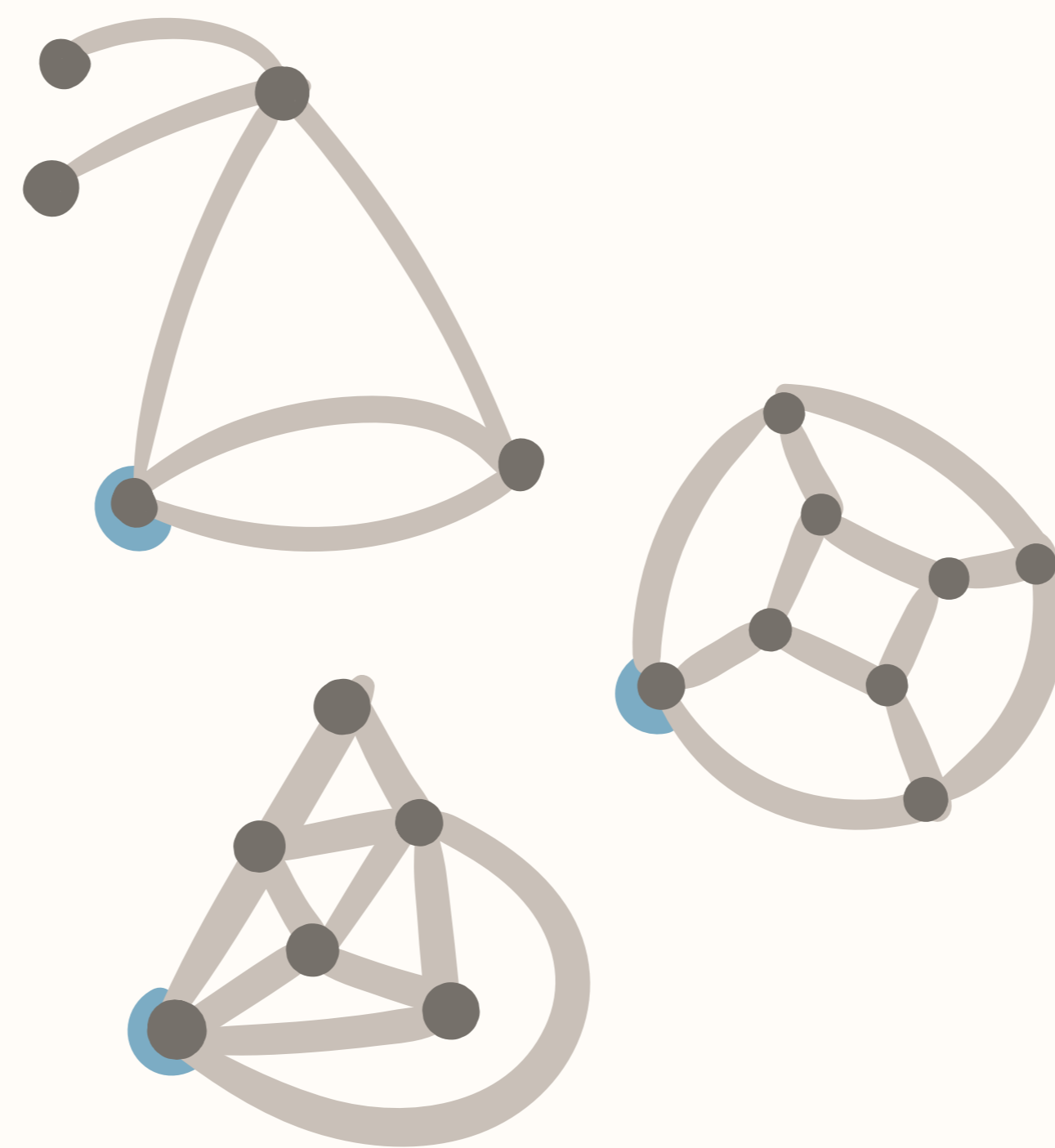
Introduction

Maps

graph embedding on the plane

universal
critical
exponent

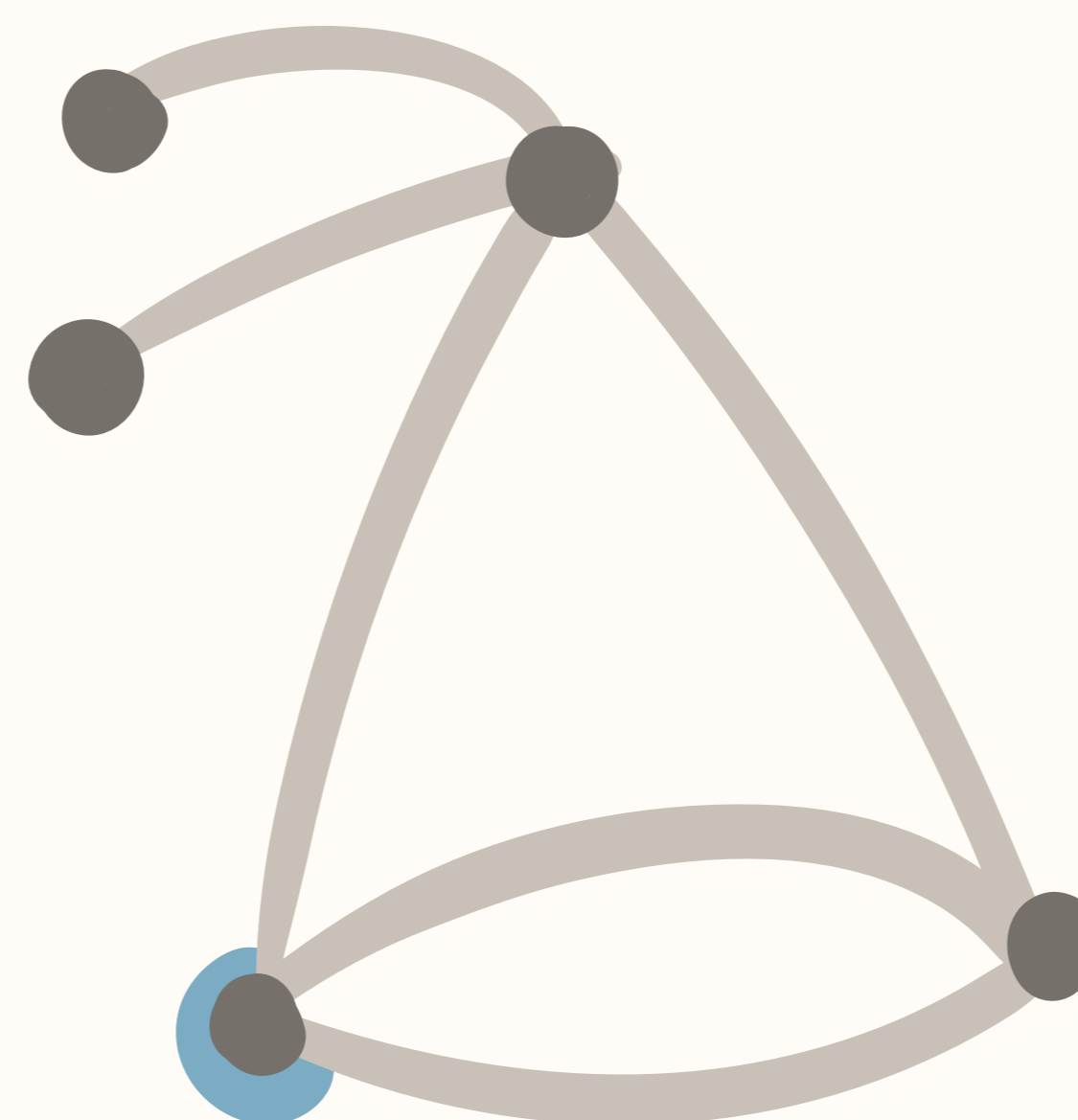
$$\kappa \cdot \gamma^n n^{-5/2}$$



maps
with
 n edges

$$2 \cdot 3^n \frac{(2n)!}{n!(n+2)!}$$

$$\sim (2/\sqrt{\pi}) \cdot 12^n n^{-5/2}$$



→ 2, 9, 54, ...

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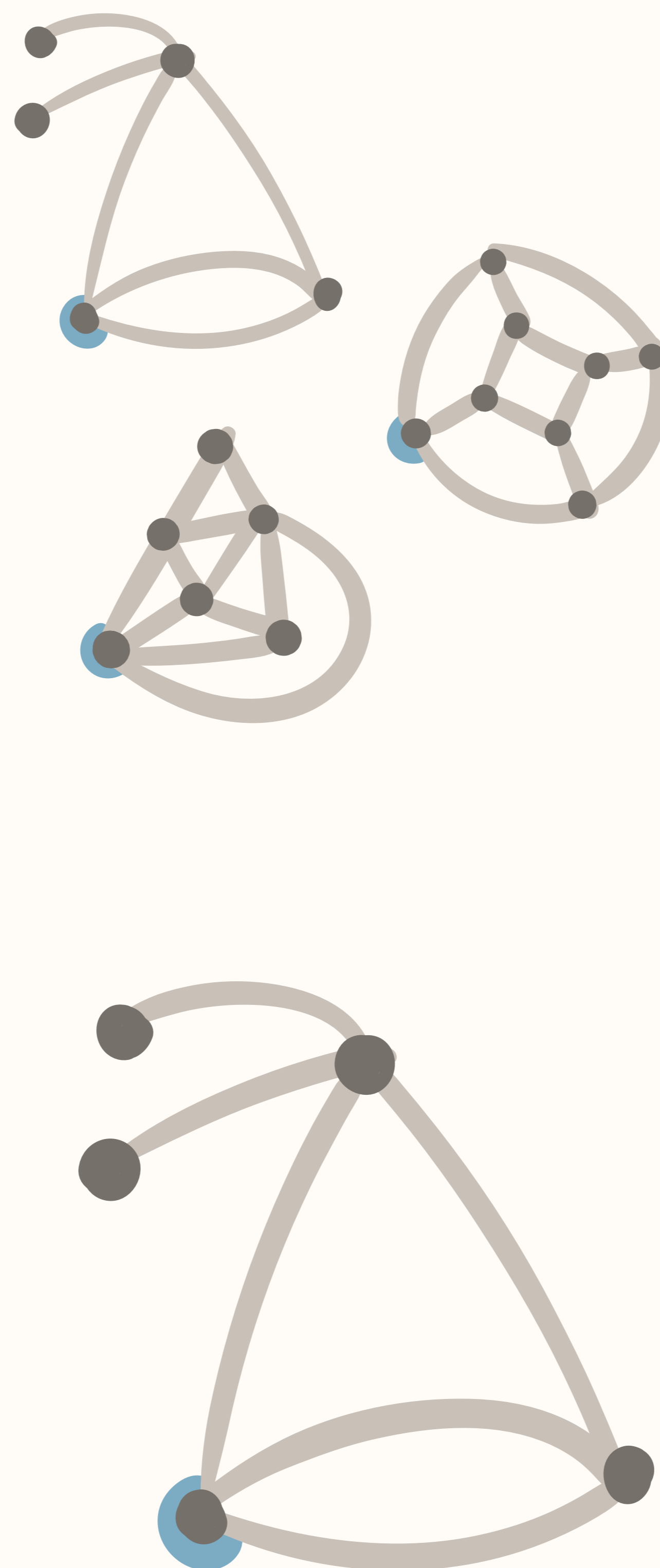
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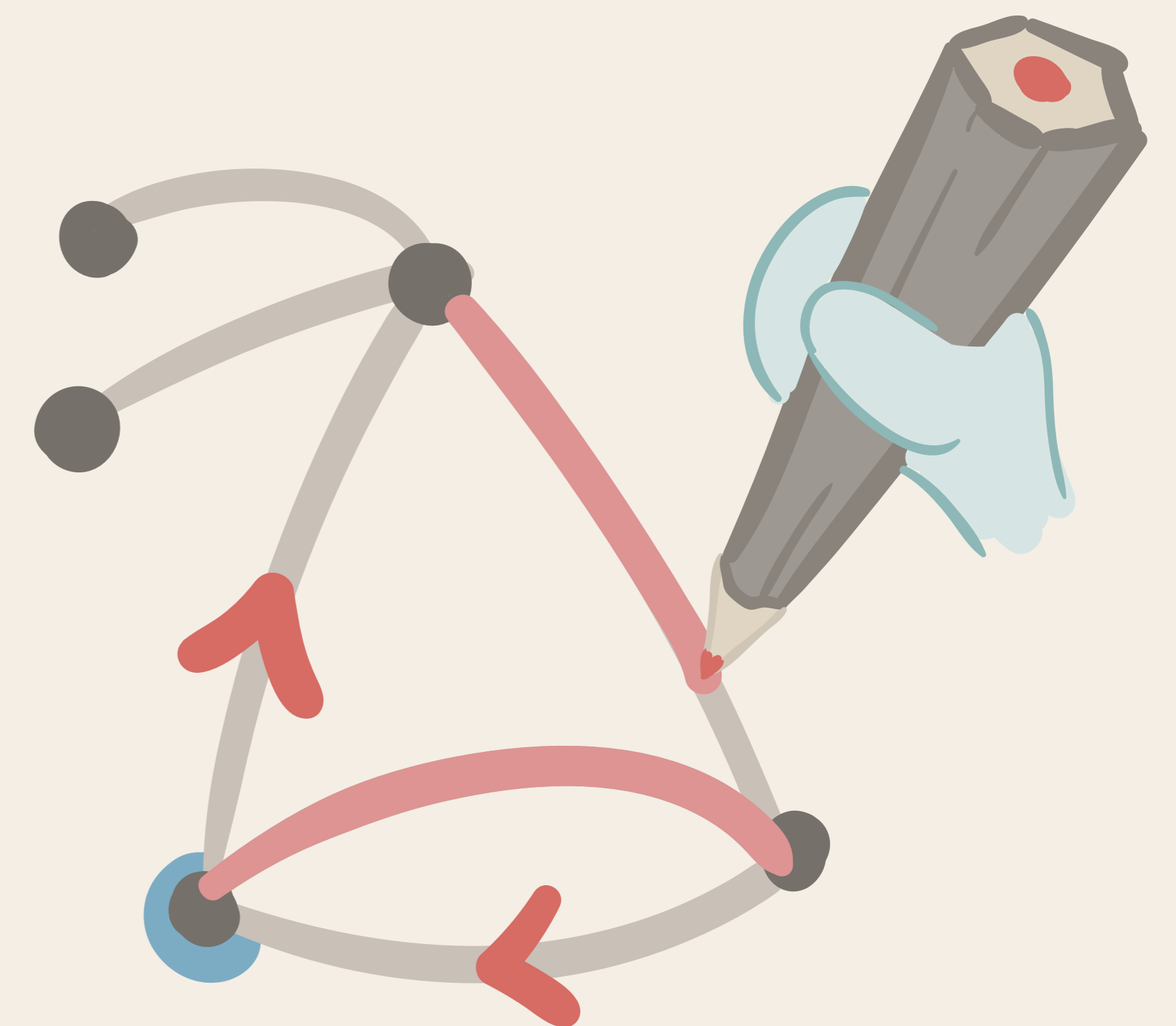
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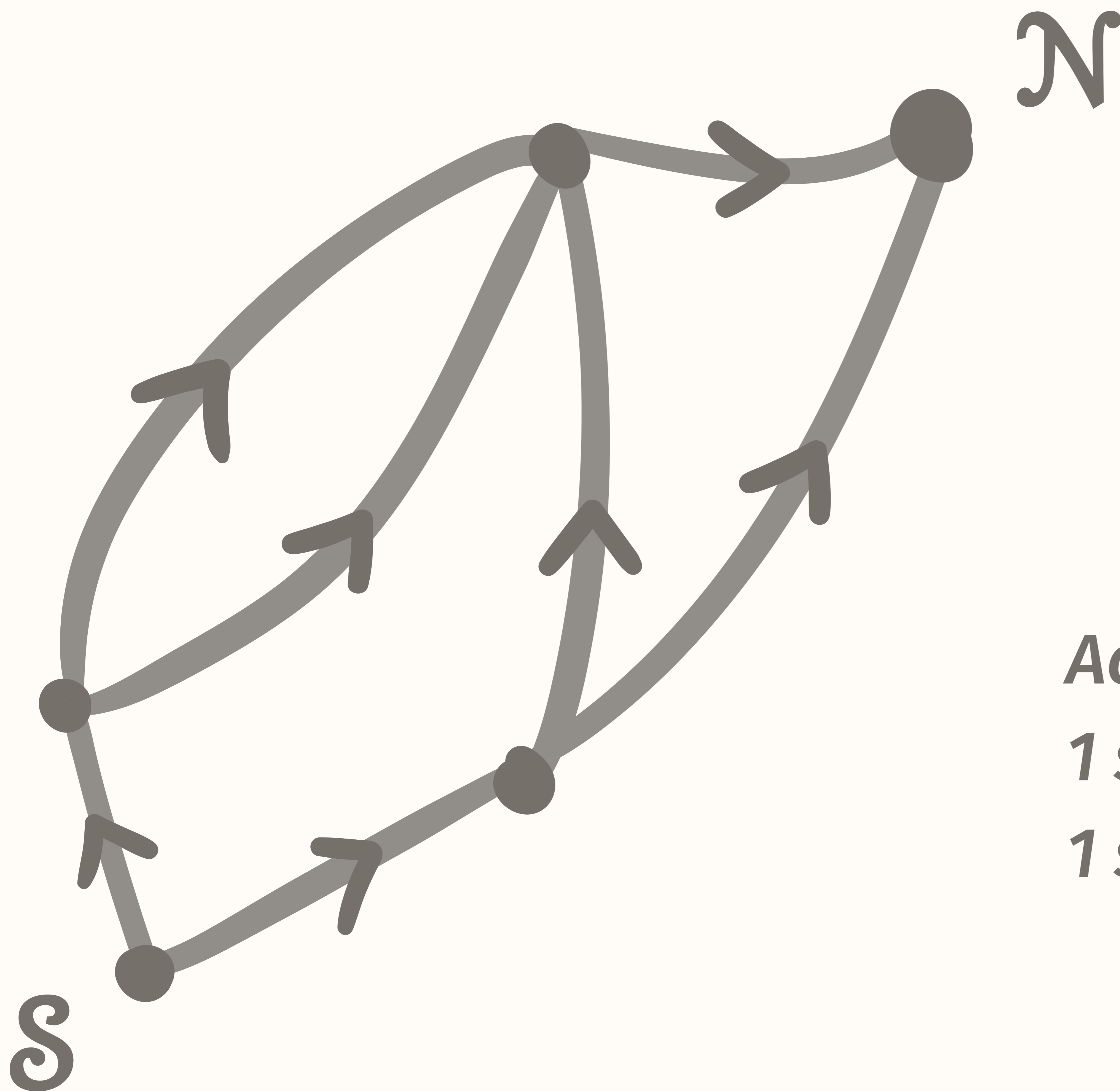


Decorated maps
orientation, coloration, etc.



Plane bipolar orientation

an example of decorated map



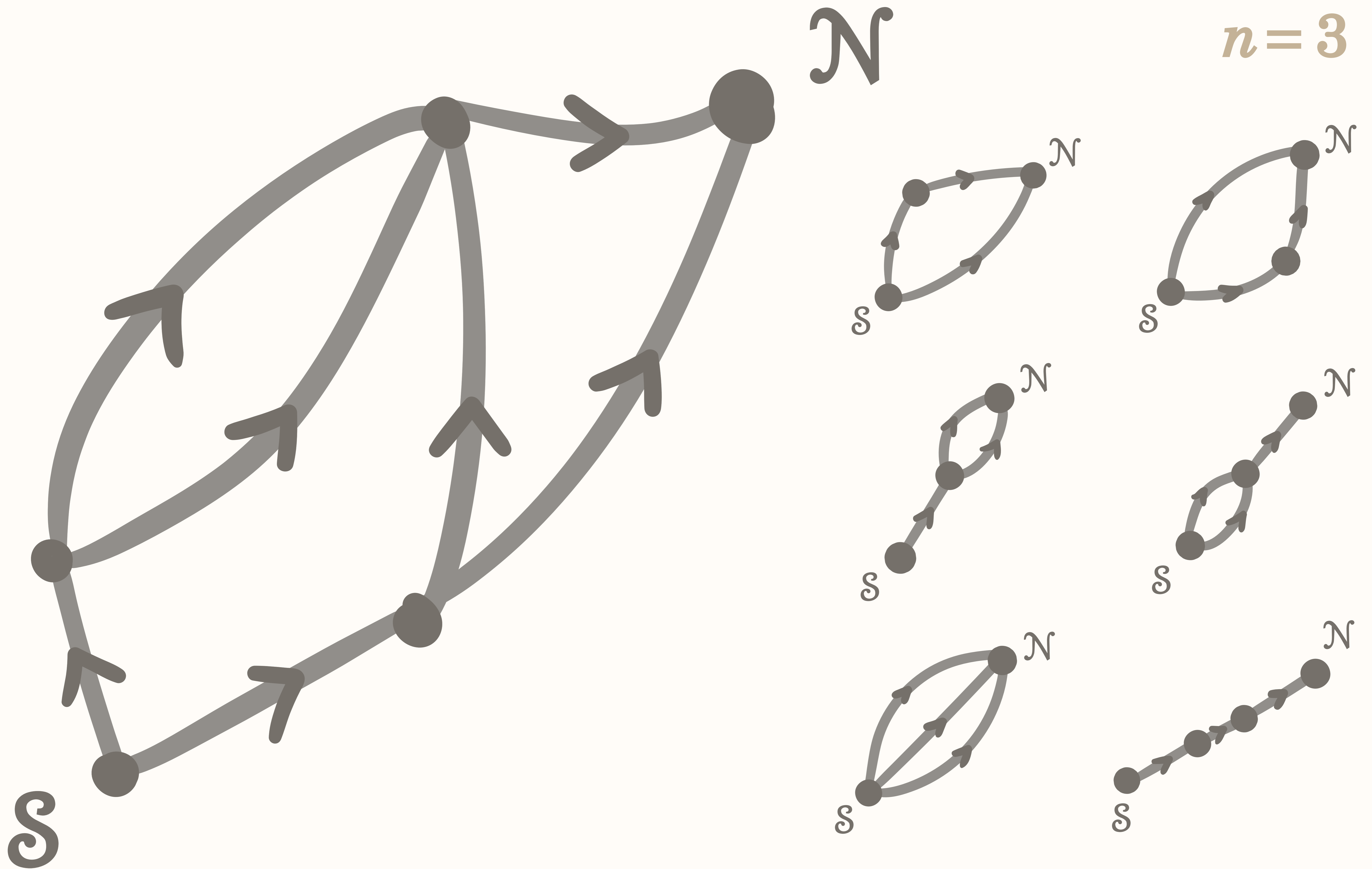
Acyclic

1 single source \mathcal{S}

1 single sink \mathcal{N}

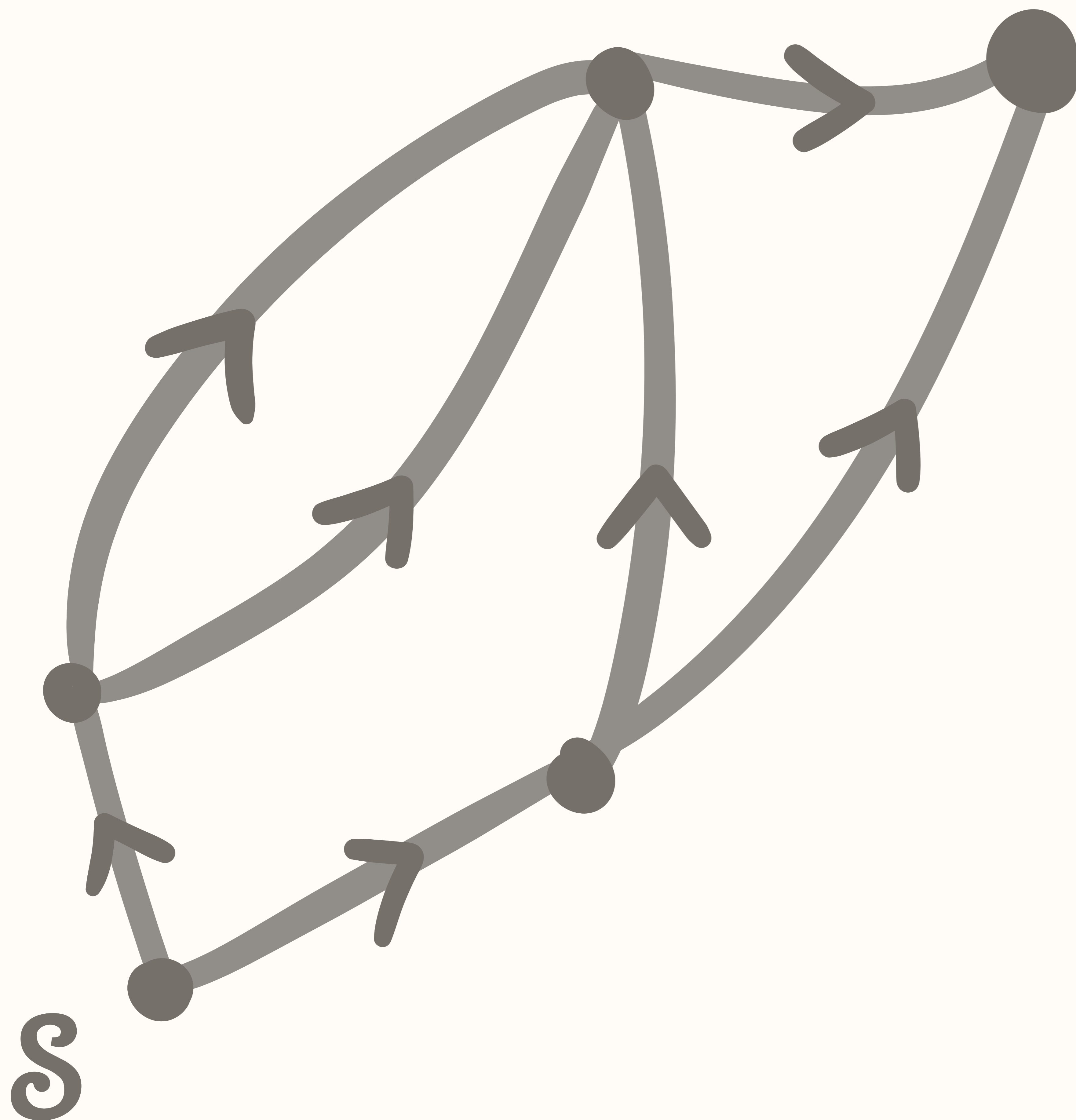
Plane bipolar orientation

an example of decorated map



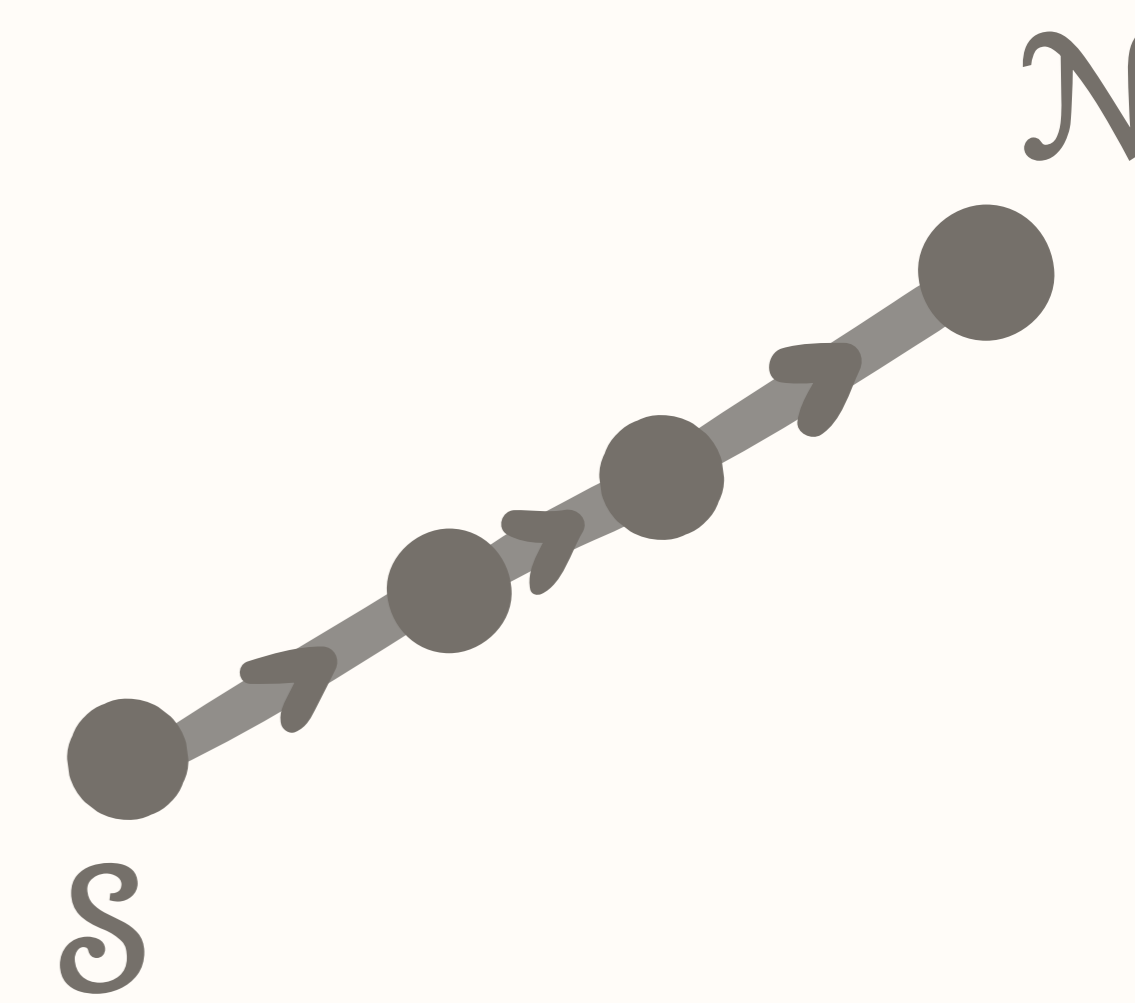
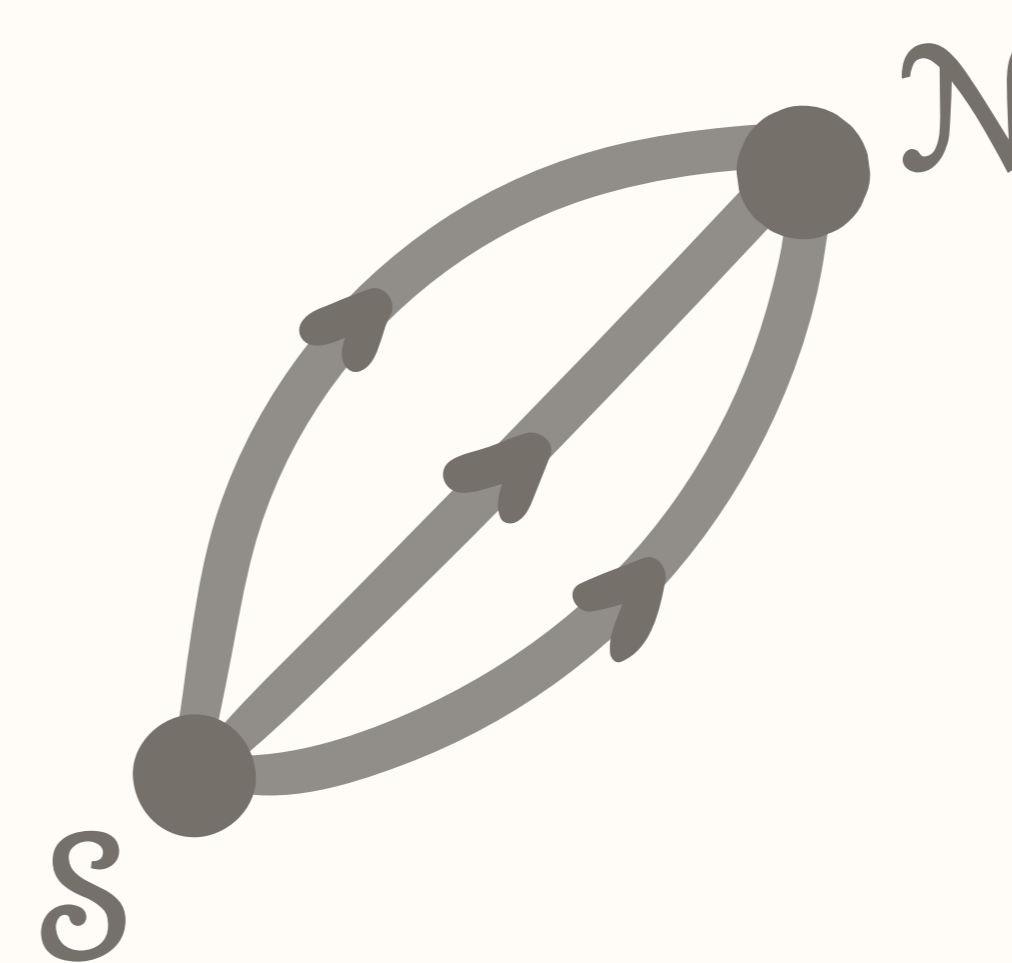
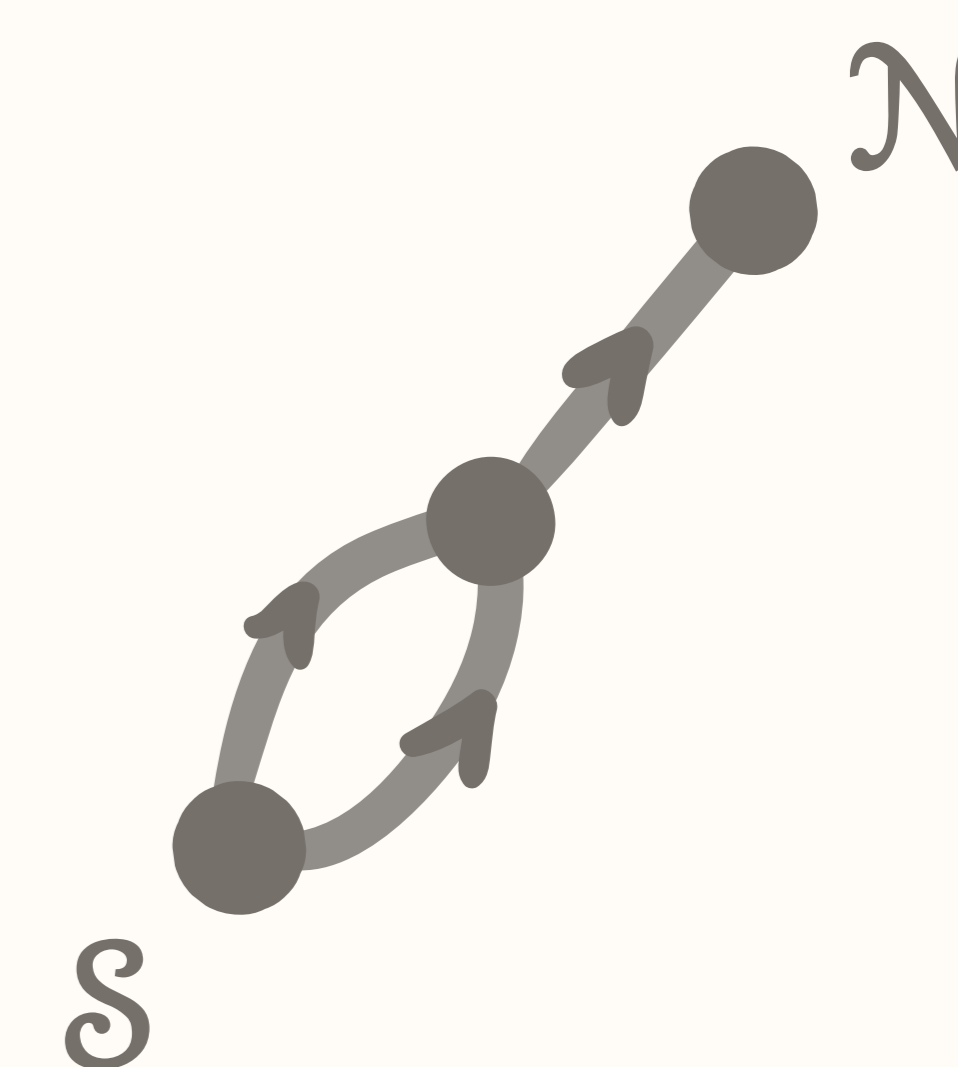
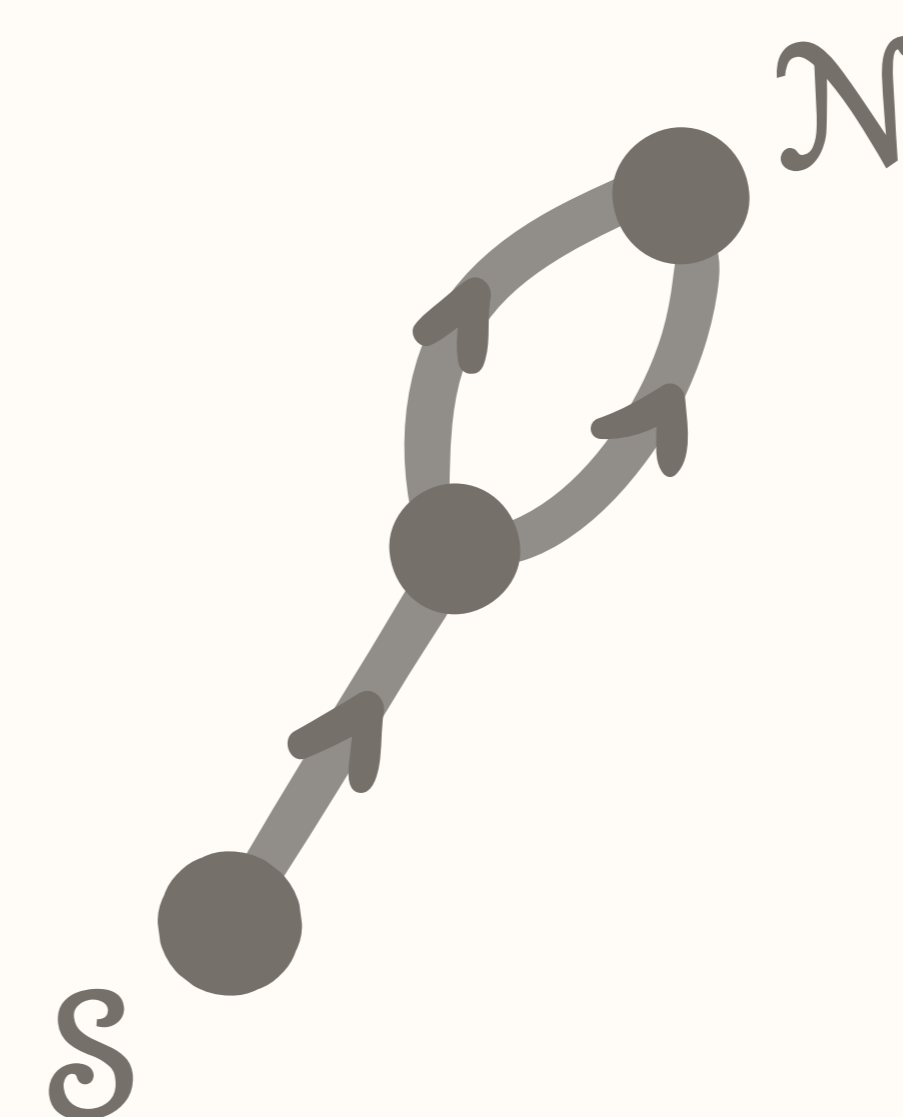
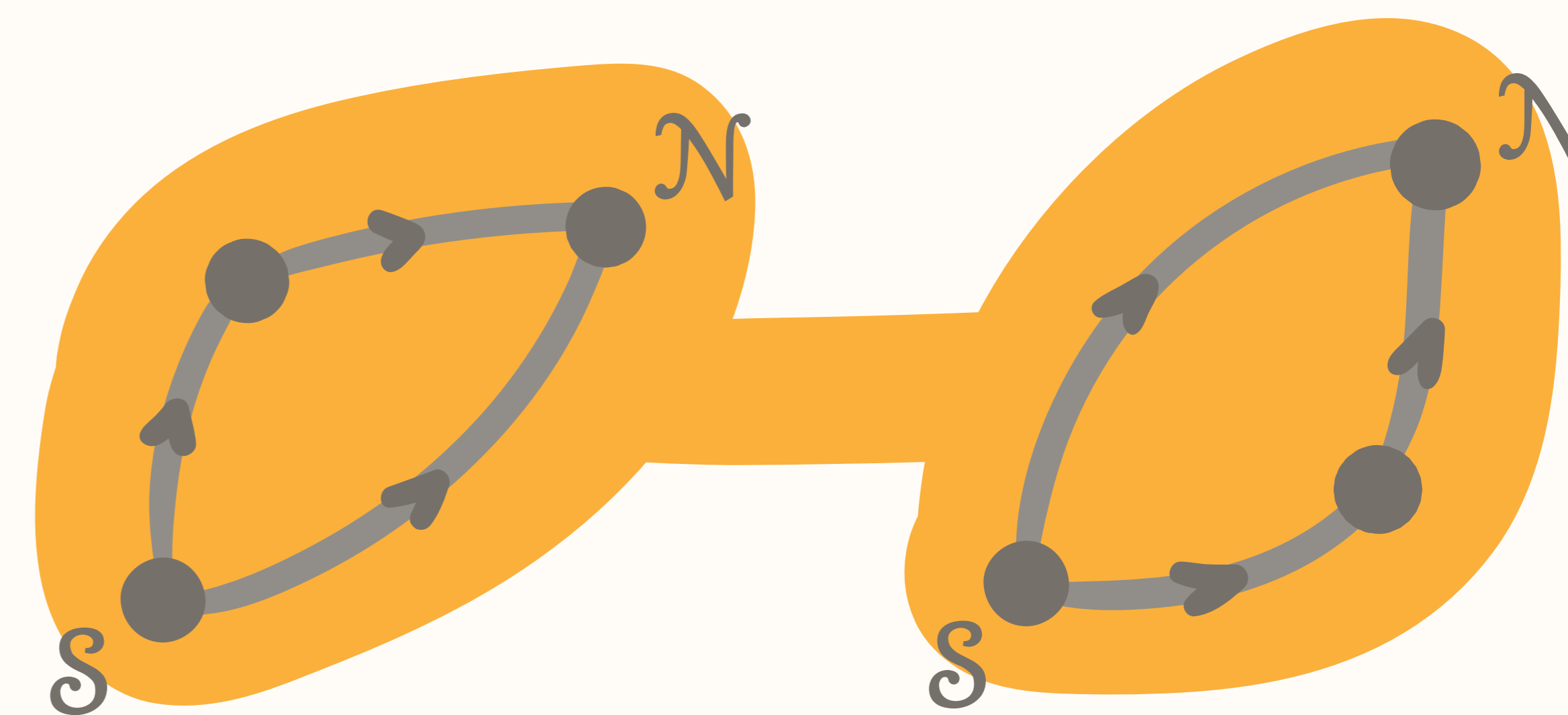
Plane bipolar orientation

an example of decorated map



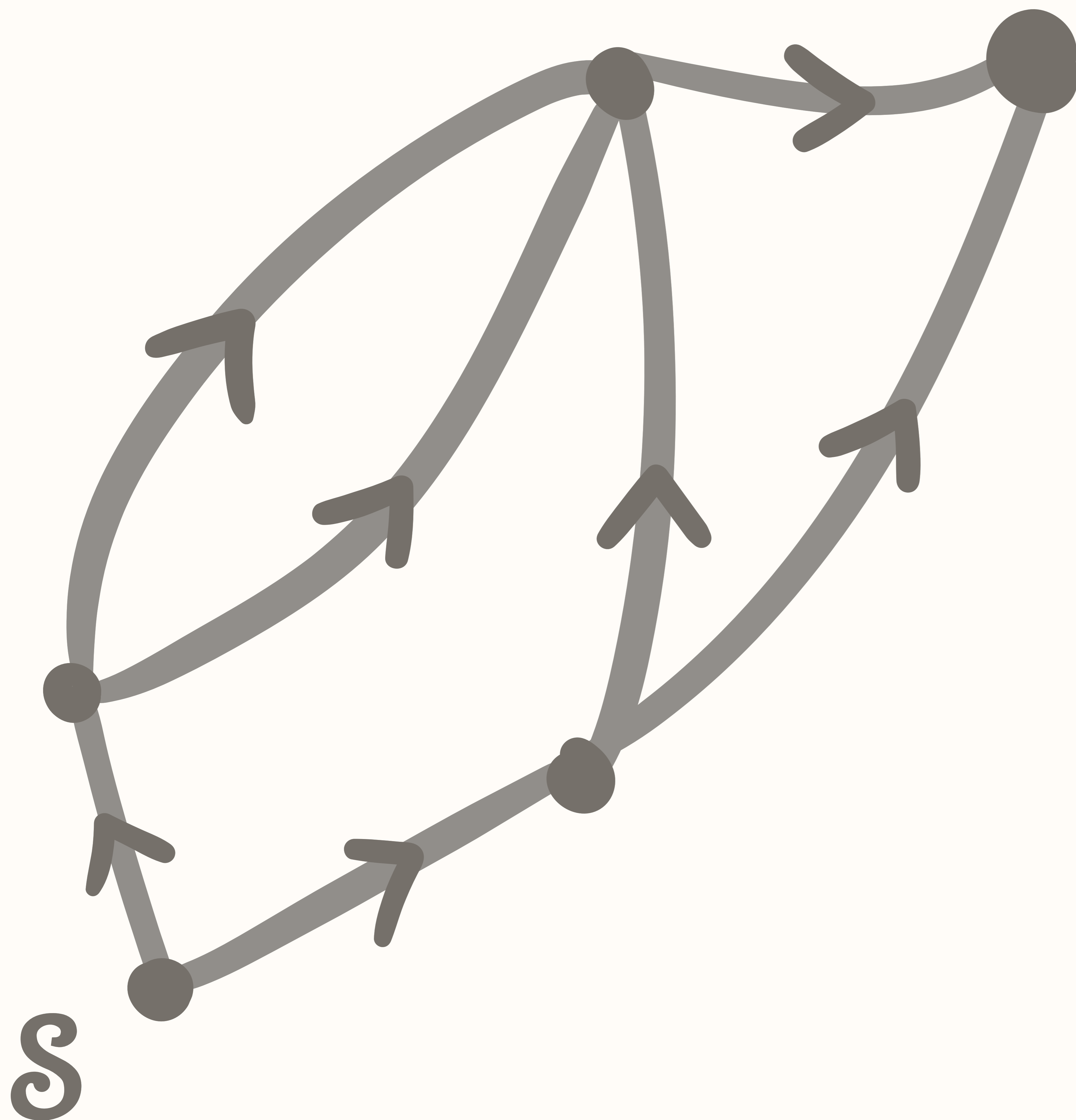
\mathcal{N}

$n = 3$



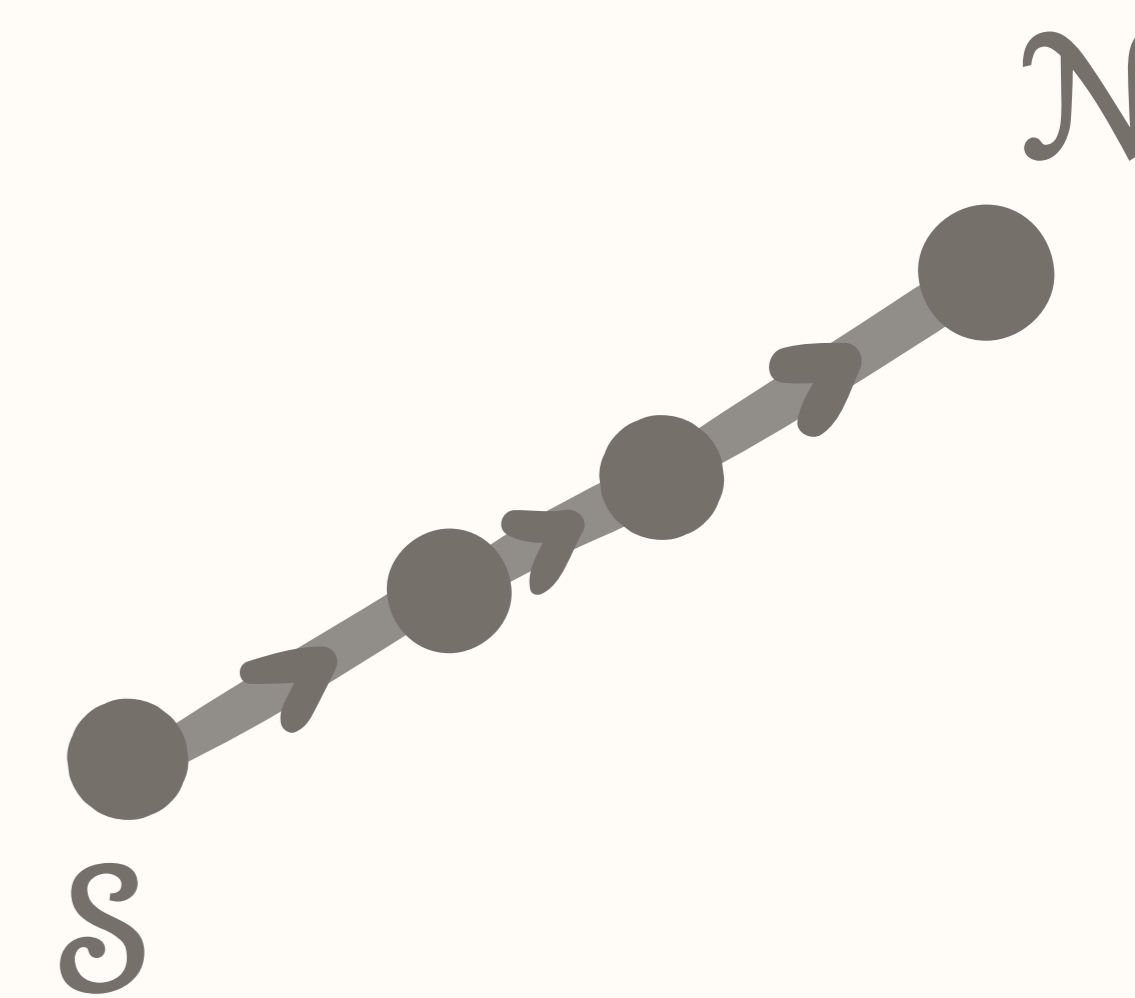
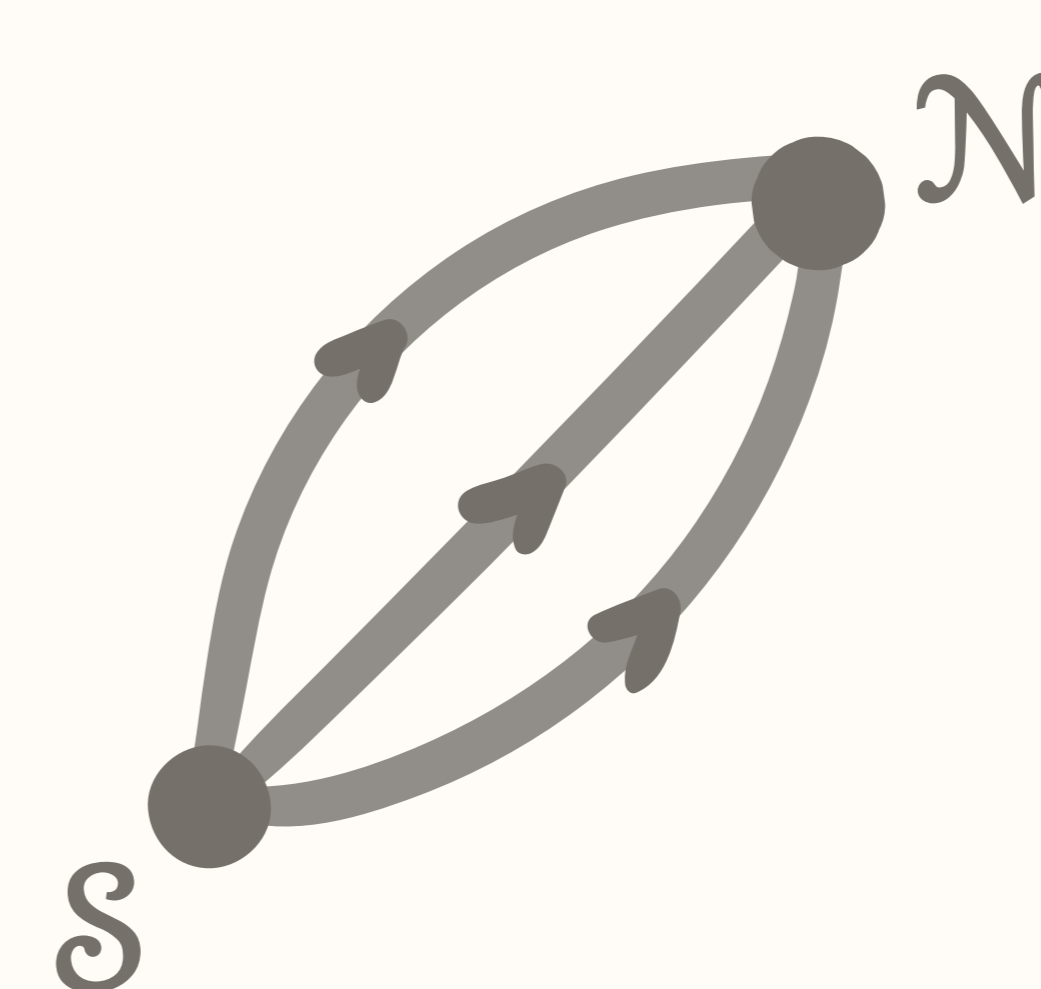
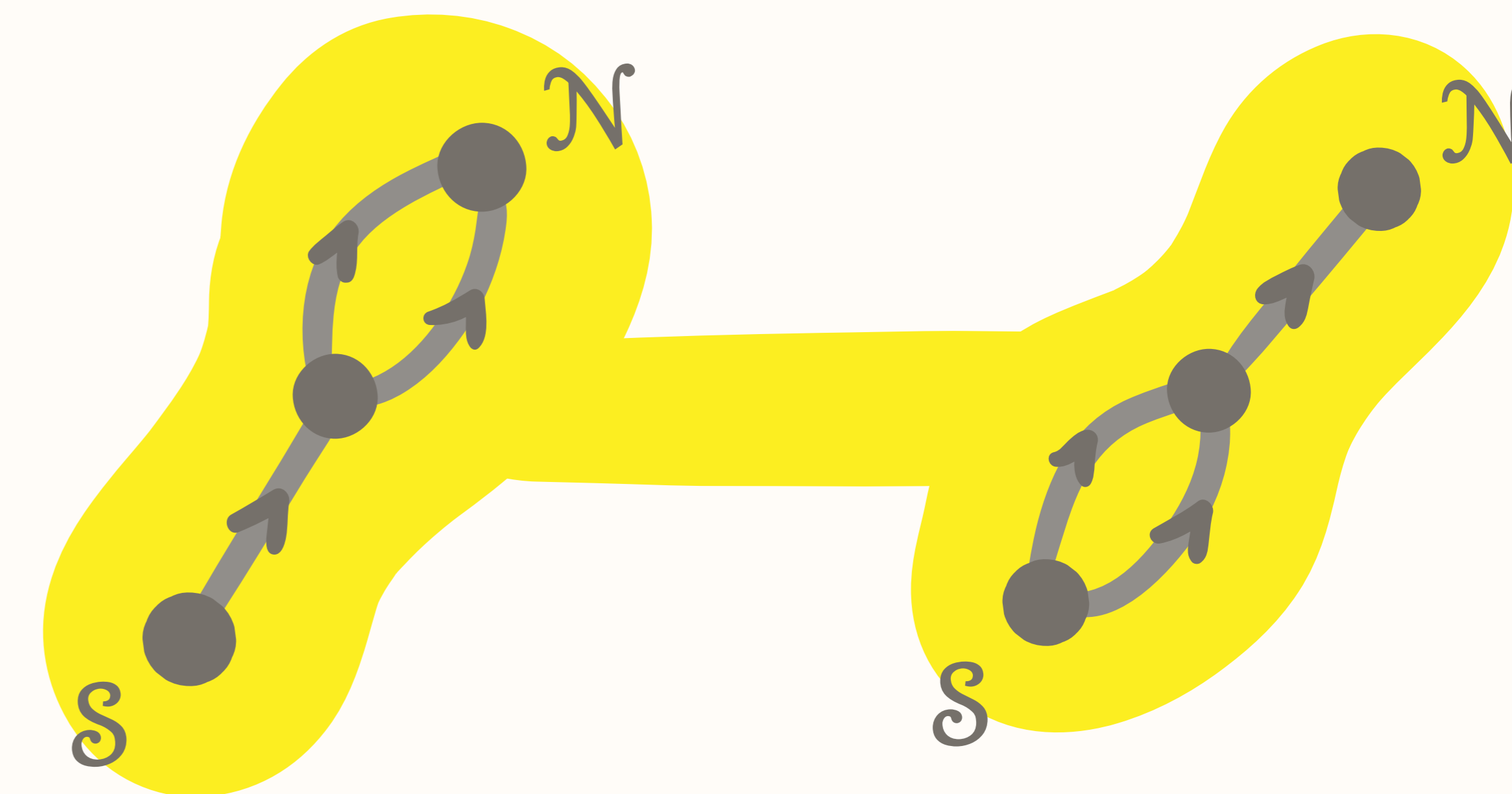
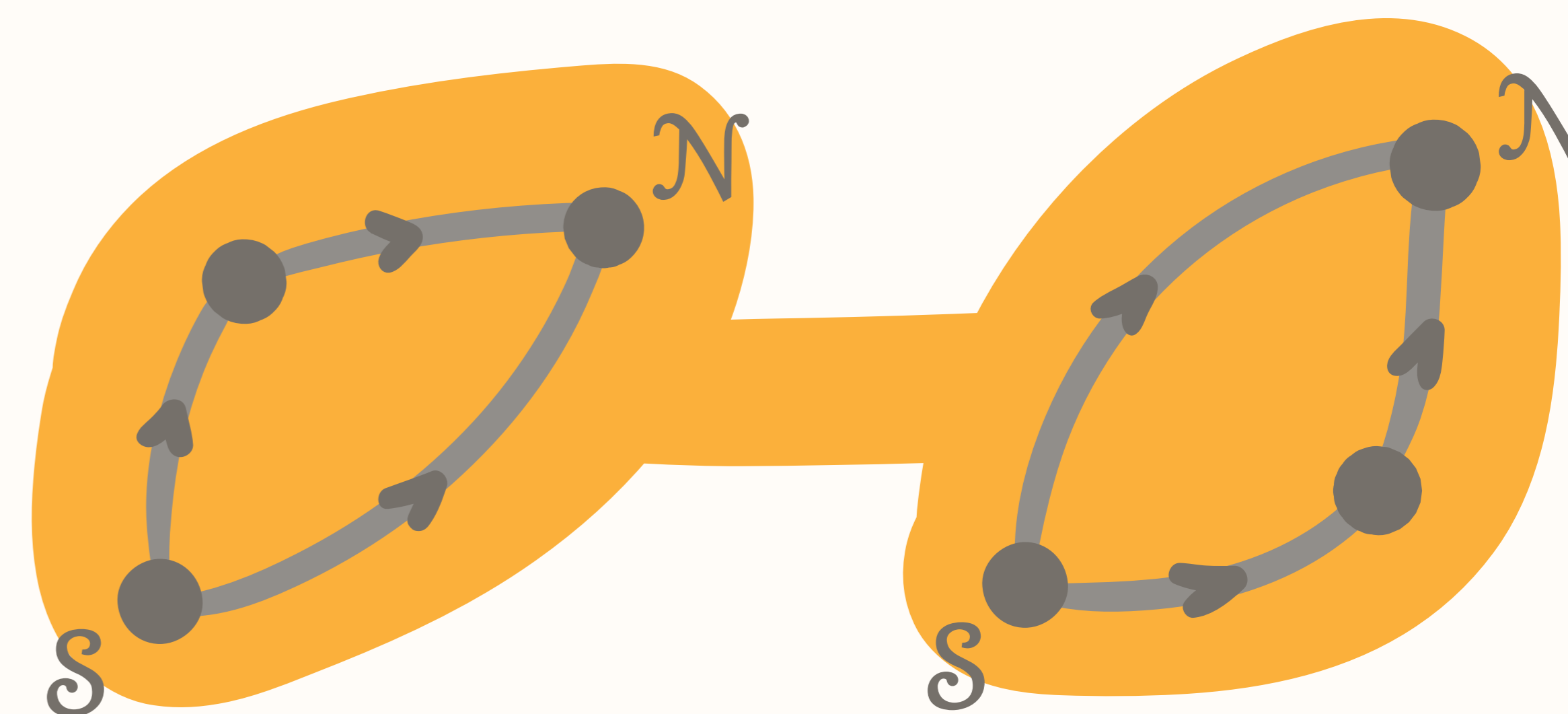
Plane bipolar orientation

an example of decorated map



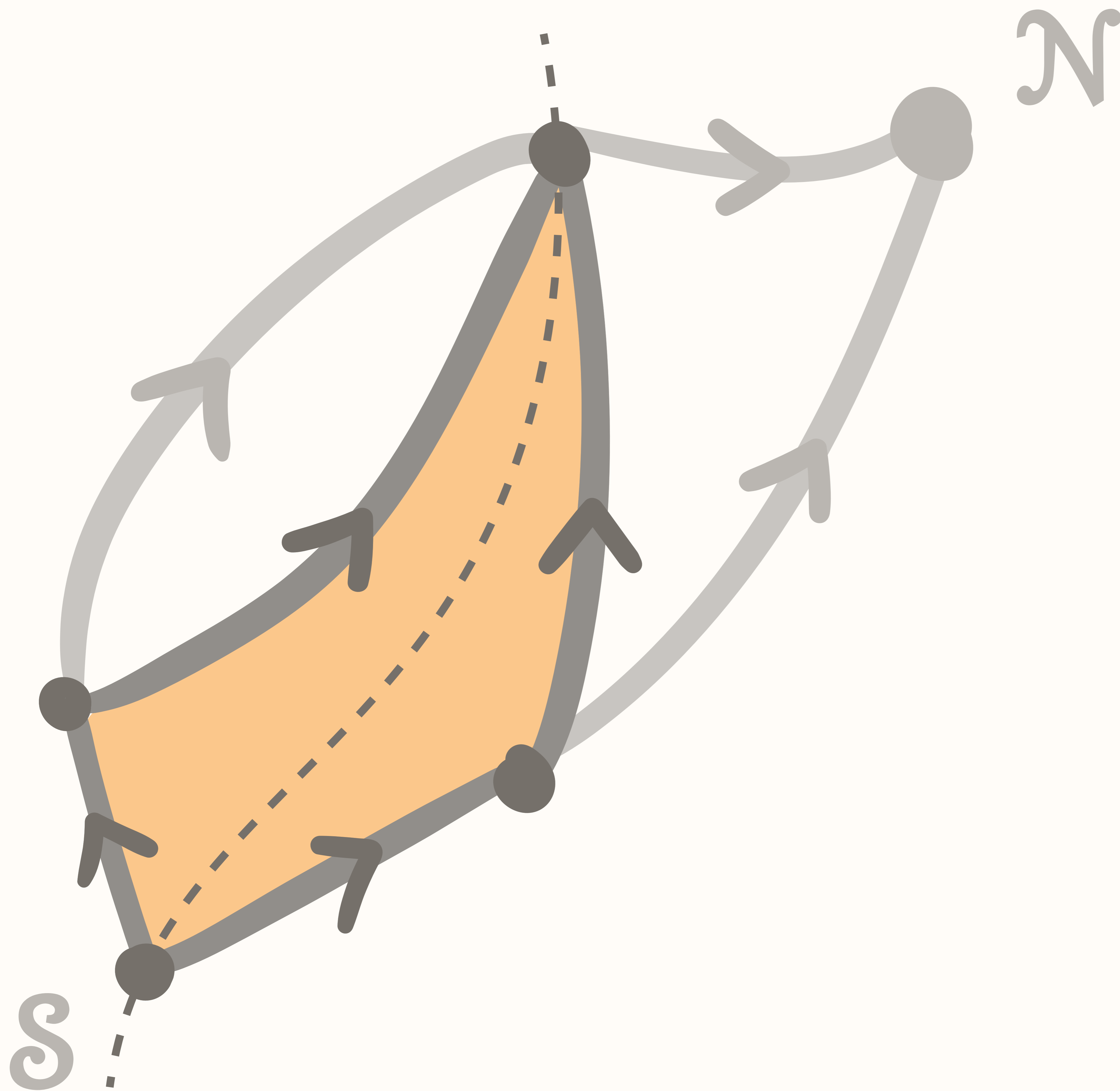
\mathcal{N}

$n = 3$



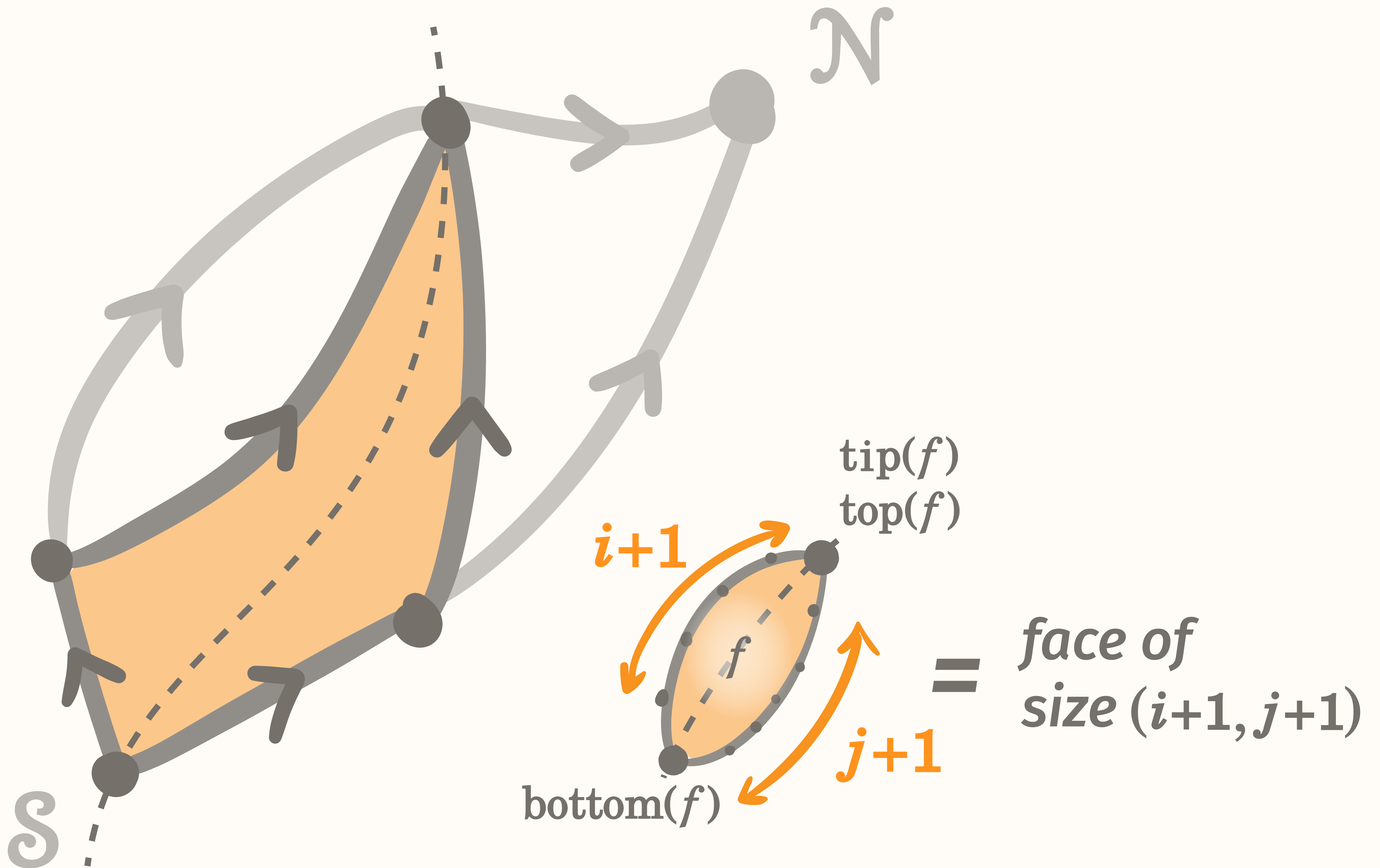
Plane bipolar orientation

an example of decorated map



Plane bipolar orientation

an example of decorated map



Summary

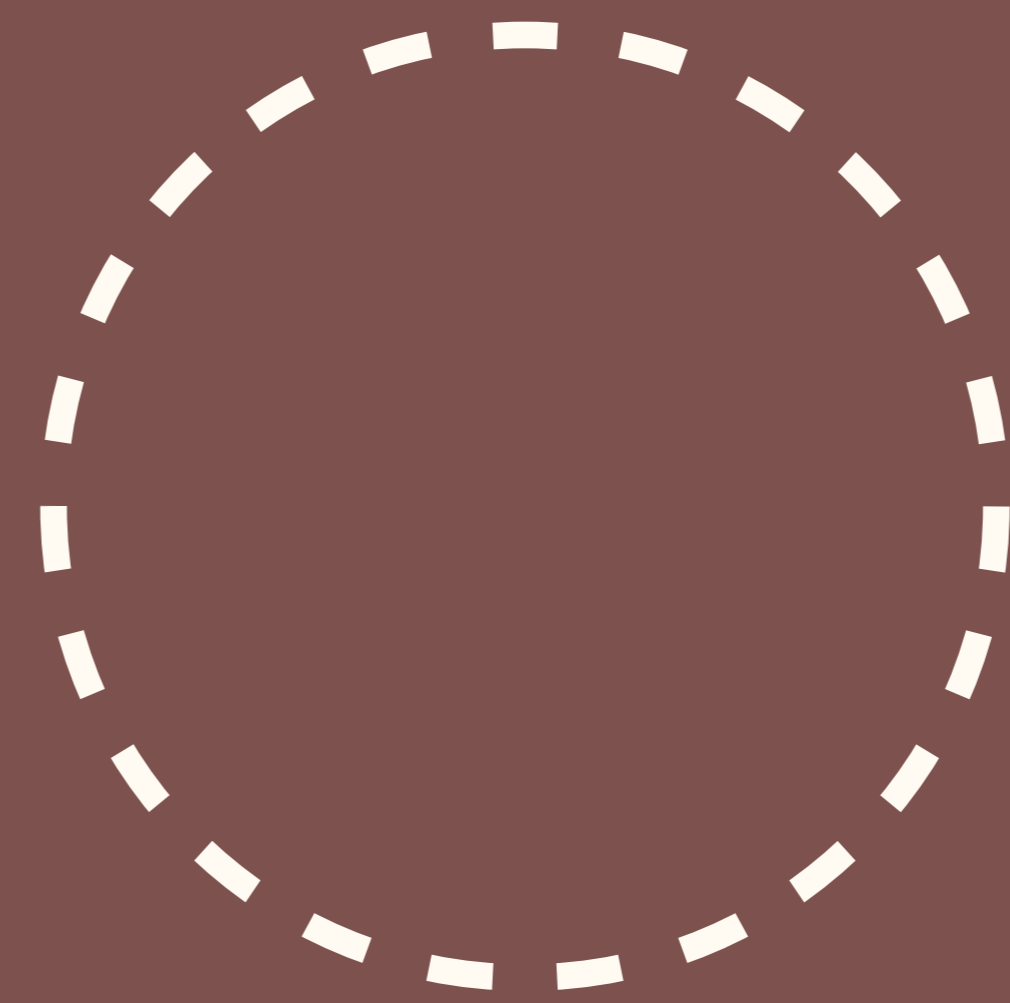
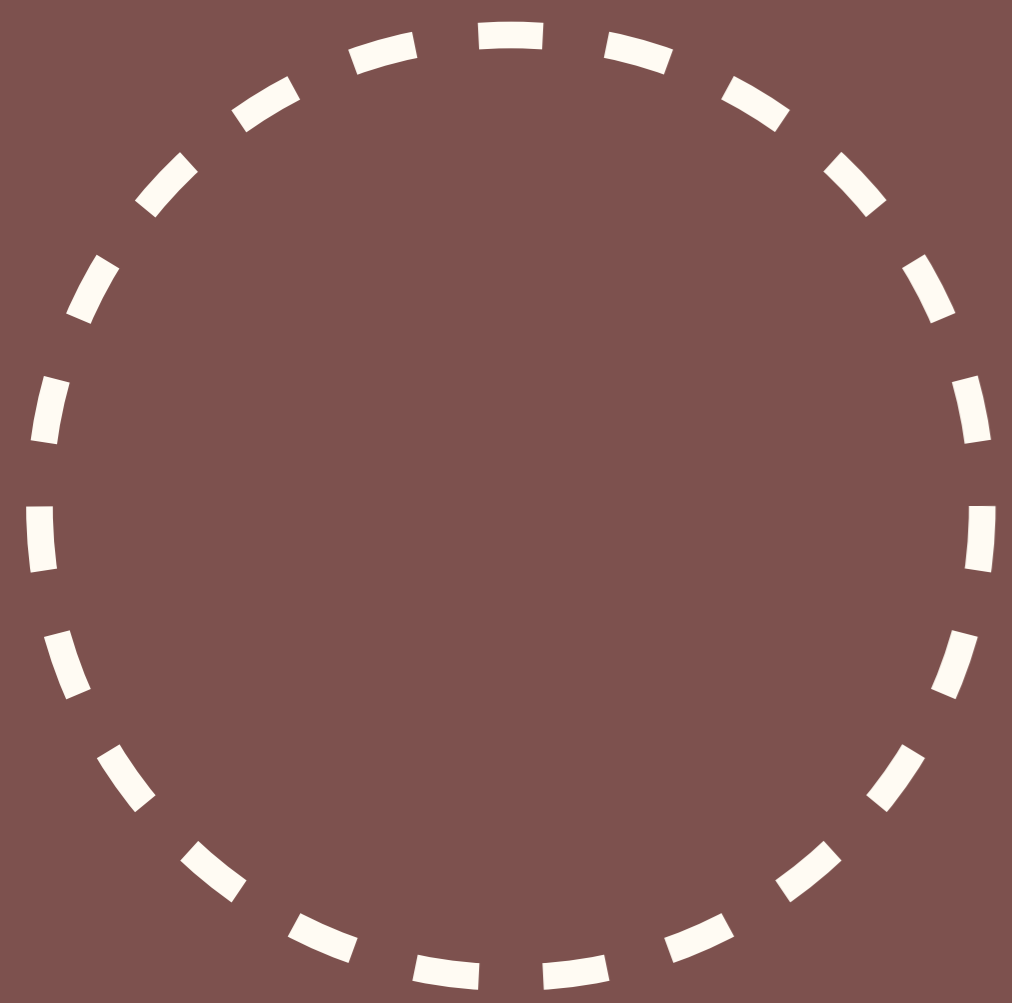
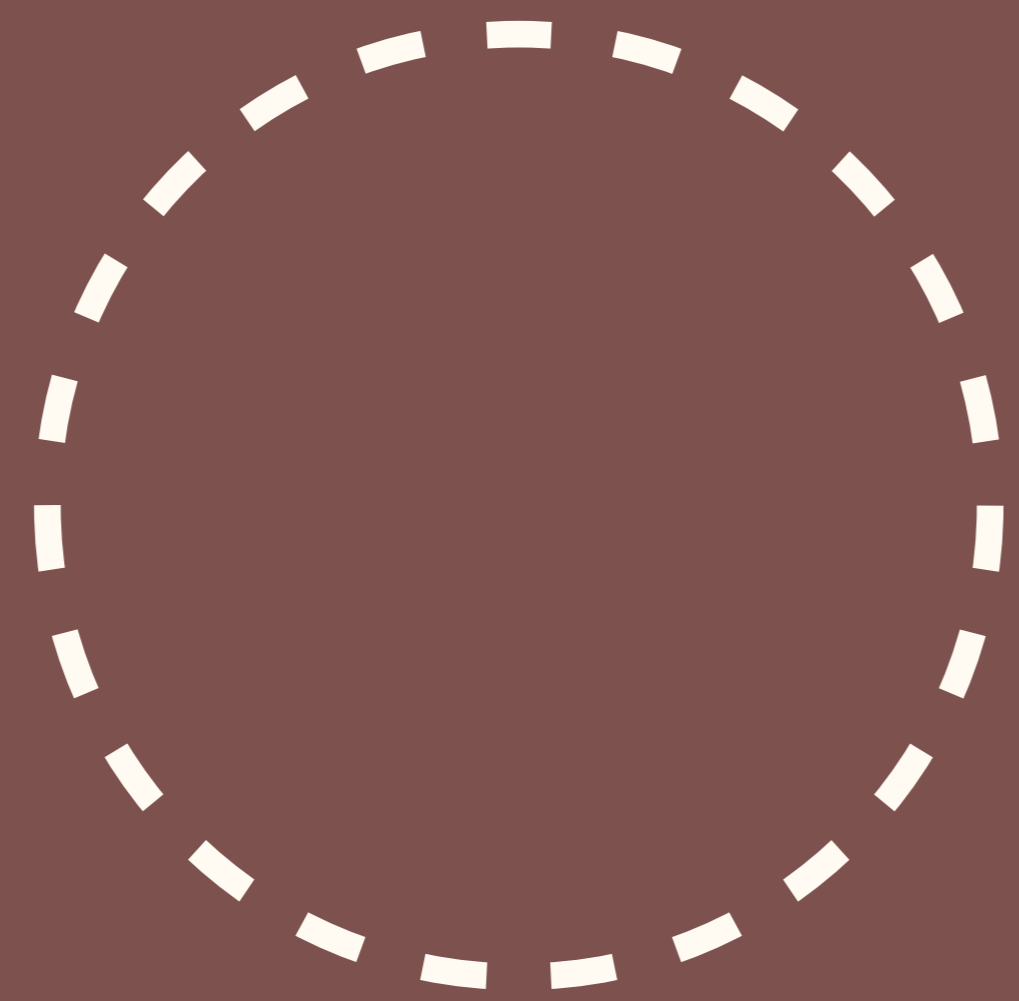


Asymptotics

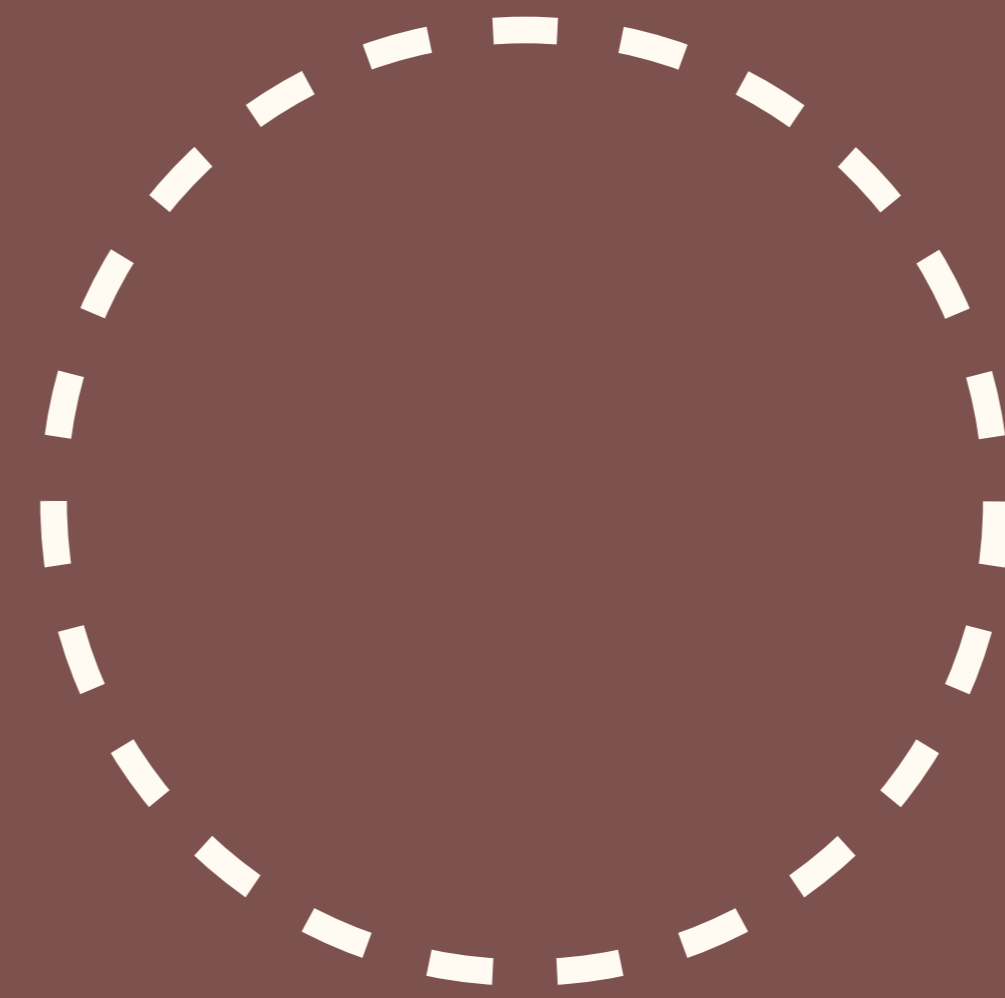
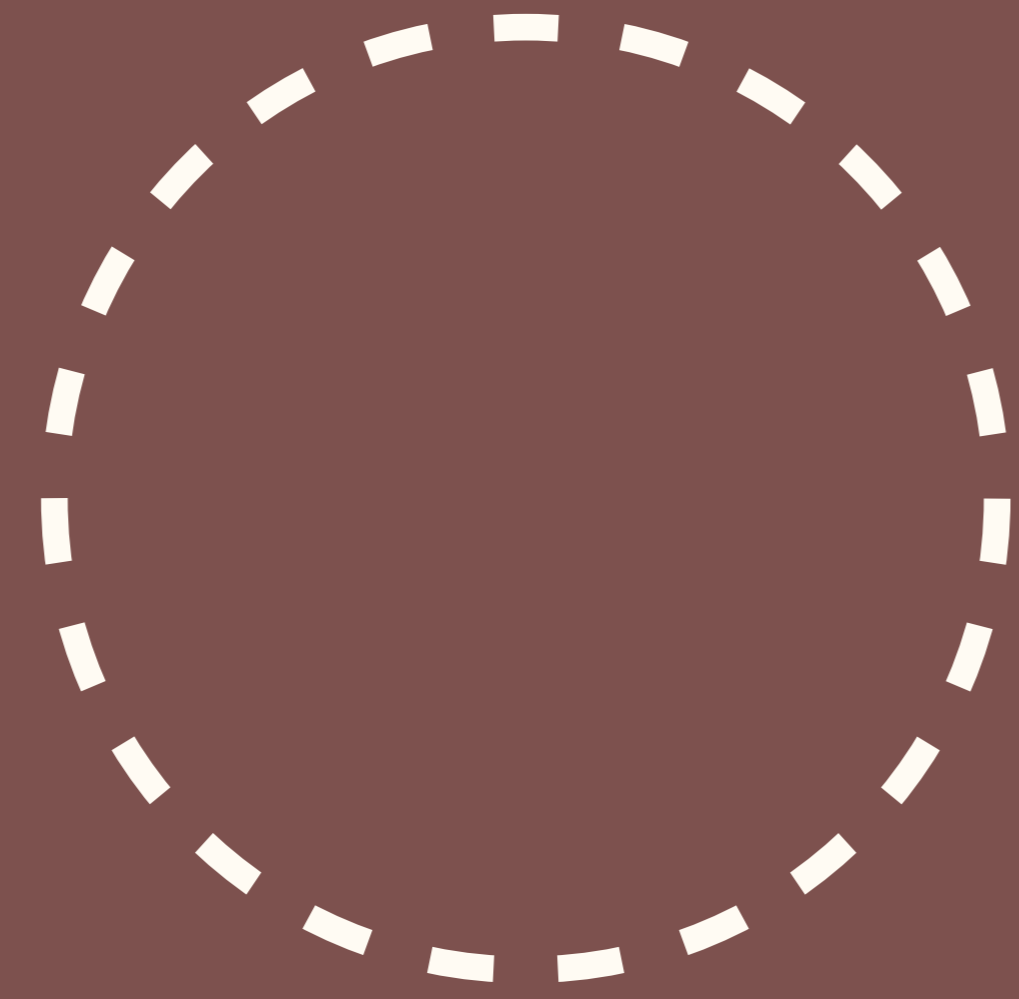
Summary



Summary



Summary



Decorated bipolar orientations

Summary



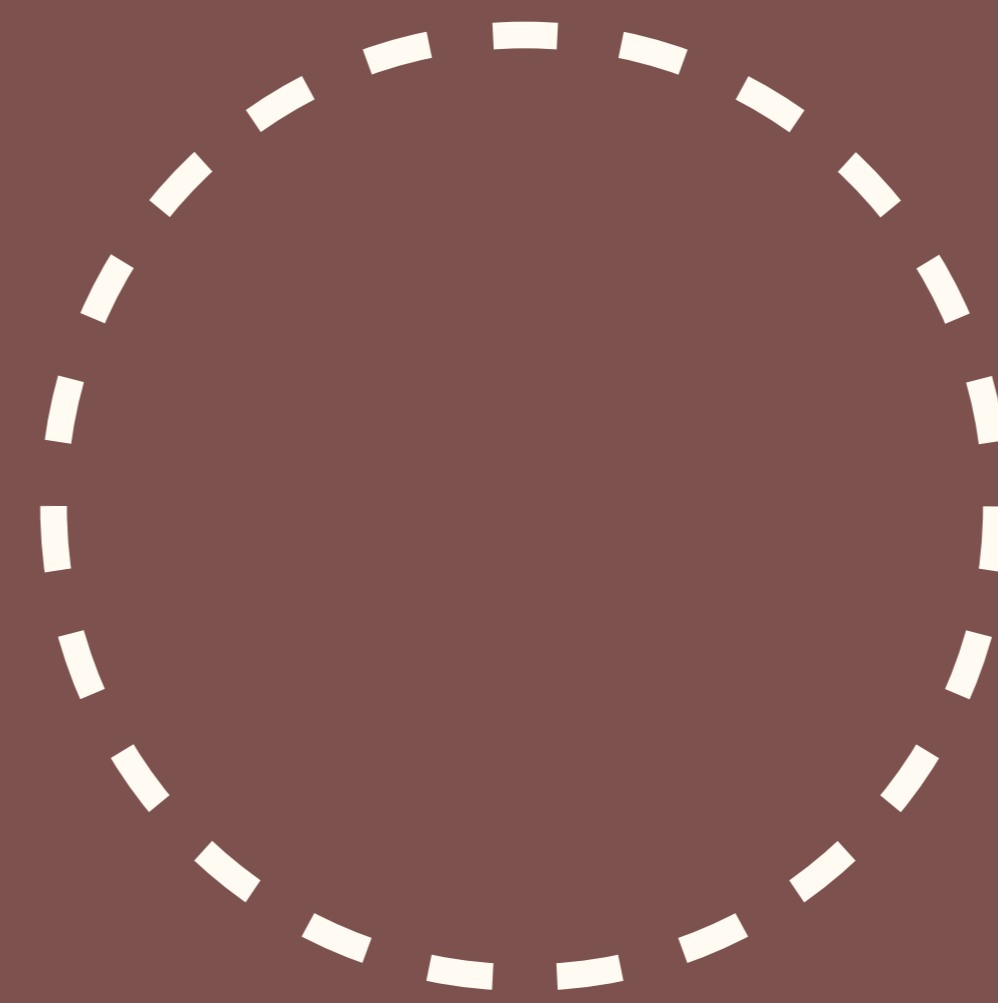
Quadrant
tandem walks



Asymptotics



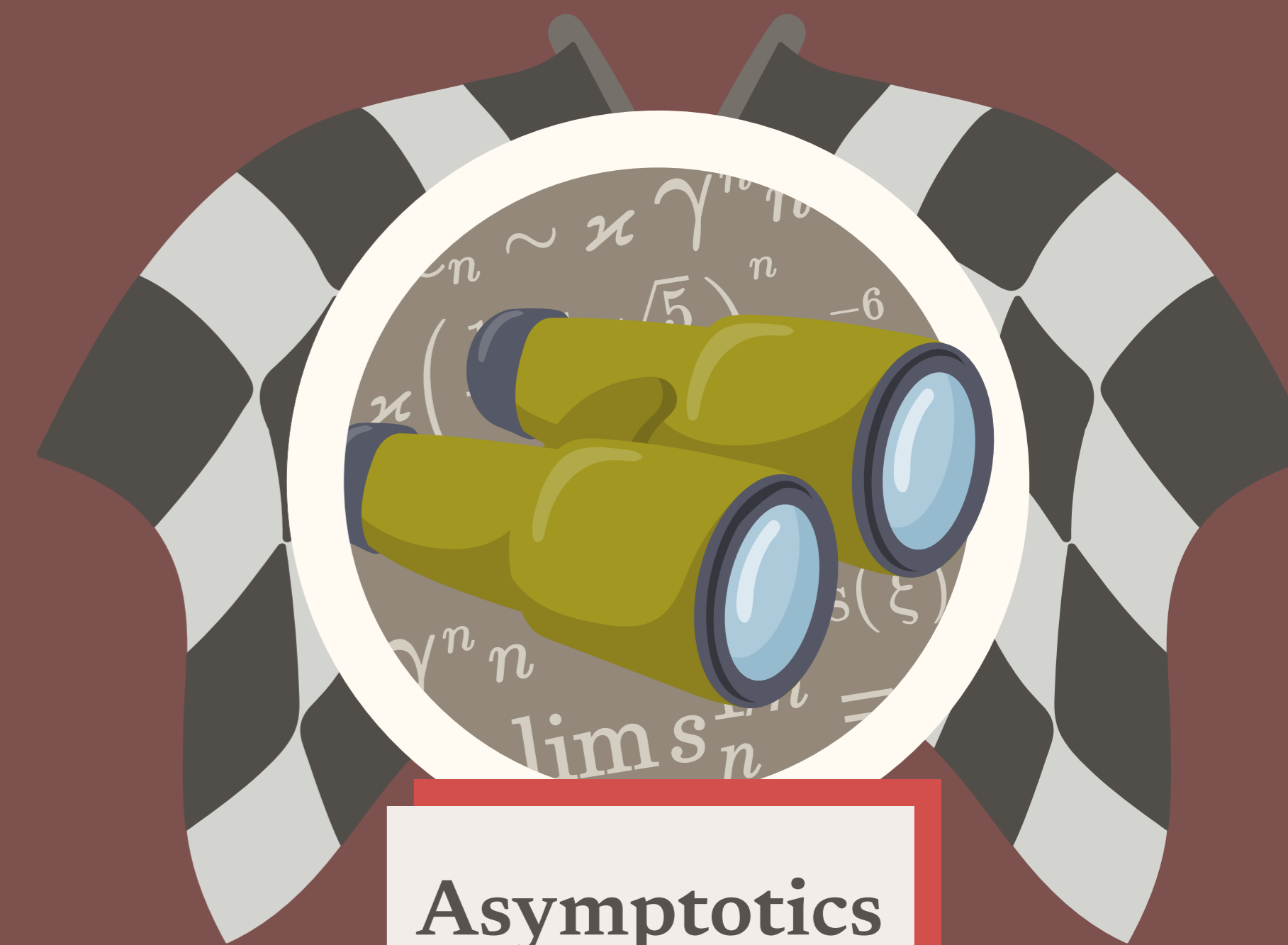
Decorated bipolar
orientations



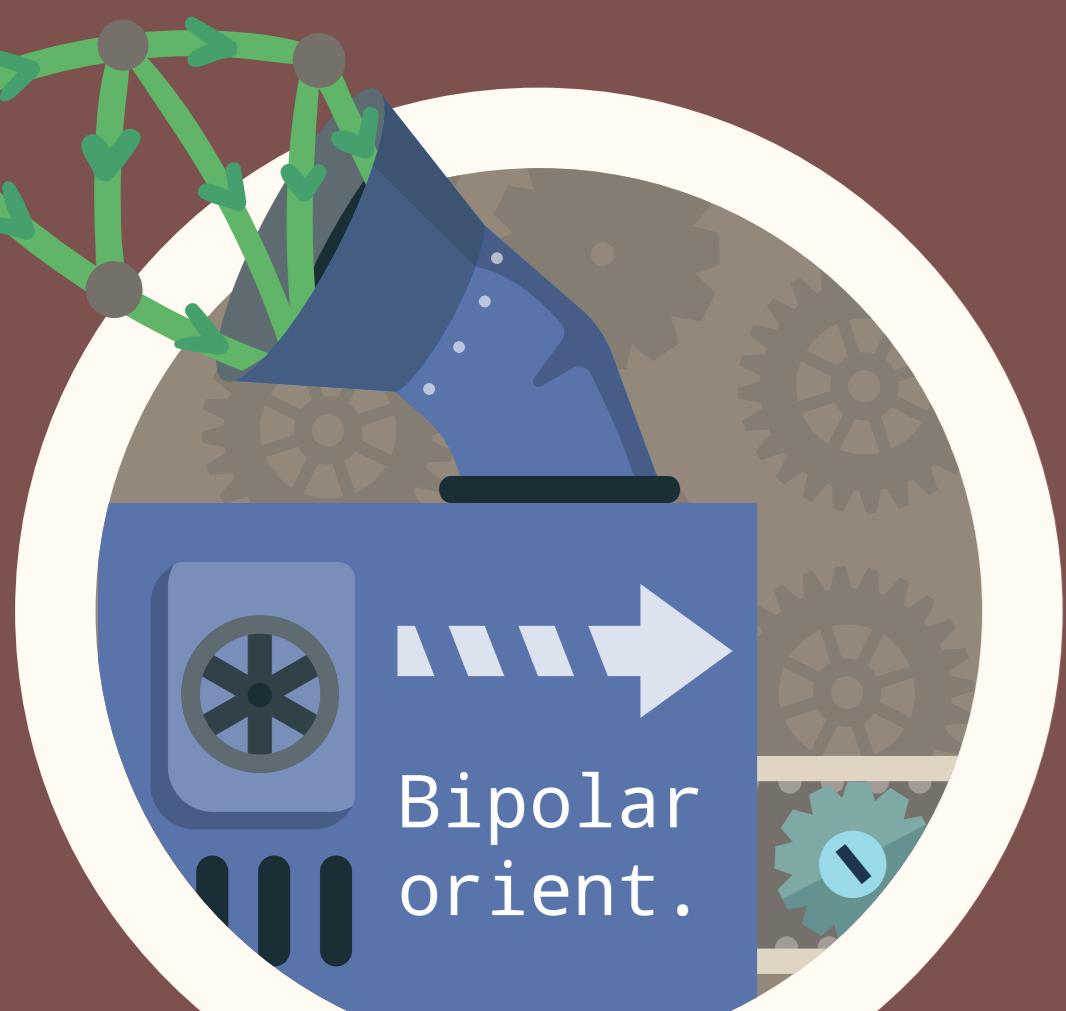
Summary



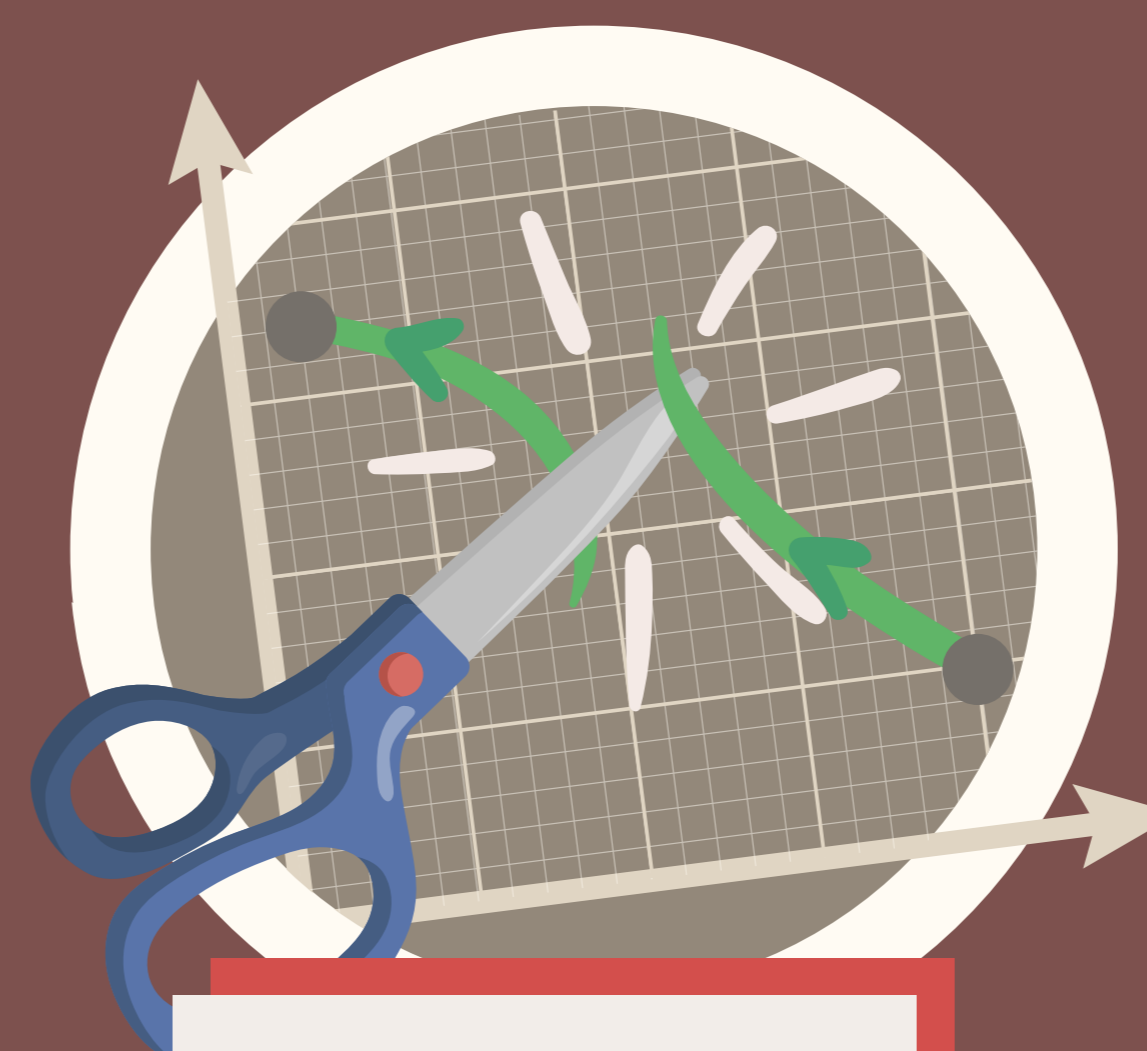
Quadrant
tandem walks



Asymptotics

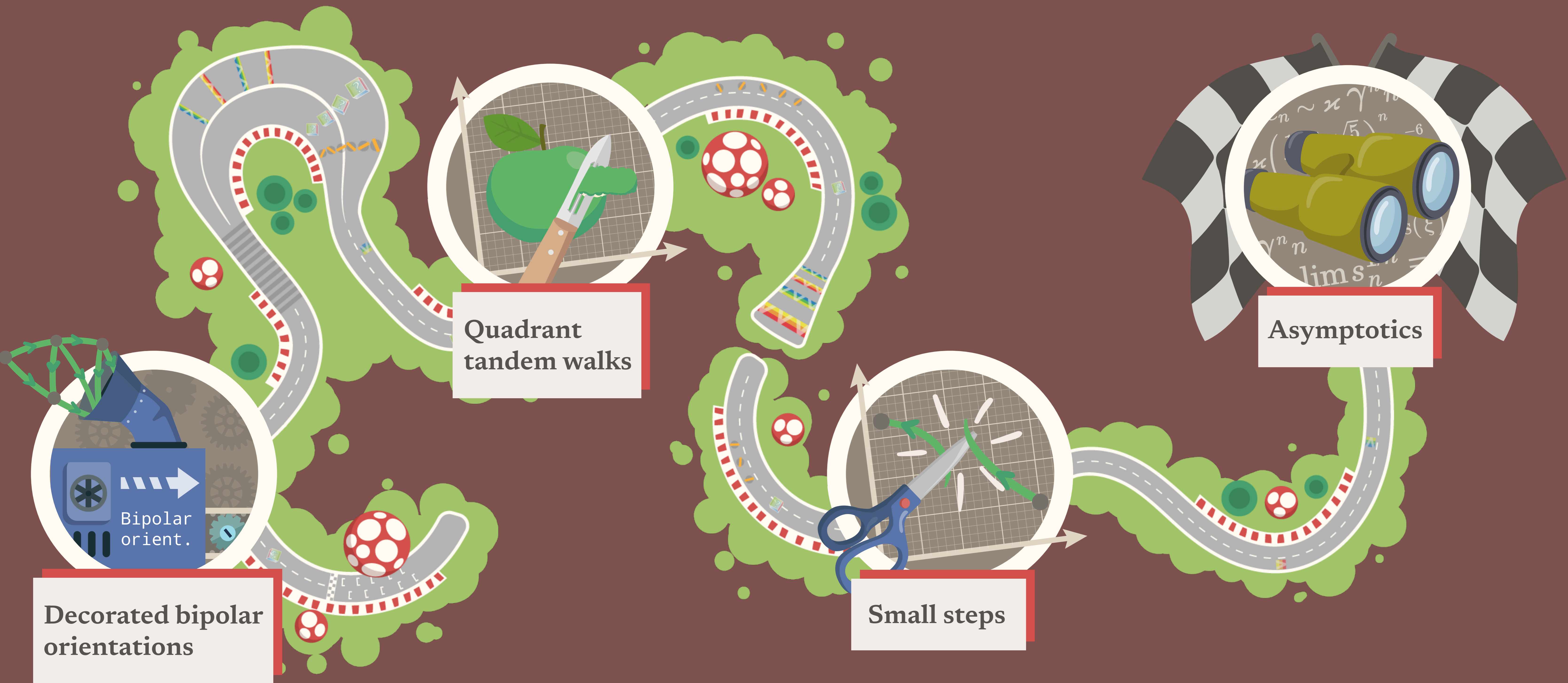


Decorated bipolar
orientations

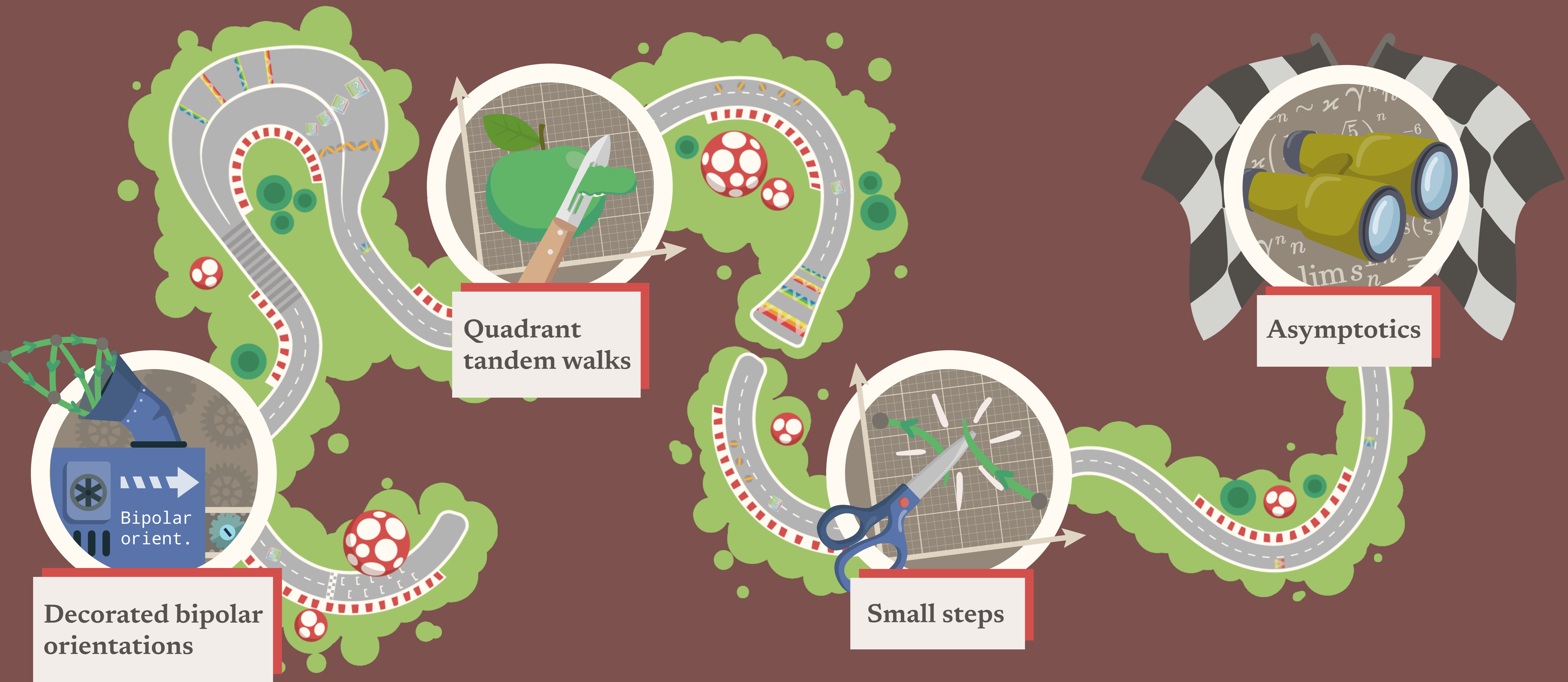


Small steps

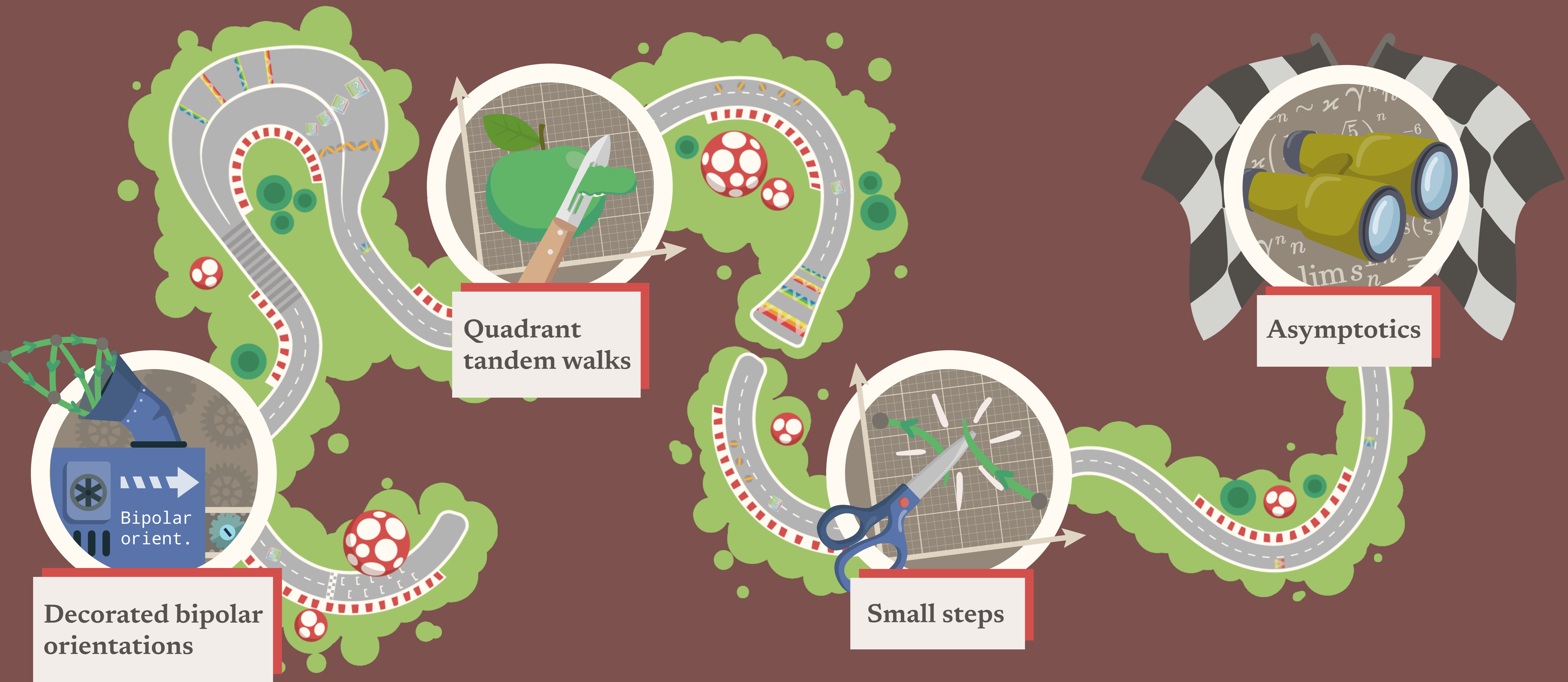
Summary



Summary



Summary



Decorated bipolar orientations

Quadrant tandem walks

Small steps

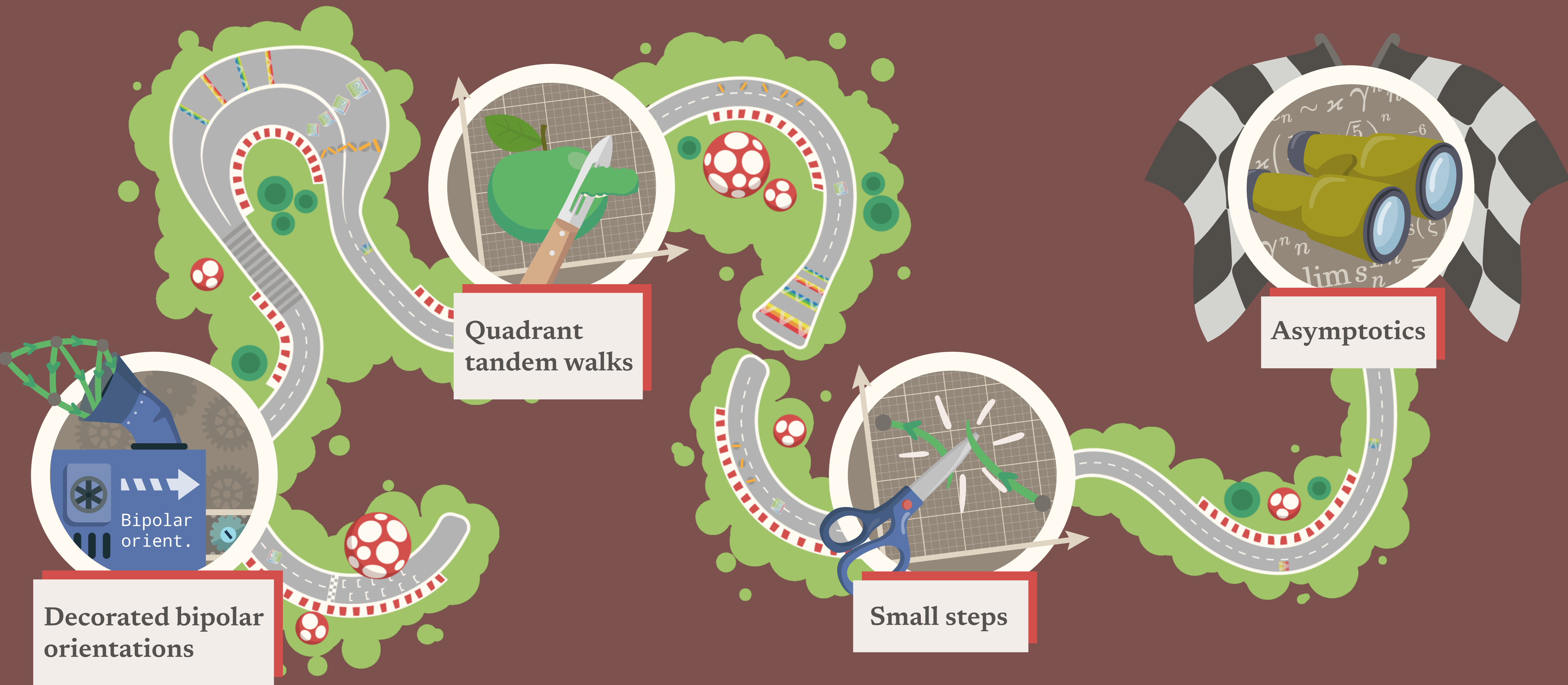
Asymptotics



*bipolar posets
by vertices*



Summary



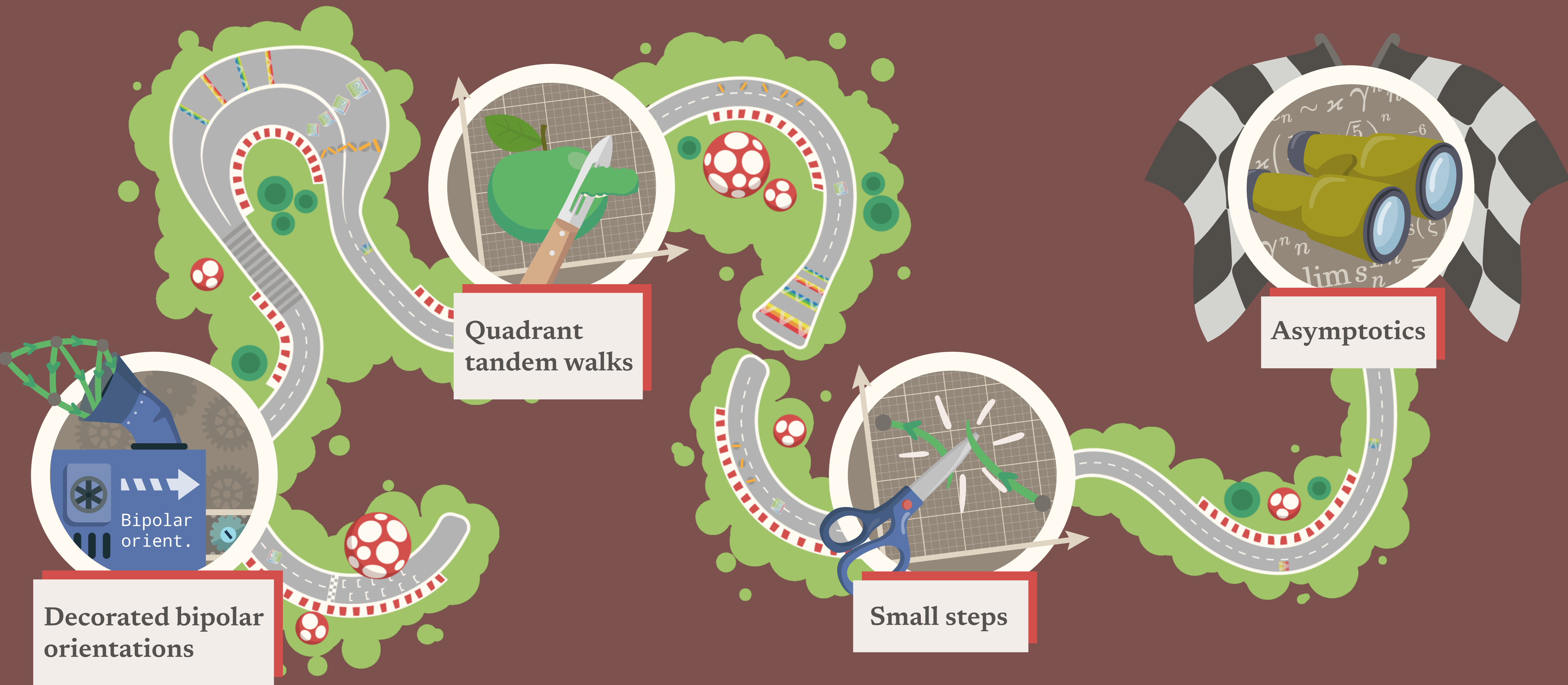
*bipolar posets
by vertices*



*bipolar posets
by edges*



Summary



*bipolar posets
by vertices*



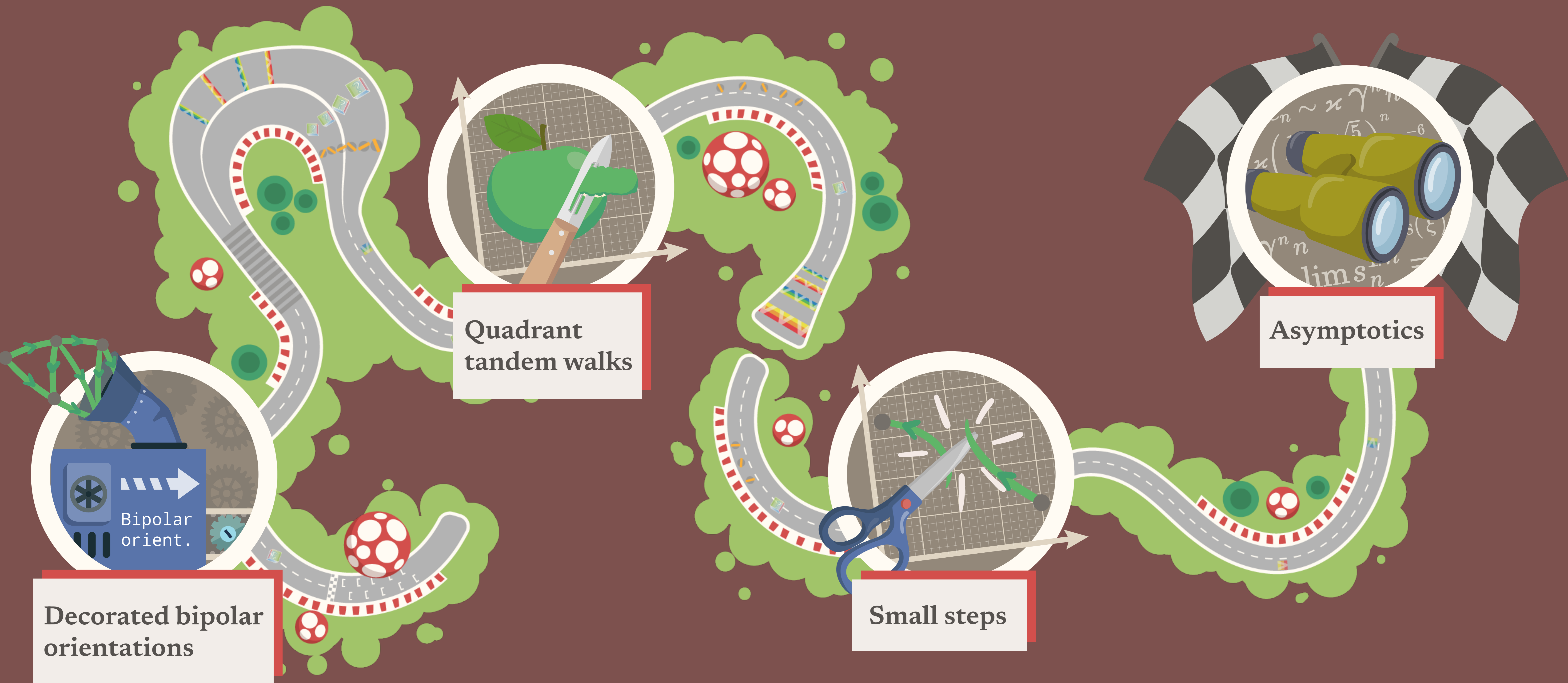
*bipolar posets
by edges*



*polyhedral
orientations*



Summary



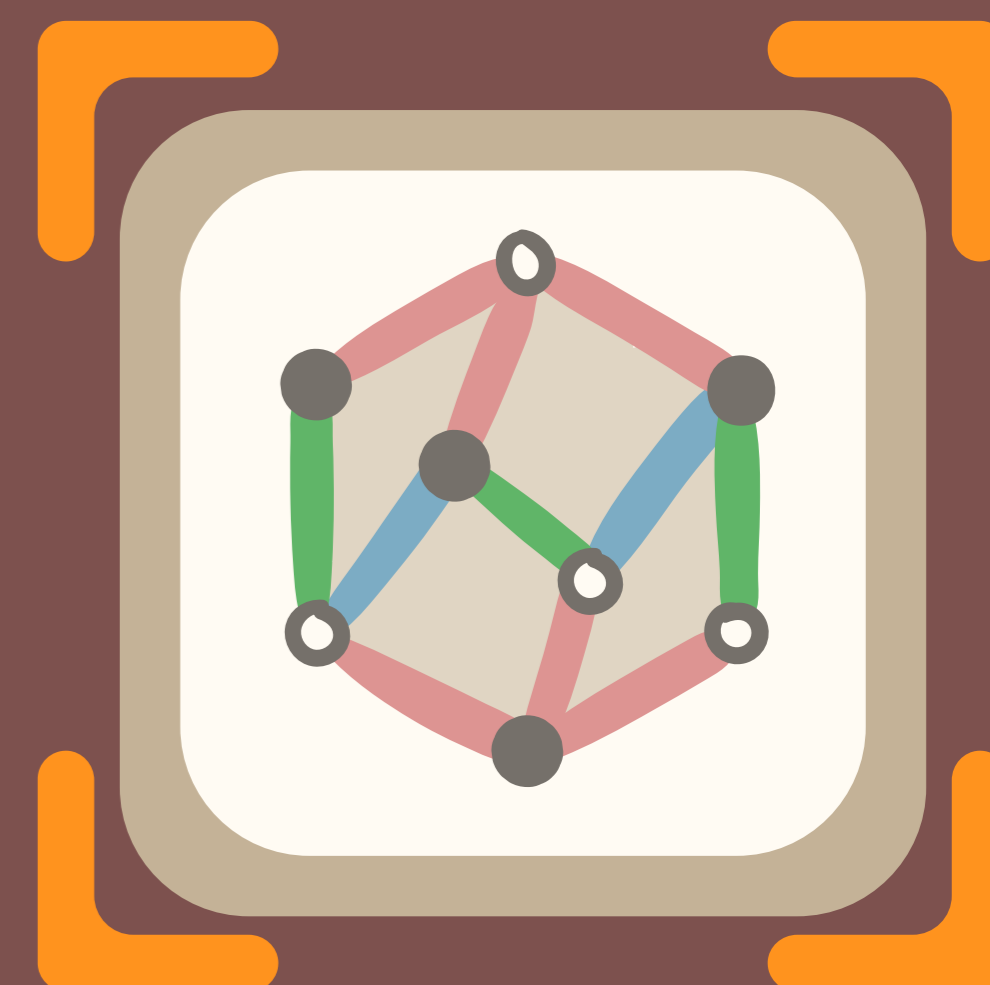
*bipolar posets
by vertices*



*bipolar posets
by edges*

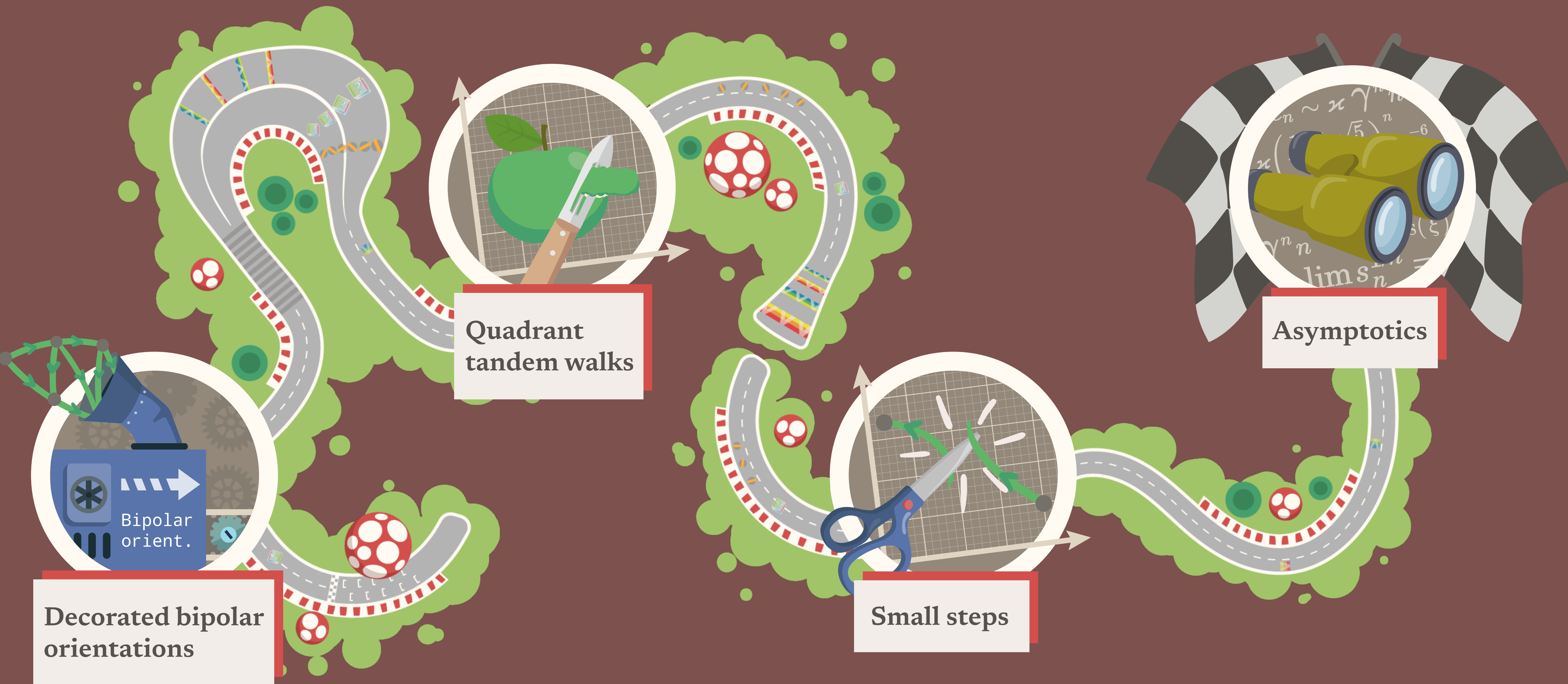


*polyhedral
orientations*



*Schnyder
labelings*

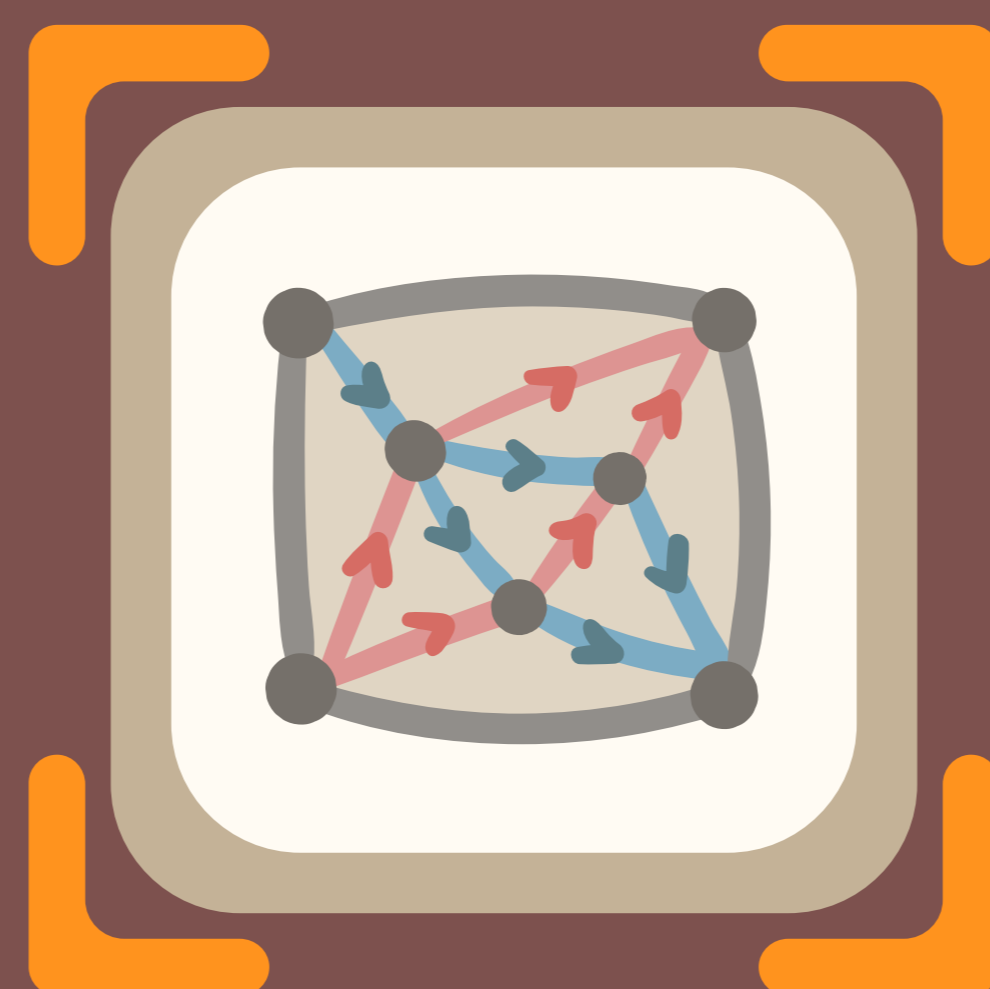
Summary



*bipolar posets
by vertices*



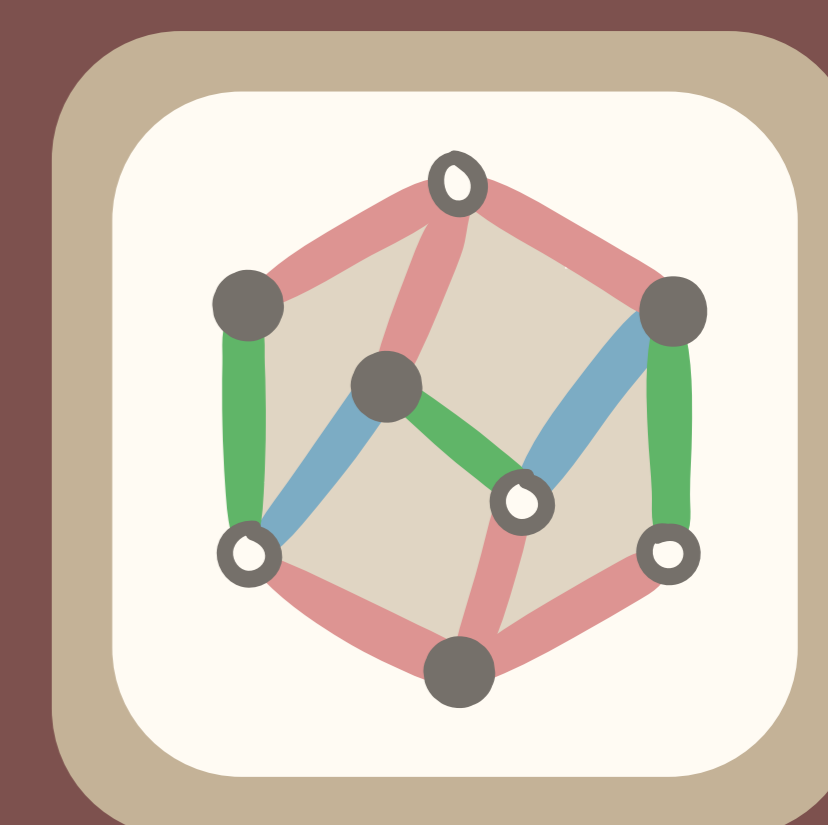
*bipolar posets
by edges*



*transversal
structures*

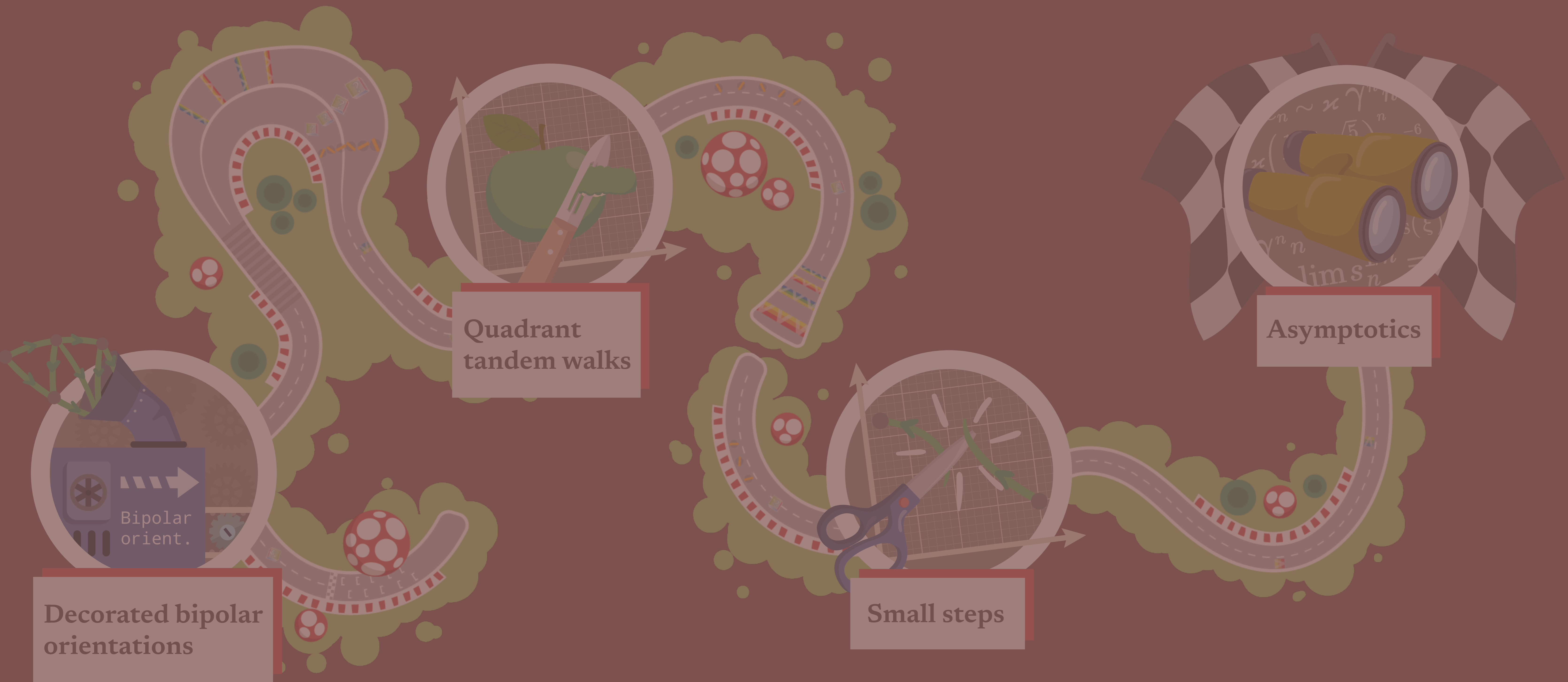


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orientations*



*Schnyder
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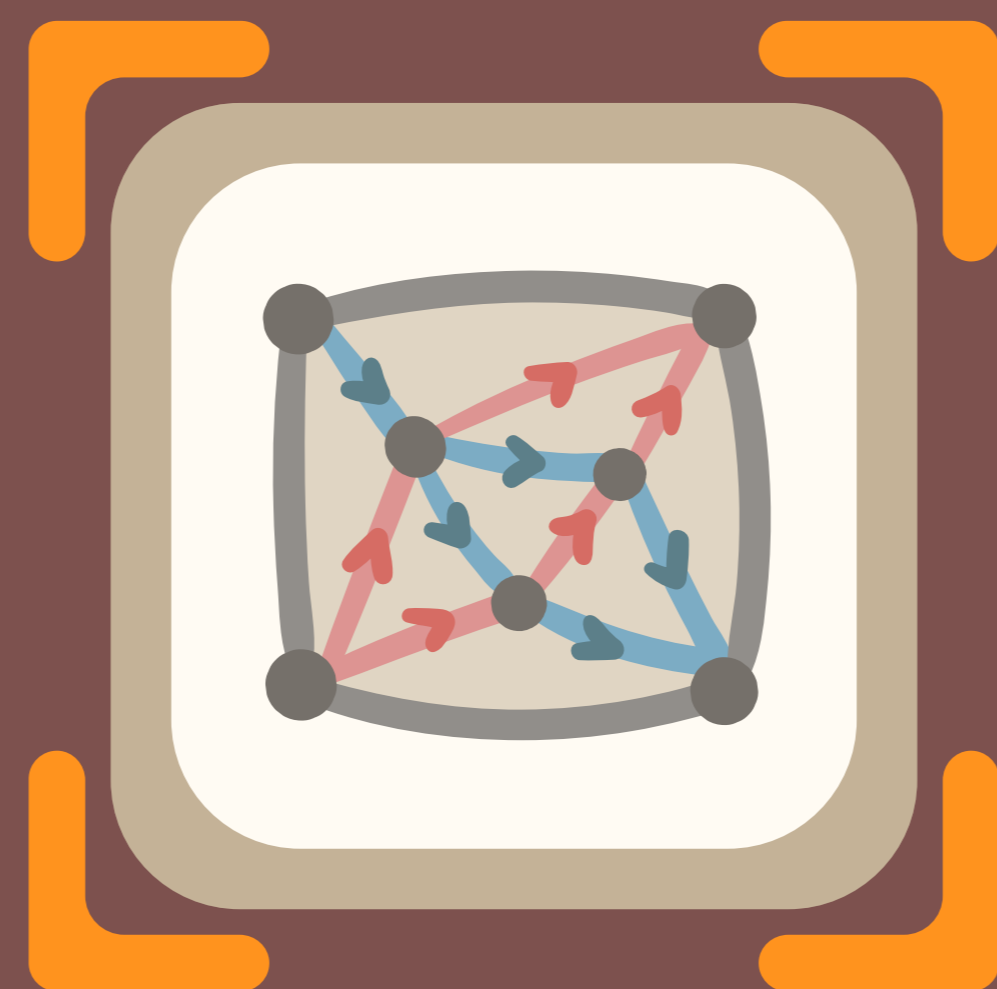
Summary



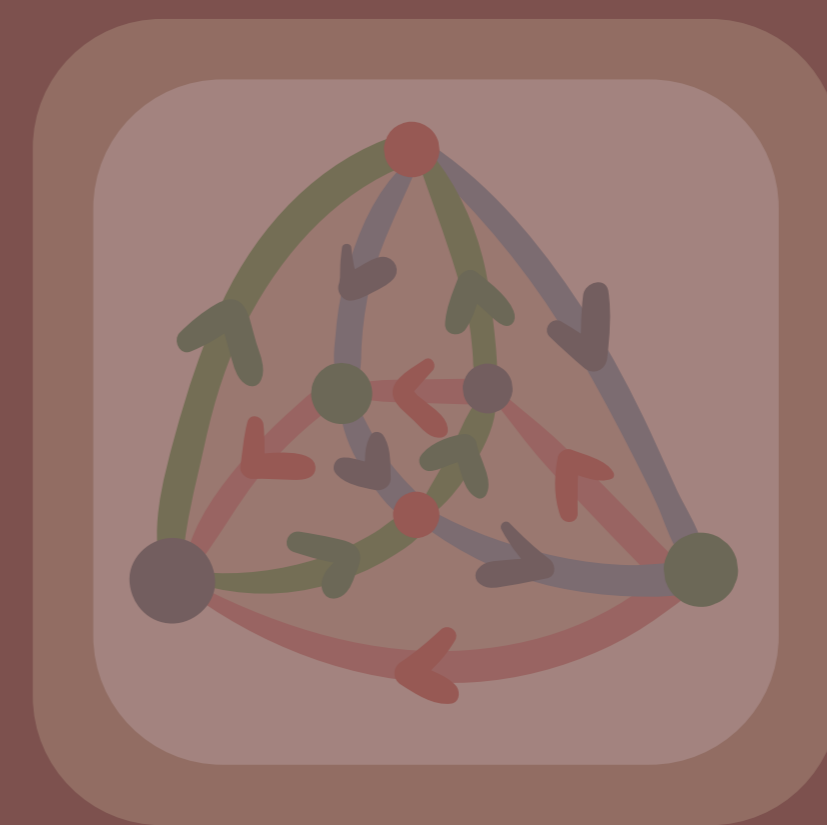
*bipolar posets
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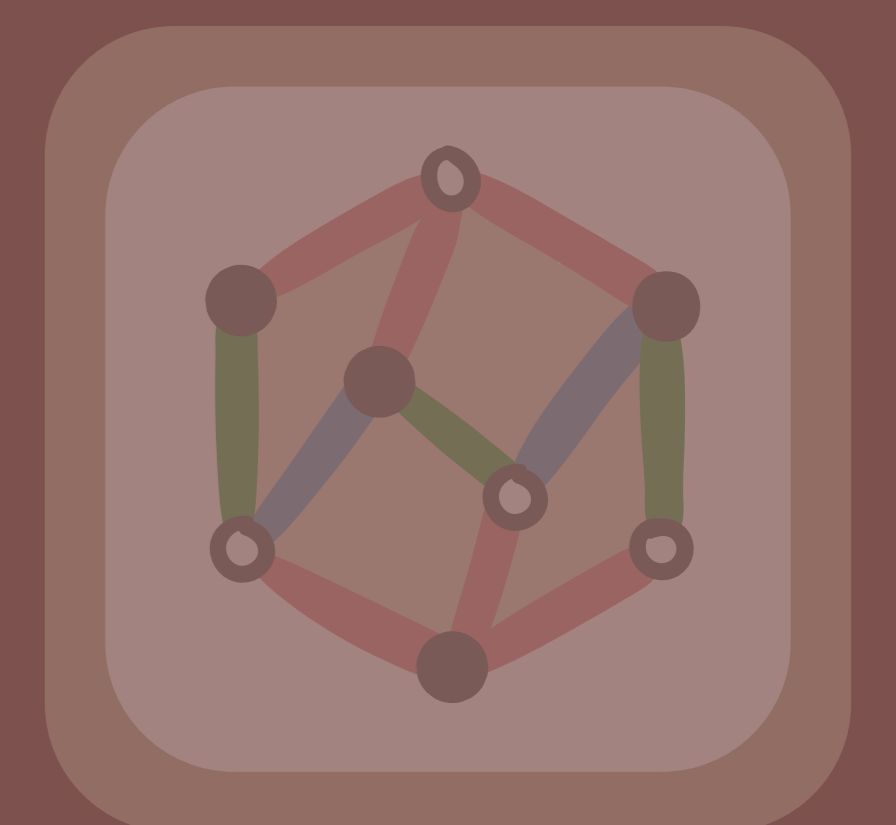
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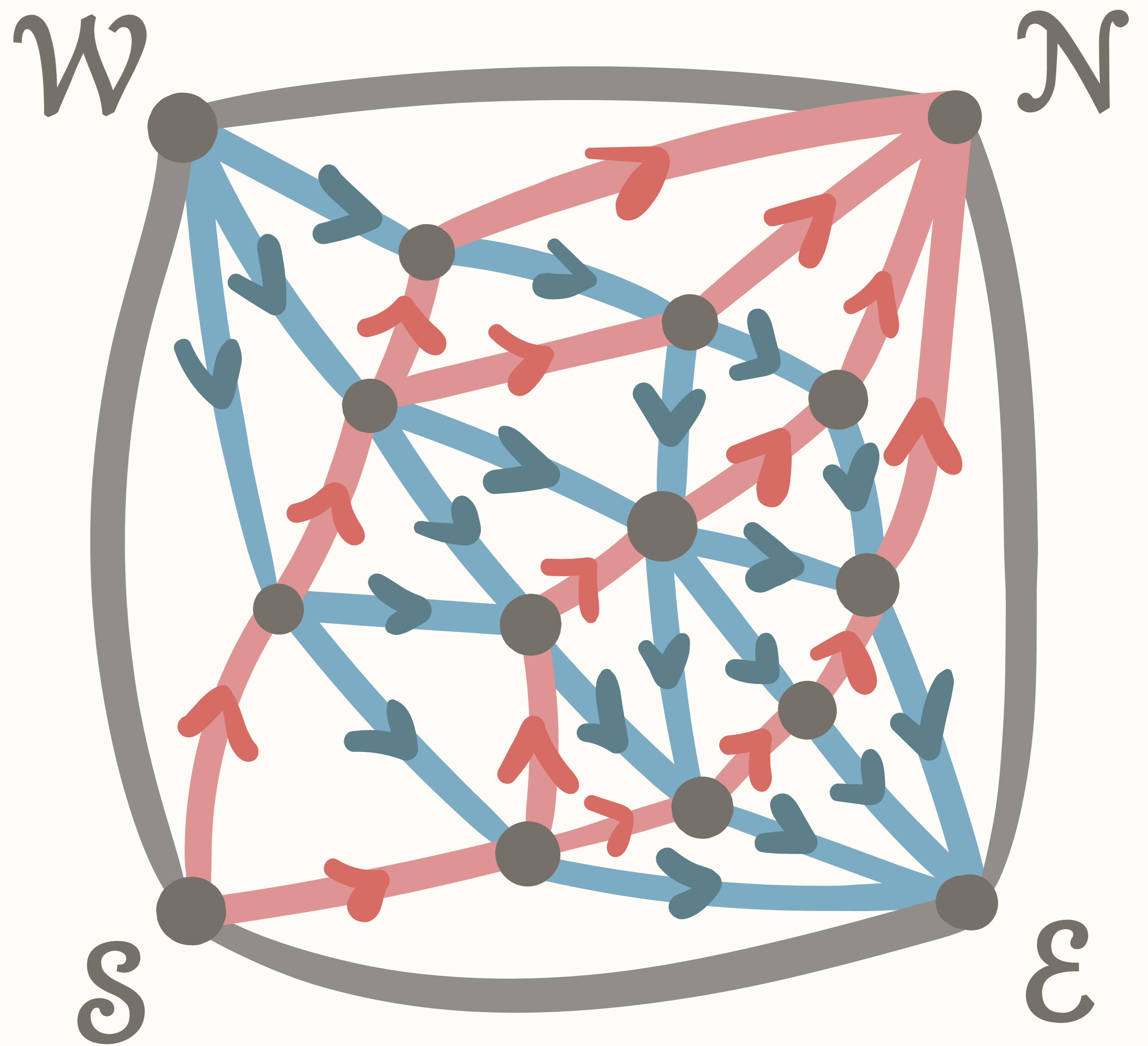


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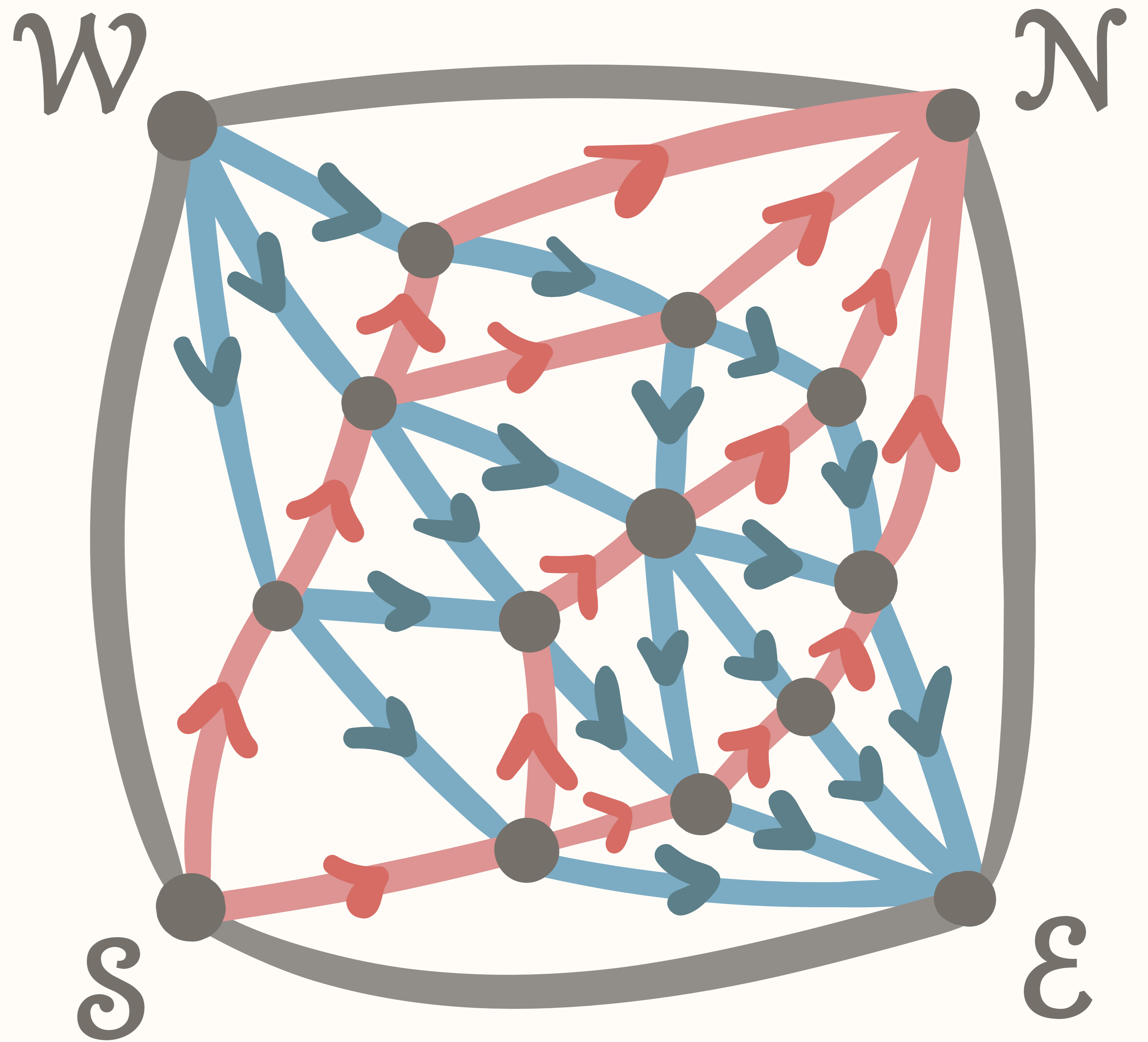
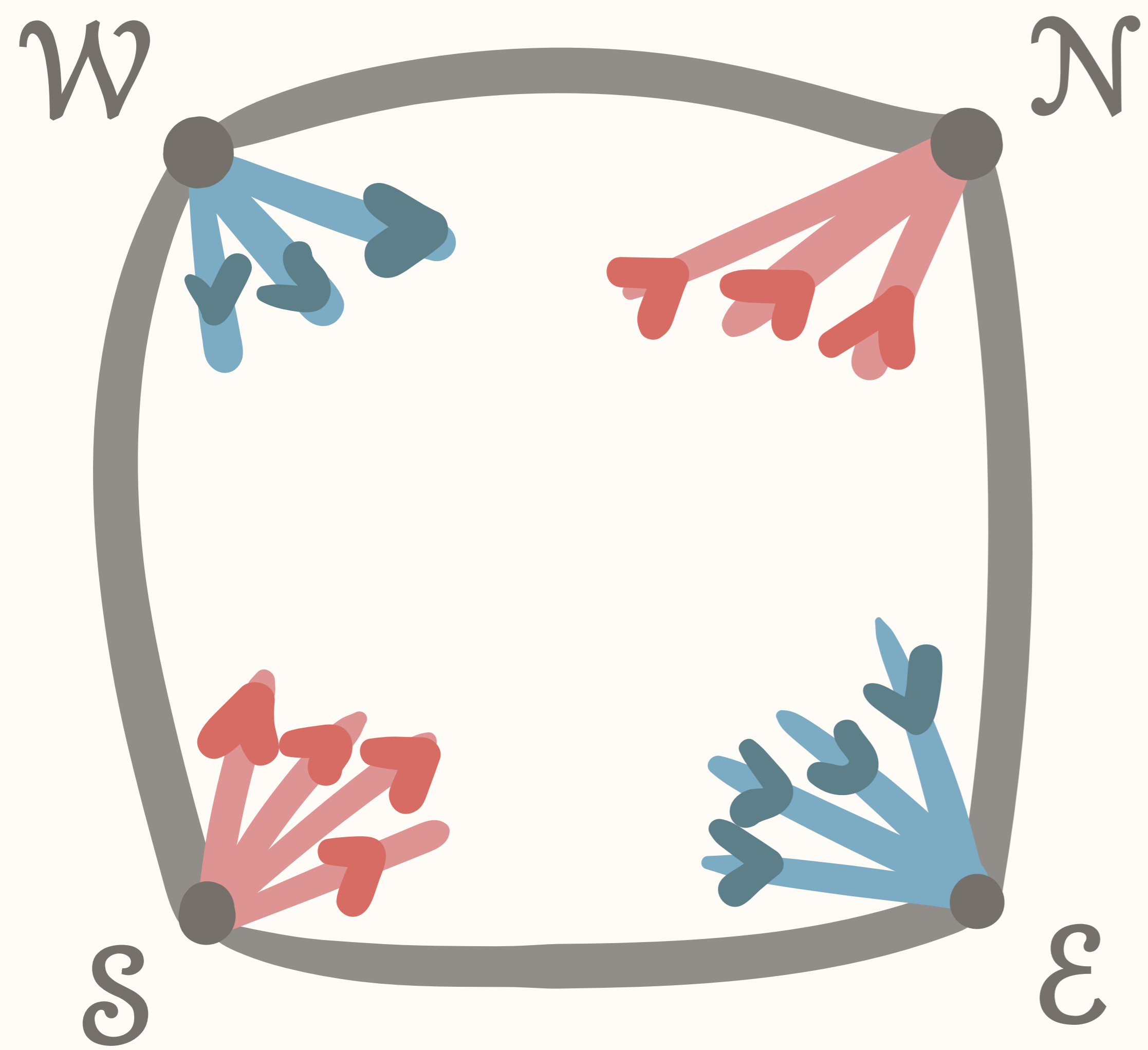


*Schnyder
labelings*

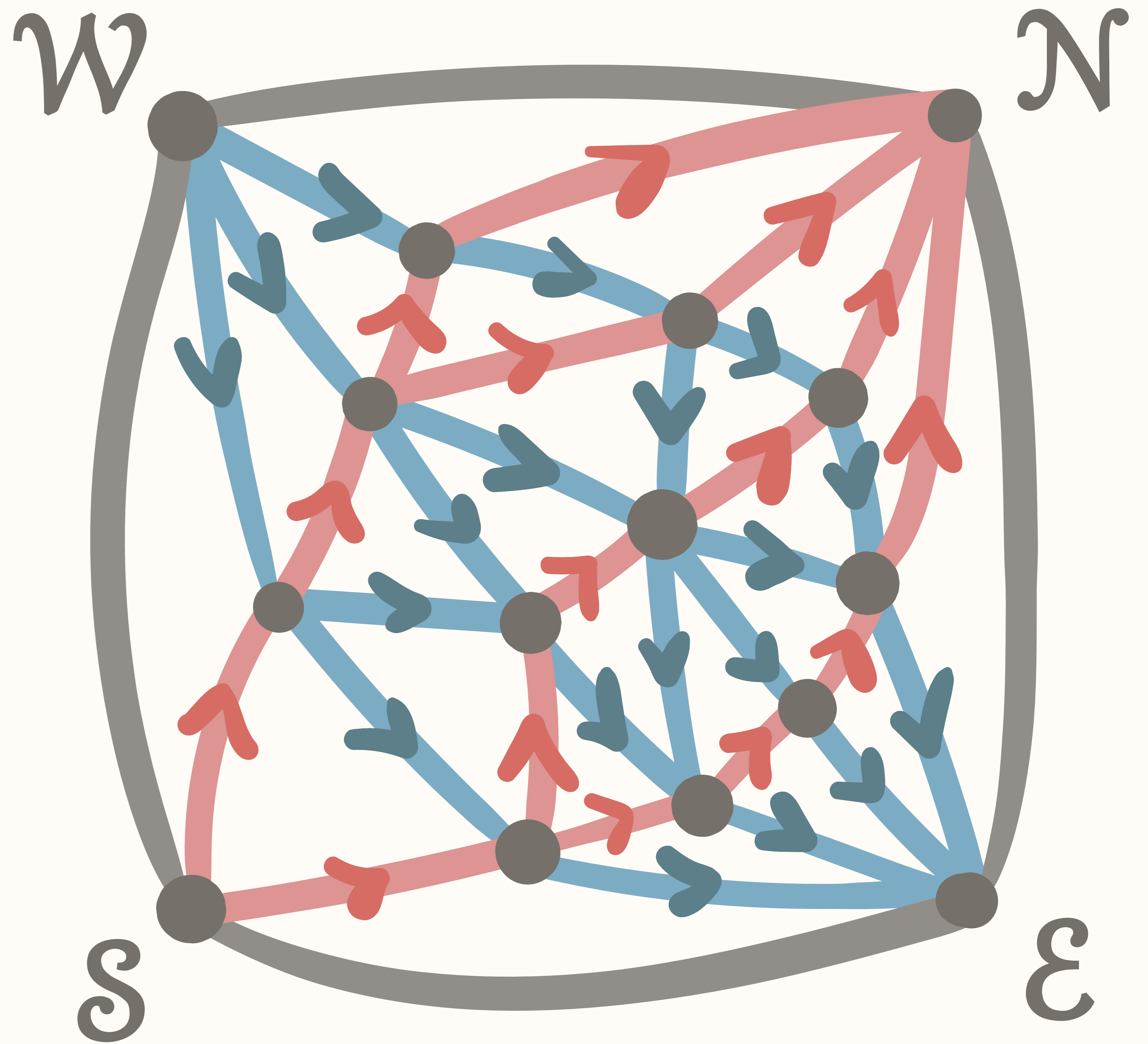
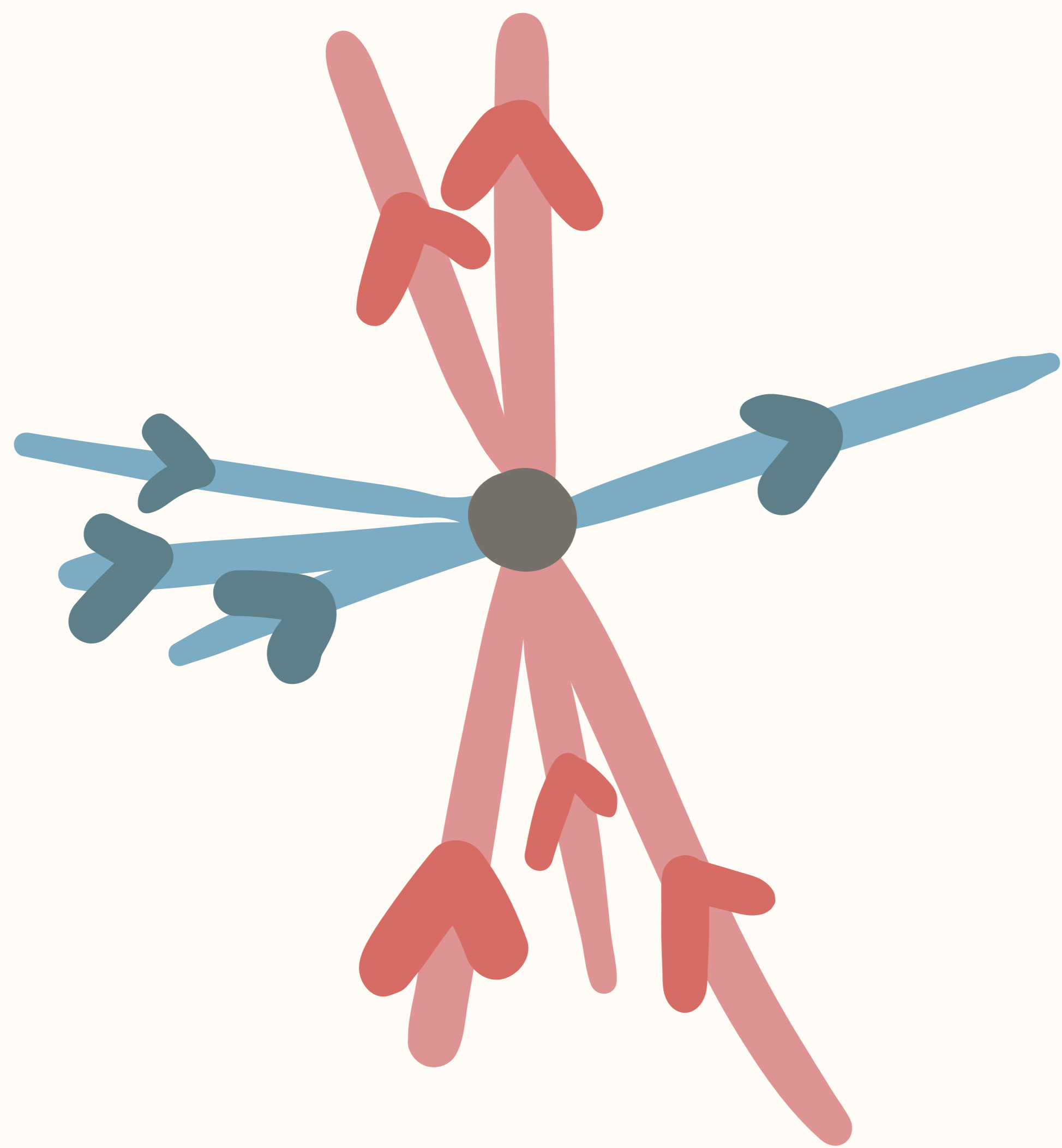
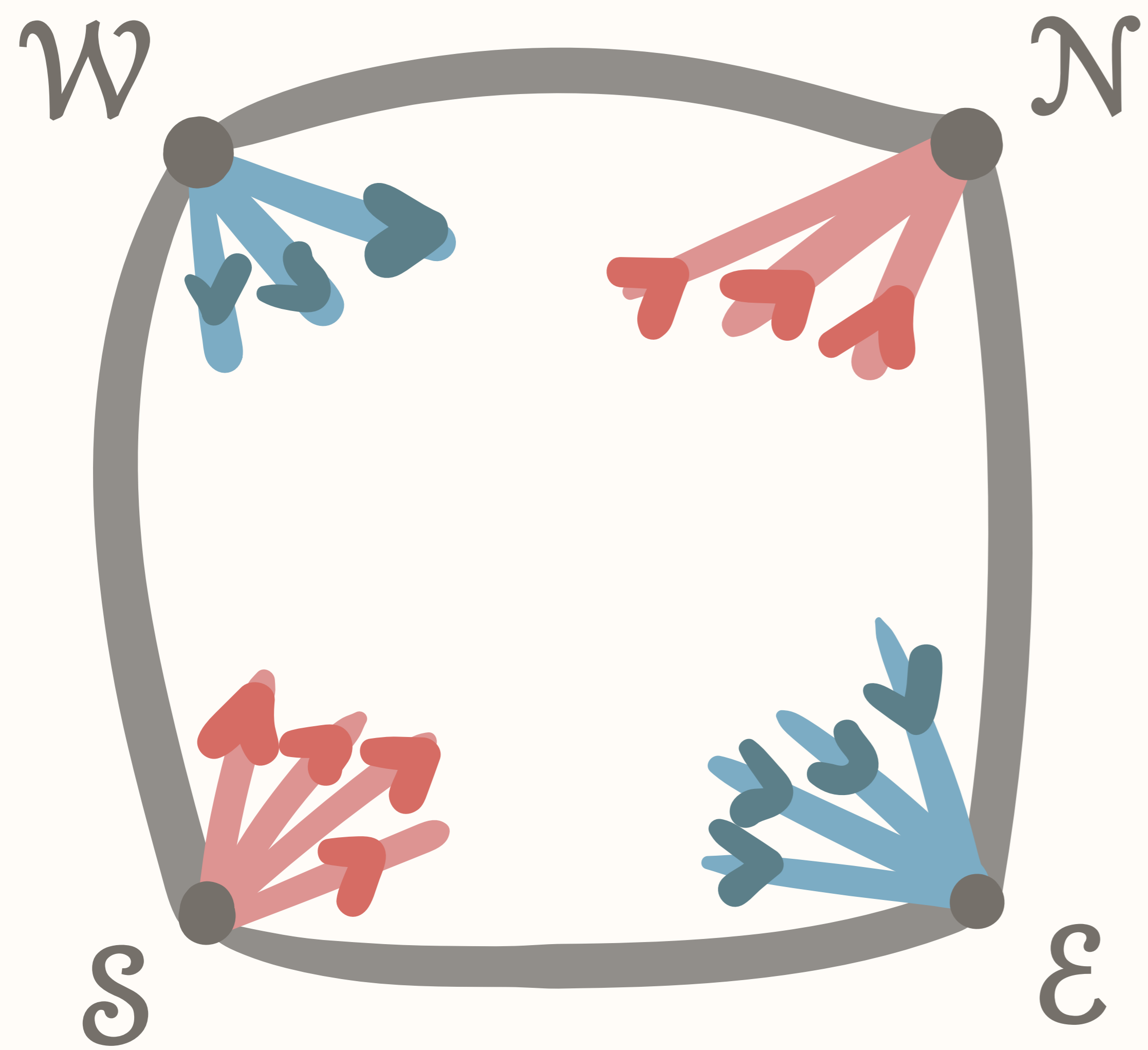
Transversal structures



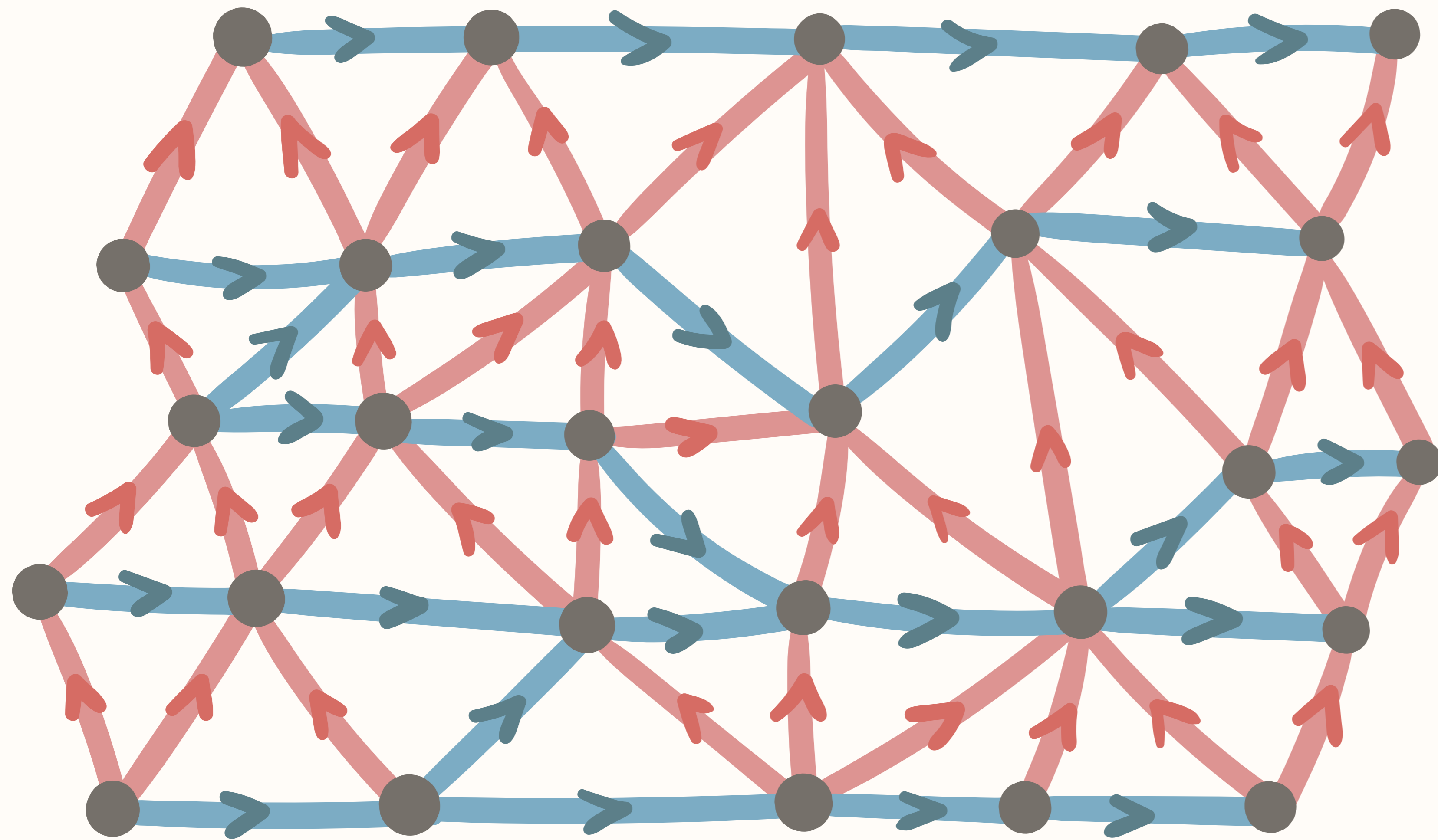
Transversal structures



Transversal structures

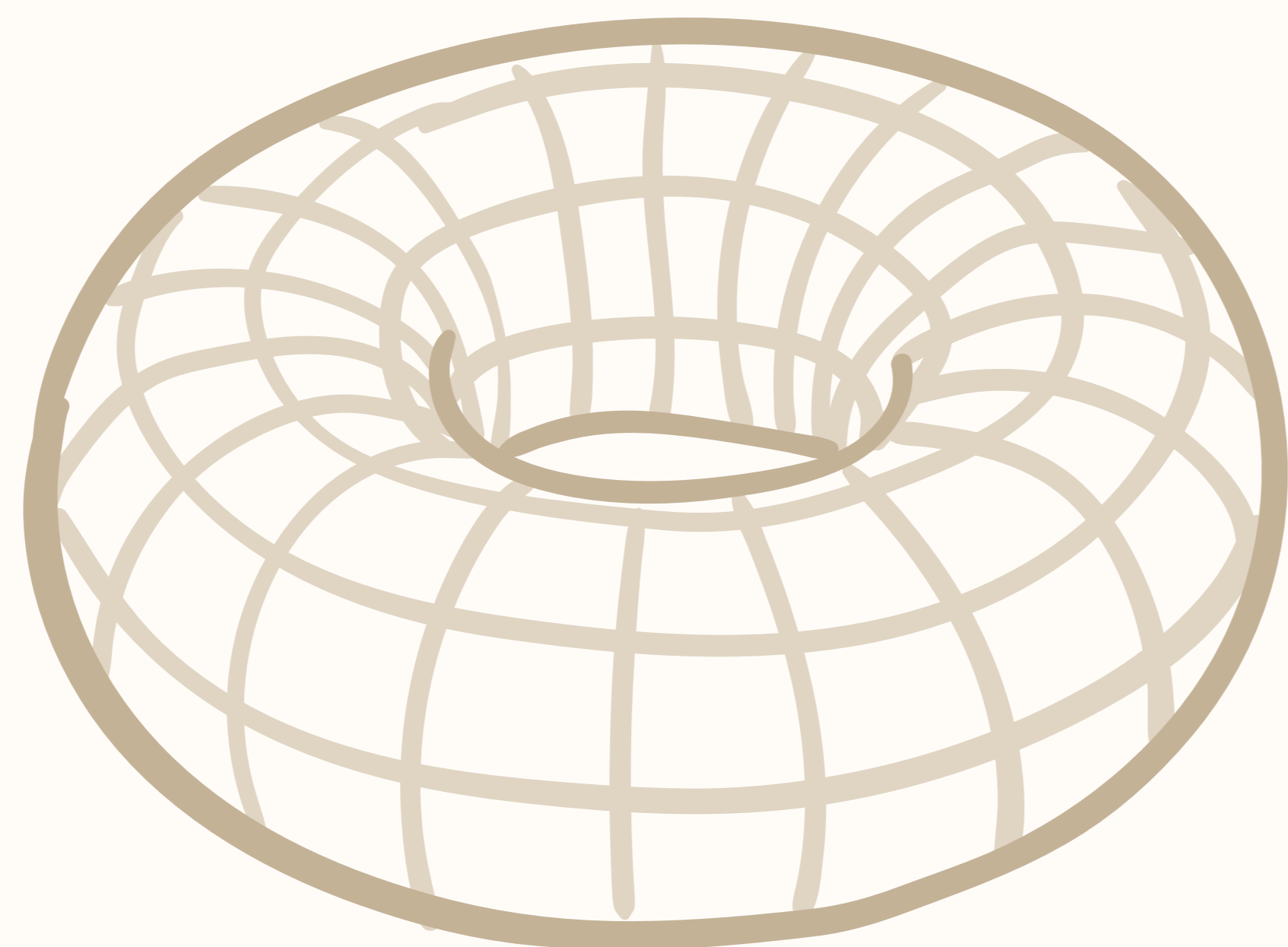
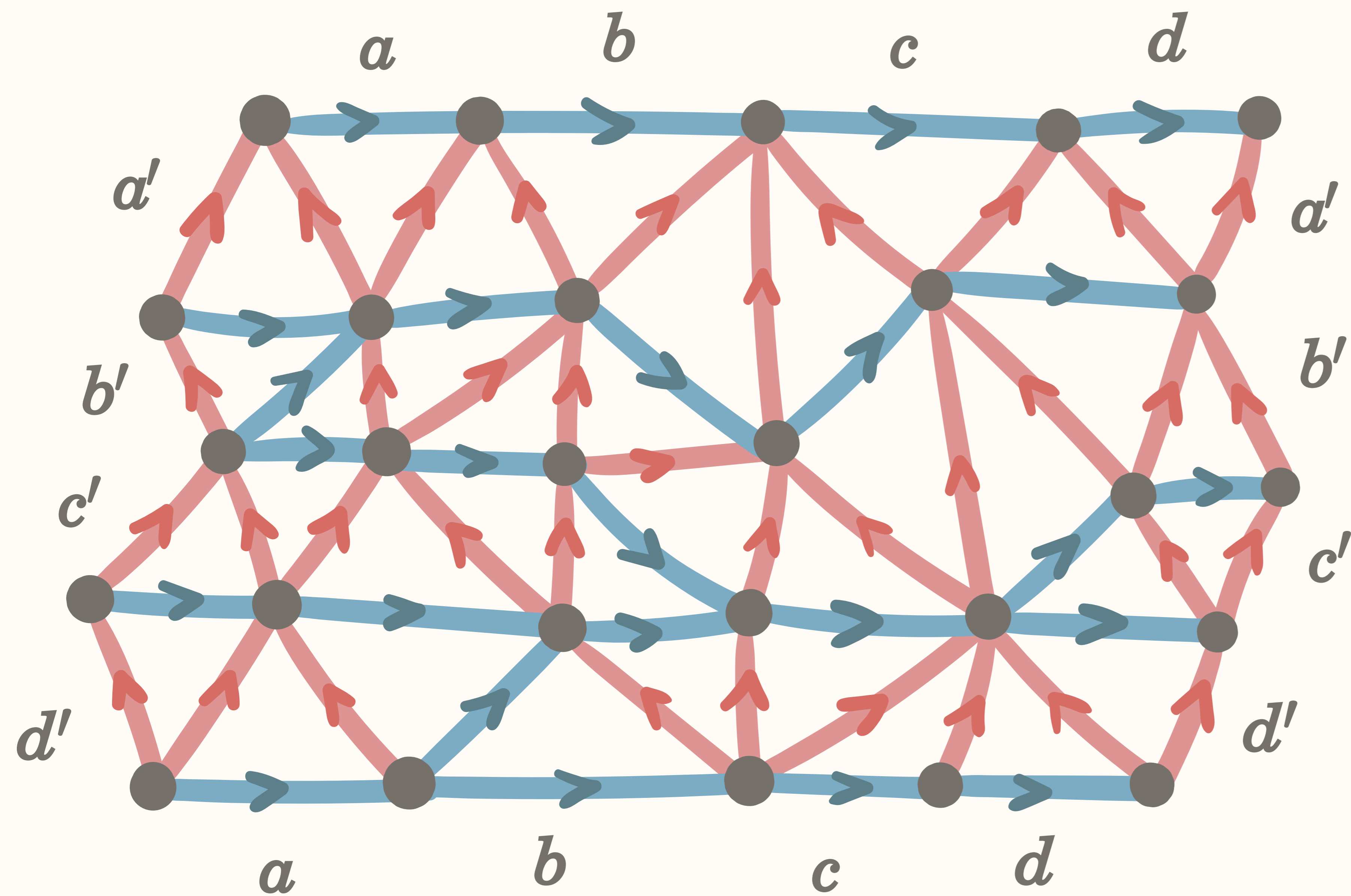


Locally causal dynamical triangulations



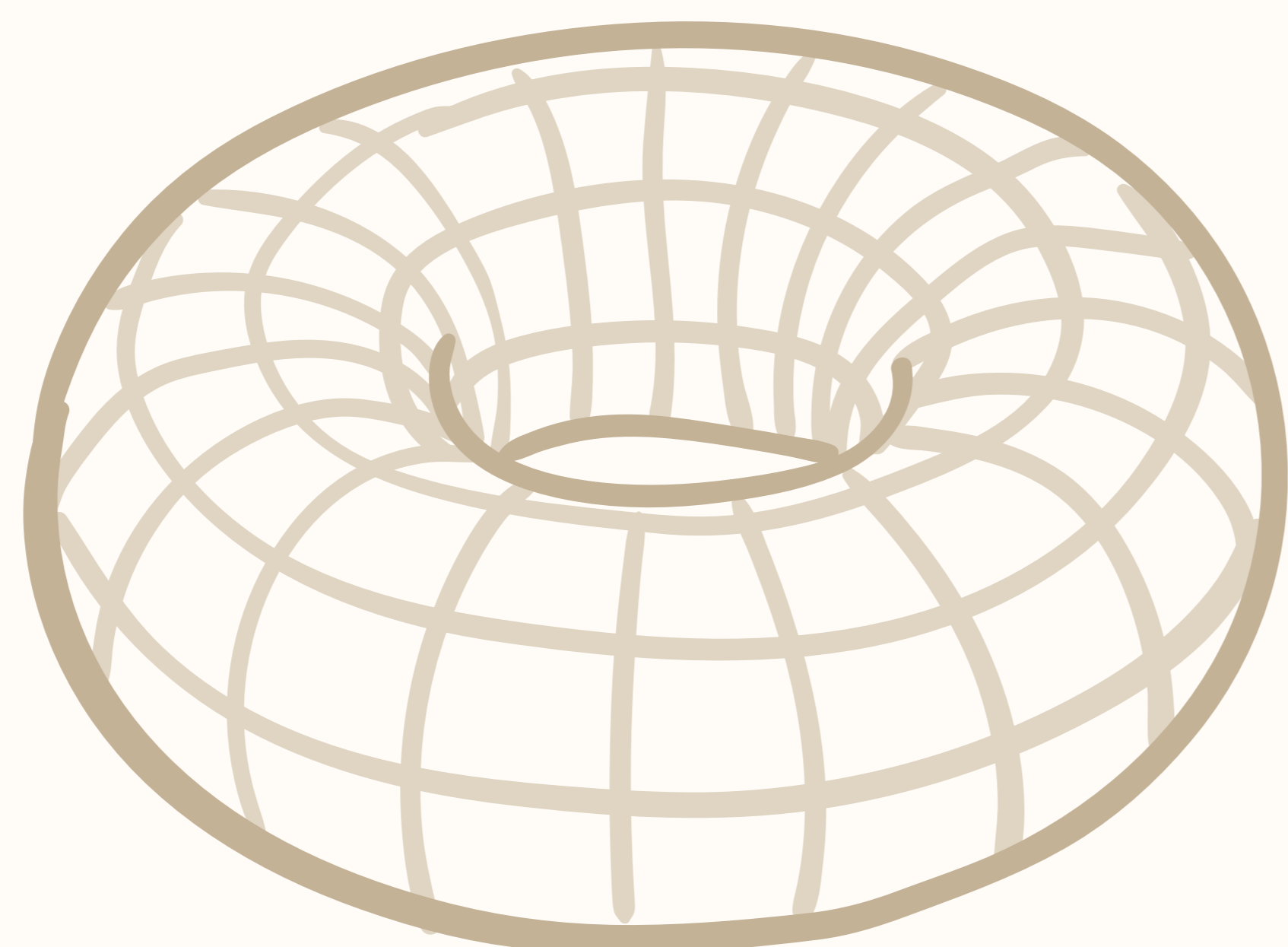
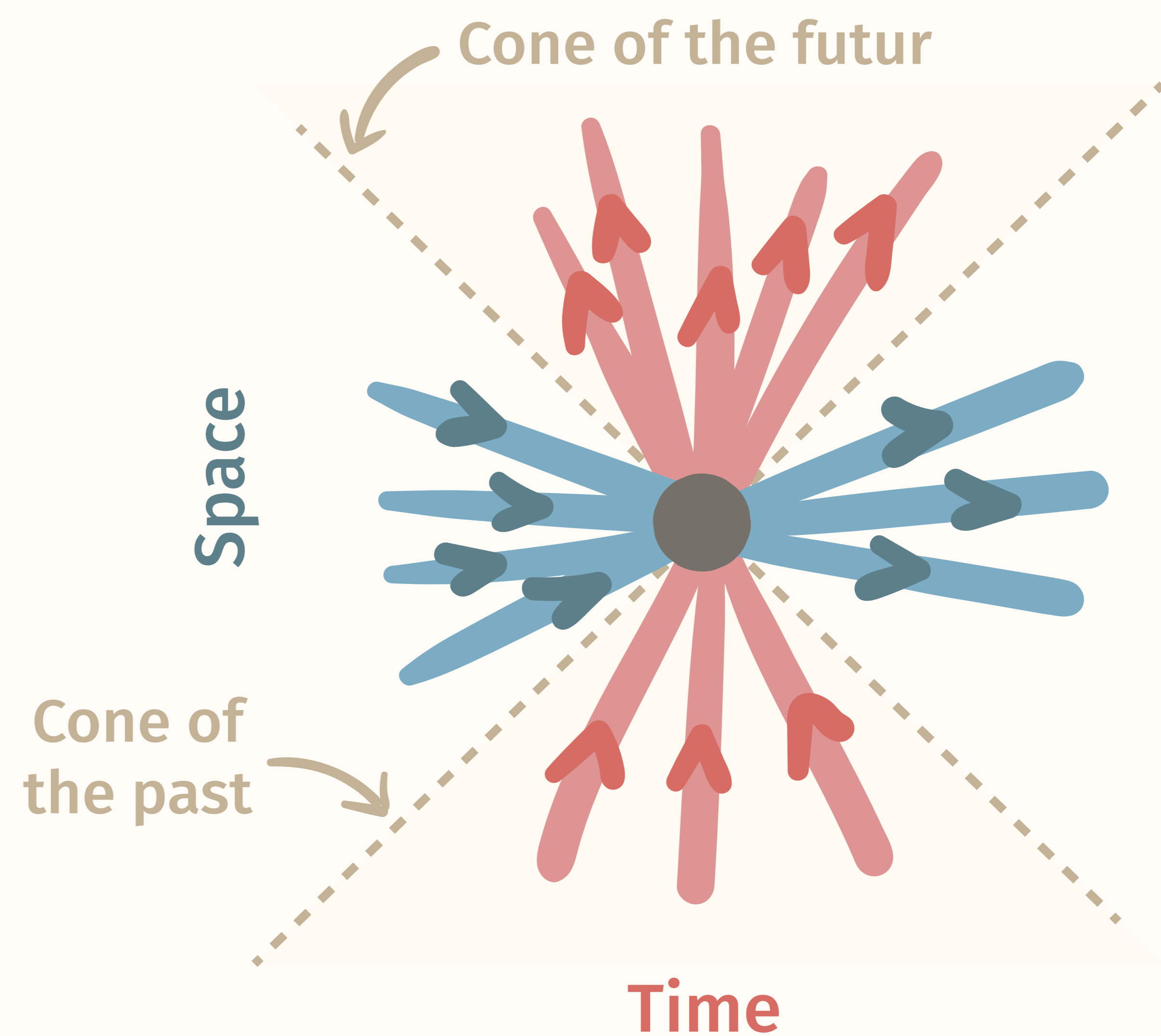
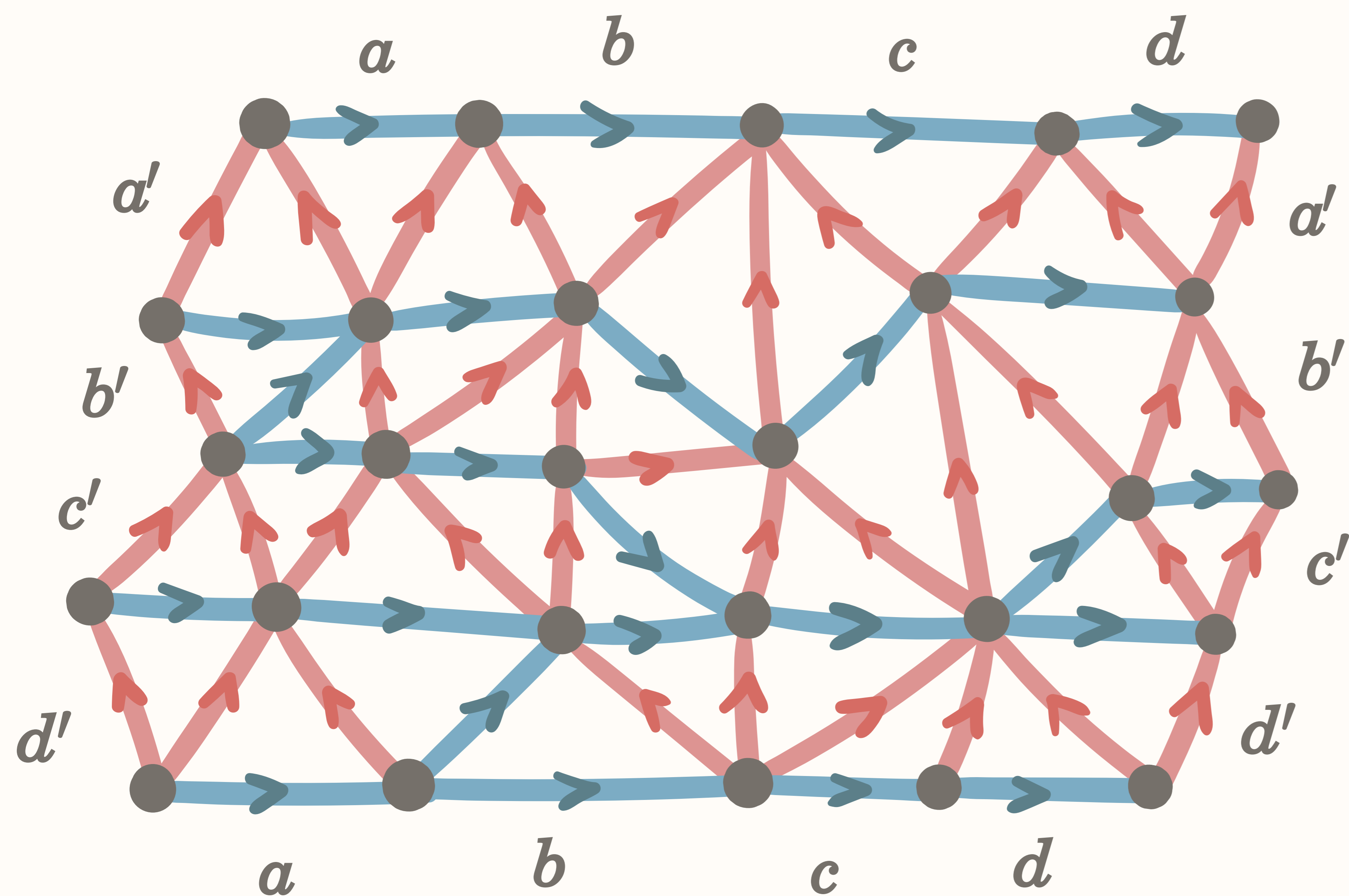
⇒ *Locally causal dynamical triangulations
in two dimensions, R. Loll & B. Ruijl (2015)*

Locally causal dynamical triangulations



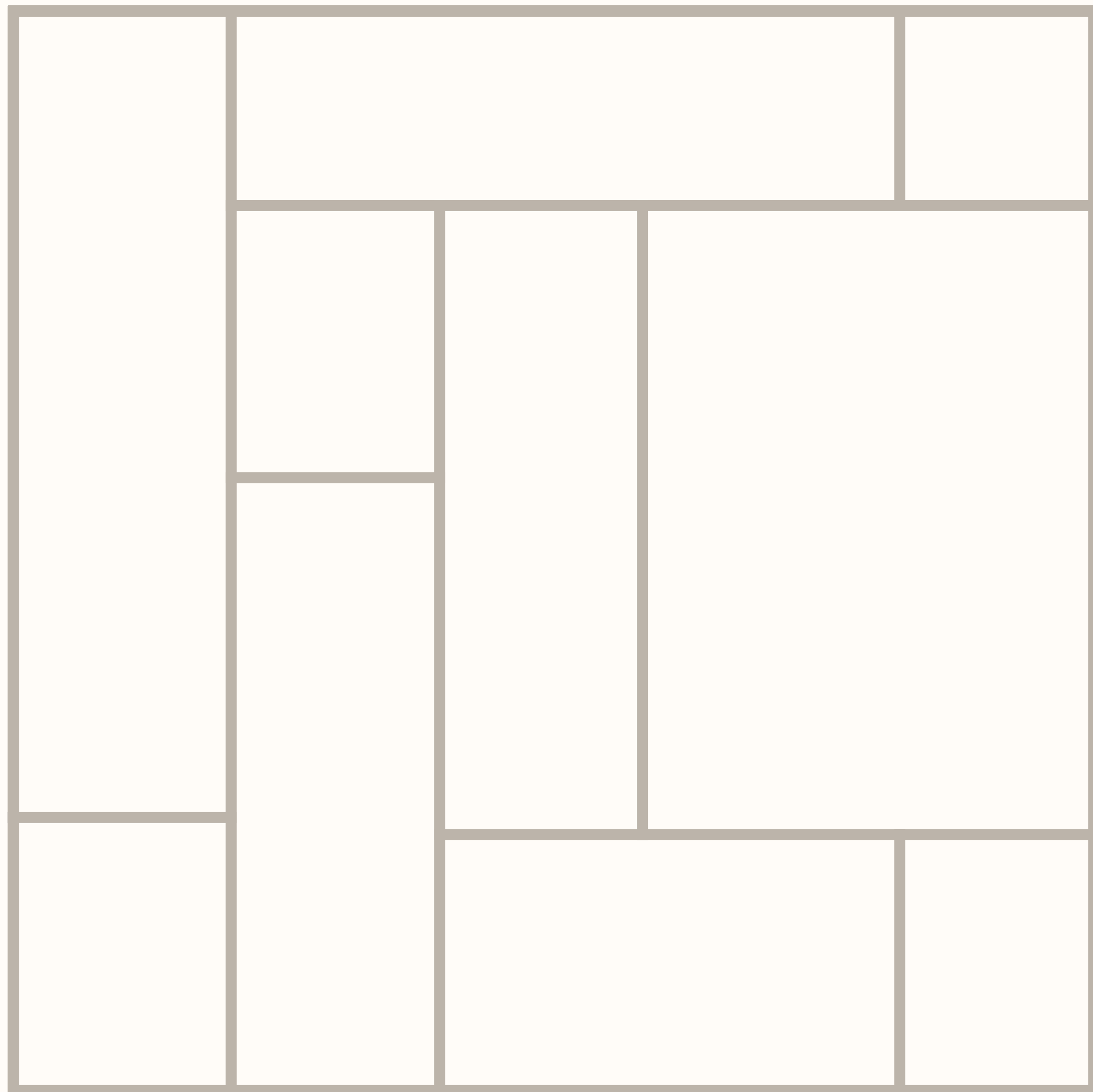
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Locally causal dynamical triangulations

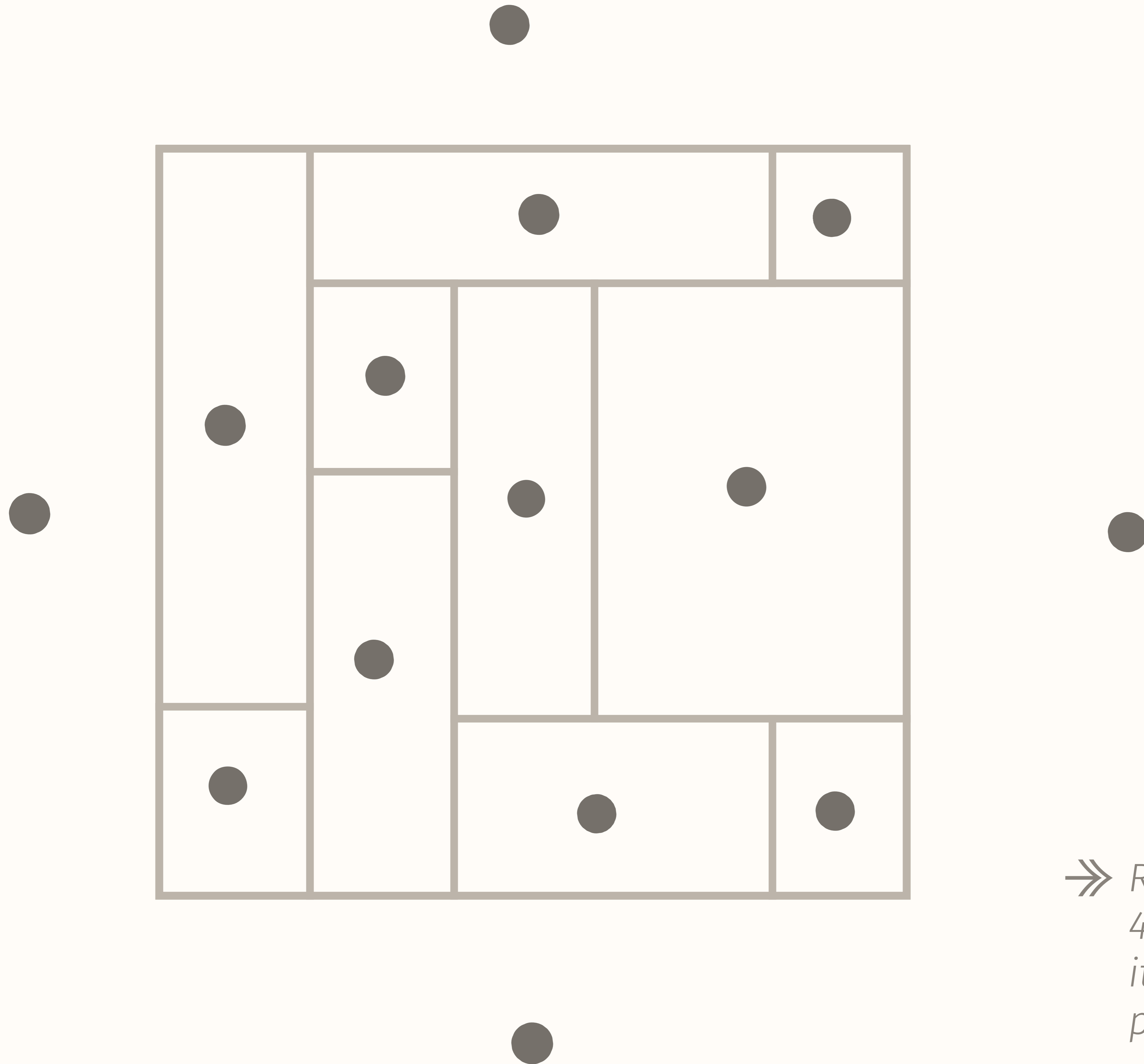


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Link with rectangular tilings

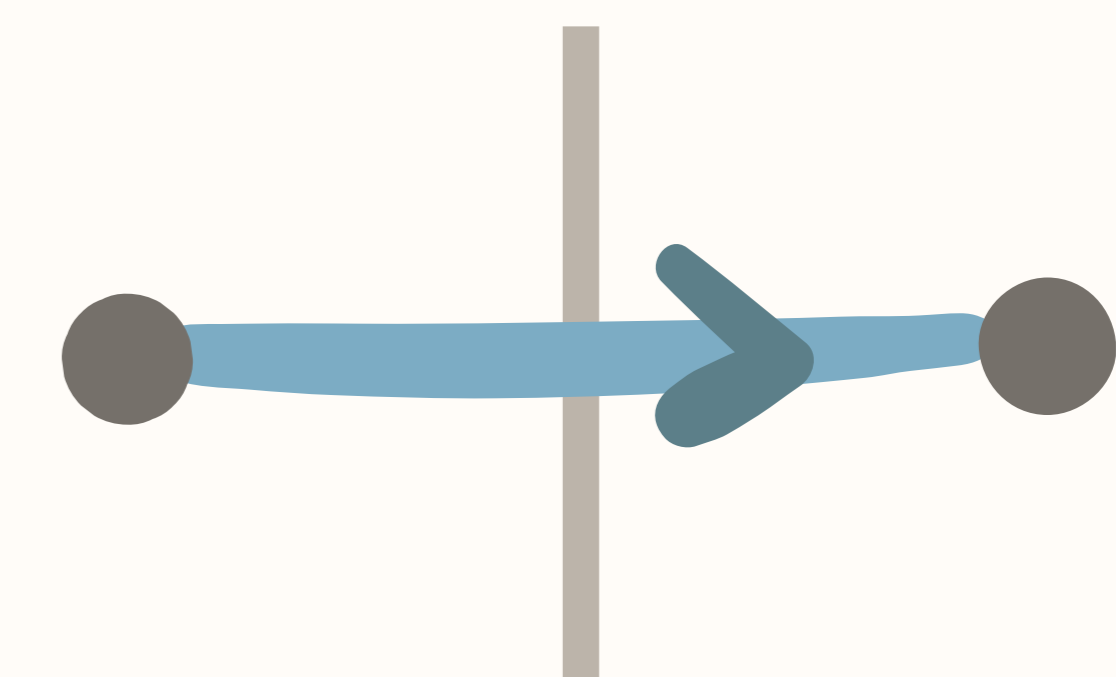
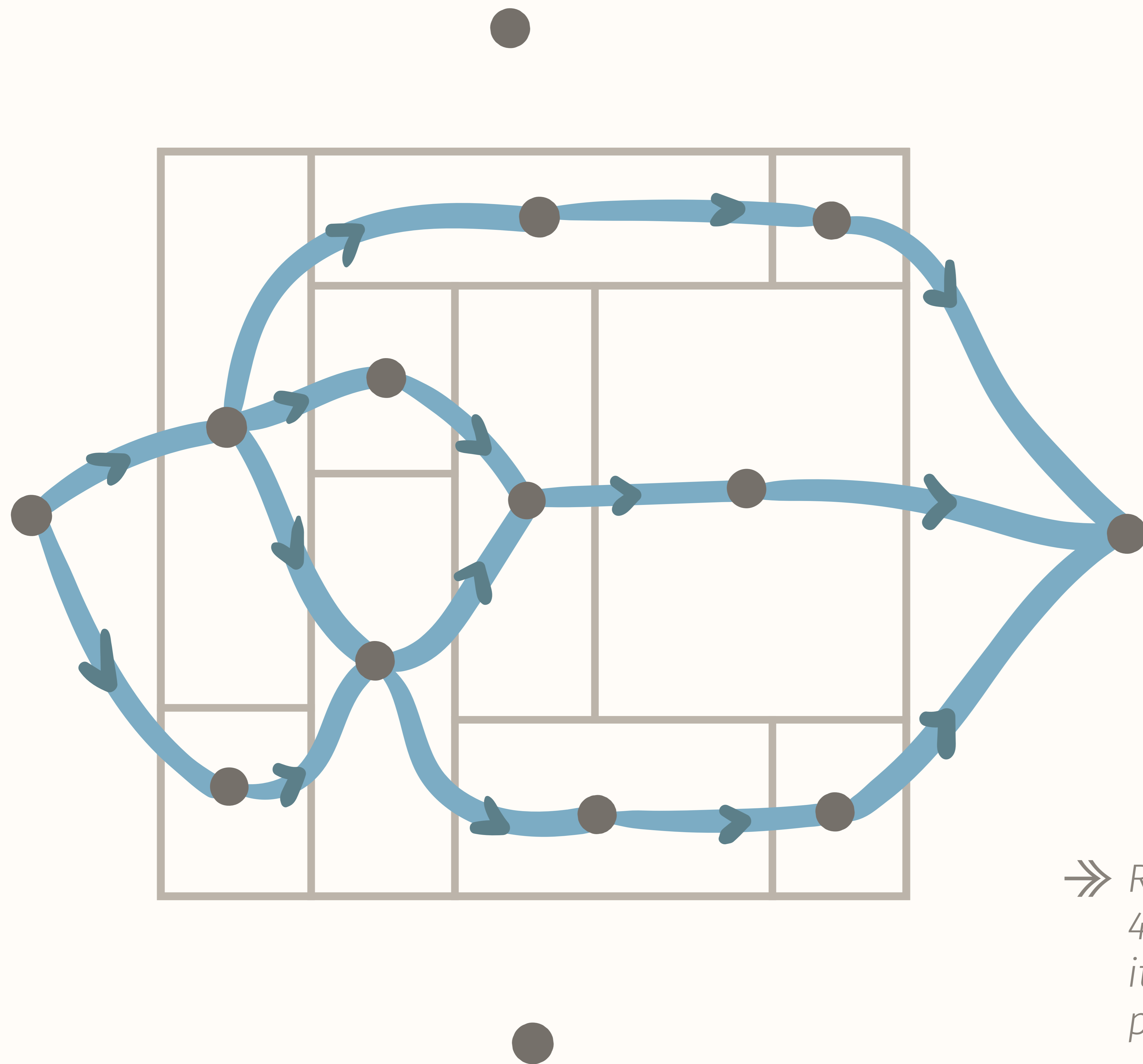


Link with rectangular tilings



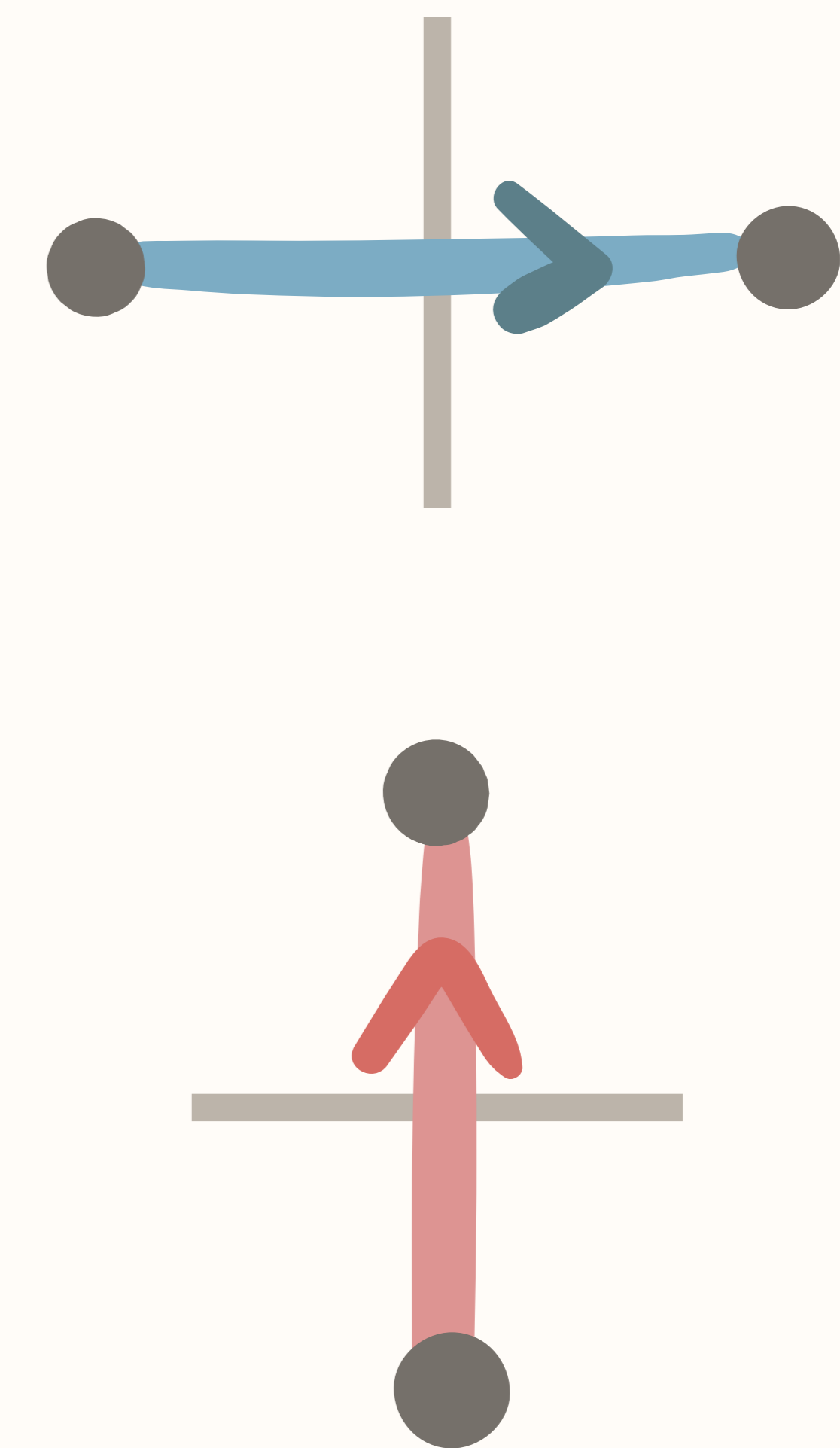
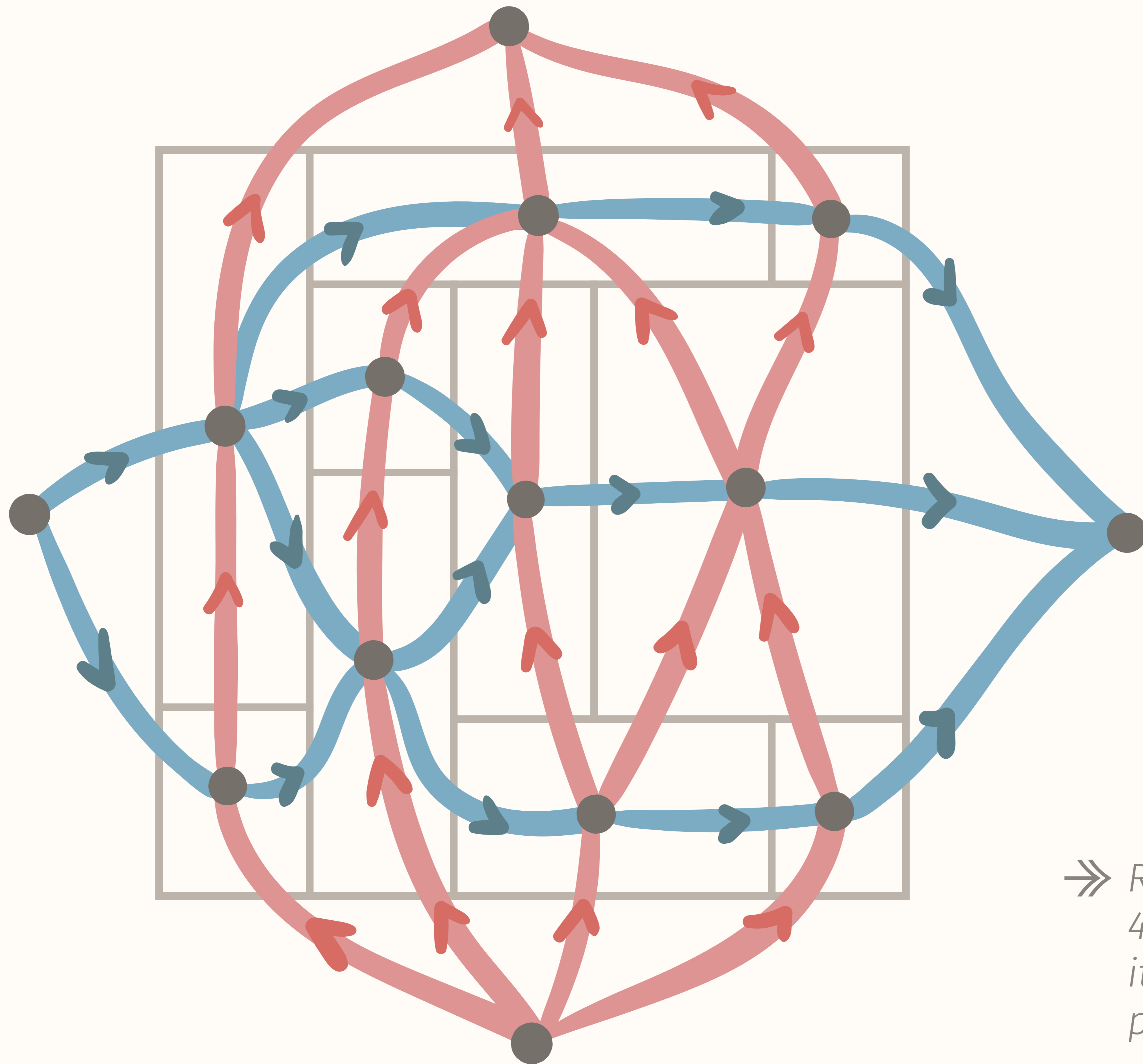
⇒ *Regular edge labellings of 4-connected plane graphs and its applications in graph drawing problems, G. Kant & X. He (1997)*

Link with rectangular tilings



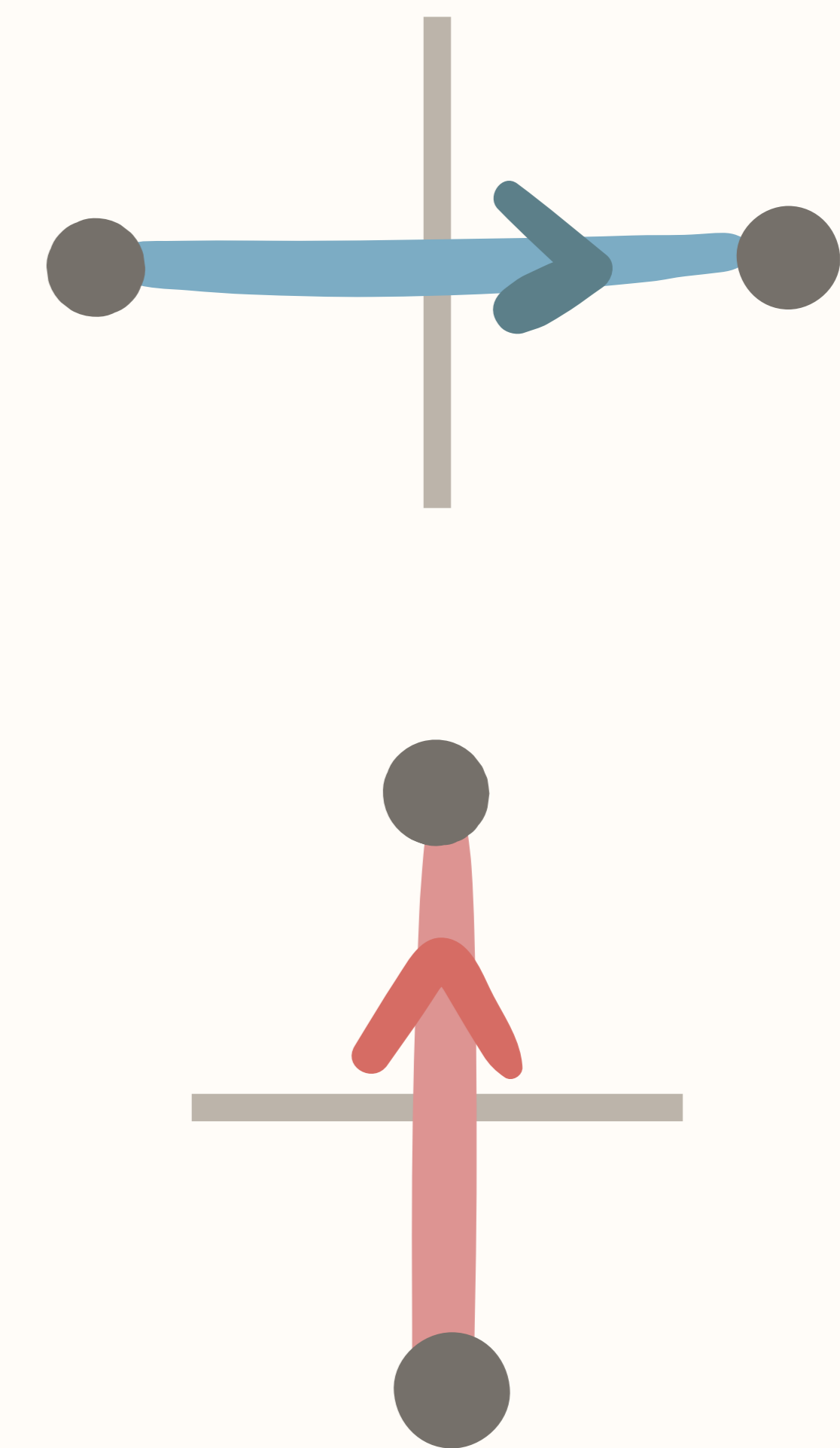
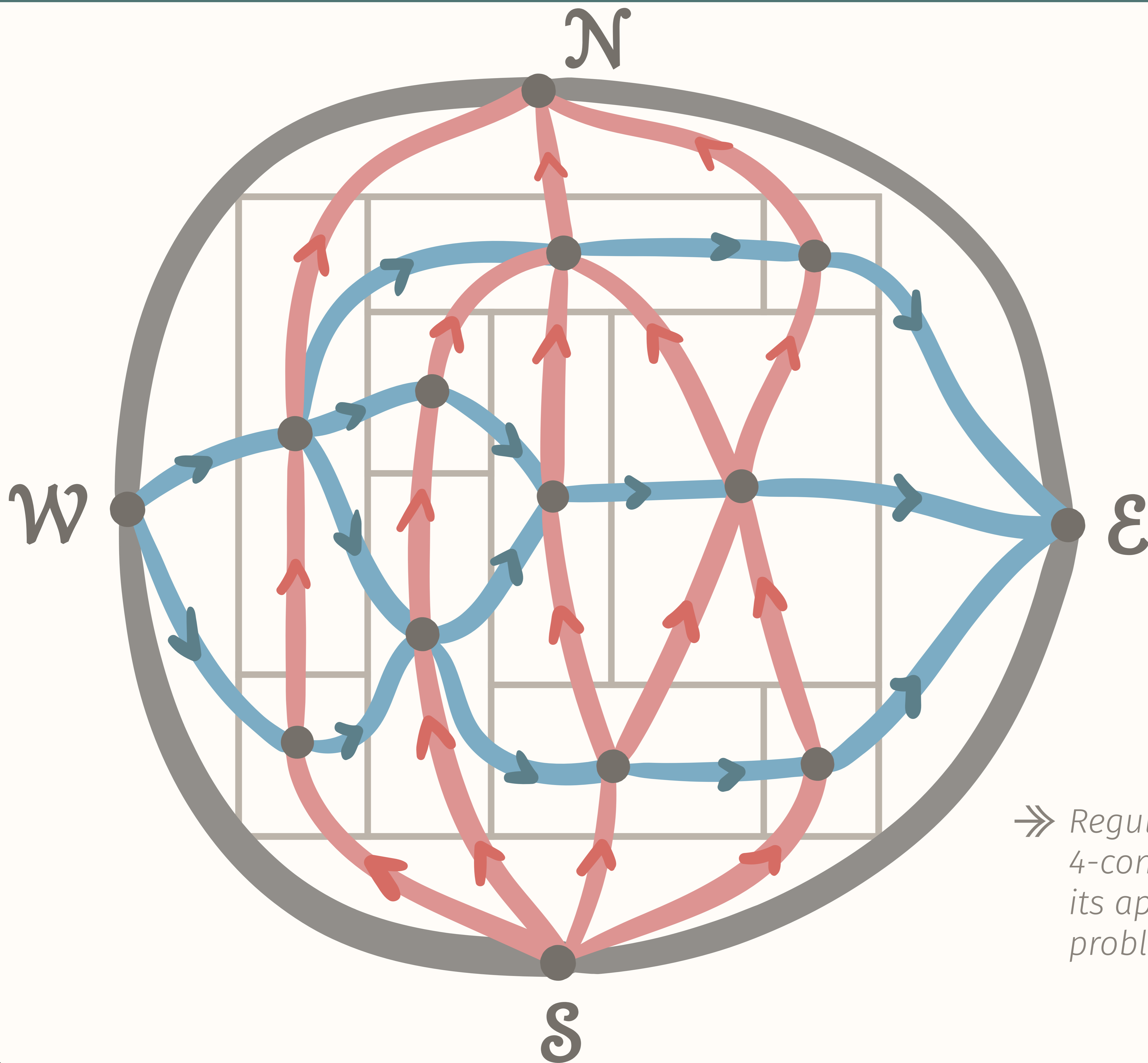
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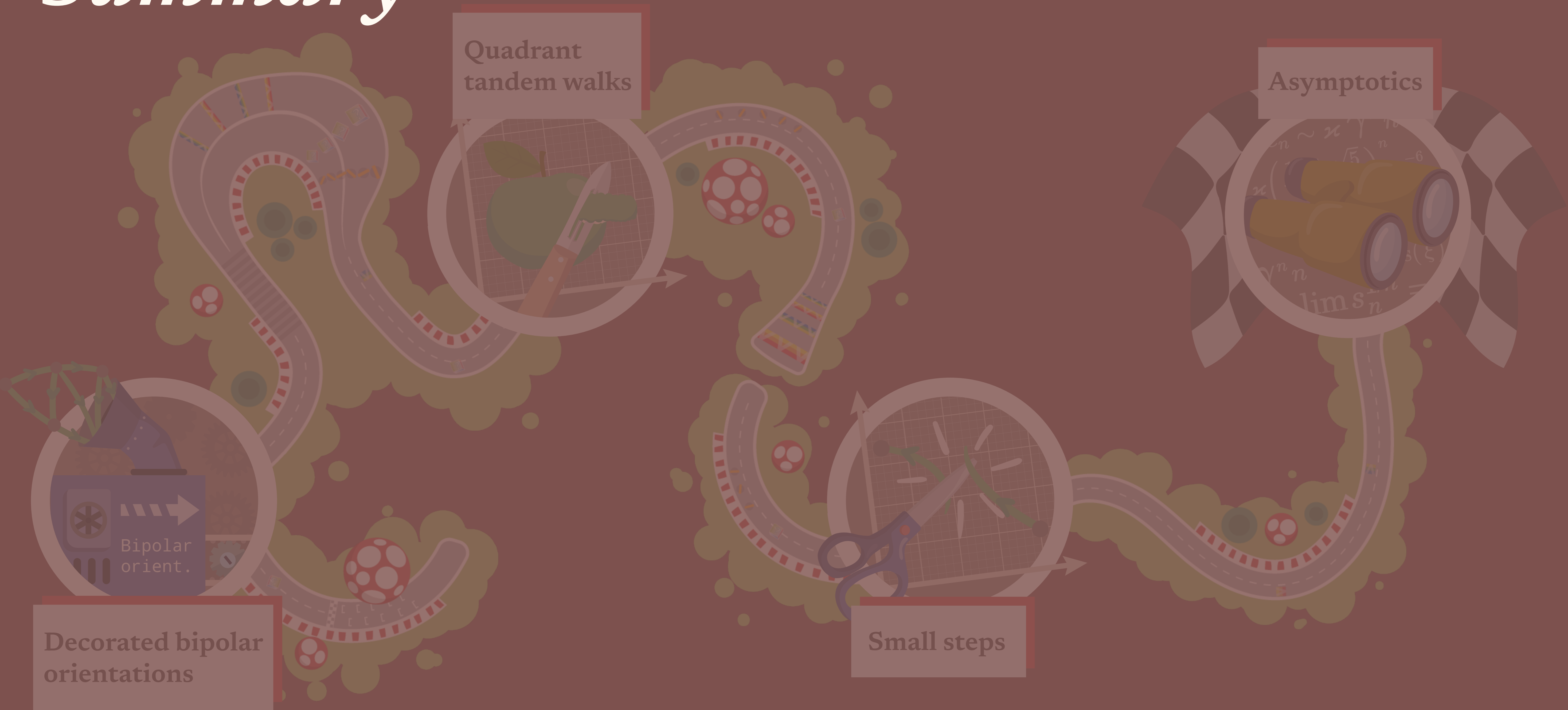
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Link with rectangular tilings



⇒ Regular edge labellings of 4-connected plane graphs and its applications in graph drawing problems, G. Kant & X. He (1997)

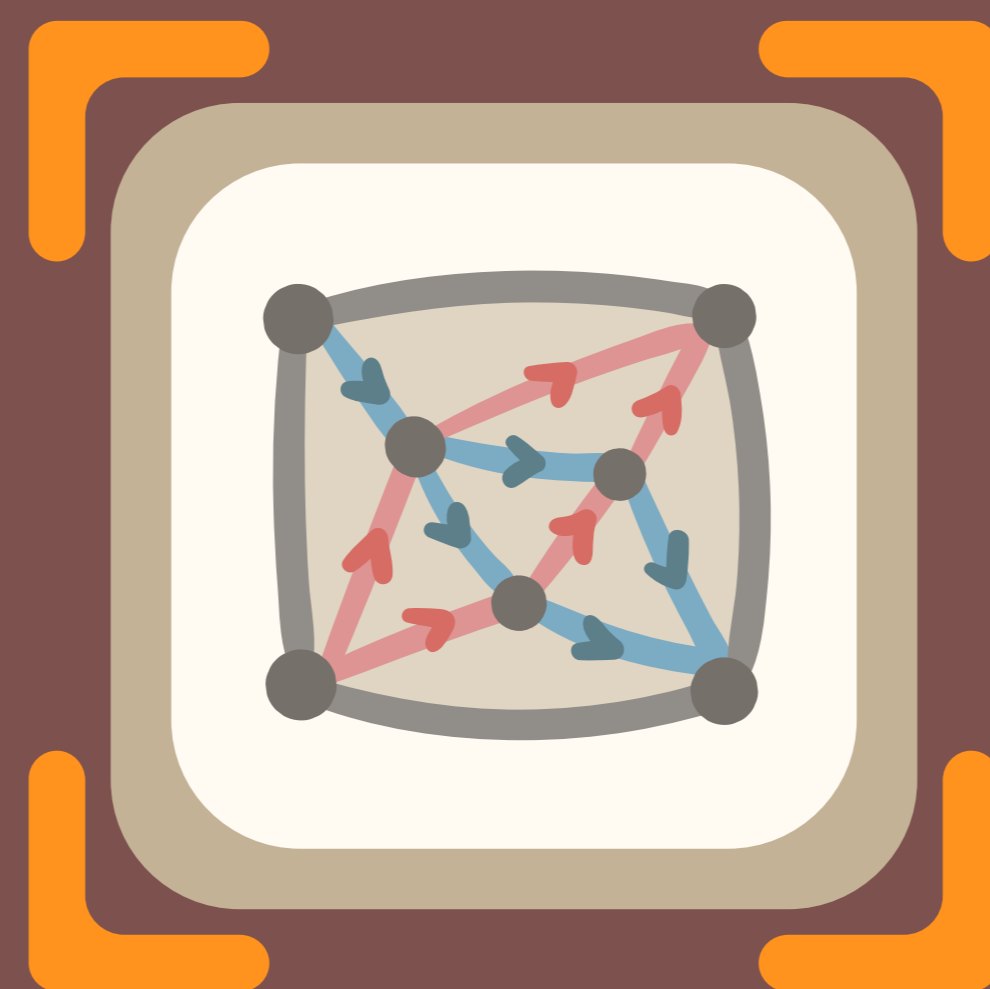
Summary



*bipolar posets
by vertices*



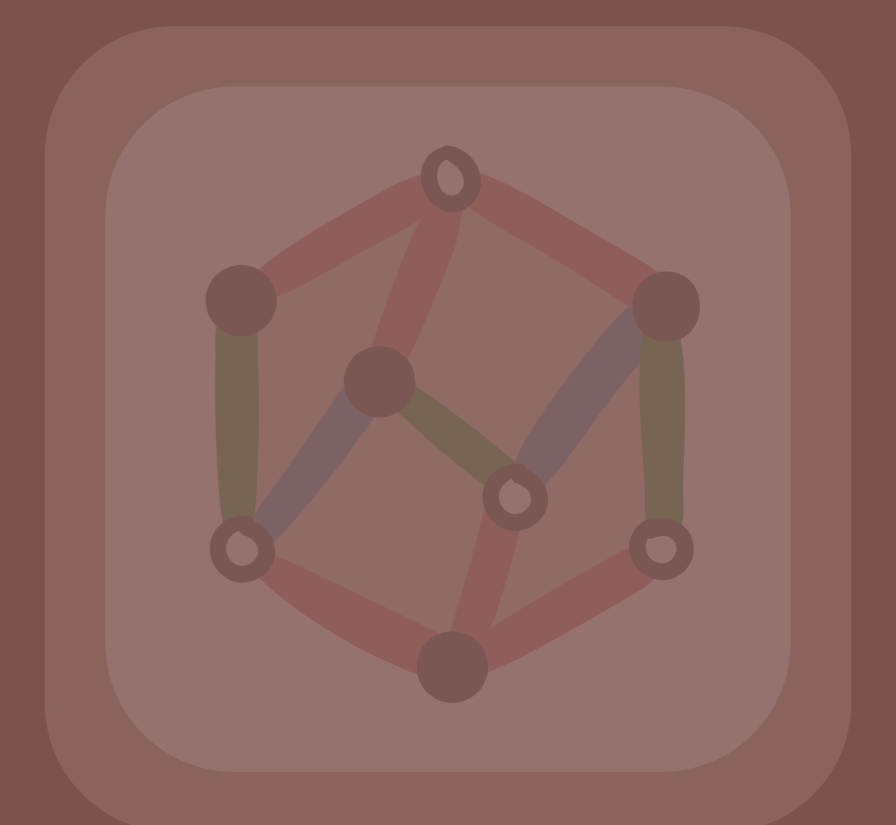
*bipolar posets
by edges*



*transversal
structures*



*polyhedral
orientations*



*Schnyder
labelings*

Summary

Quadrant
tandem walks

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Bipolar
orient.

Decorated bipolar
orientations

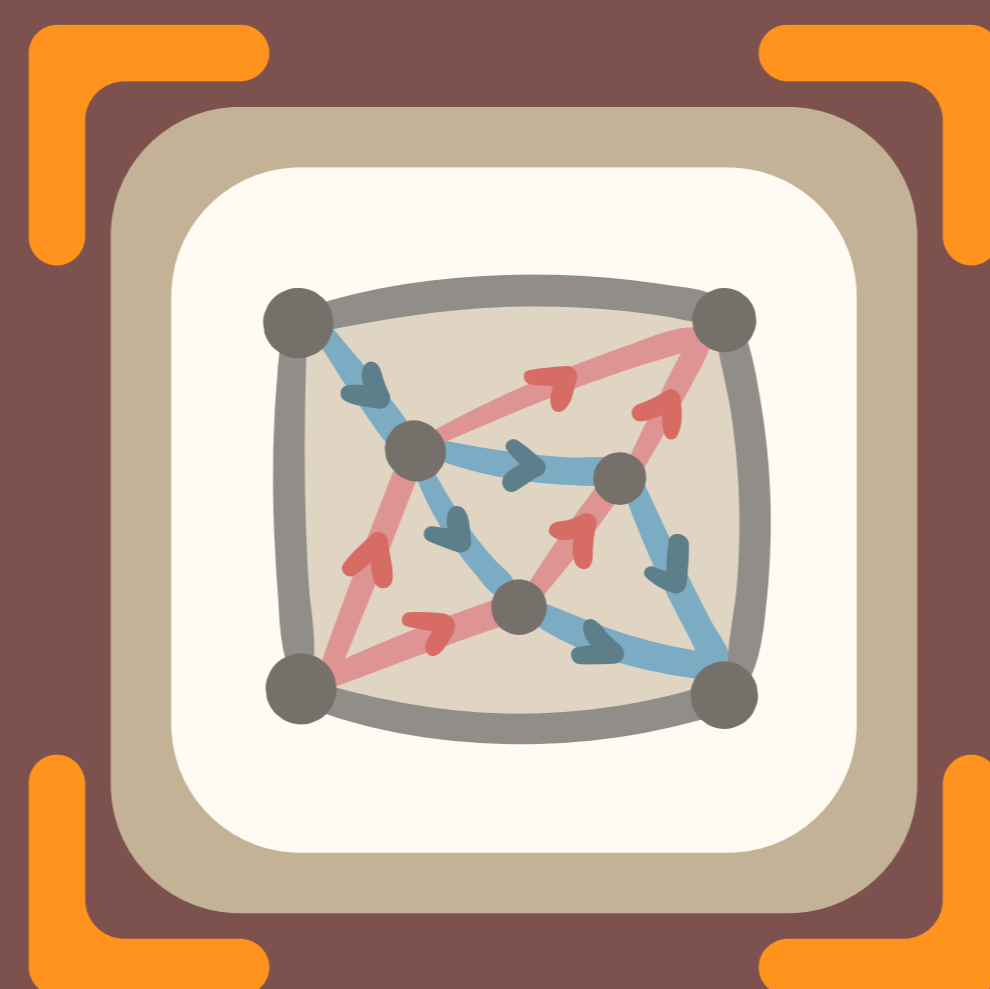
Small steps



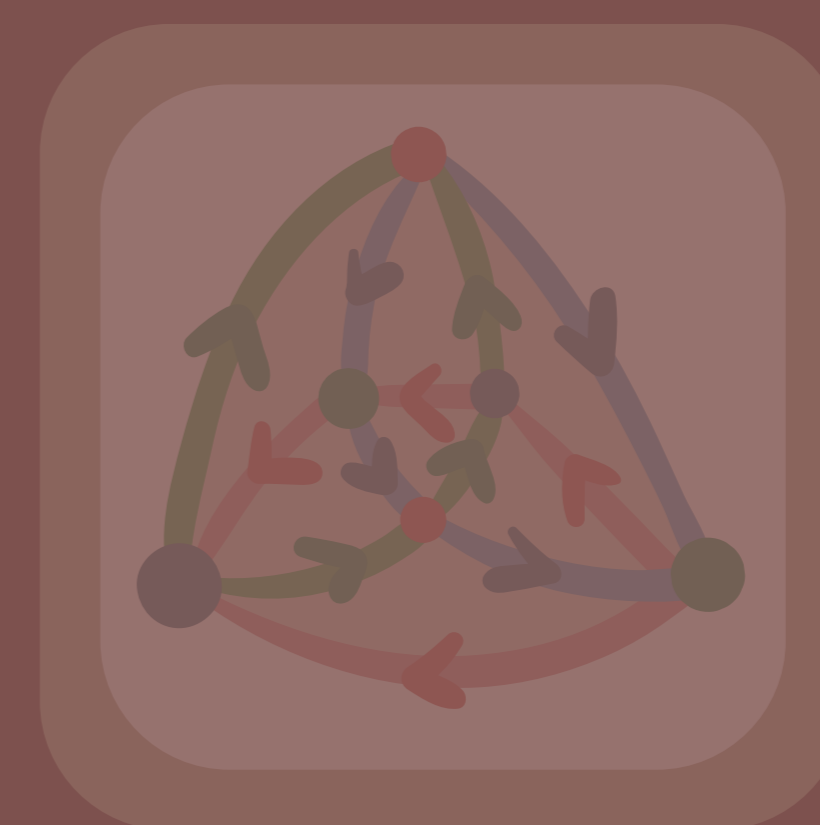
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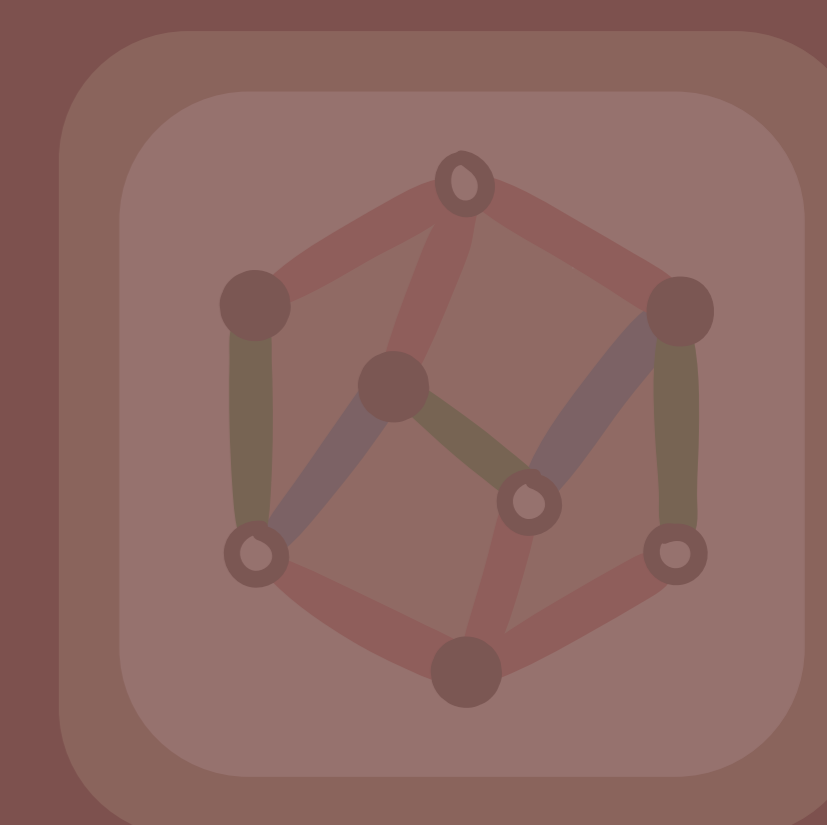
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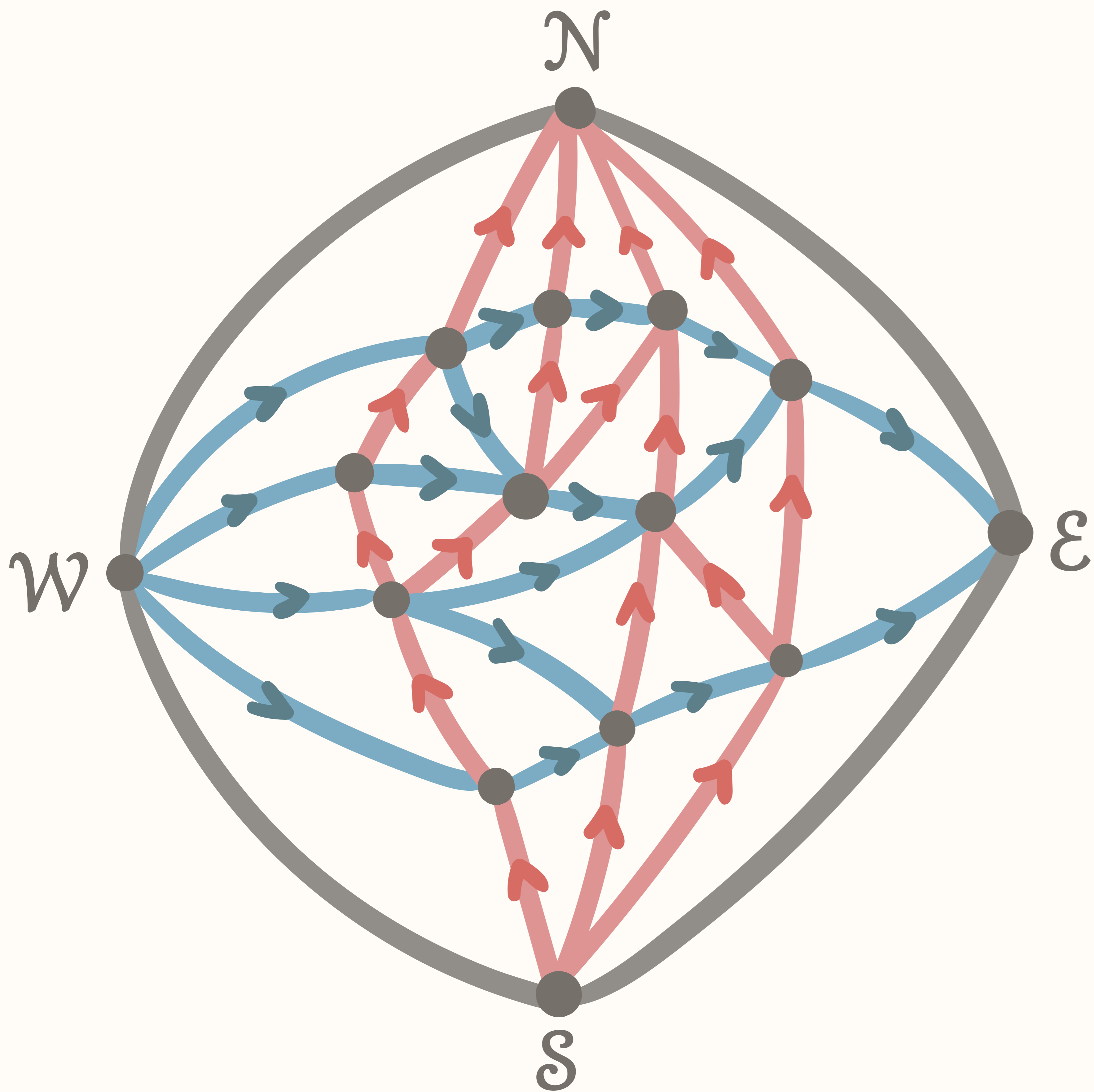


*polyhedral
orientations*

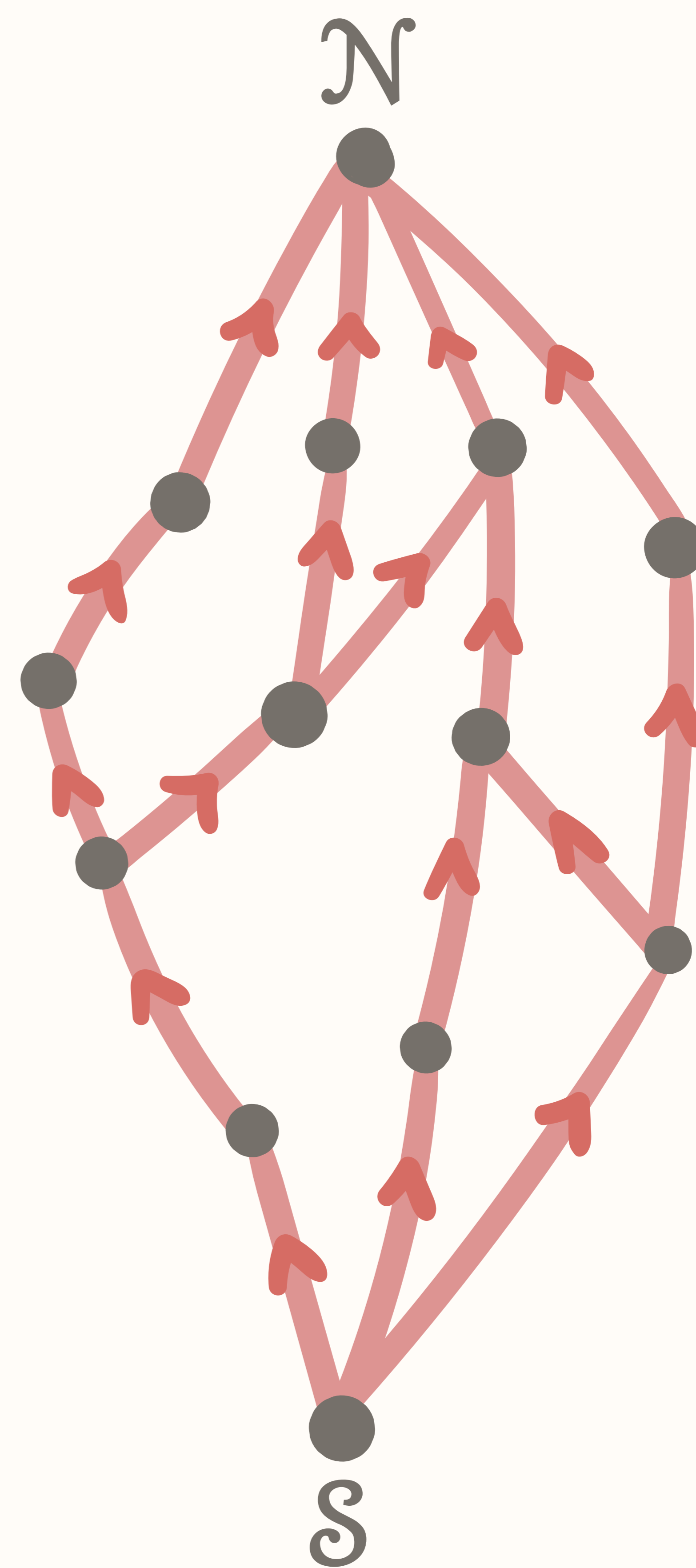
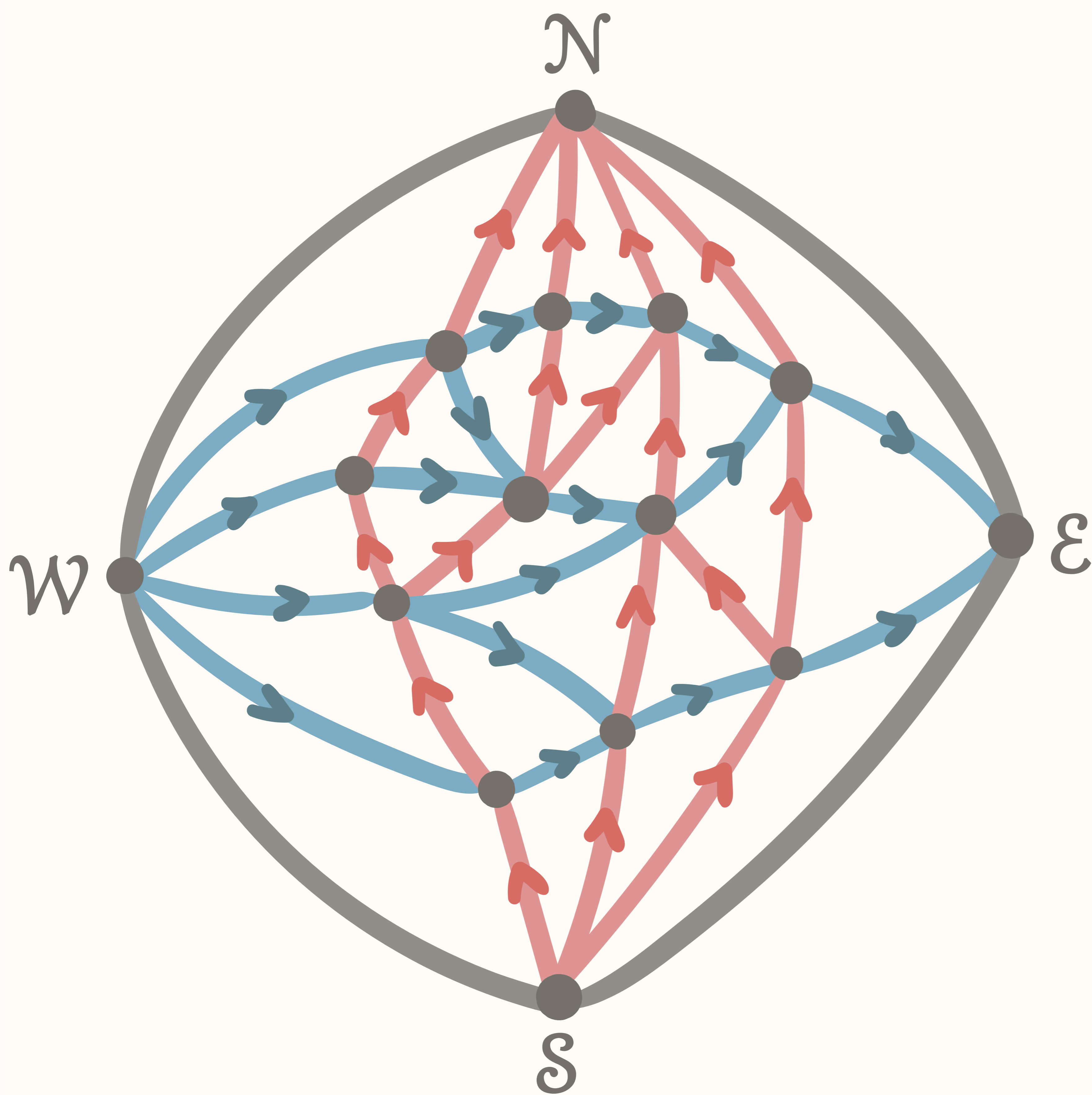


*Schnyder
labelings*

Encoding with bipolar orientations

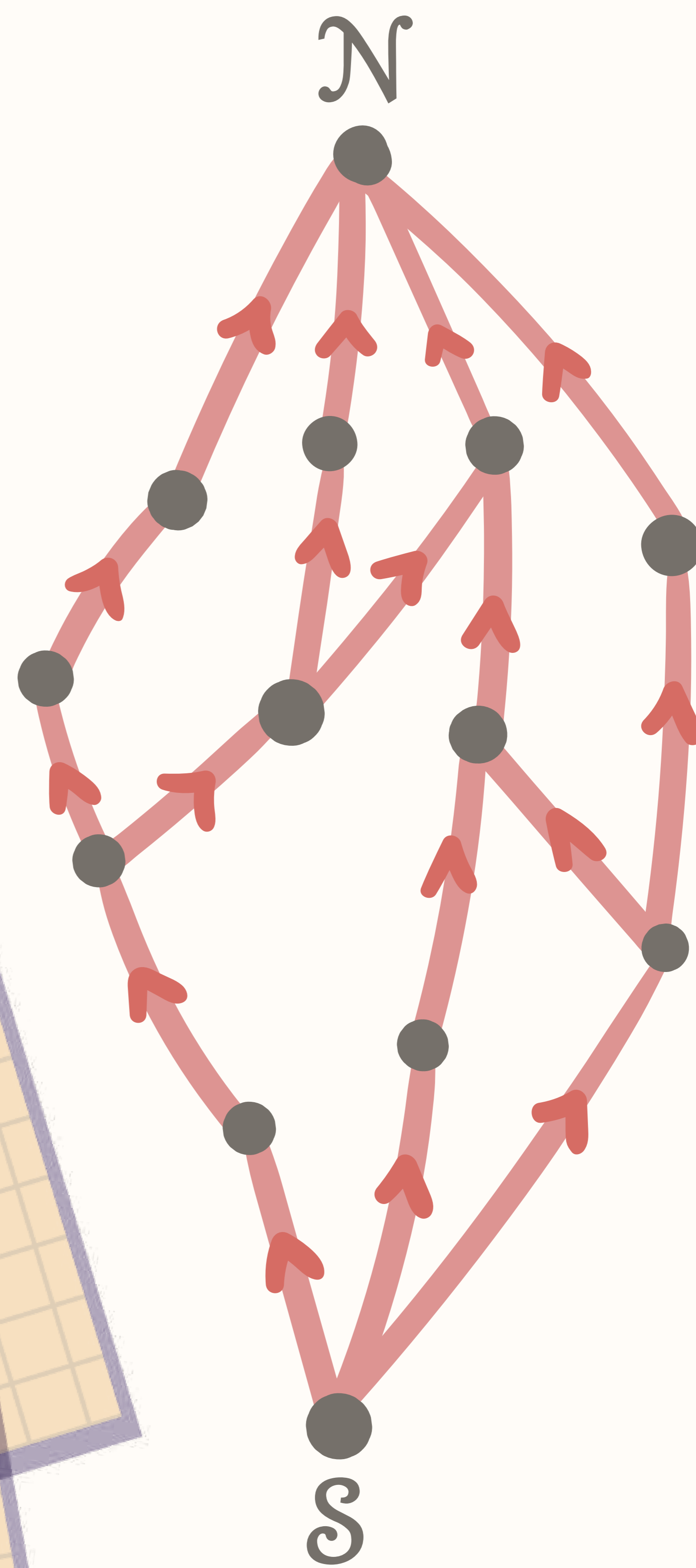
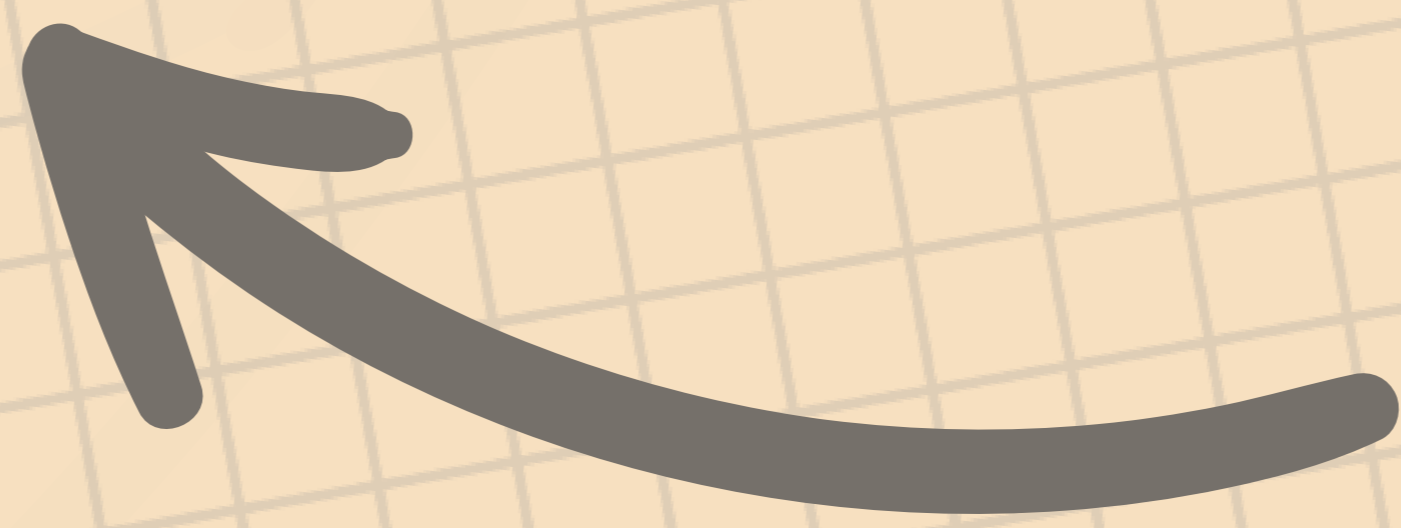


Encoding with bipolar orientations

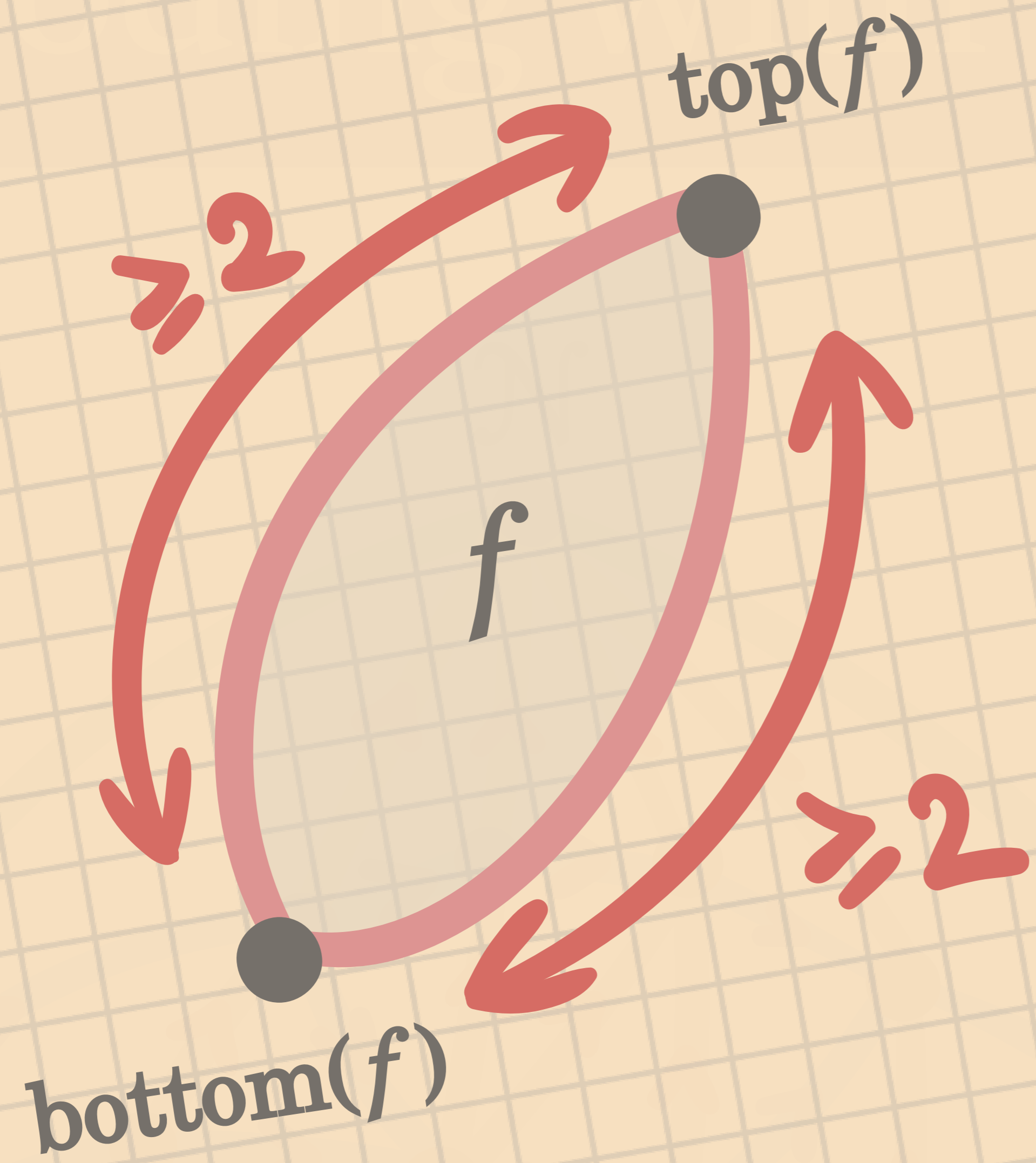


orientations

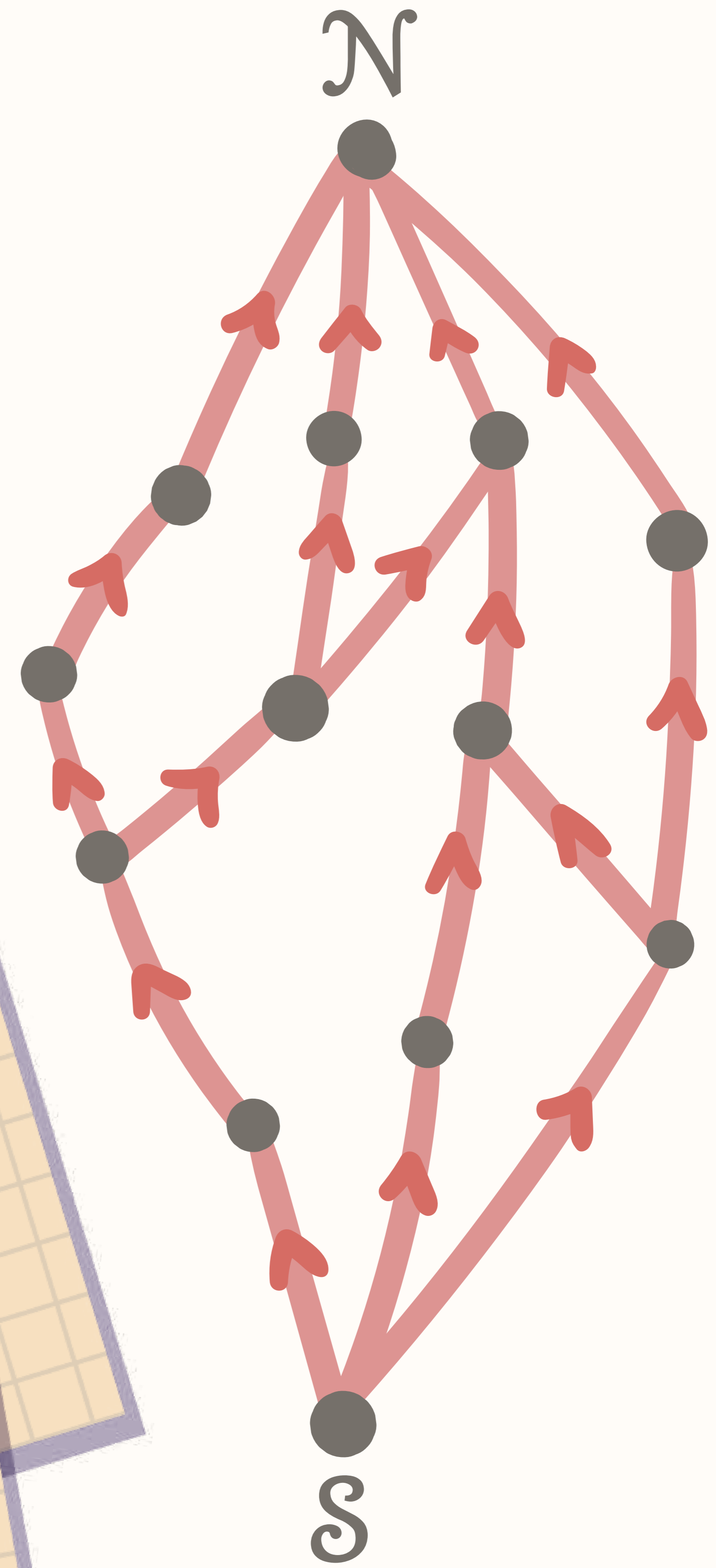
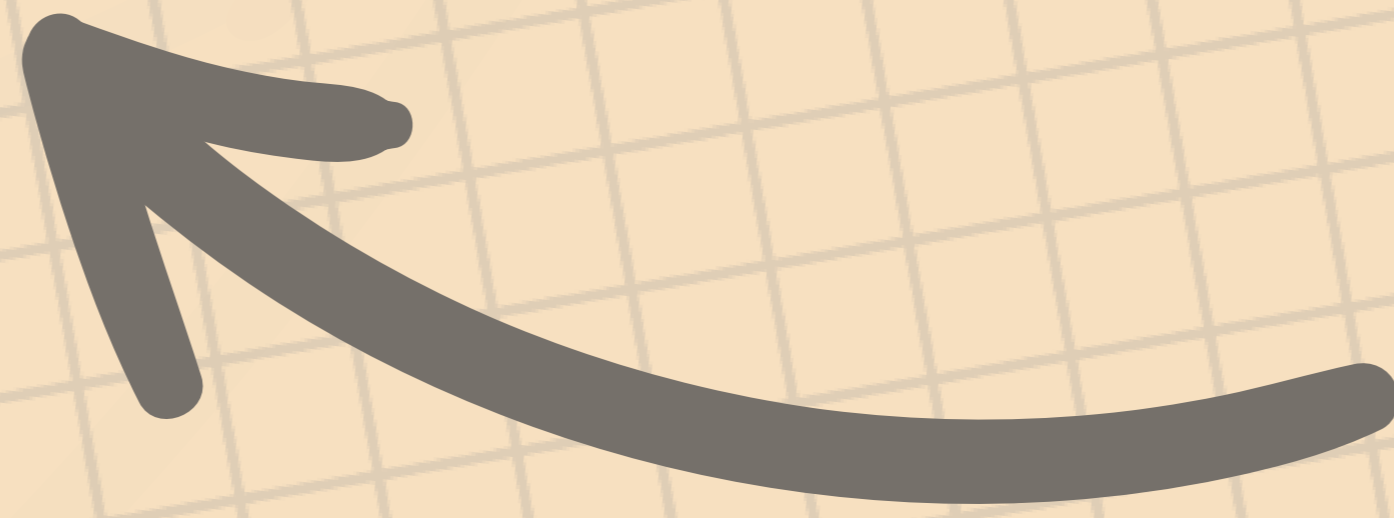
planar poset



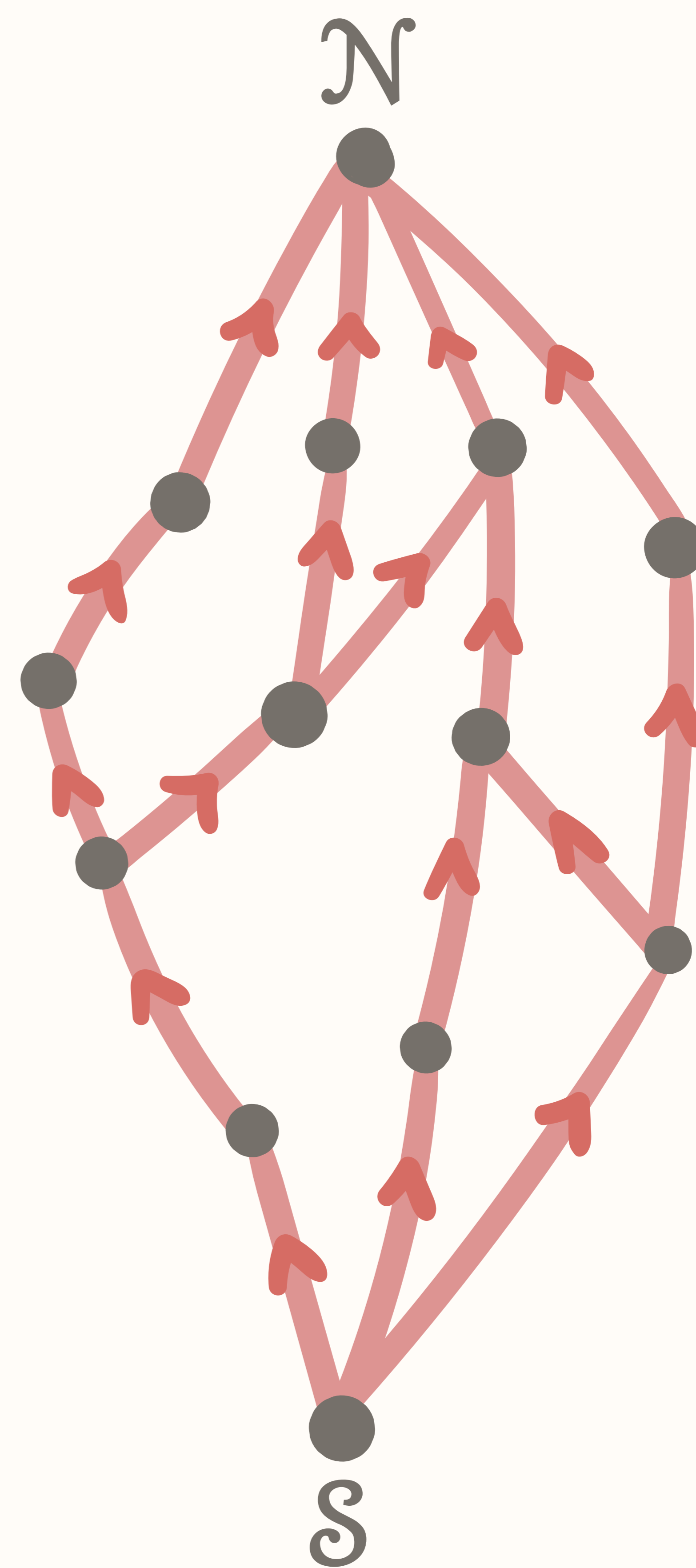
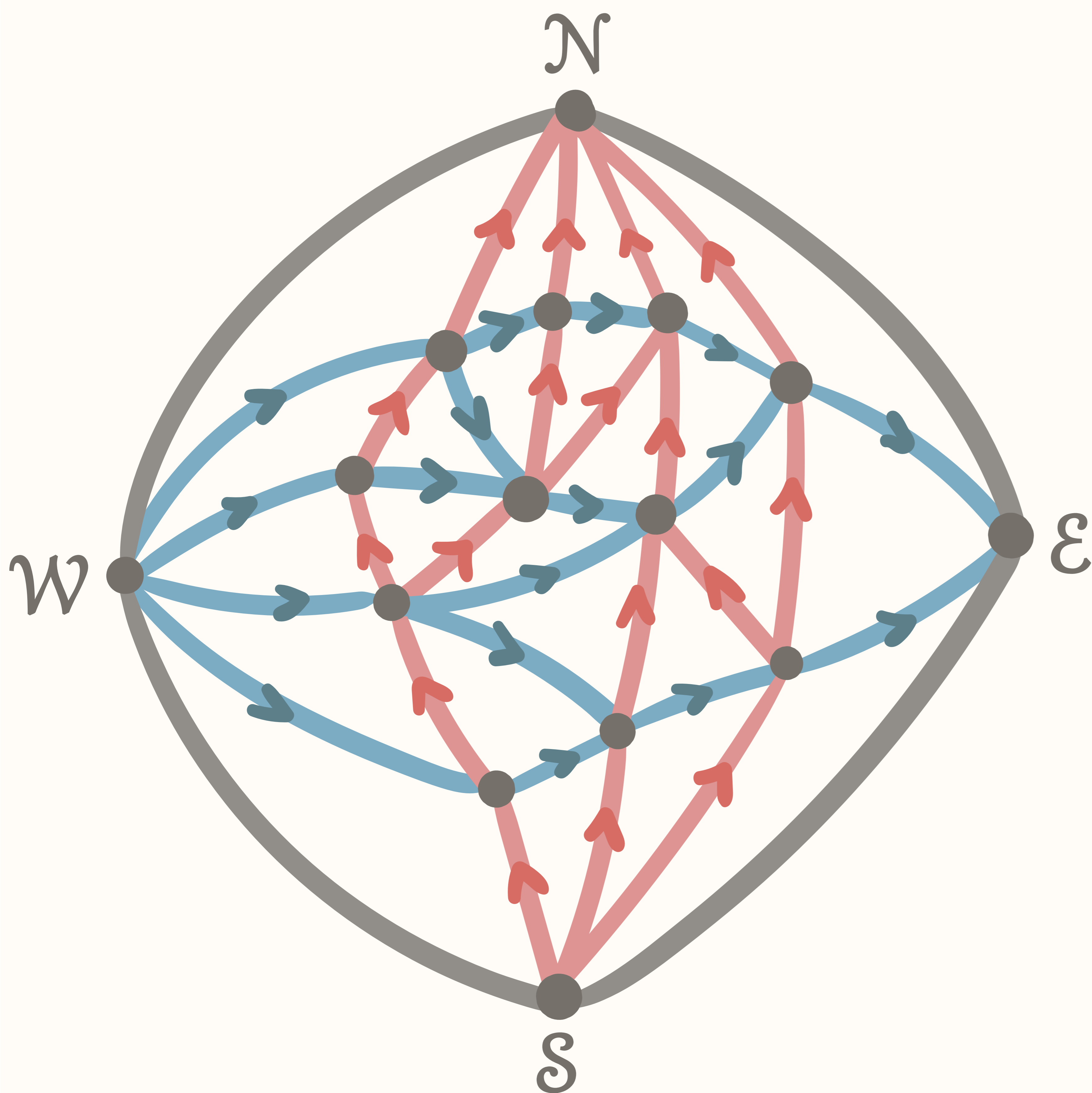
orientations



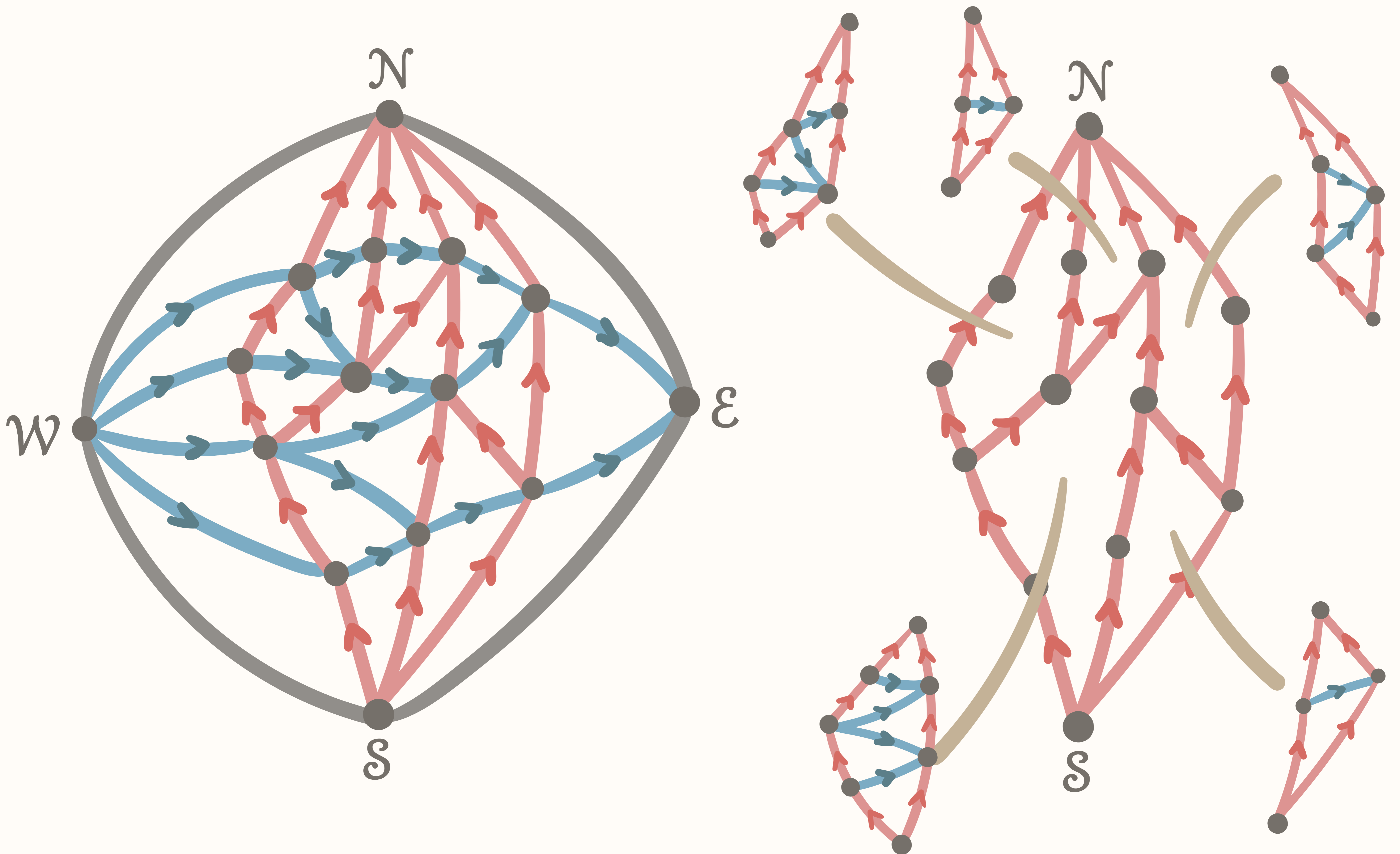
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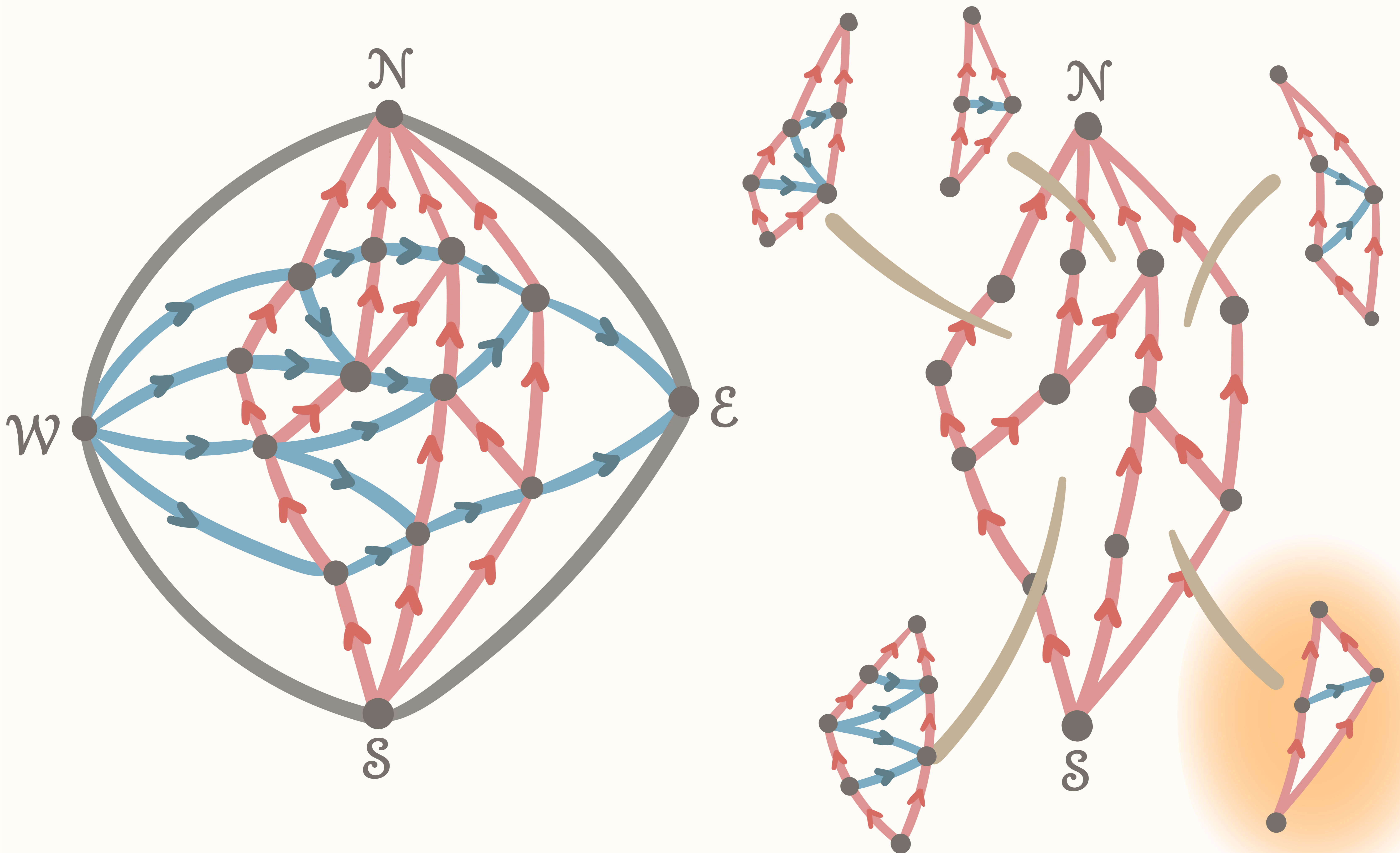
Encoding with bipolar orientations



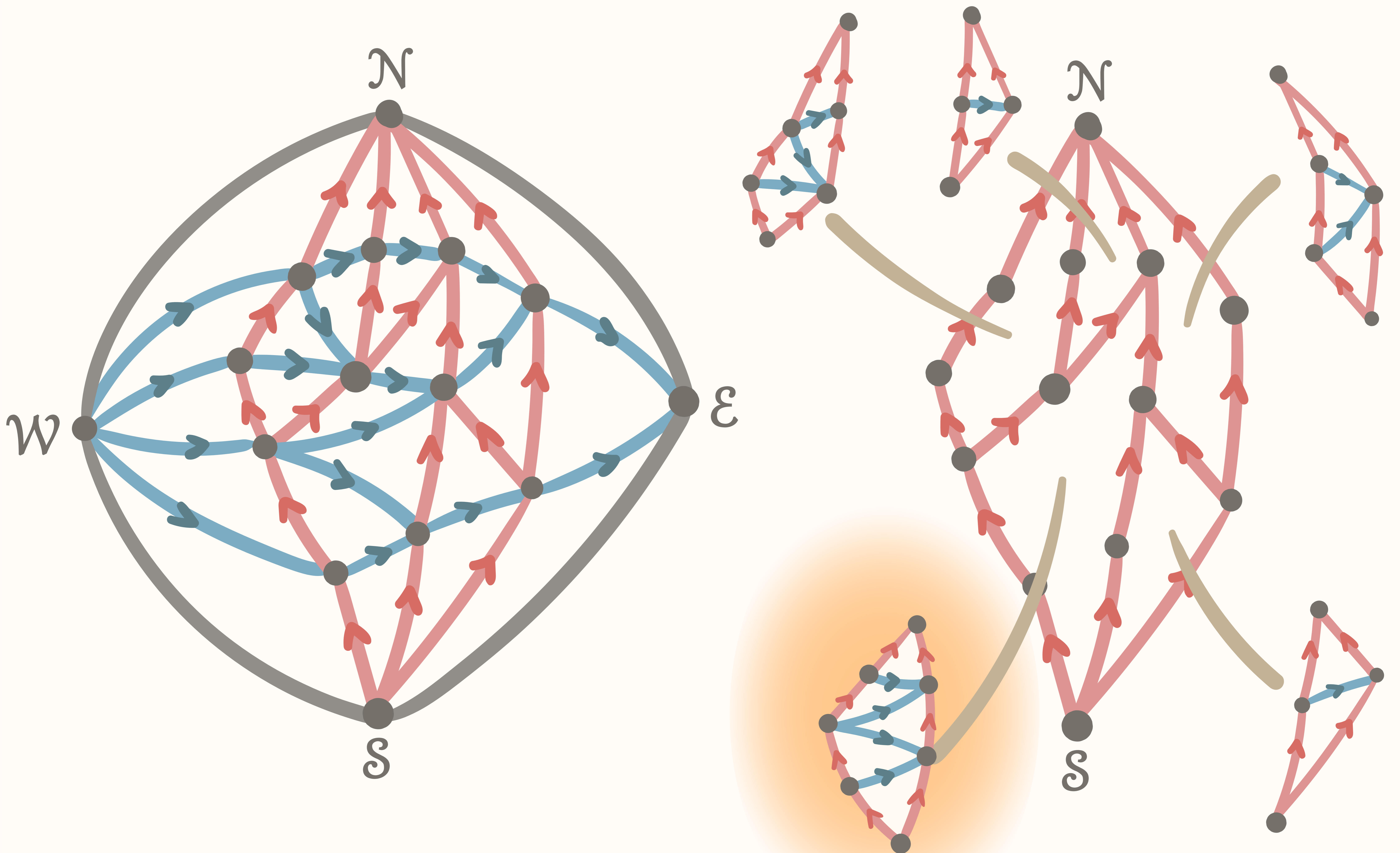
Encoding with bipolar orientations



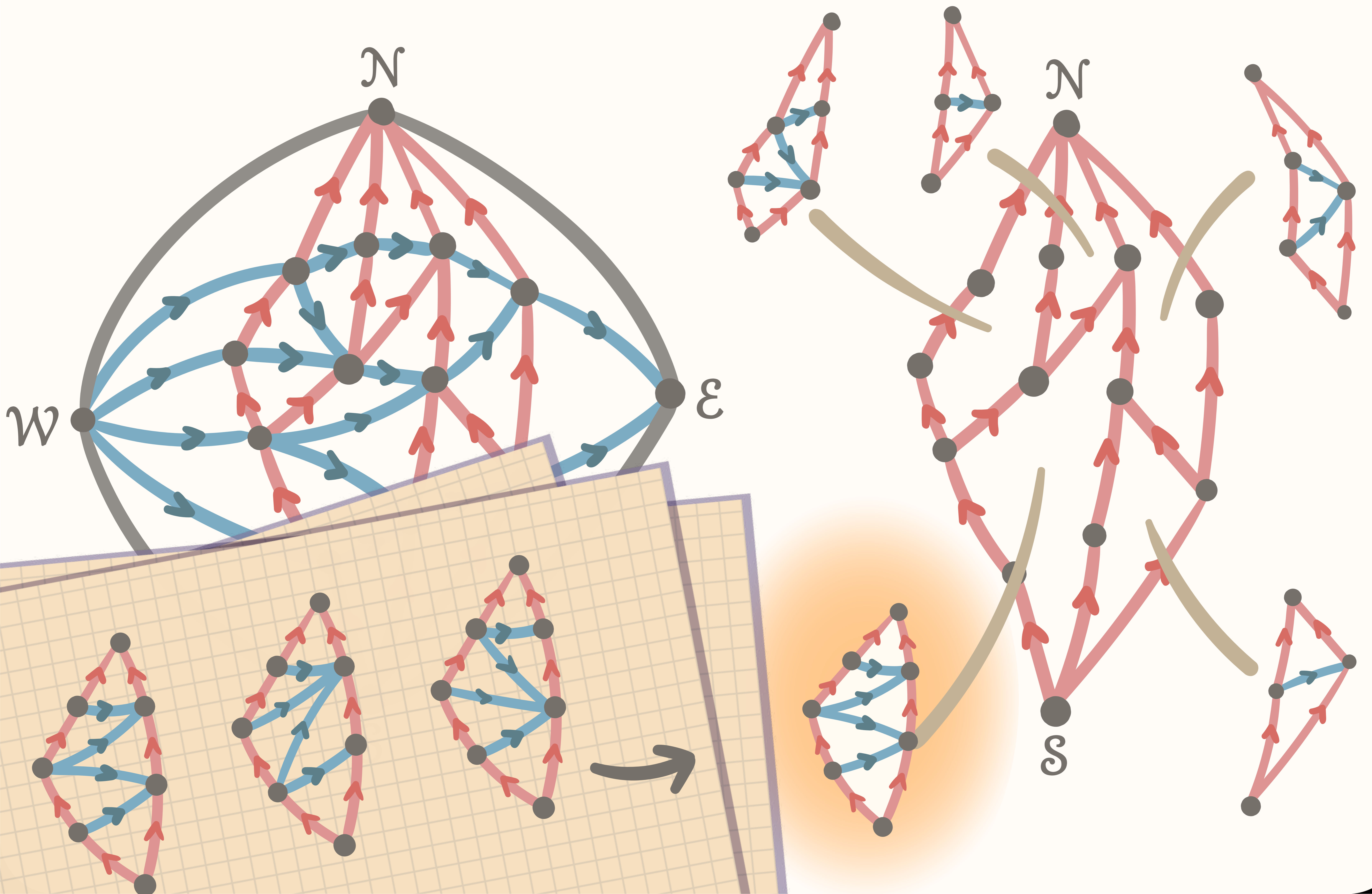
Encoding with bipolar orientations



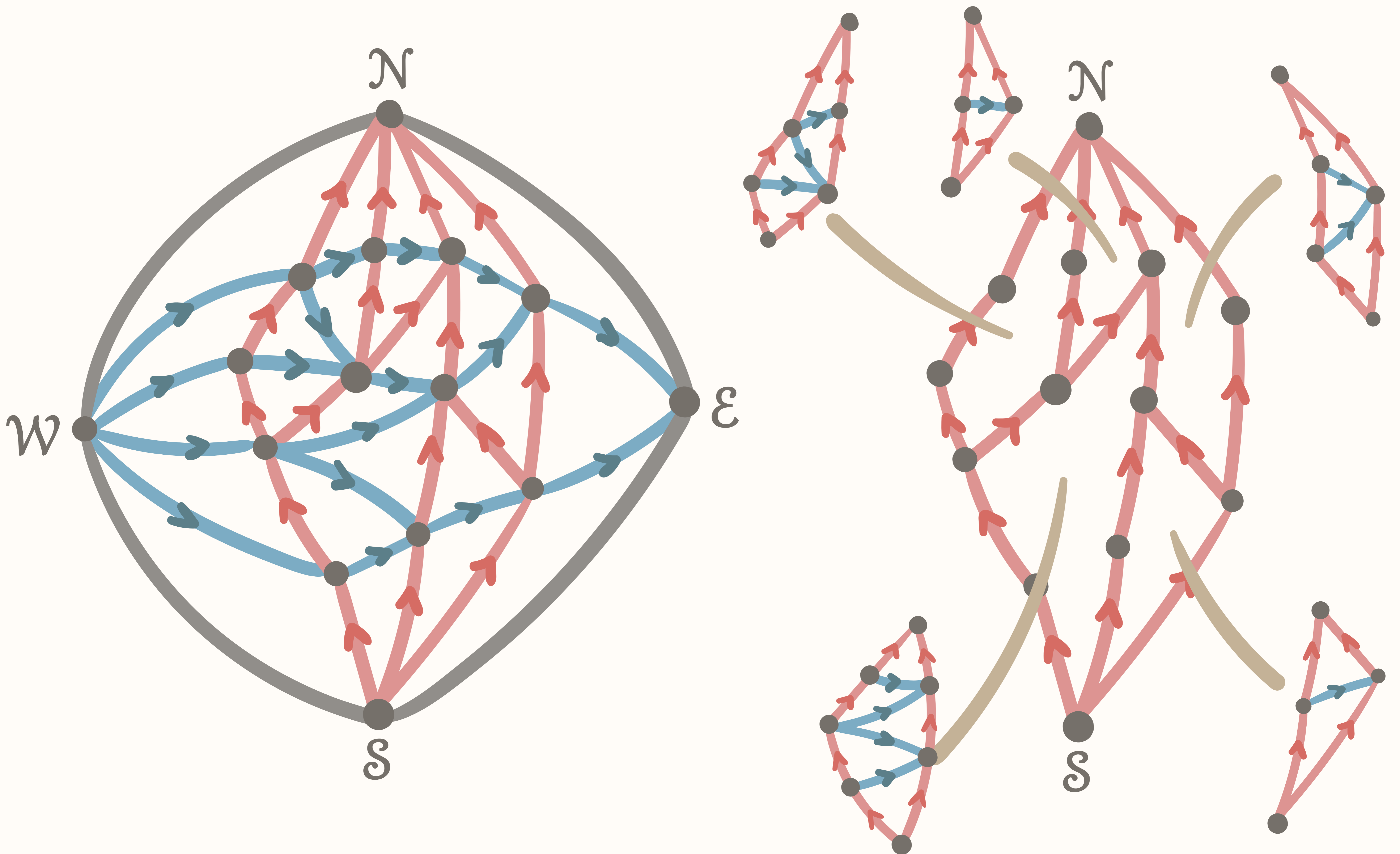
Encoding with bipolar orientations



Encoding with bipolar orientations

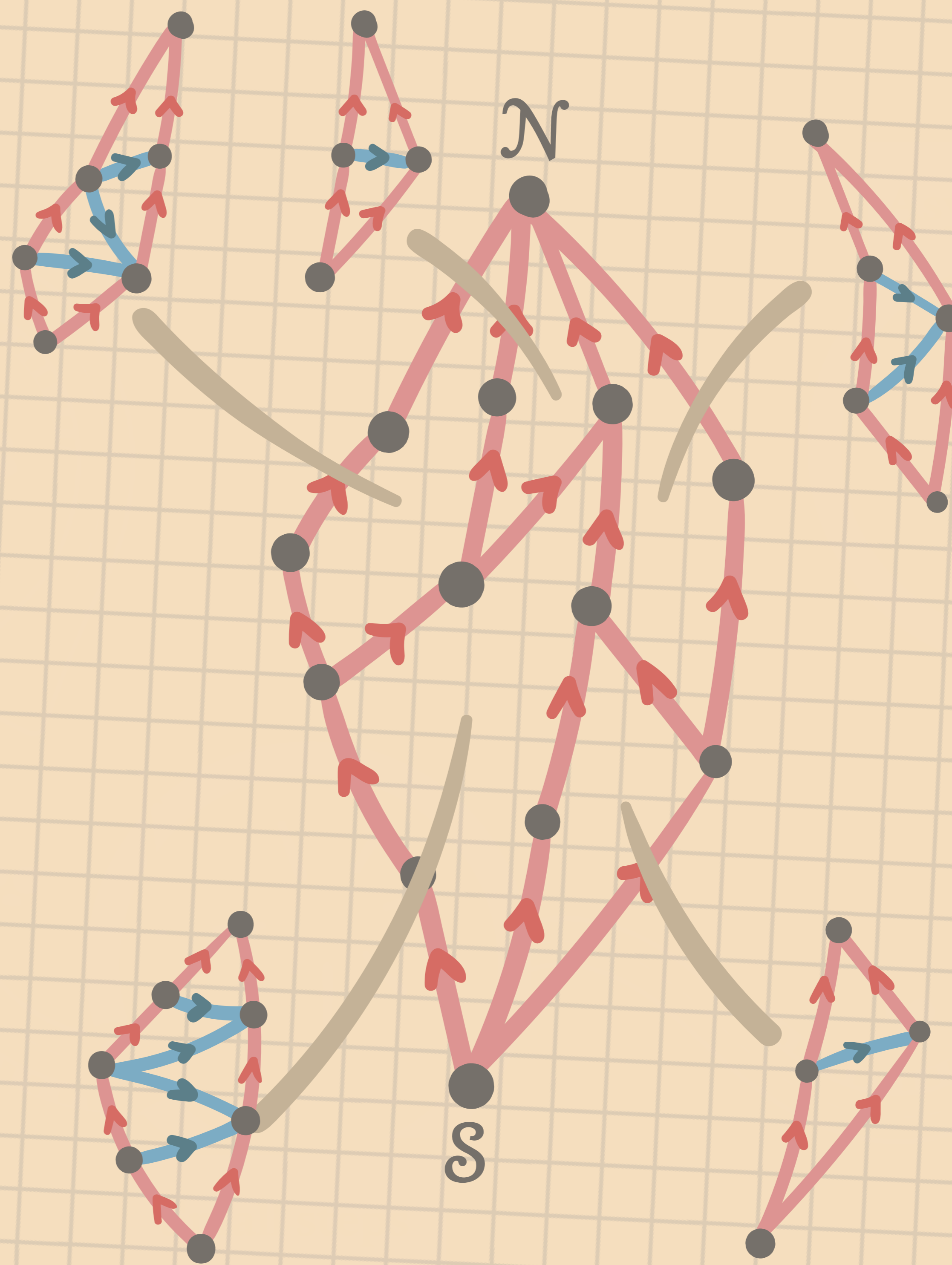


Encoding with bipolar orientations



Summary

Quadrant
tander



Decorated bipolar orientations



*bipolar posets
by vertices*



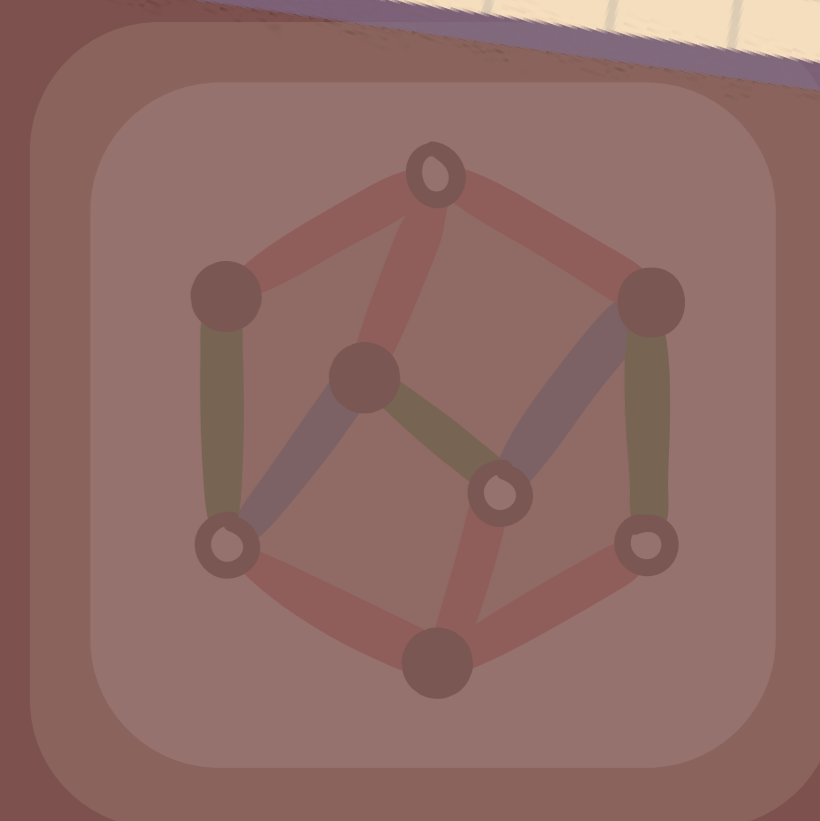
*bipolar posets
by edges*



*transversal
structures*



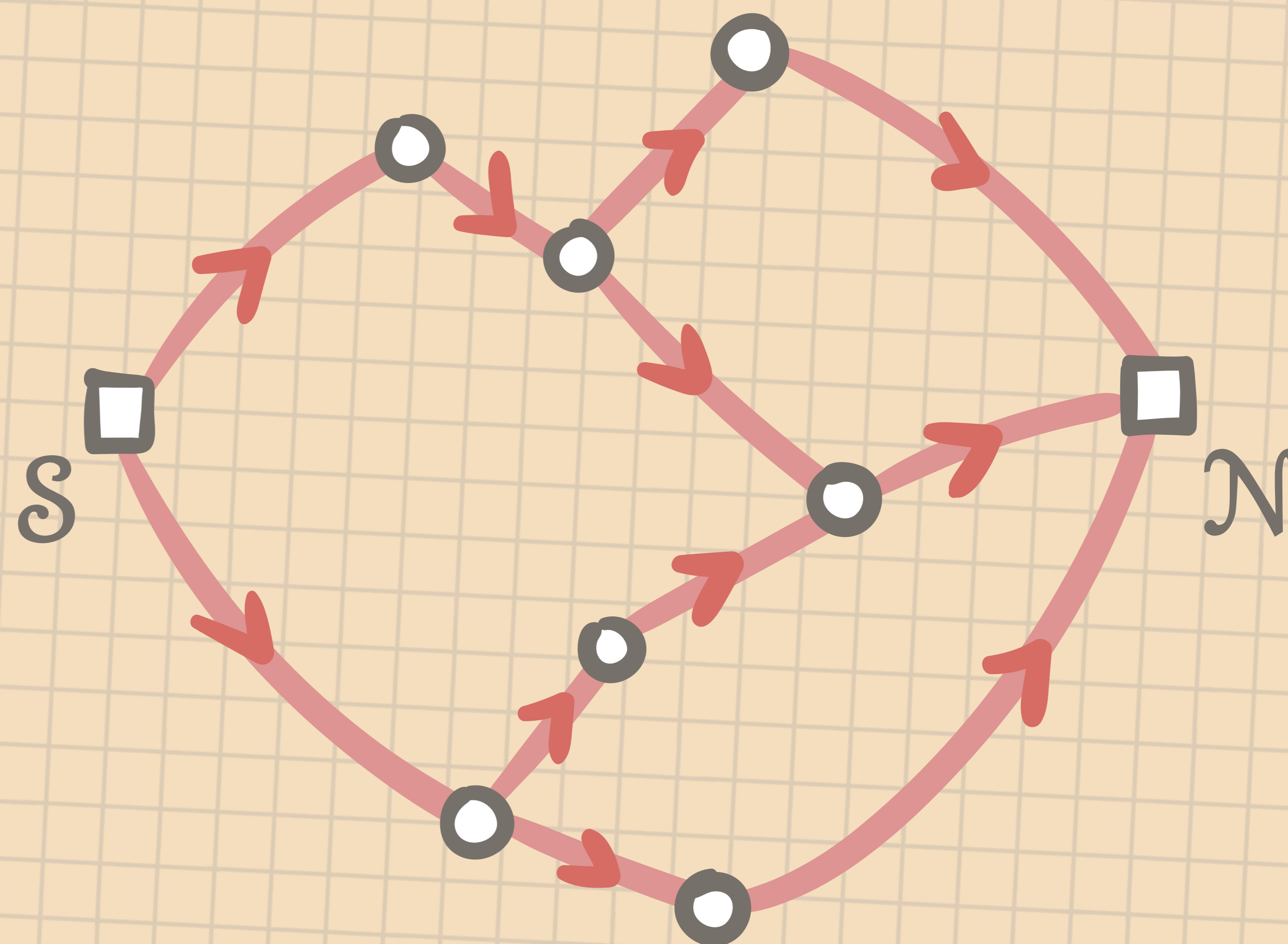
*polyhedral
orientations*



*Schnyder
labelings*

Summary

Quadrant
tander



Decorated bipolar orientations



*bipolar posets
by vertices*



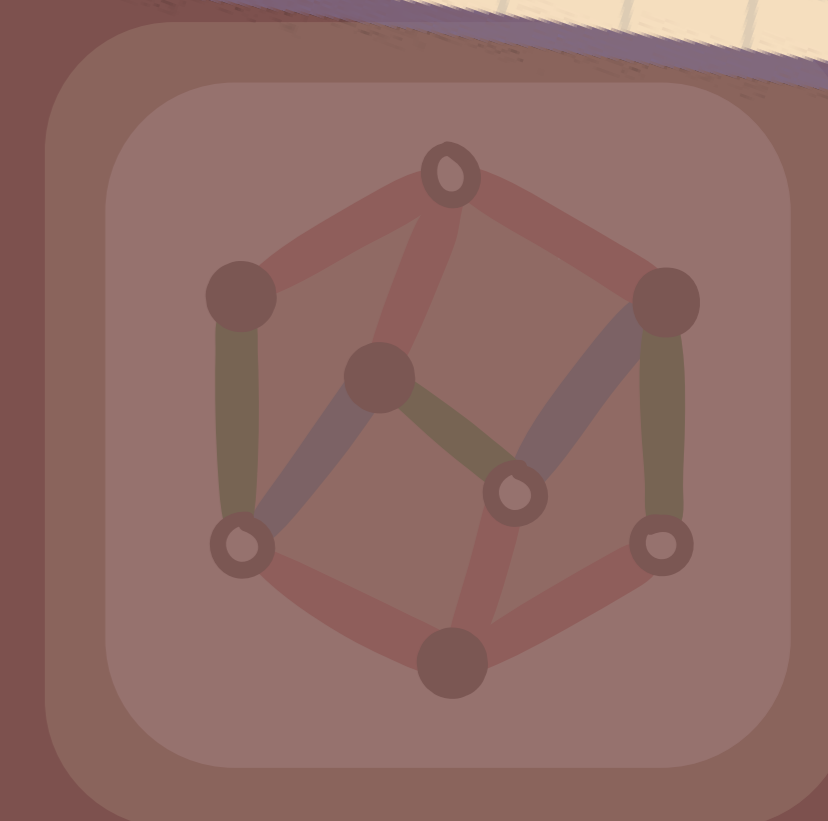
*bipolar posets
by edges*



*transversal
structures*



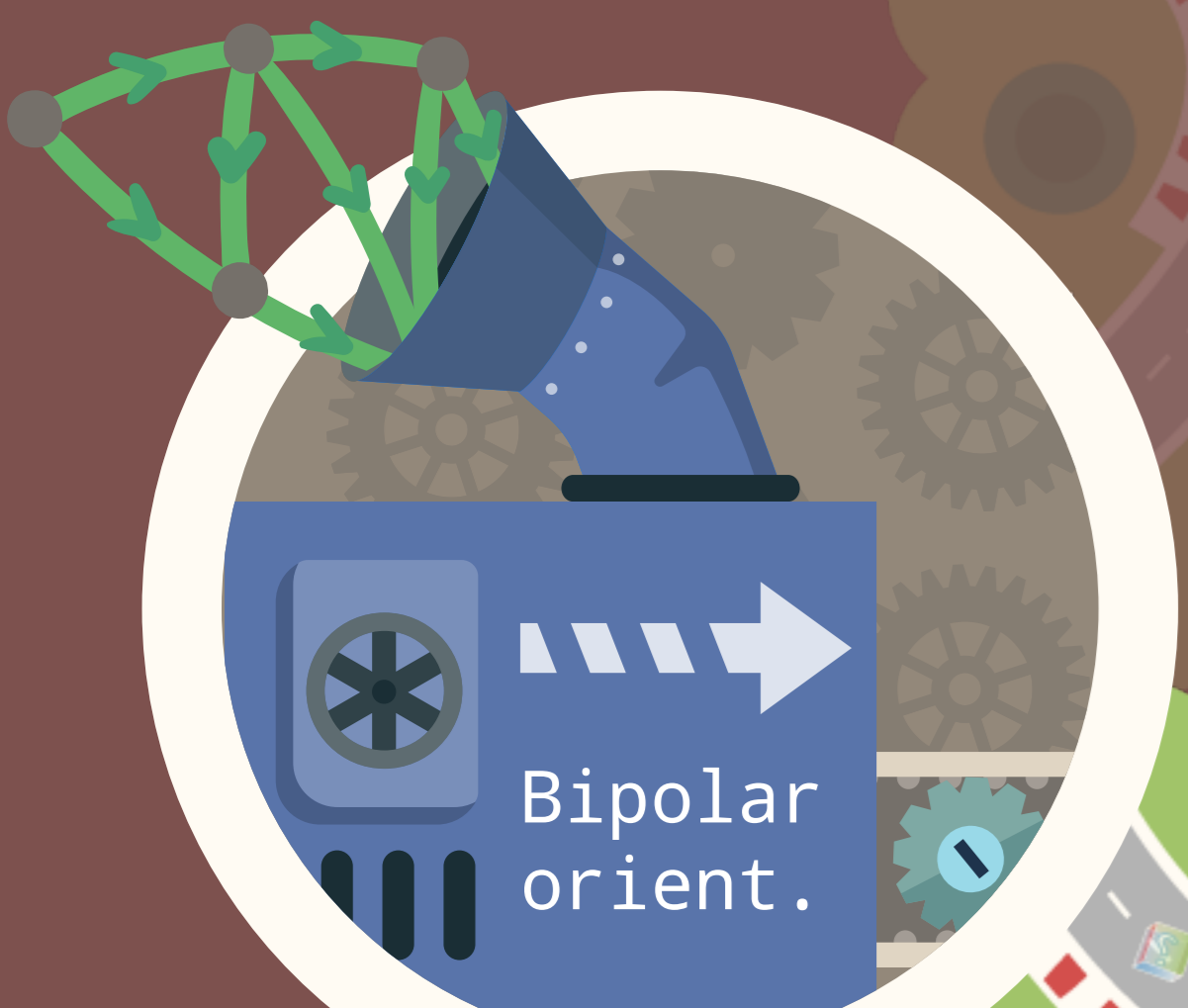
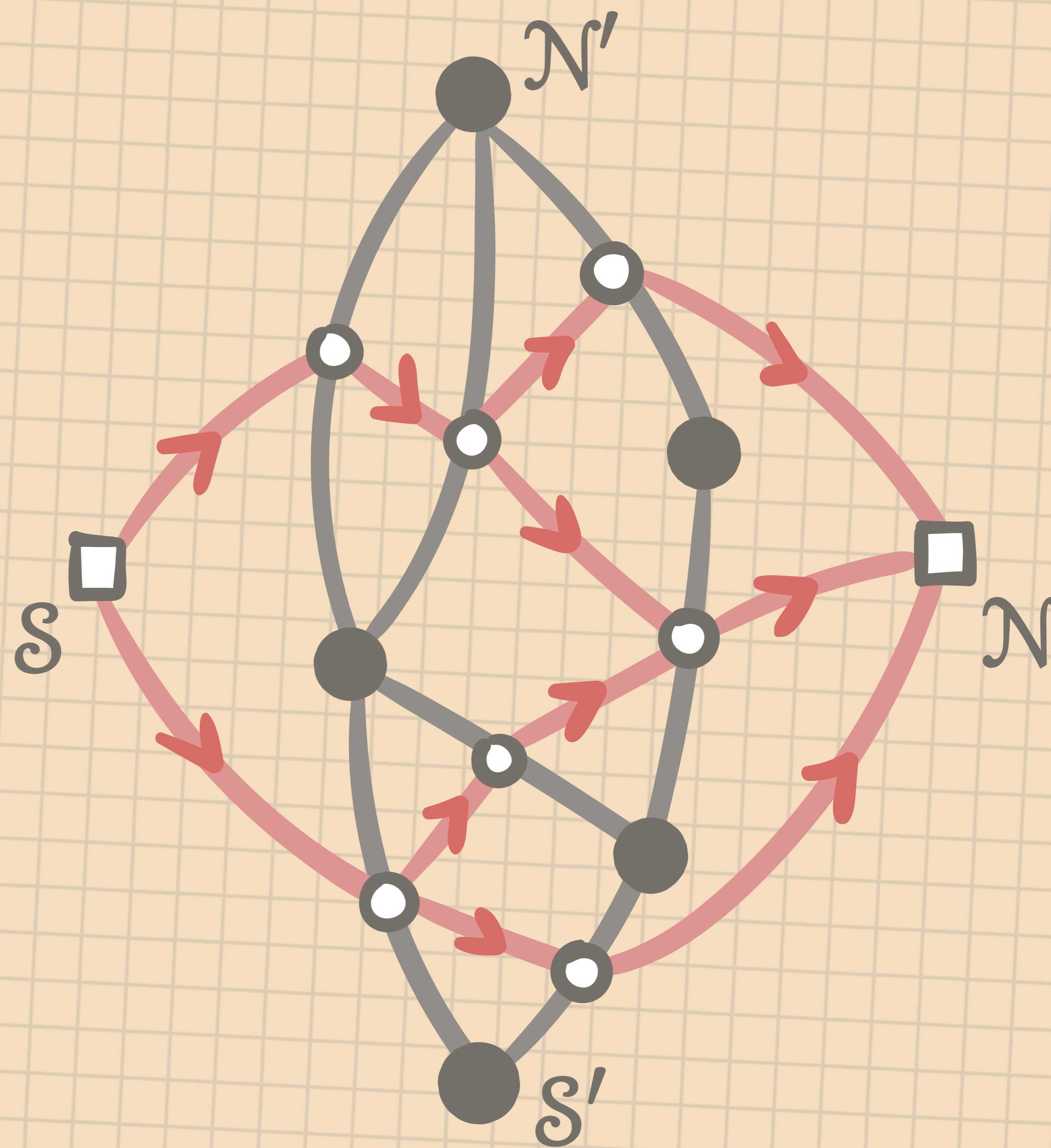
*polyhedral
orientations*



*Schnyder
labelings*

Summary

Quadrant
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Decorated bipolar orientations



*bipolar posets
by vertices*



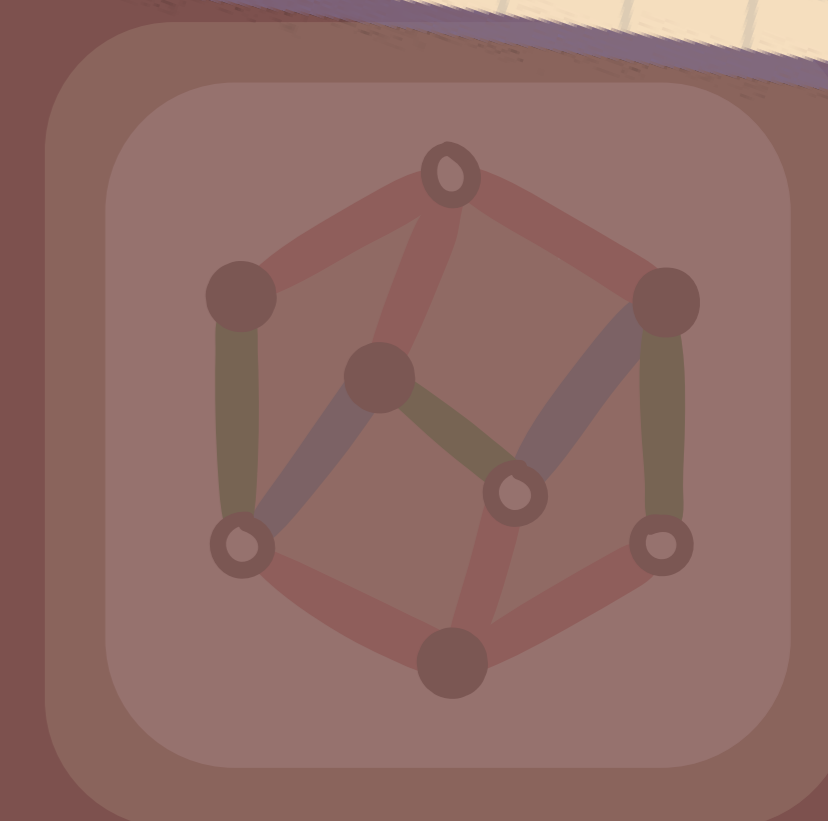
*bipolar posets
by edges*



*transversal
structures*



*polyhedral
orientations*



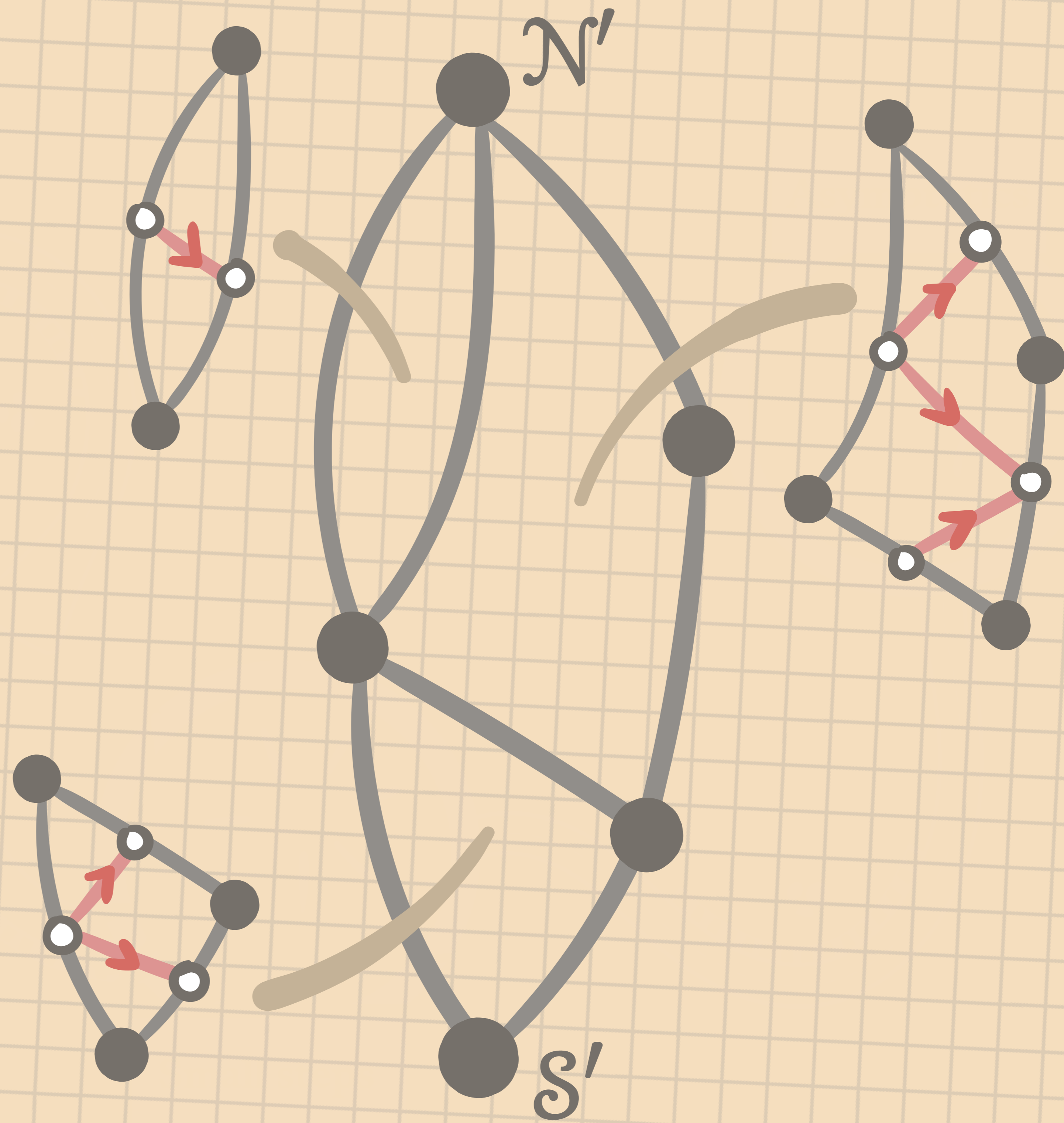
*Schnyder
labelings*

Summary

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tander



Decorated bipolar orientations



bipolar posets
by vertices



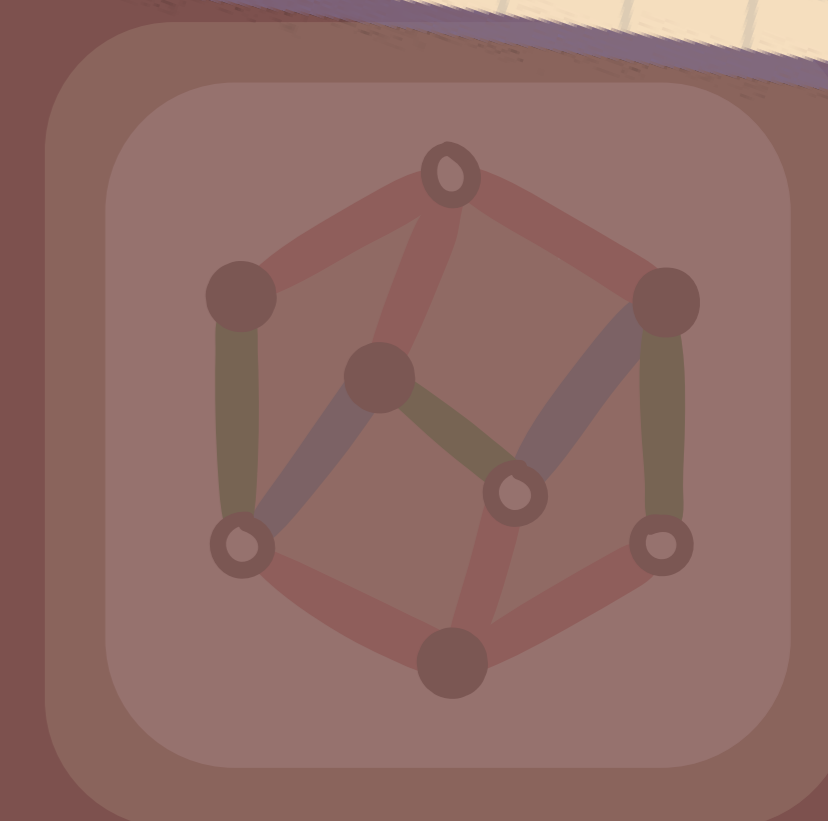
bipolar posets
by edges



transversal
structures



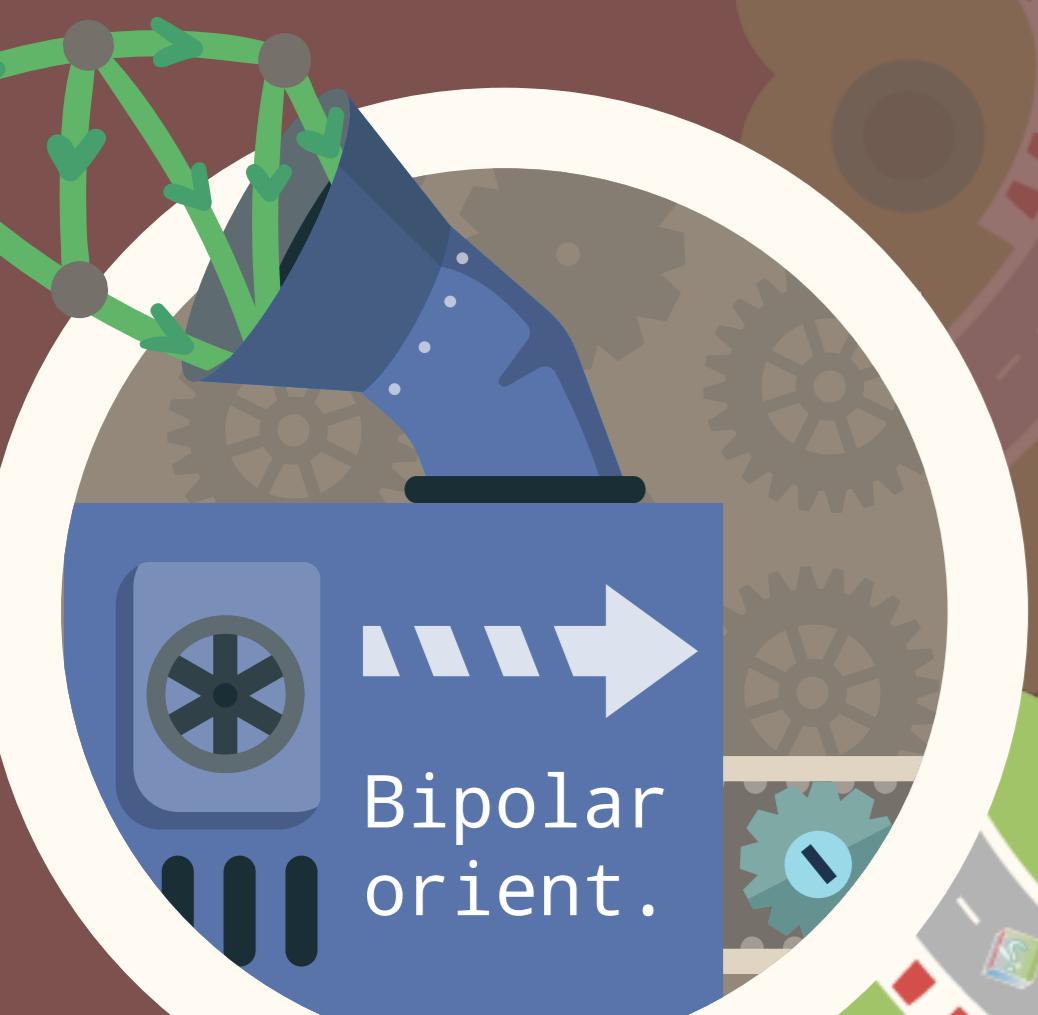
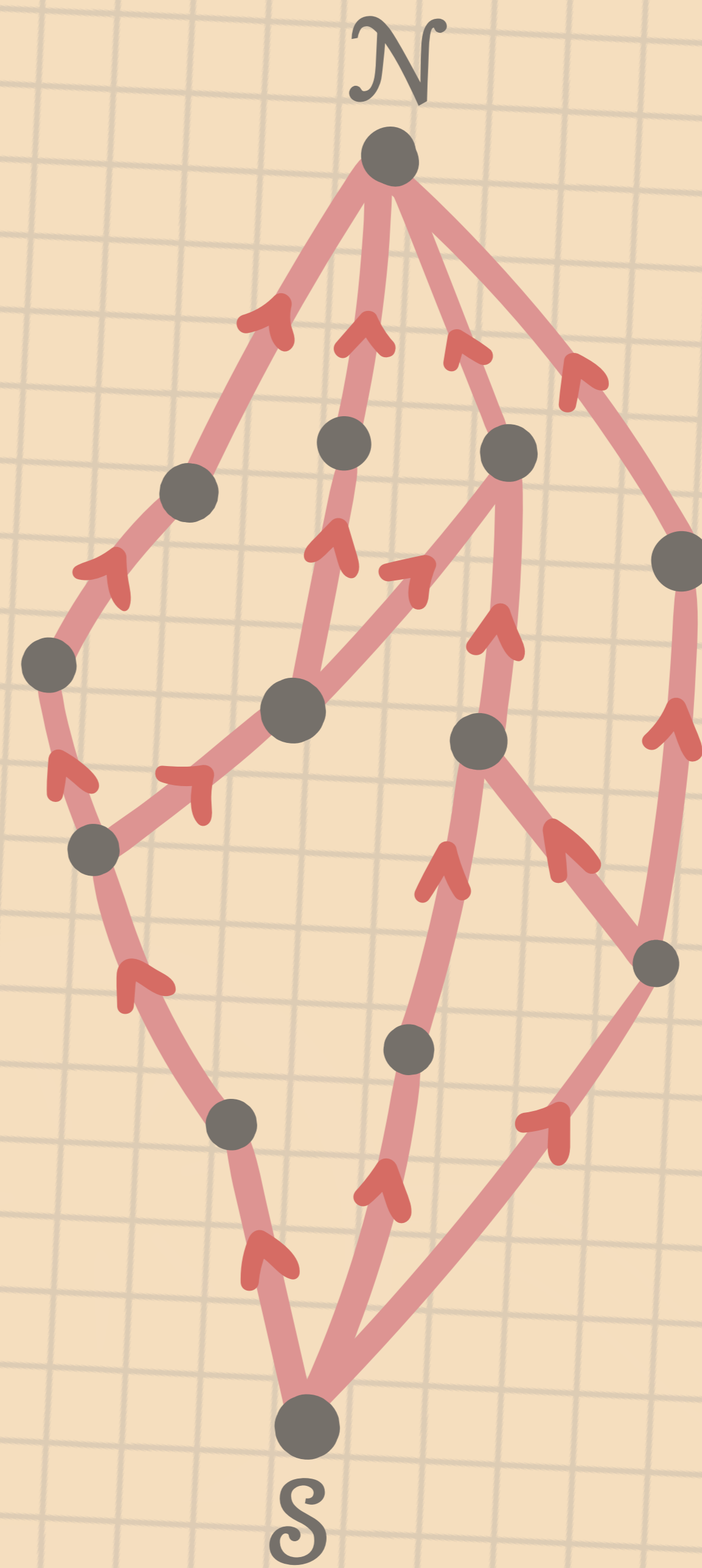
polyhedral
orientations



Schnyder
labelings

Summary

Quadrant
tander



Decorated bipolar orientations



*bipolar posets
by vertices*



*bipolar posets
by edges*



*transversal
structures*



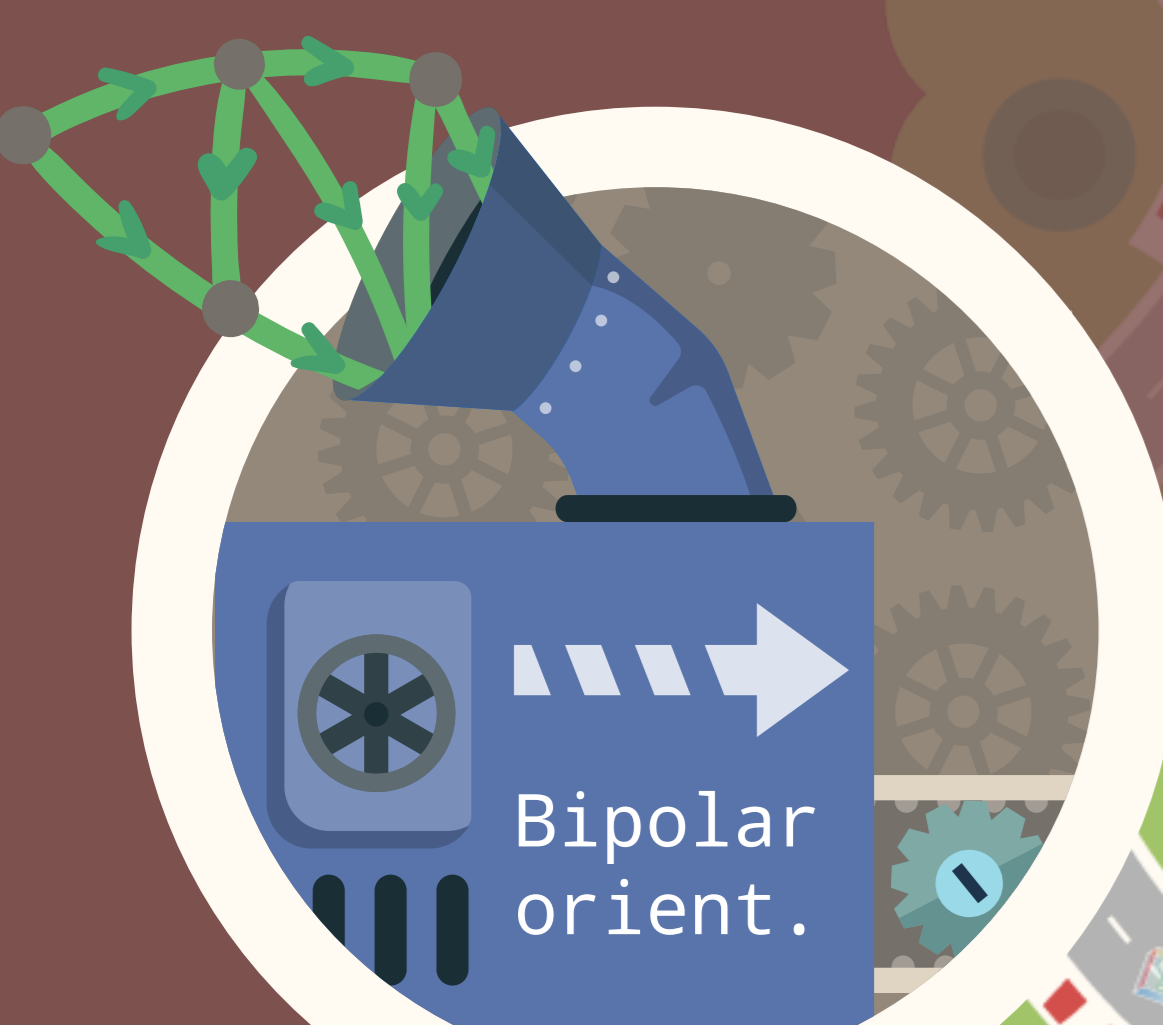
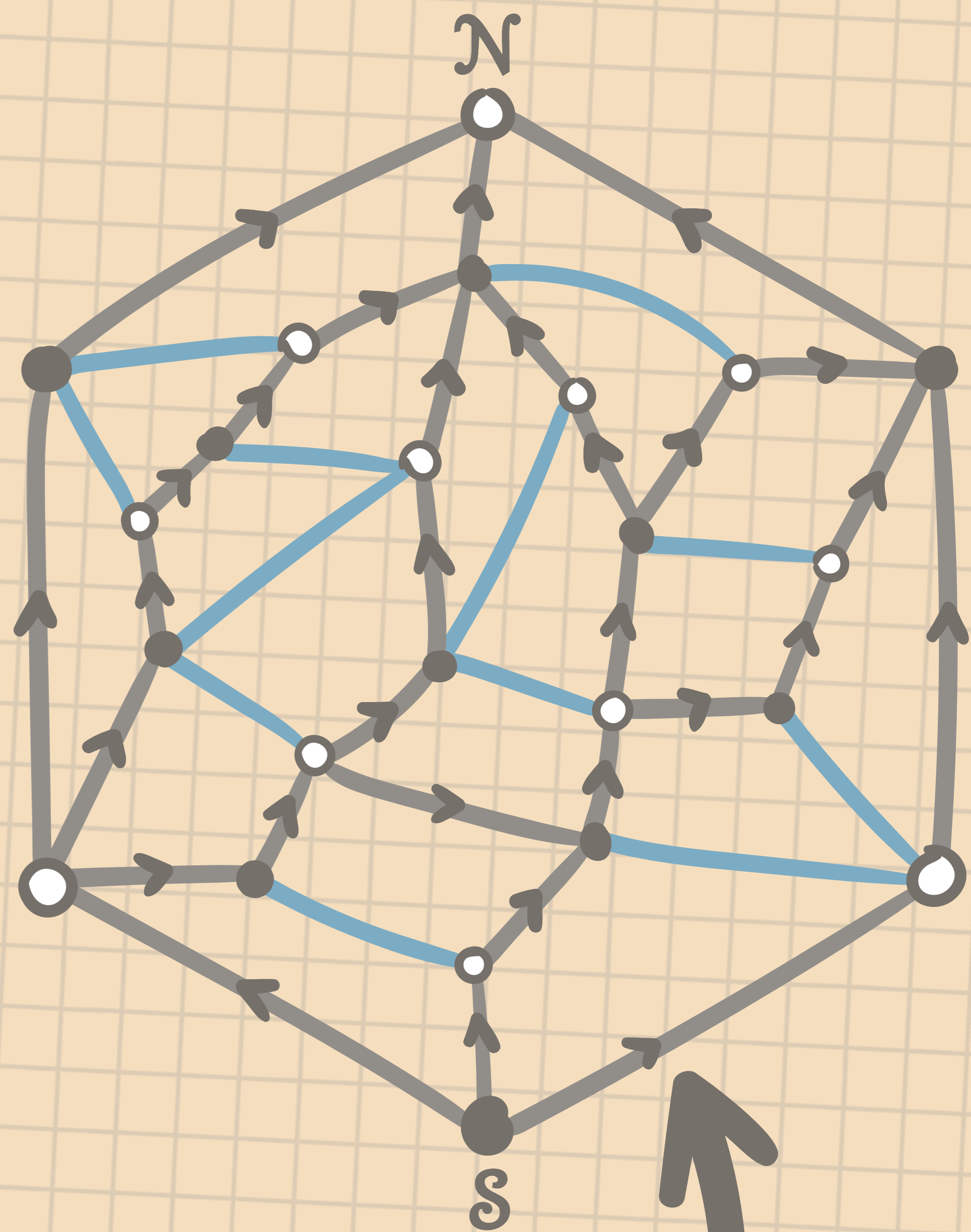
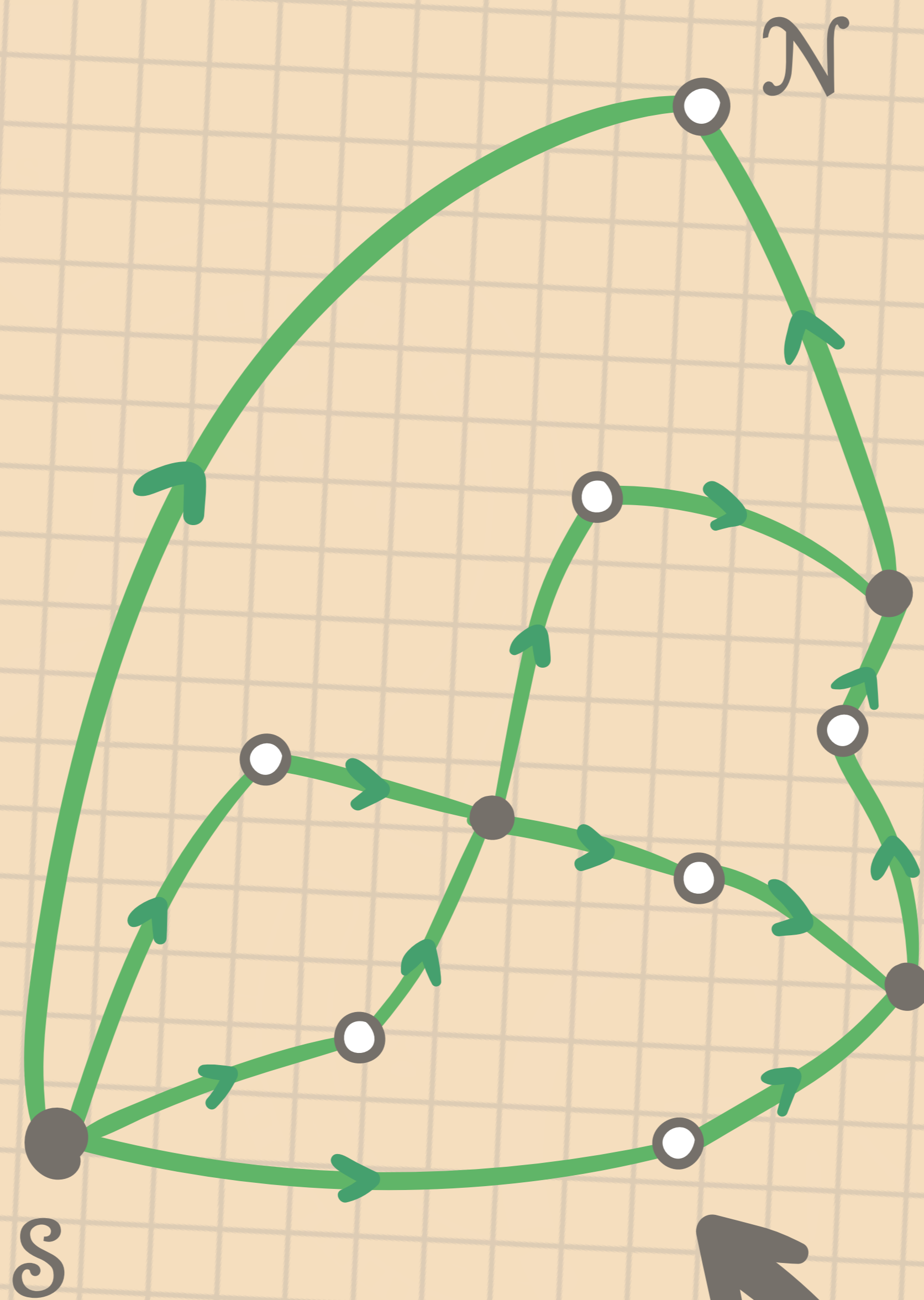
*polyhedral
orientations*



*Schnyder
labelings*

Summary

Quadrant
tander



Decorated bipolar orientations



*bipolar posets
by vertices*



*bipolar posets
by edges*



*transversal
structures*



*polyhedral
orientations*

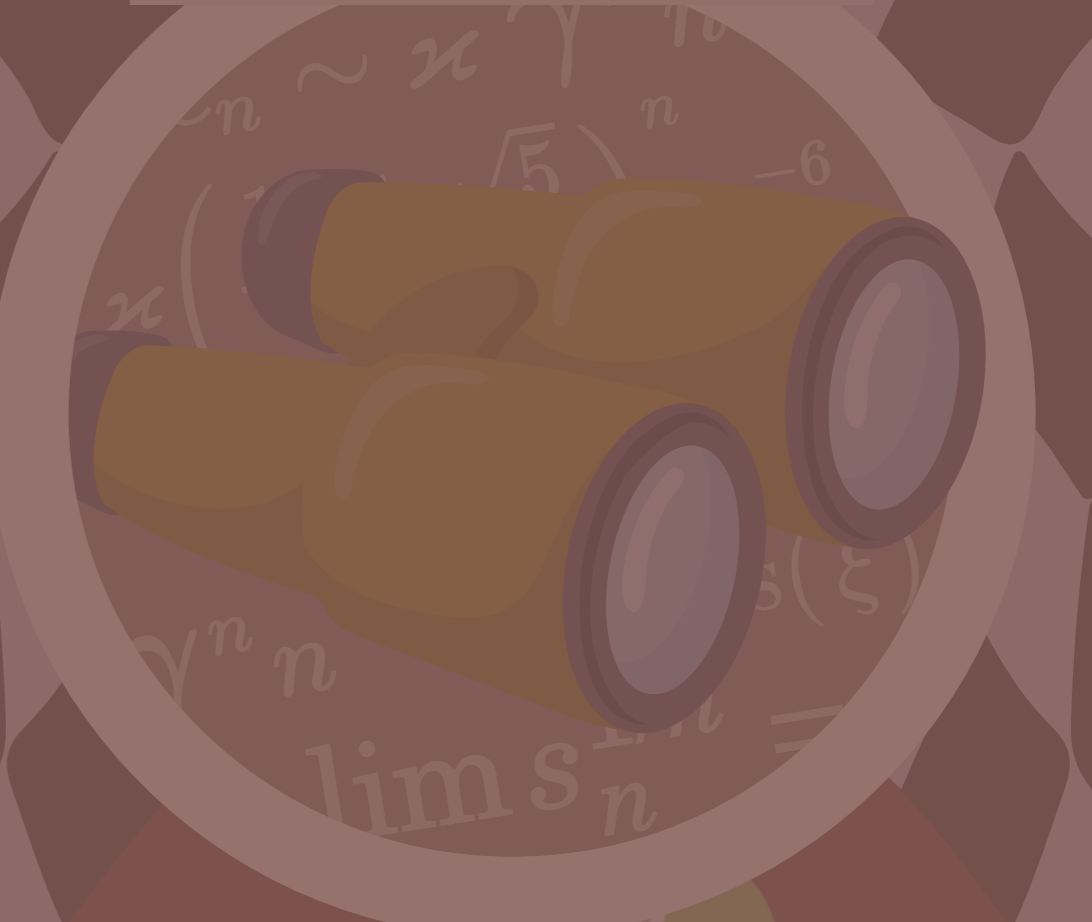
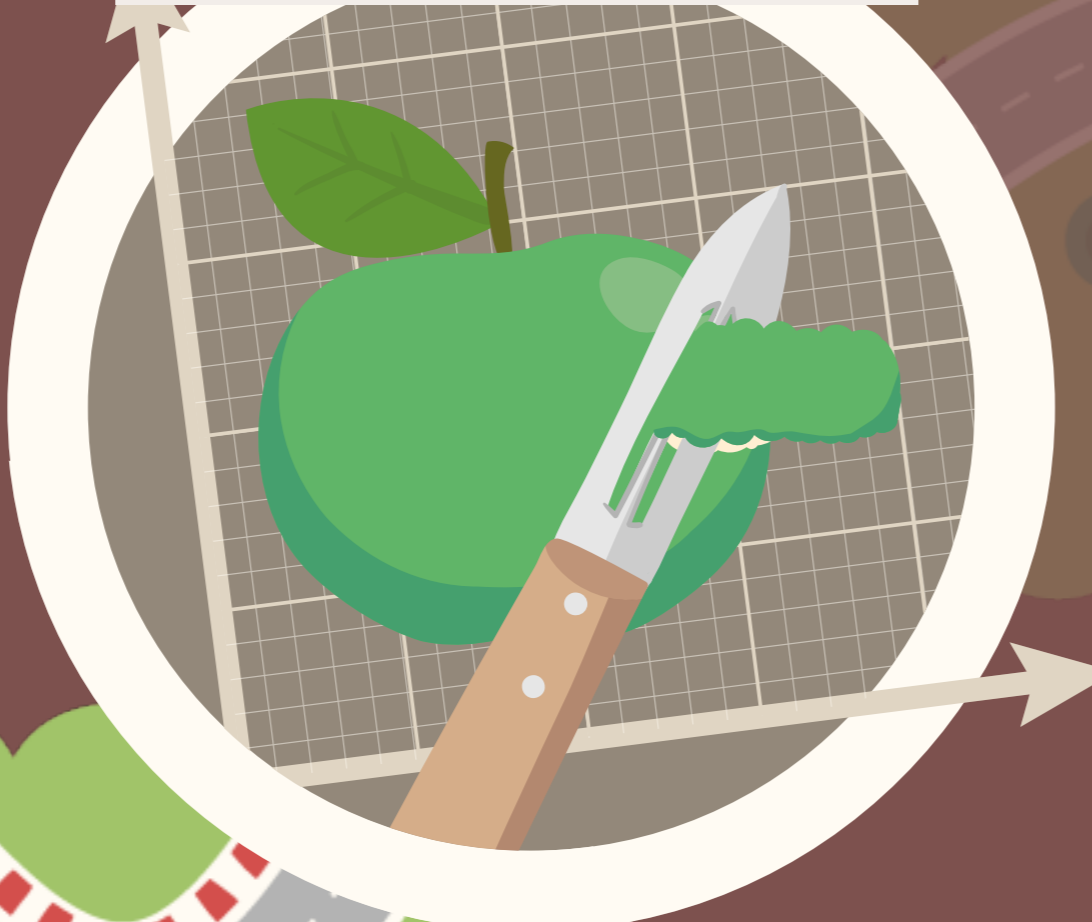


*Schnyder
labelings*

Summary

Quadrant
tandem walks

Asymptotics



Decorated bipolar
orientations

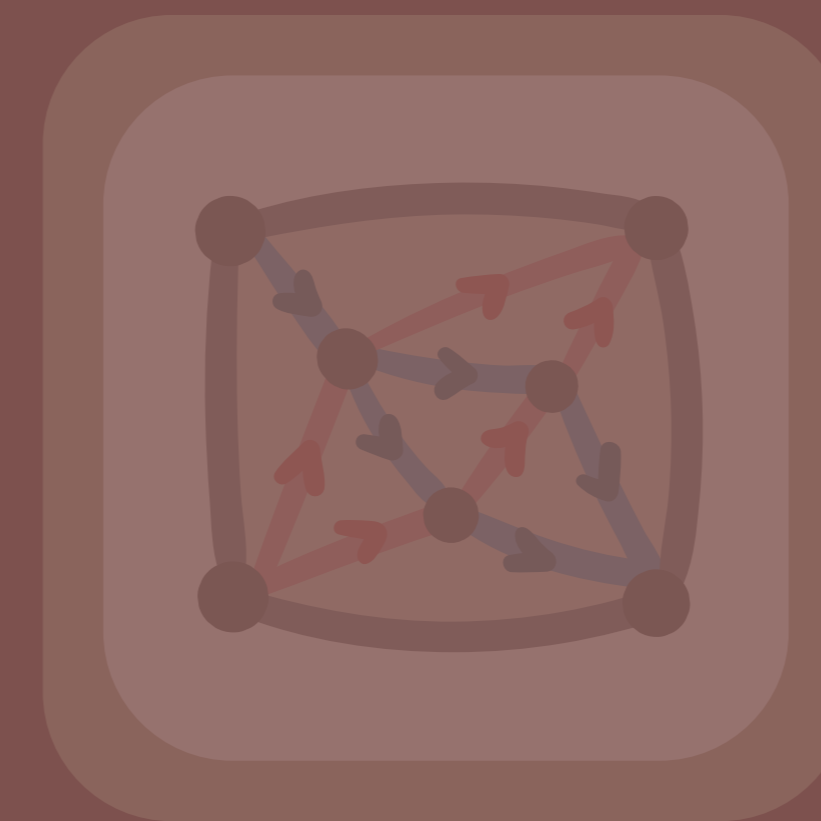
Small steps



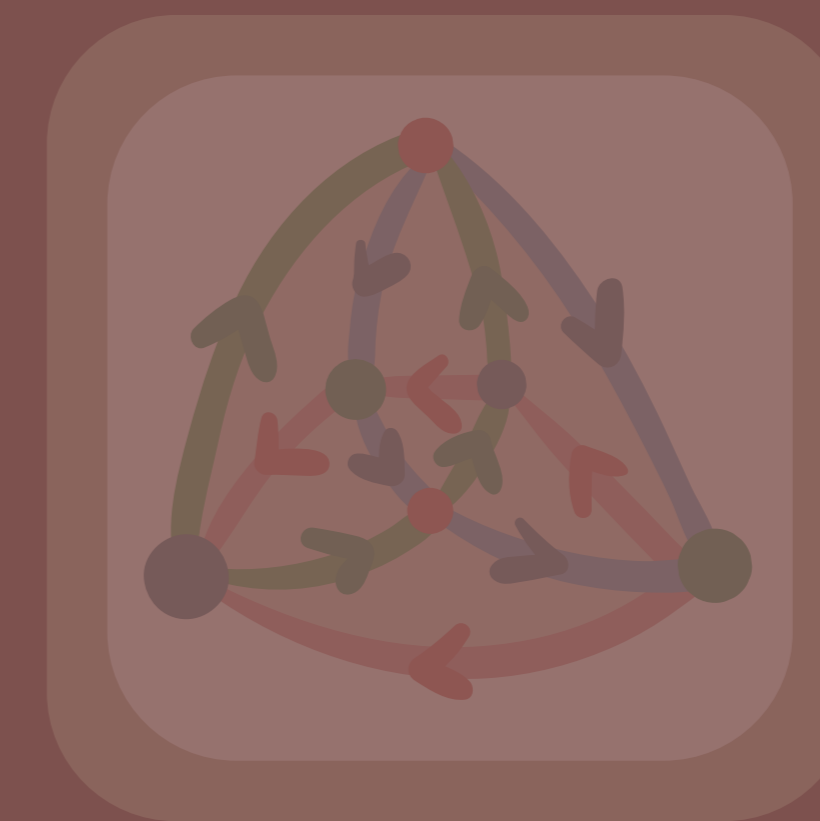
*bipolar posets
by vertices*



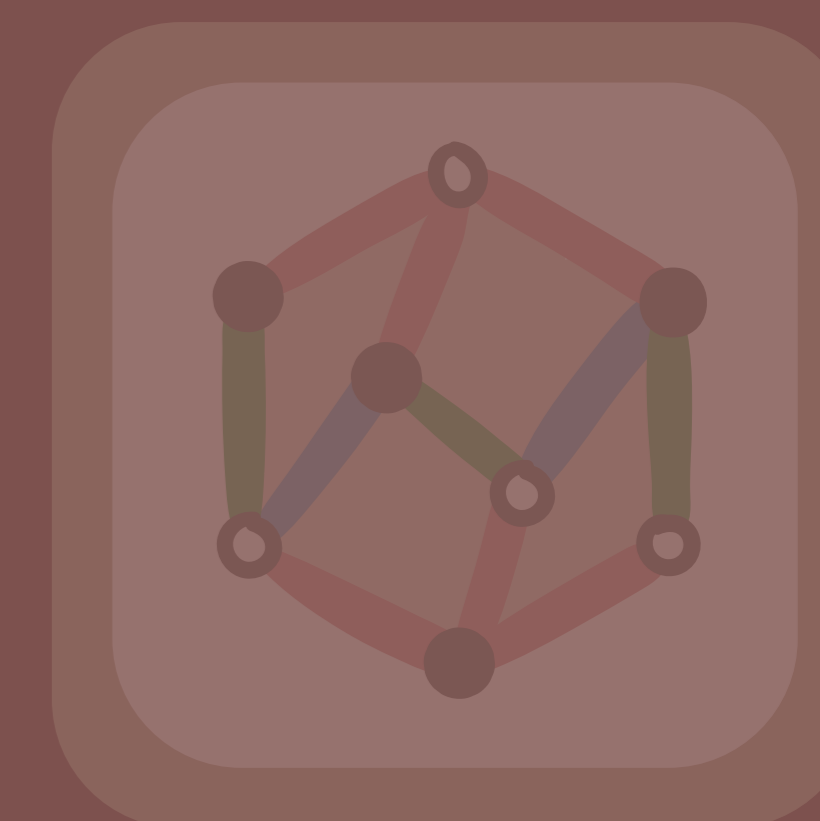
*bipolar posets
by edges*



*transversal
structures*



*polyhedral
orientations*

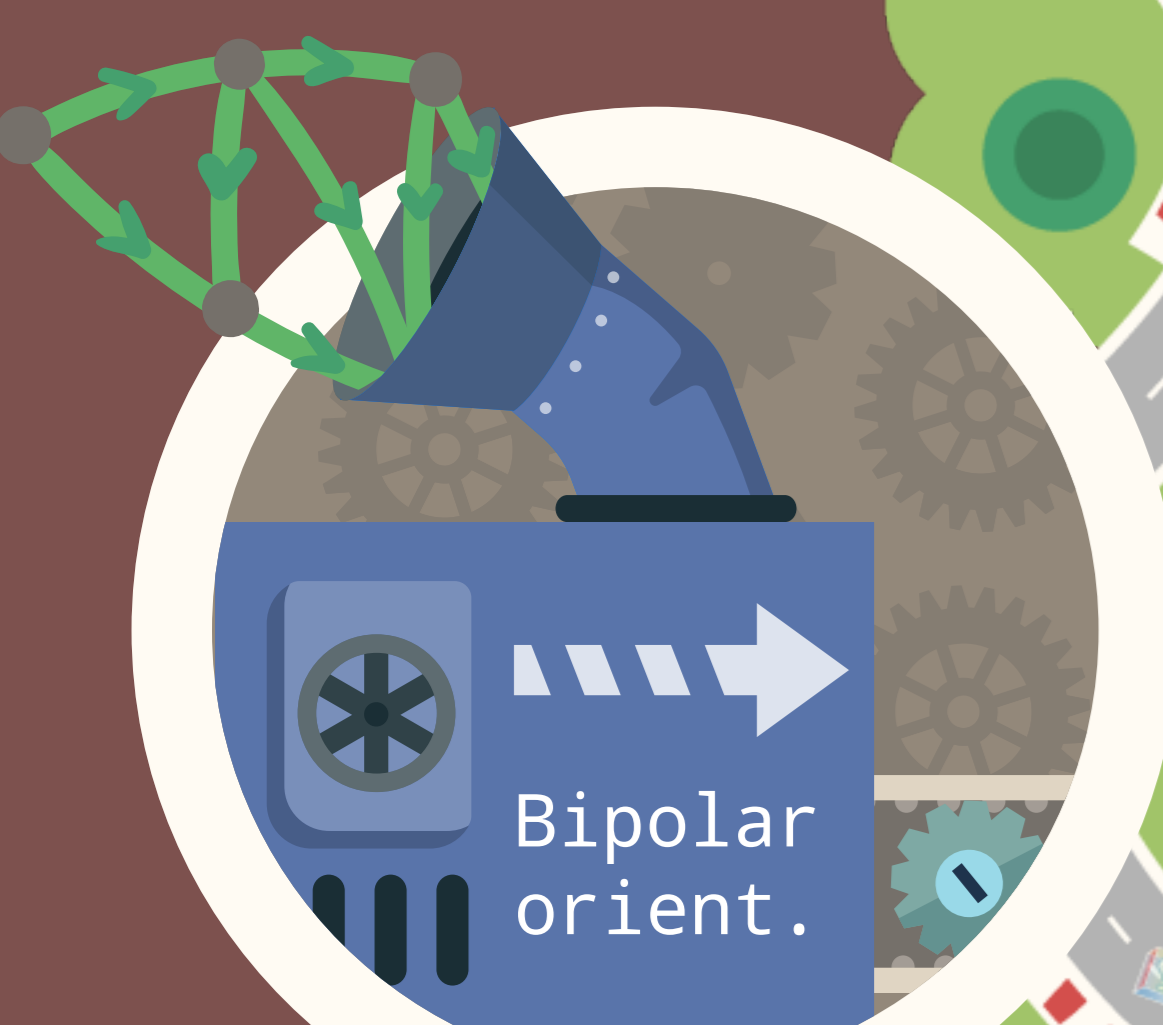


*Schnyder
labelings*

Summary

Quadrant tandem walks

Asymptotics



Decorated bipolar orientations

Small steps

a. The KMSW bijection

b.  Application to transversal structures

The KMSW bijection

*Plane bipolar
orientations*



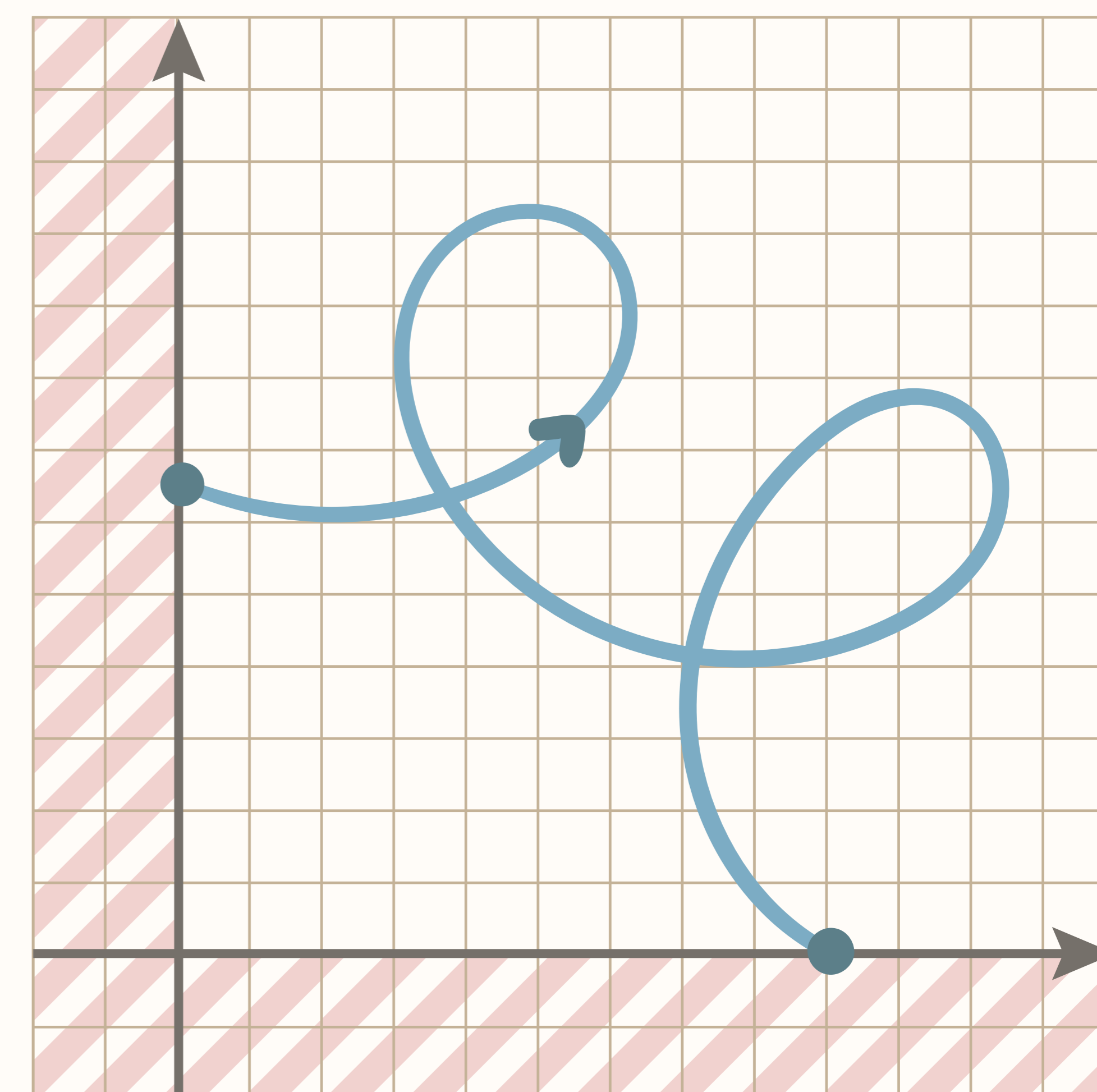
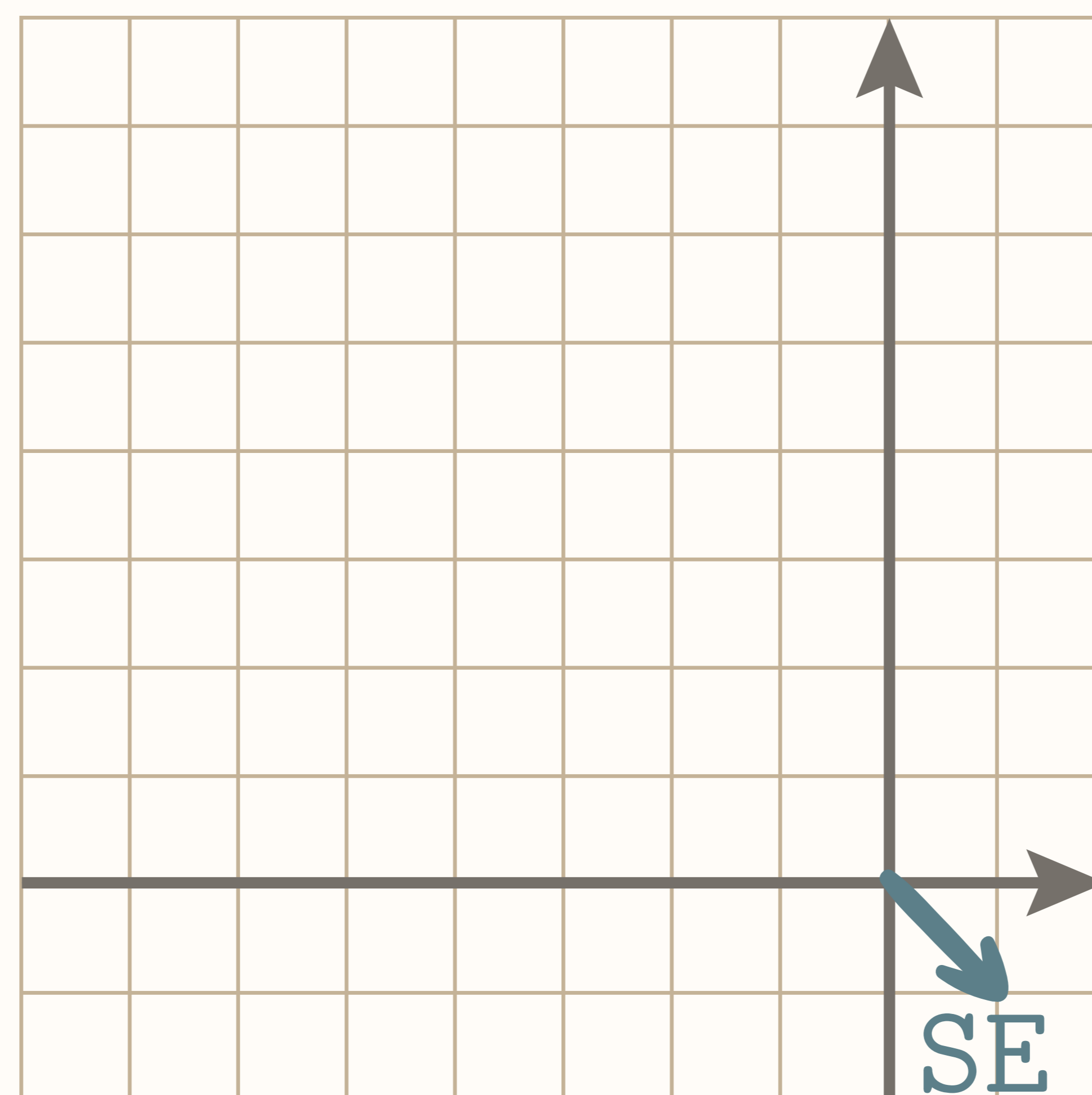
*tandems walks
in the quarter plane*

The KMSW bijection

Plane bipolar orientations



tandem walks in the quarter plane

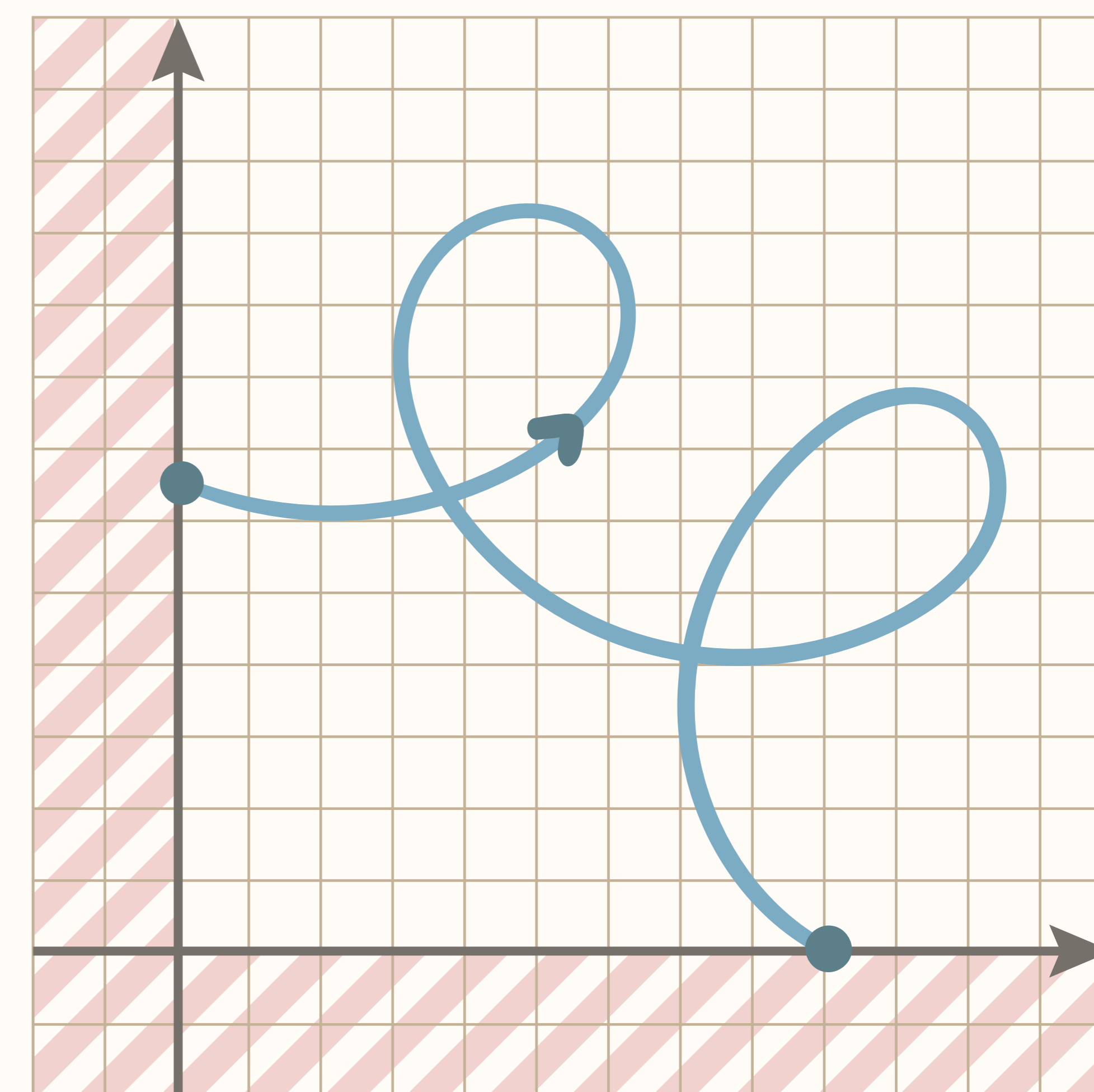
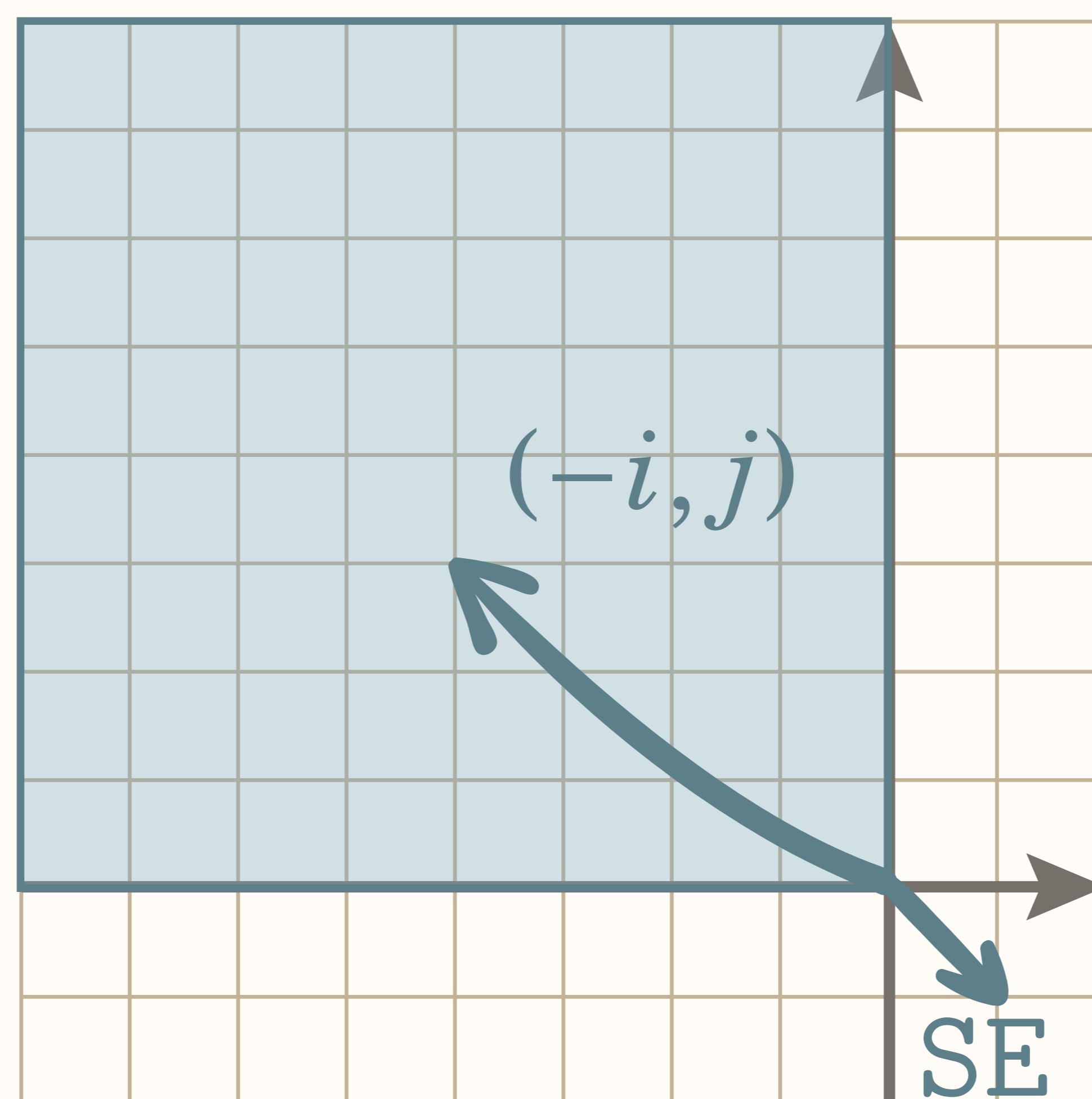


The KMSW bijection

Plane bipolar orientations



tandems walks in the quarter plane

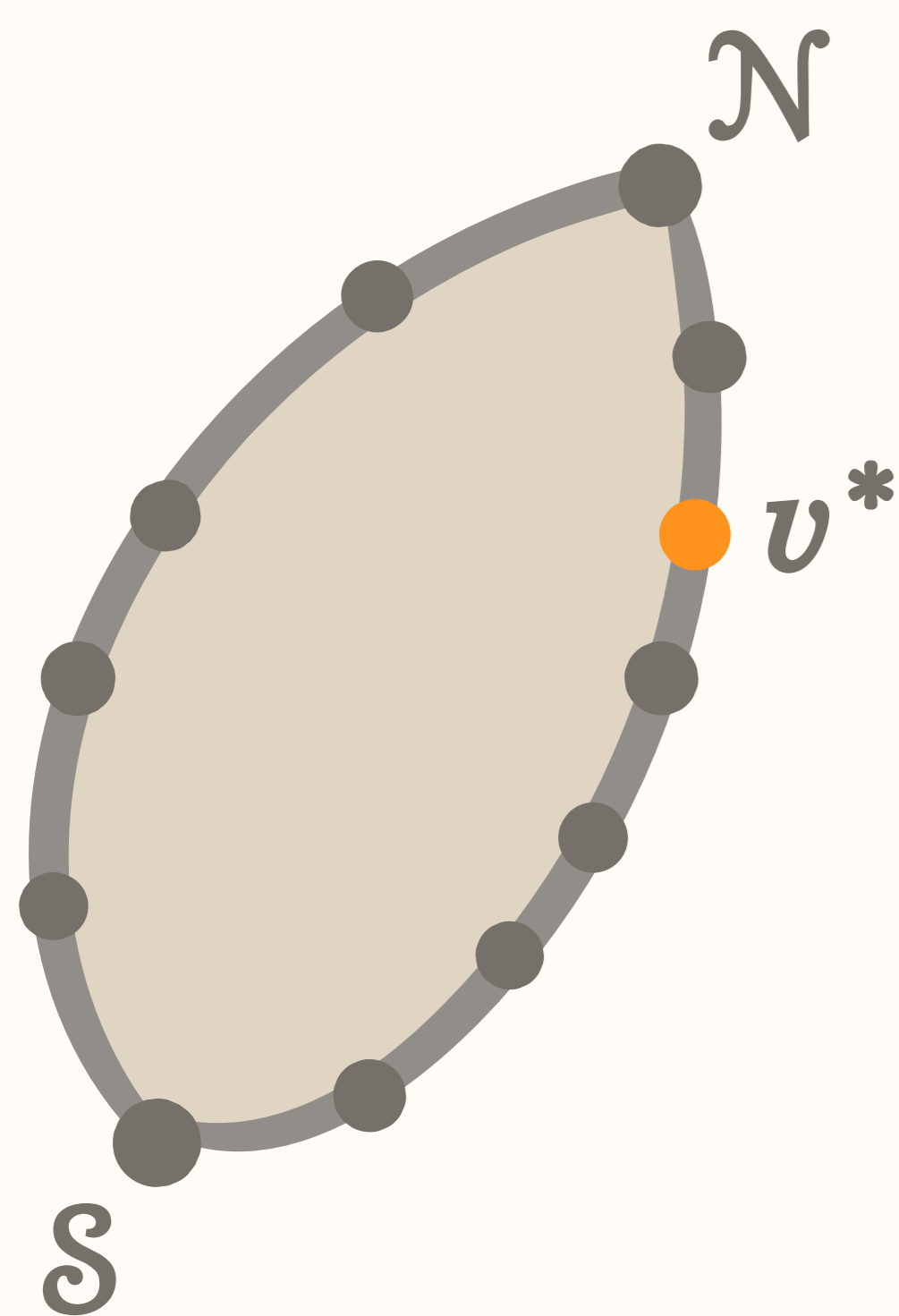


The KMSW bijection

Plane bipolar orientations



tandem walks in the quarter plane

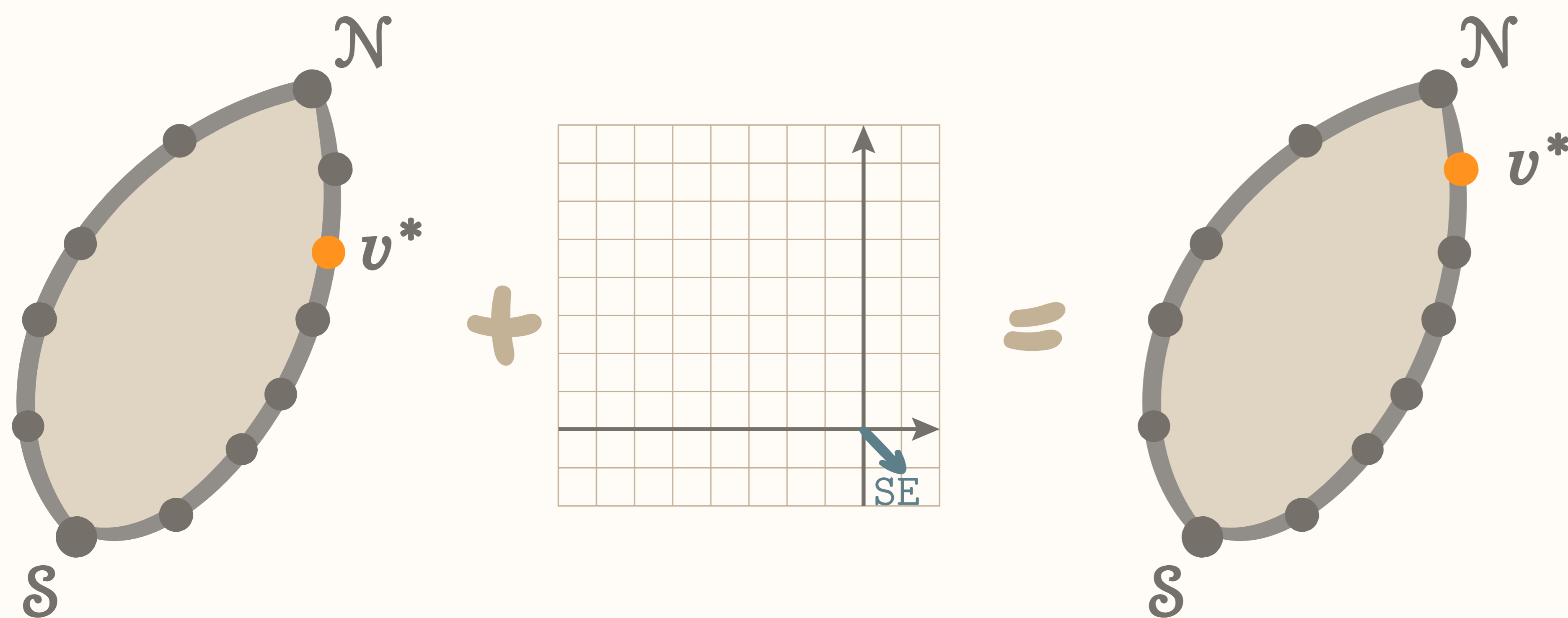


The KMSW bijection

Plane bipolar orientations



tandems walks in the quarter plane

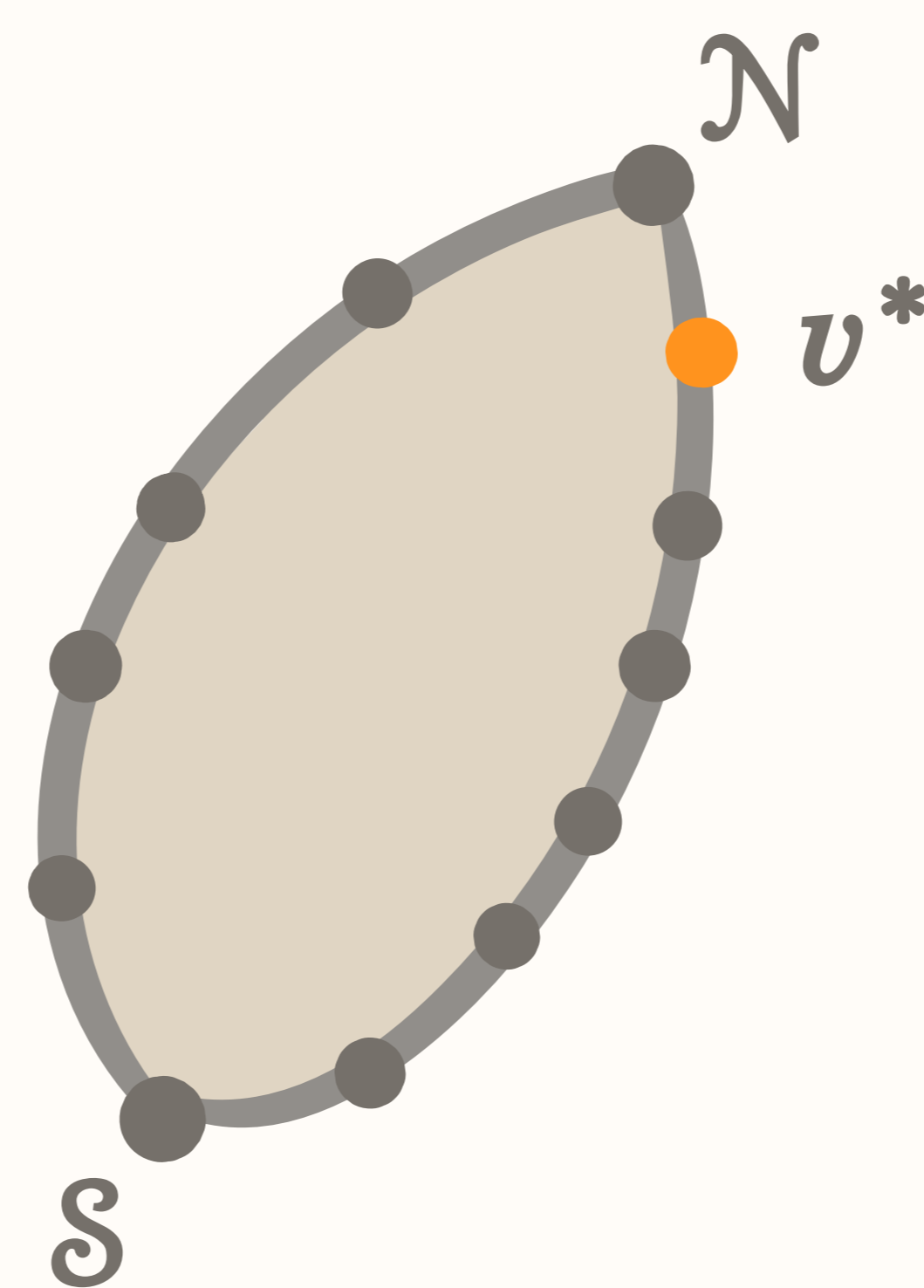
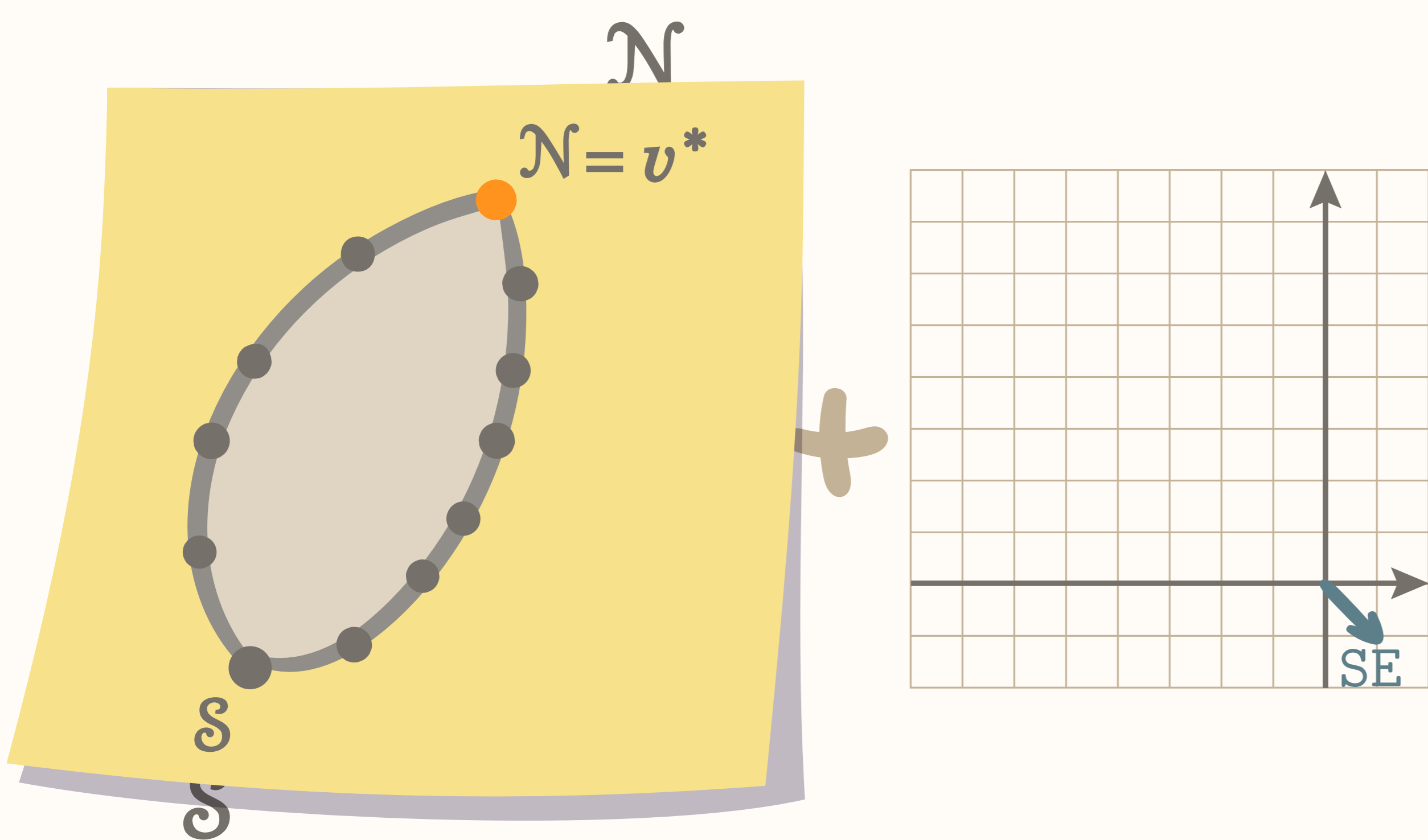


The KMSW bijection

Plane bipolar orientations



tandems walks in the quarter plane

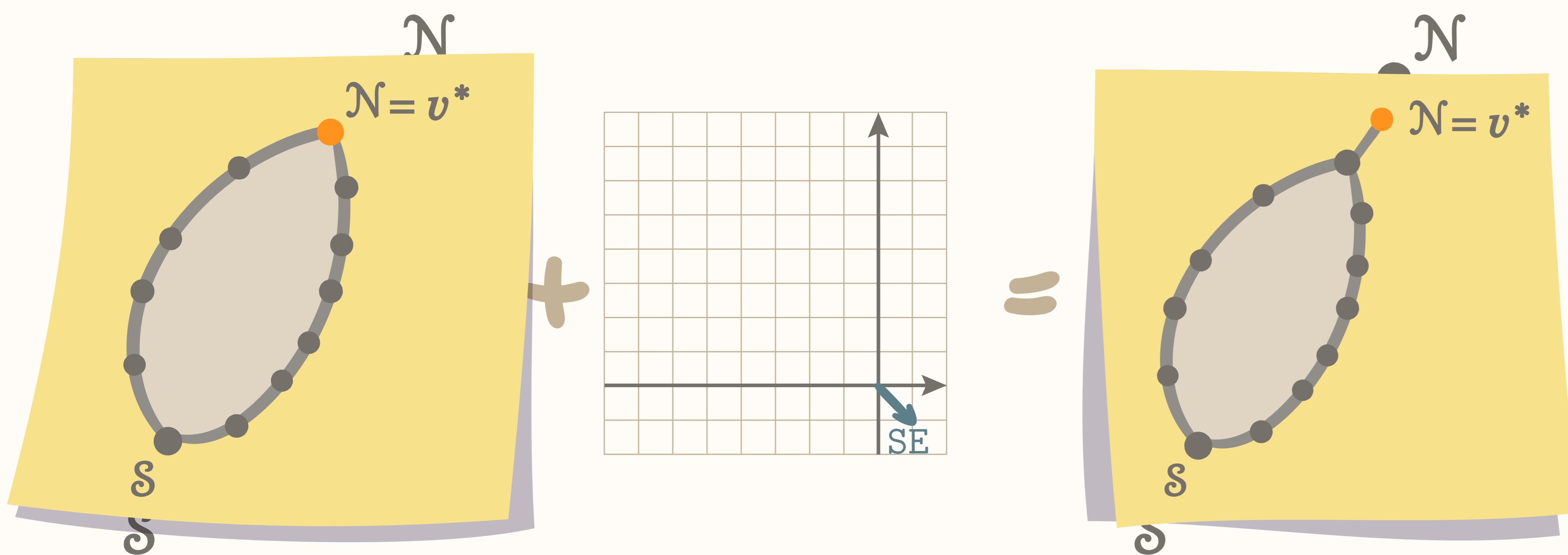


The KMSW bijection

Plane bipolar orientations



tandem walks in the quarter plane

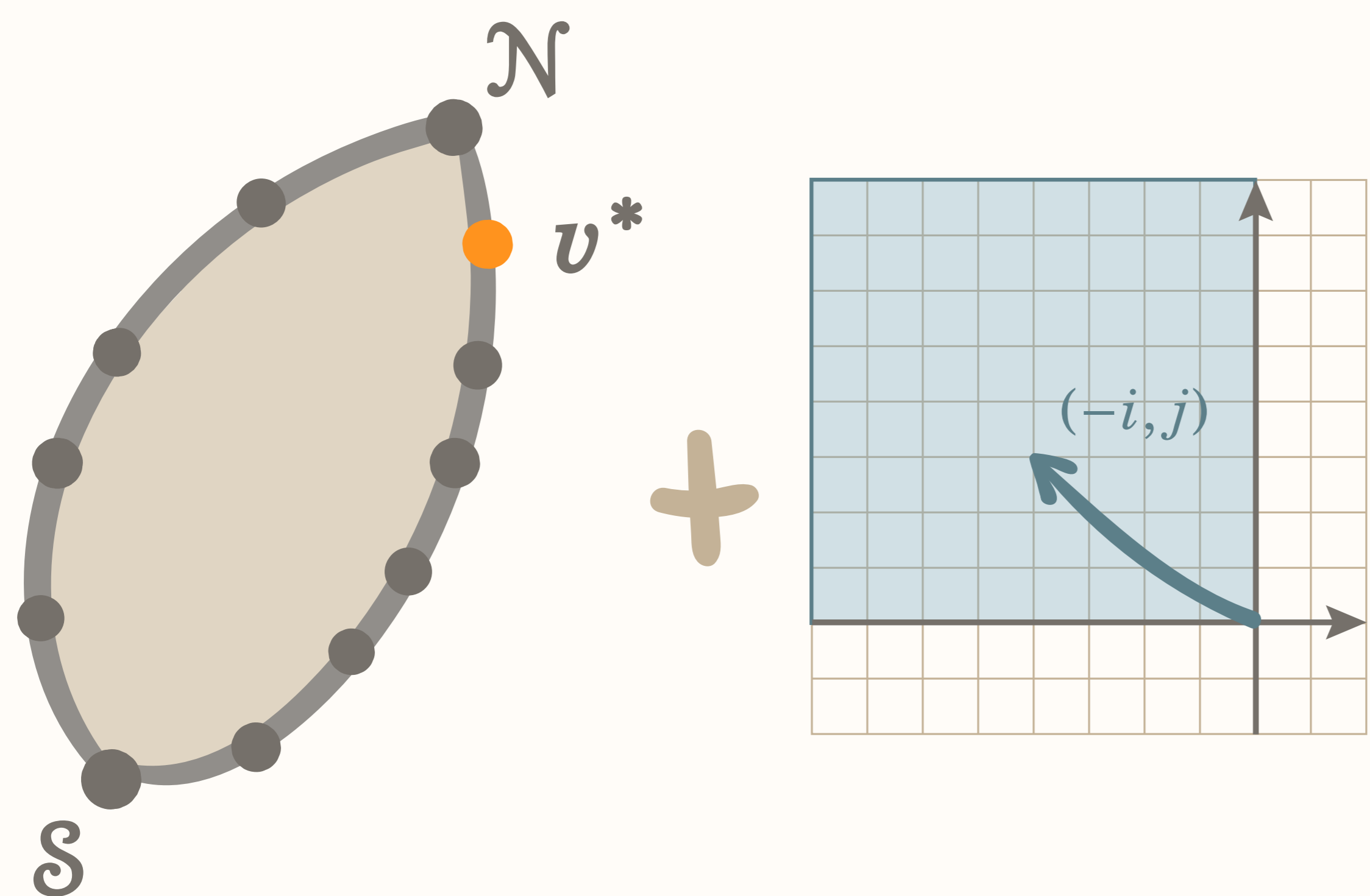
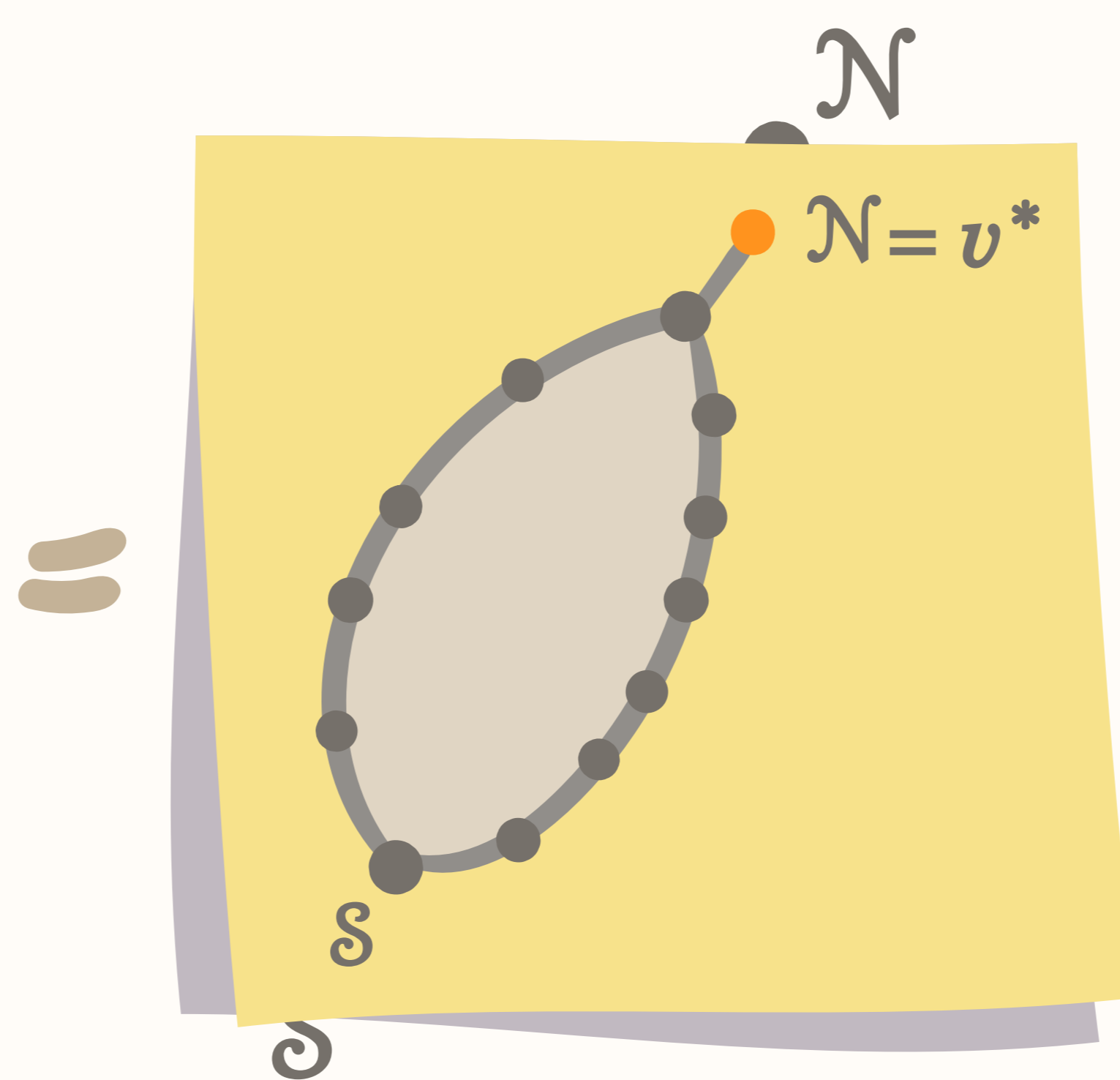
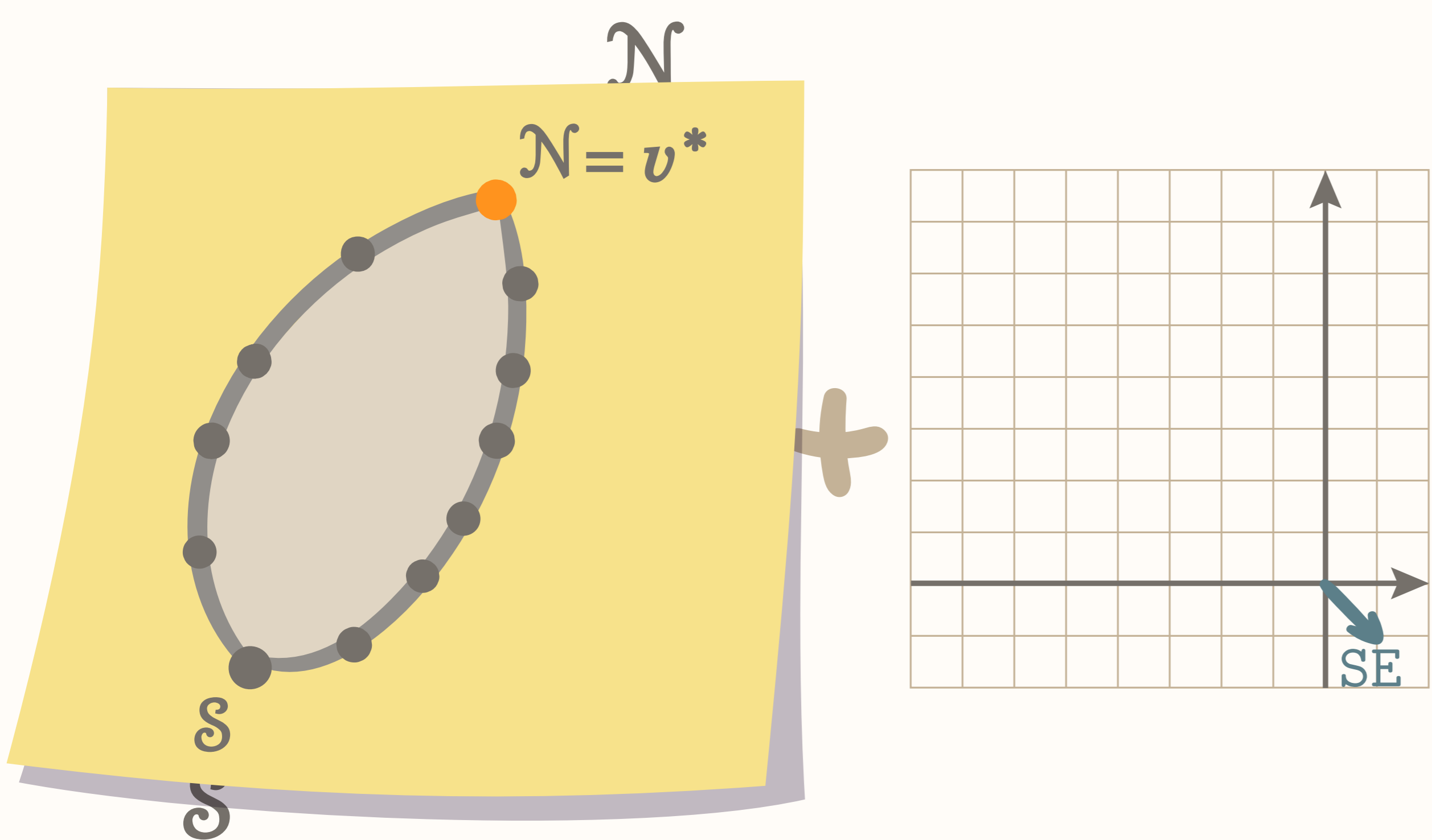


The KMSW bijection

Plane bipolar orientations



tandems walks in the quarter plane

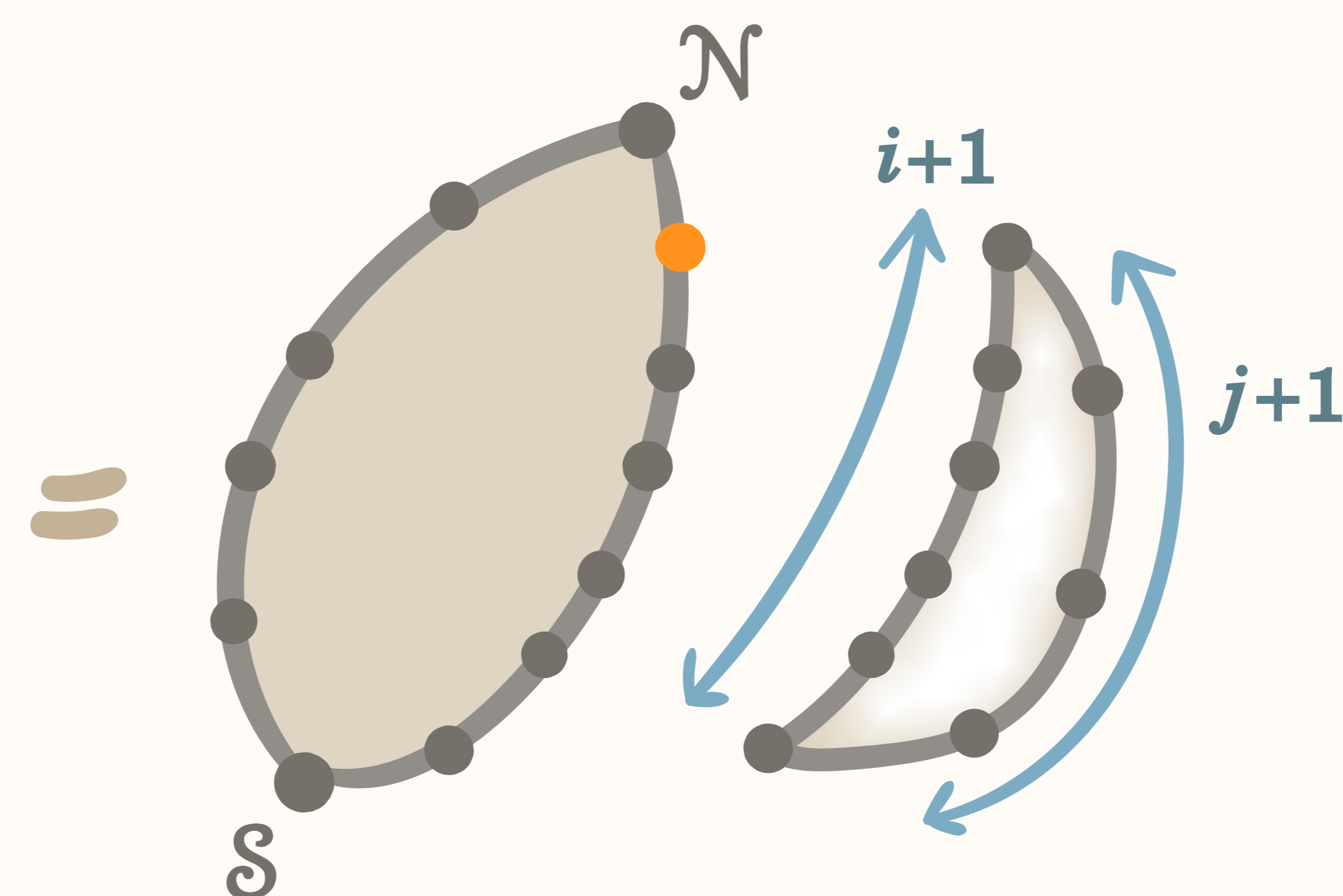
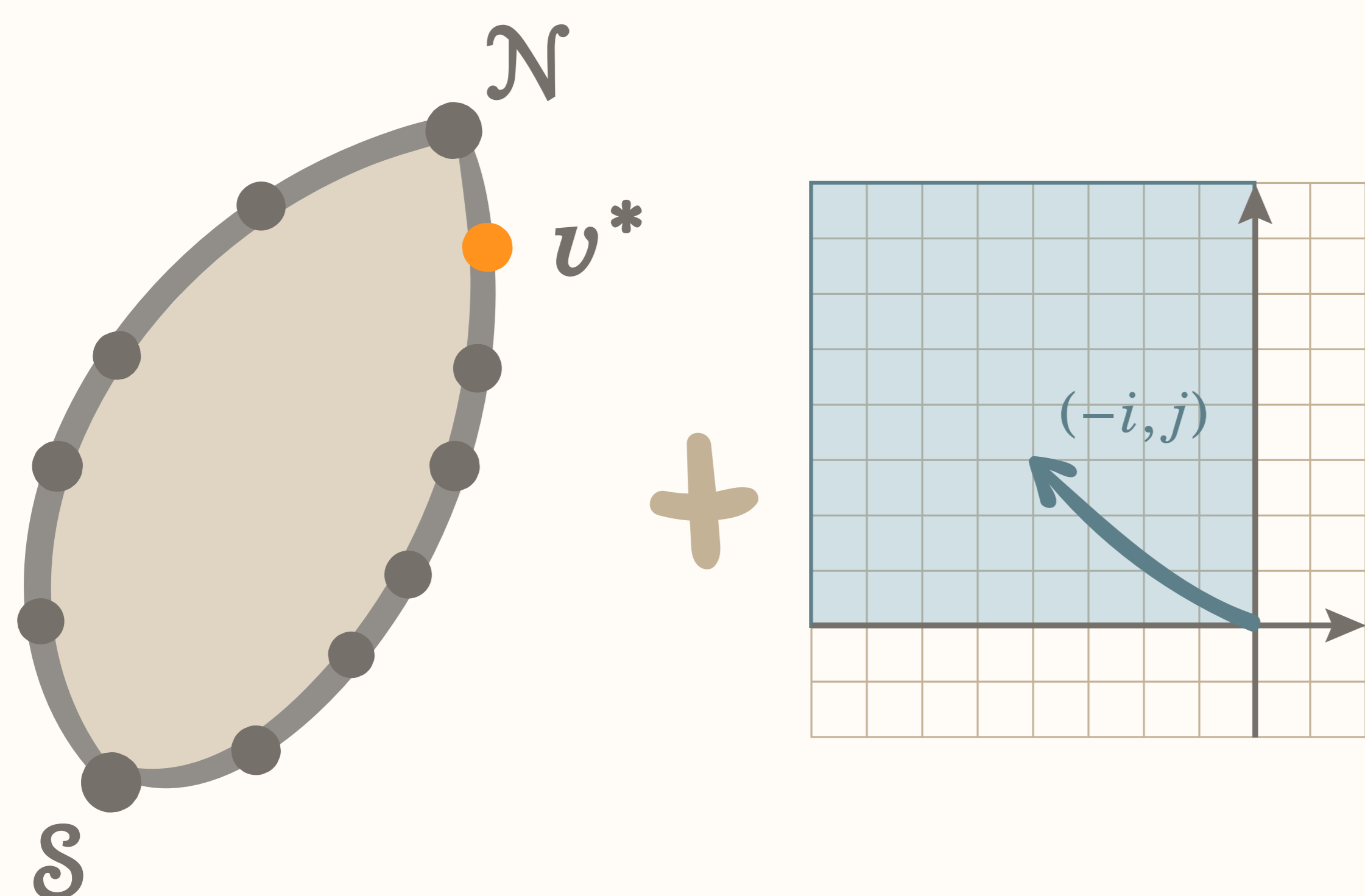
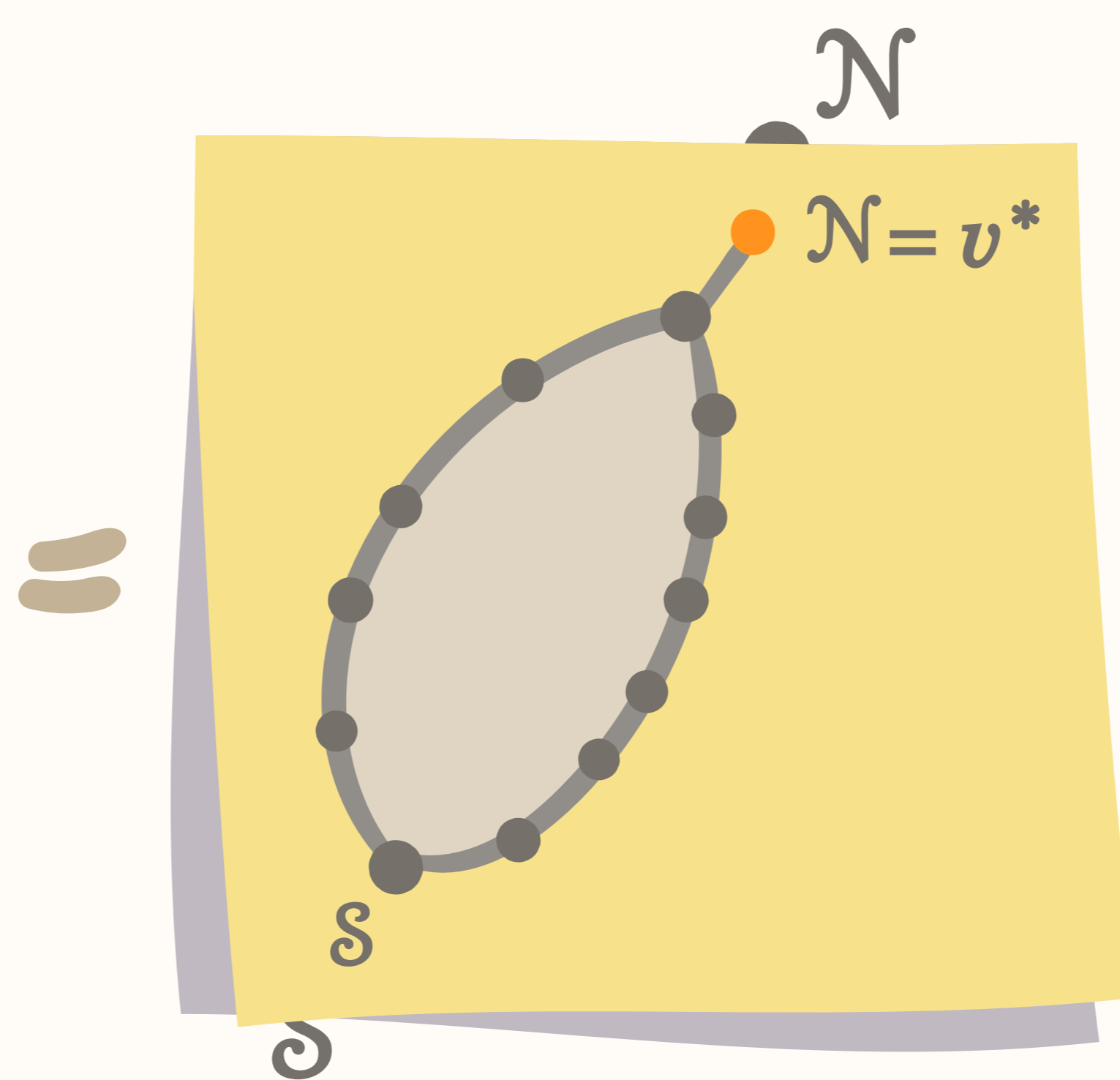
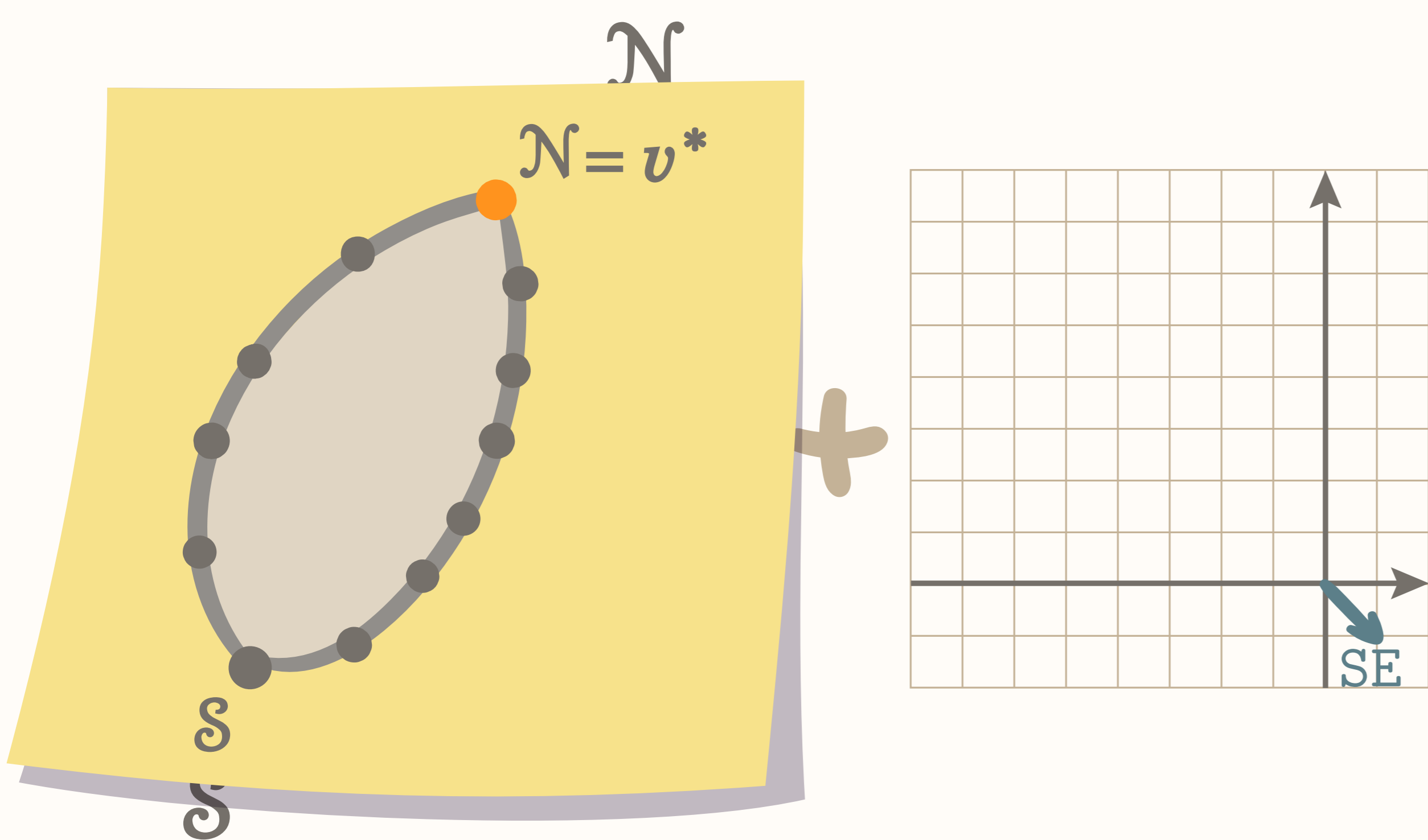


The KMSW bijection

Plane bipolar orientations



tandems walks in the quarter plane

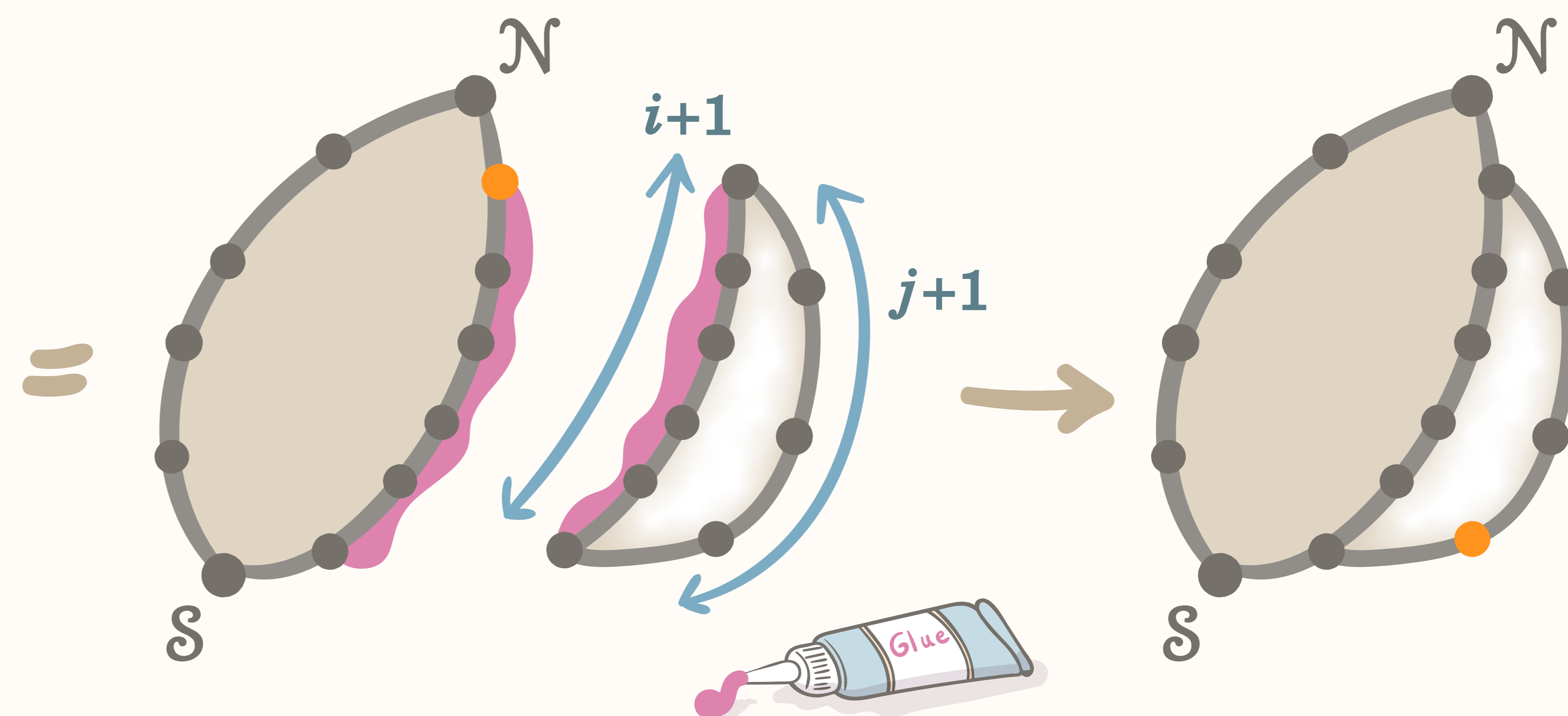
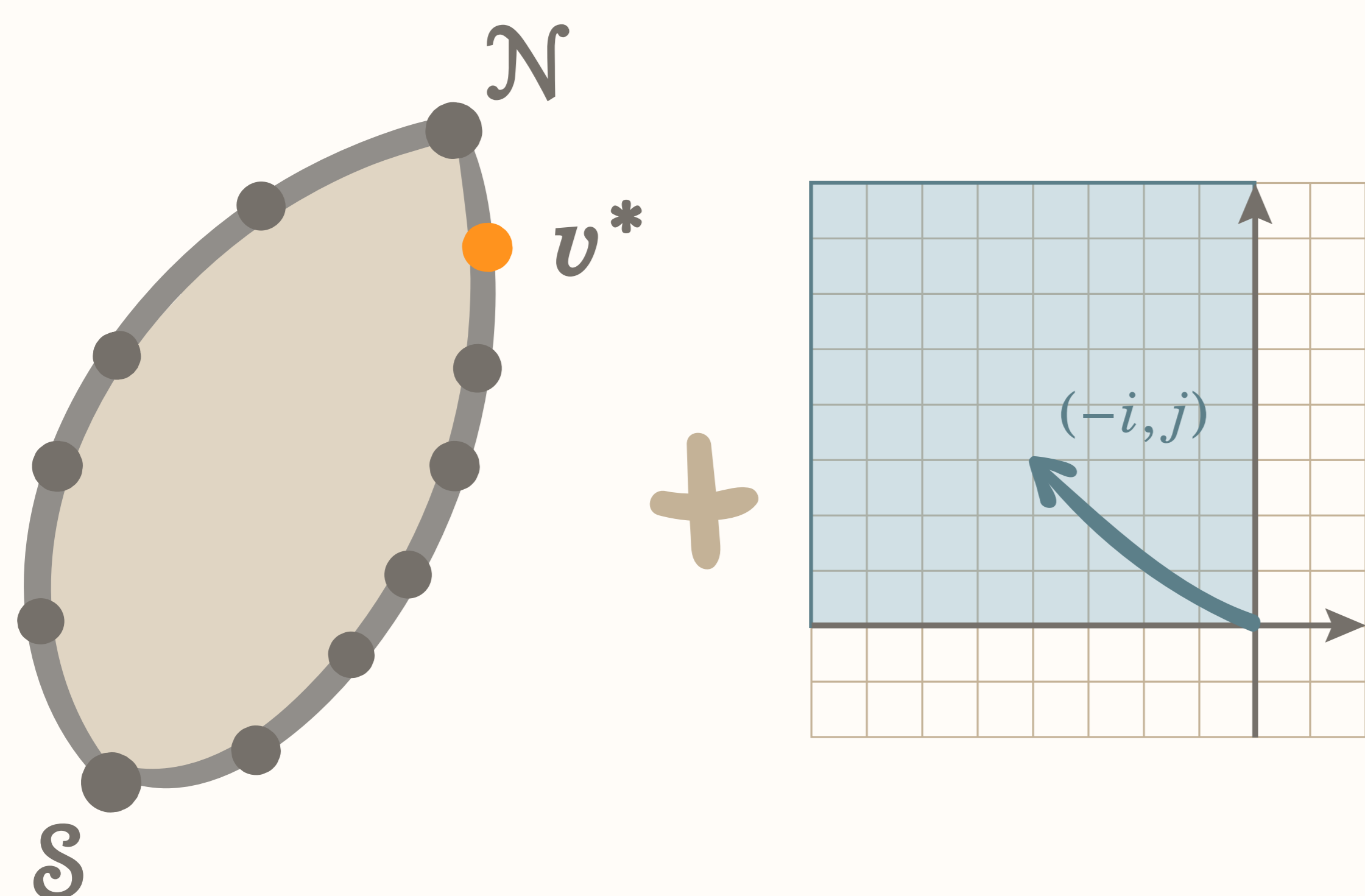
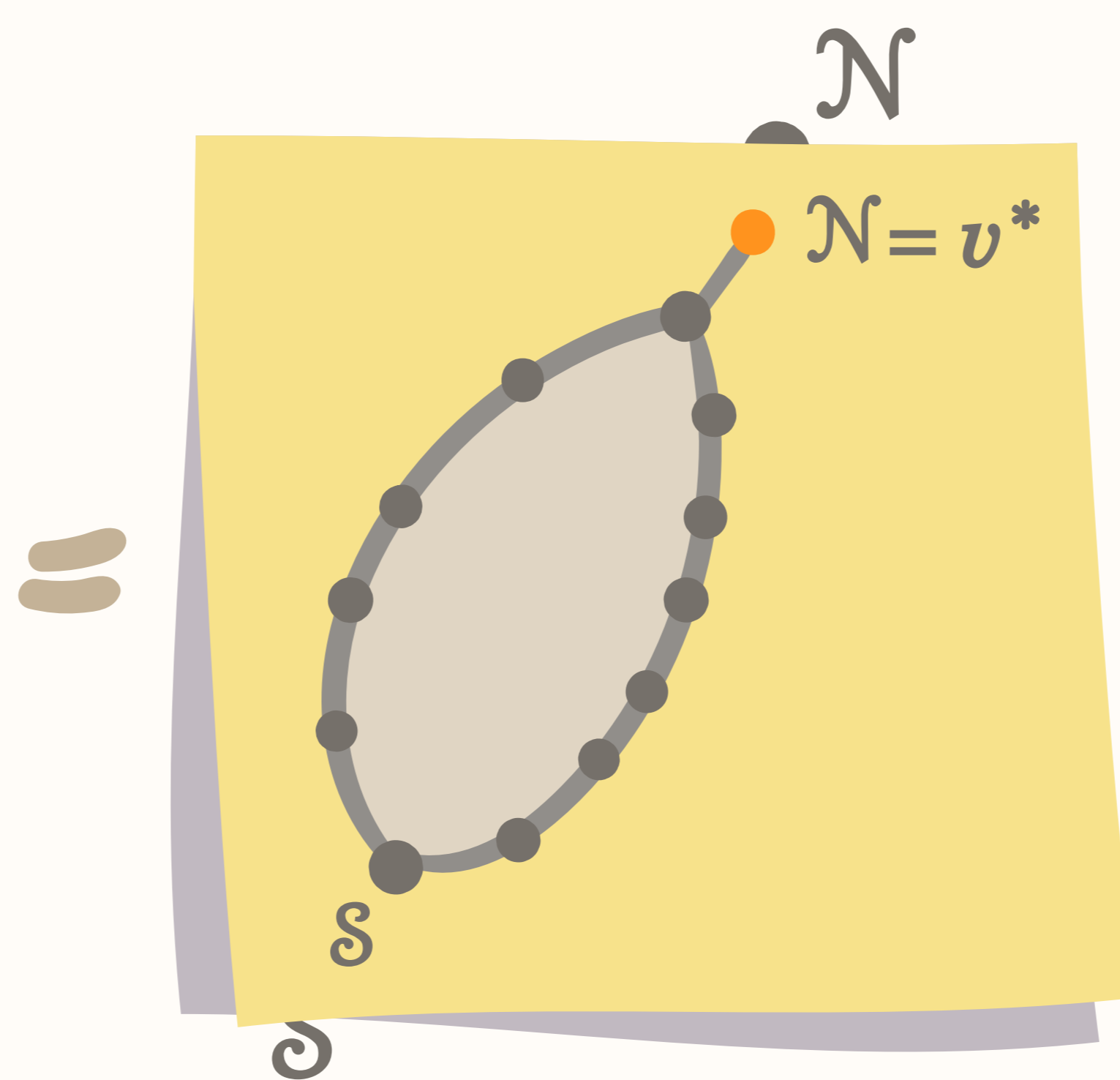
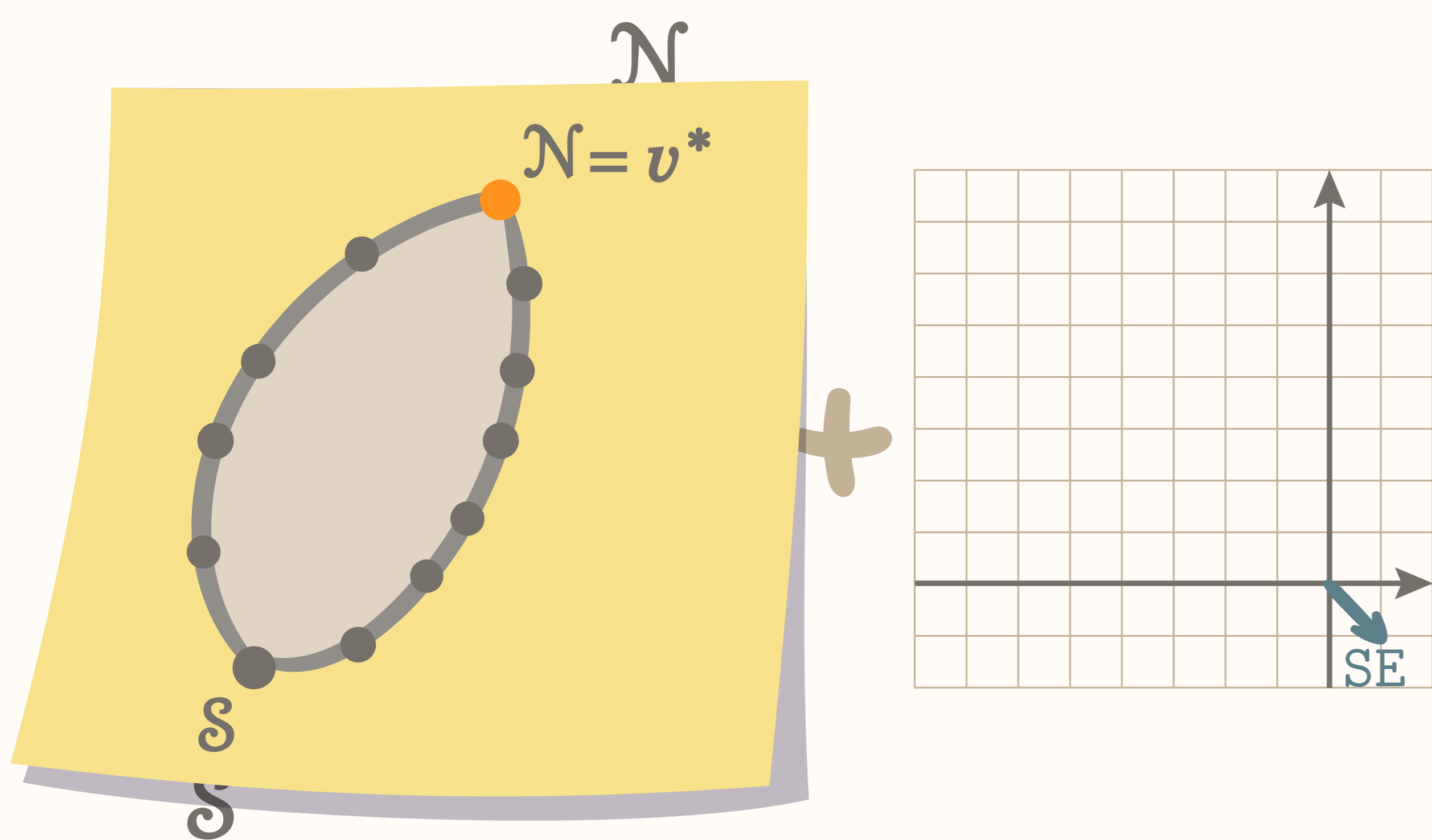


The KMSW bijection

Plane bipolar orientations



tandem walks in the quarter plane

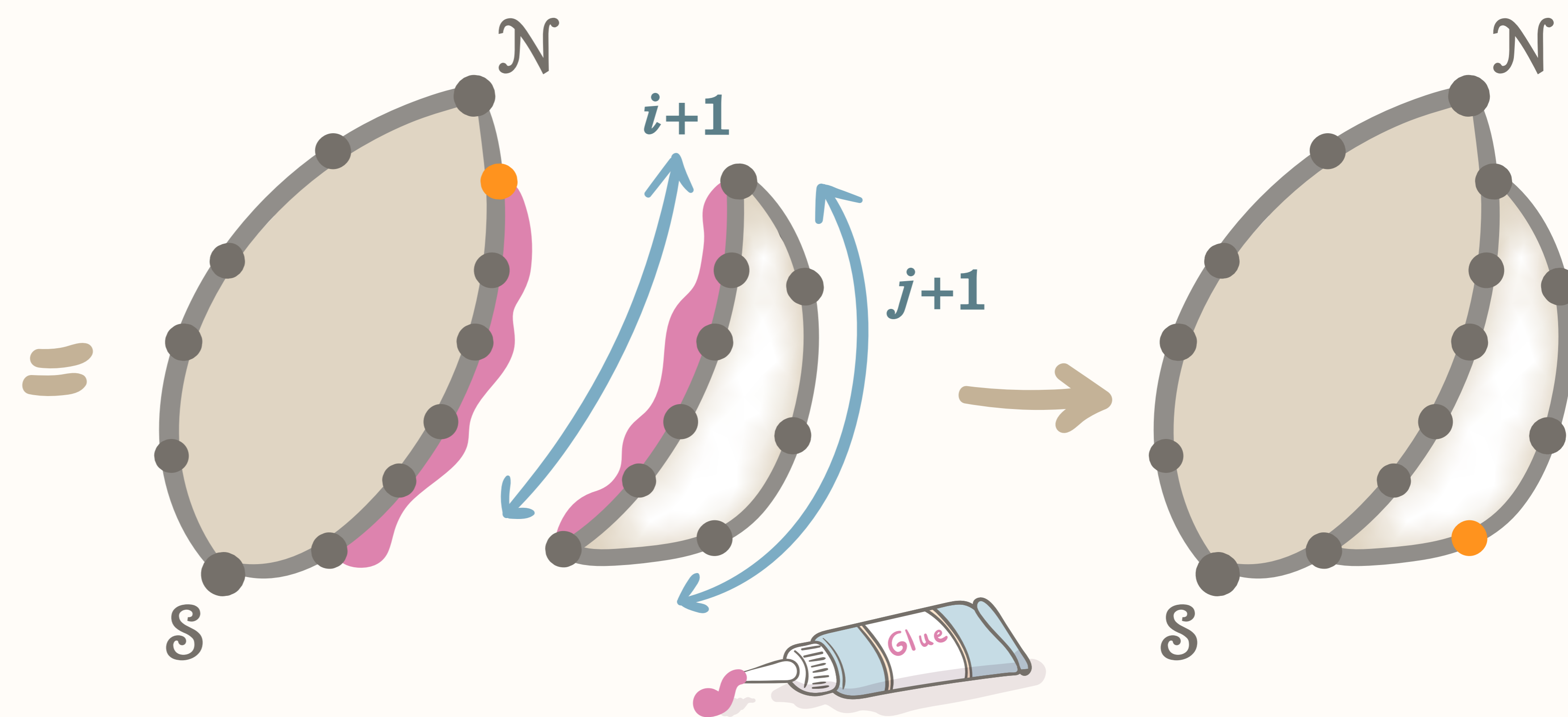
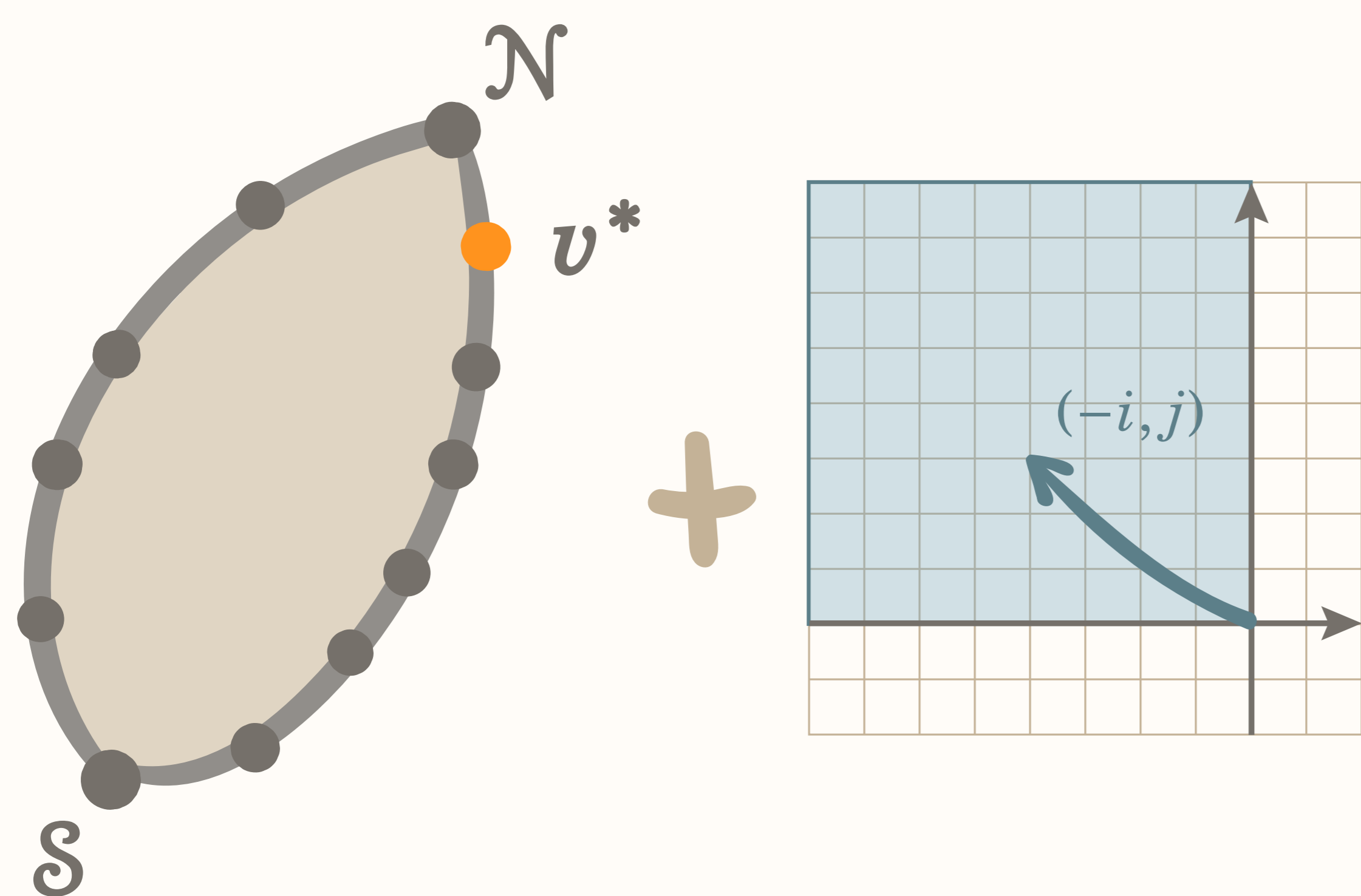
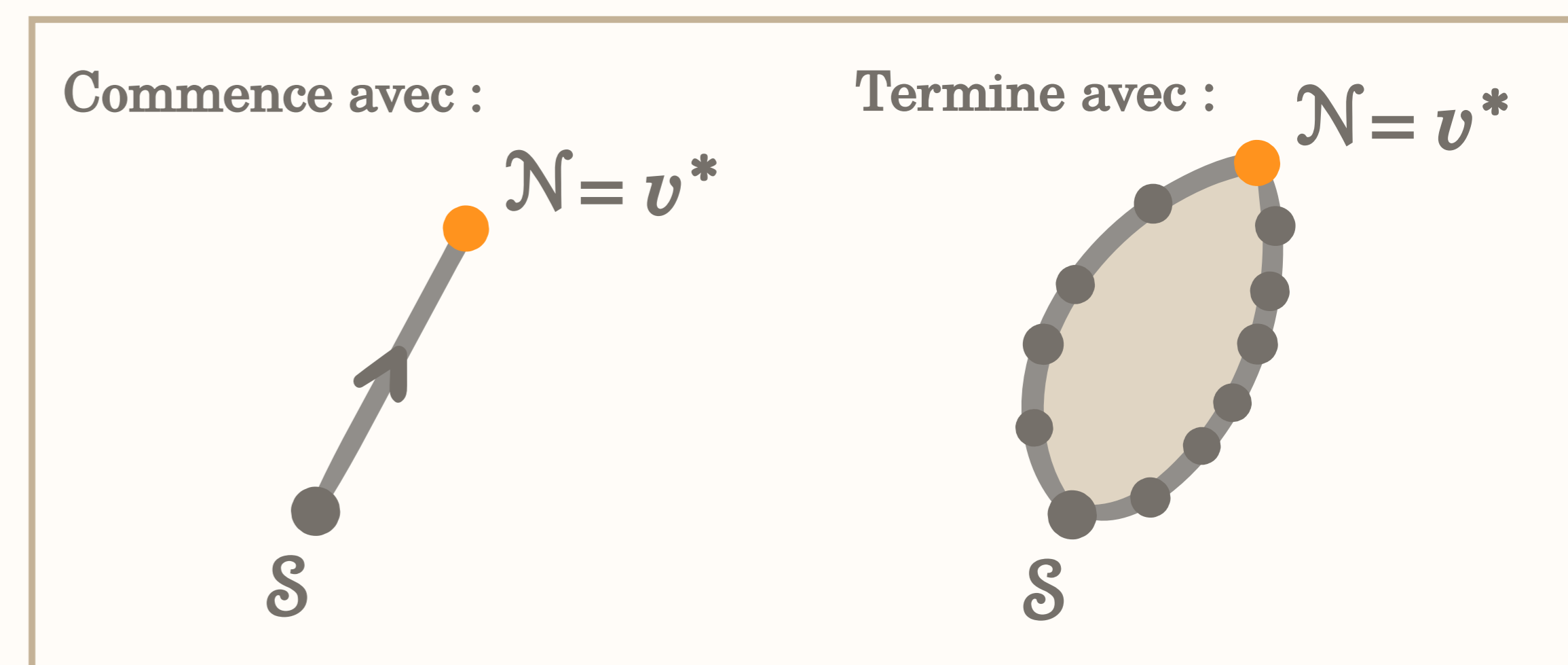
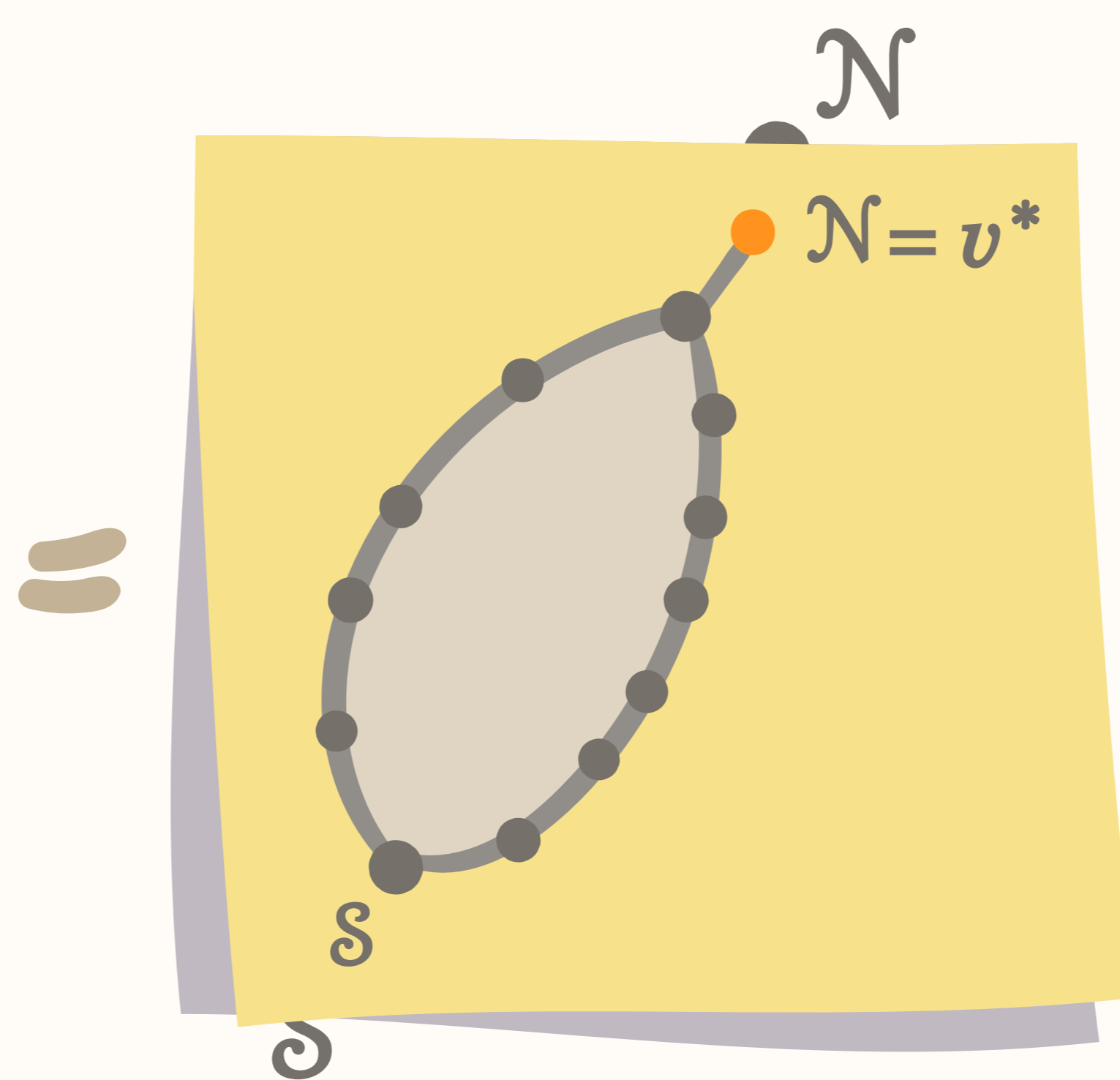
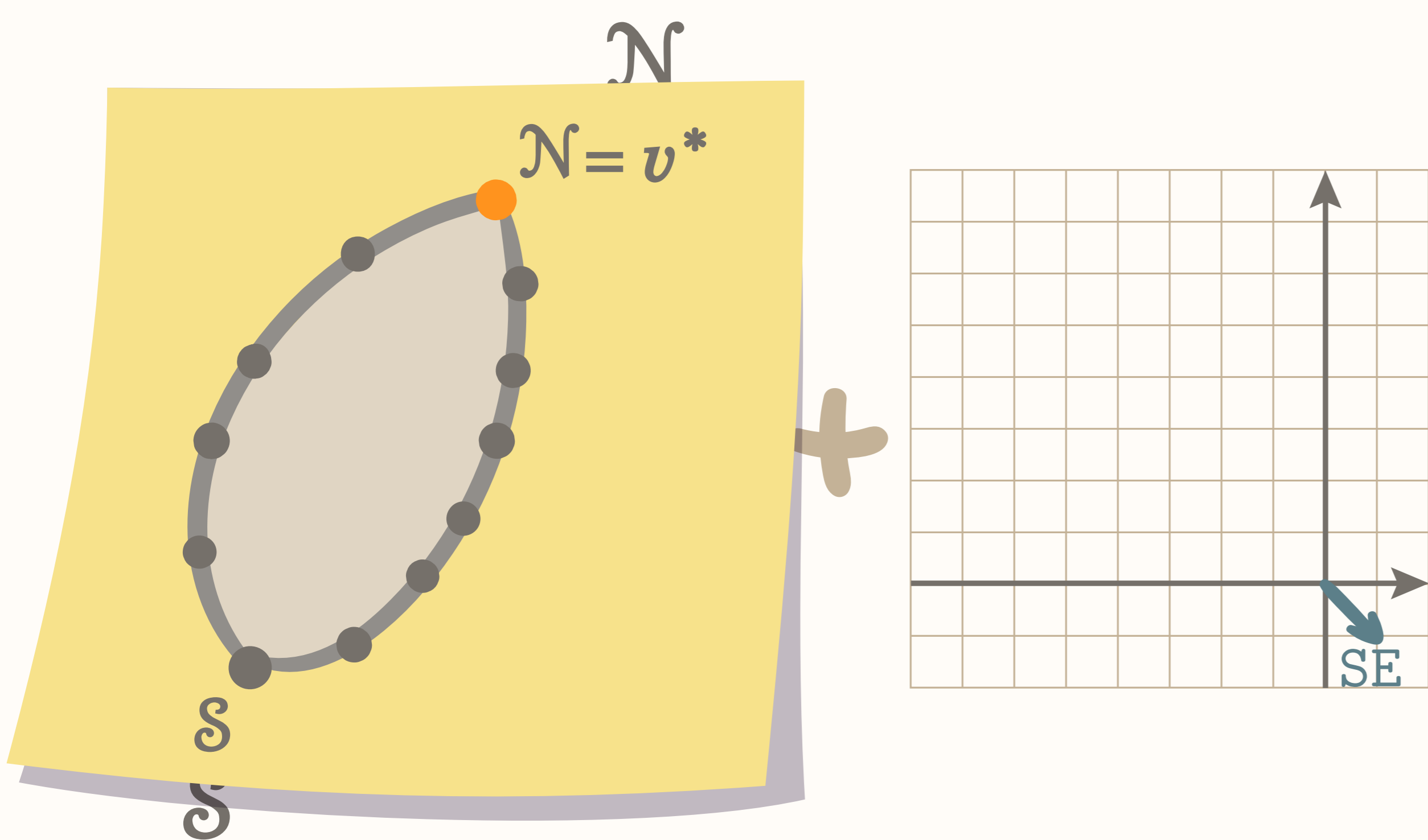


The KMSW bijection

Plane bipolar orientations



tandem walks in the quarter plane

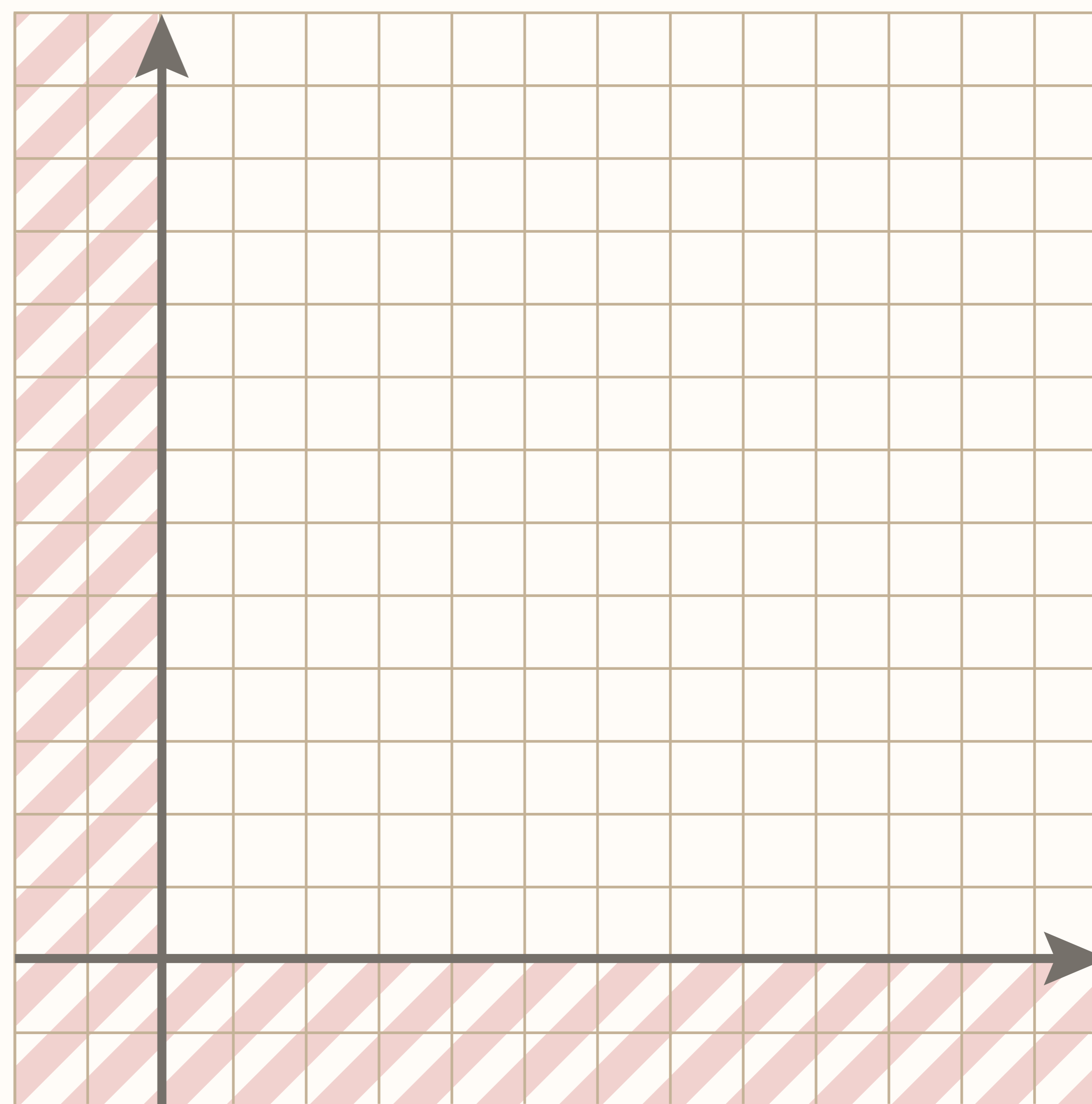
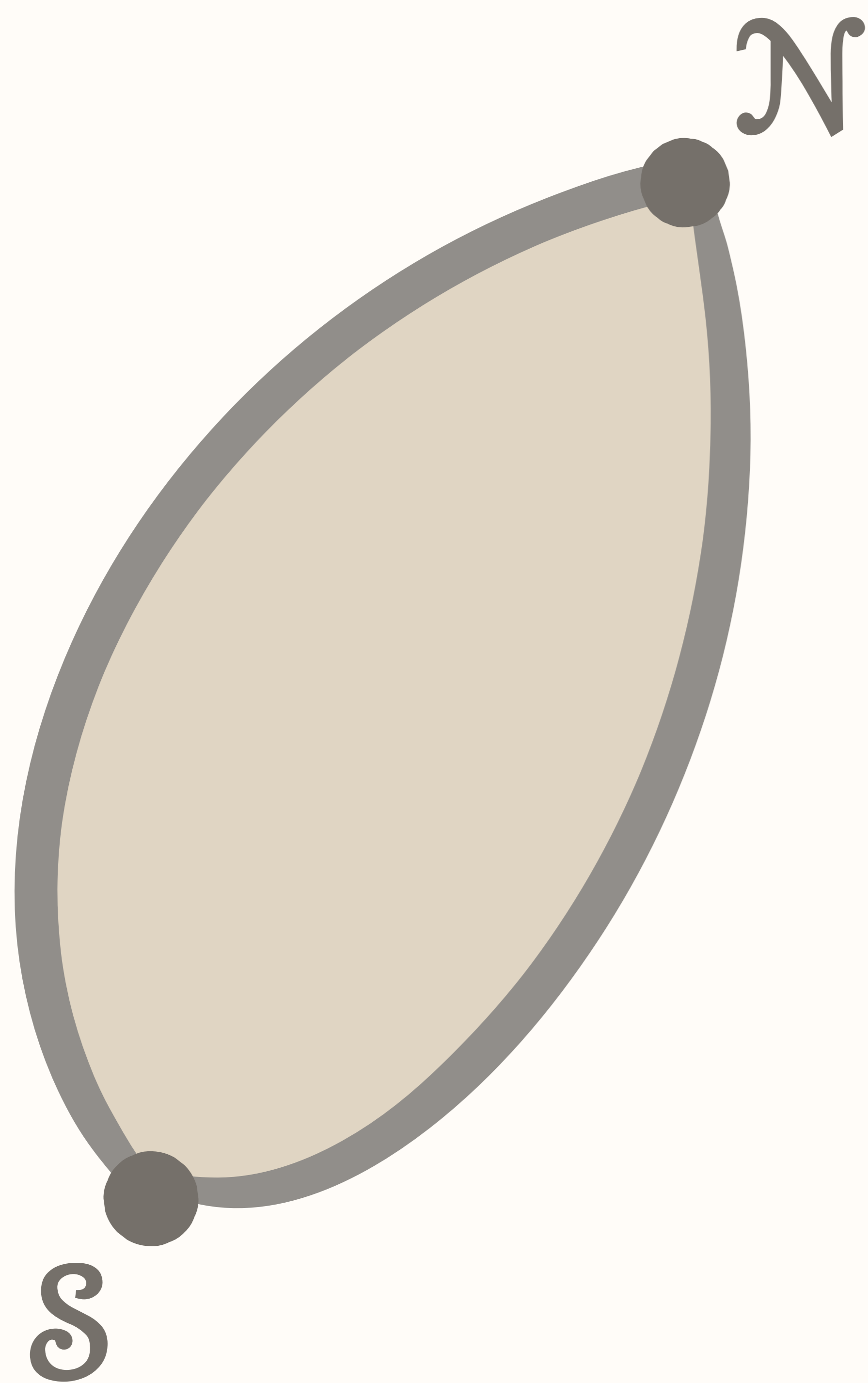


The KMSW bijection

Plane bipolar orientations



tandem walks in the quarter plane

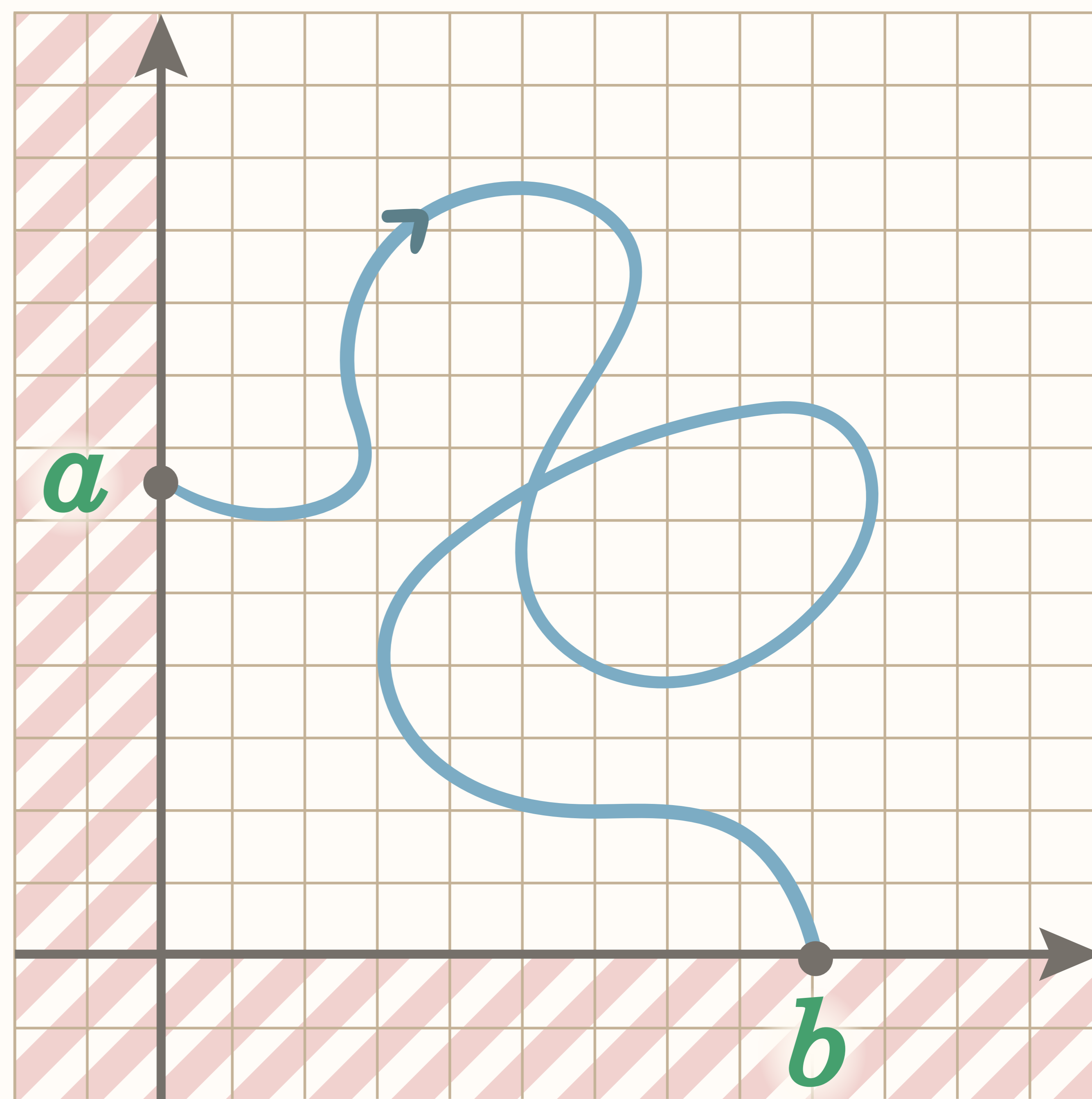
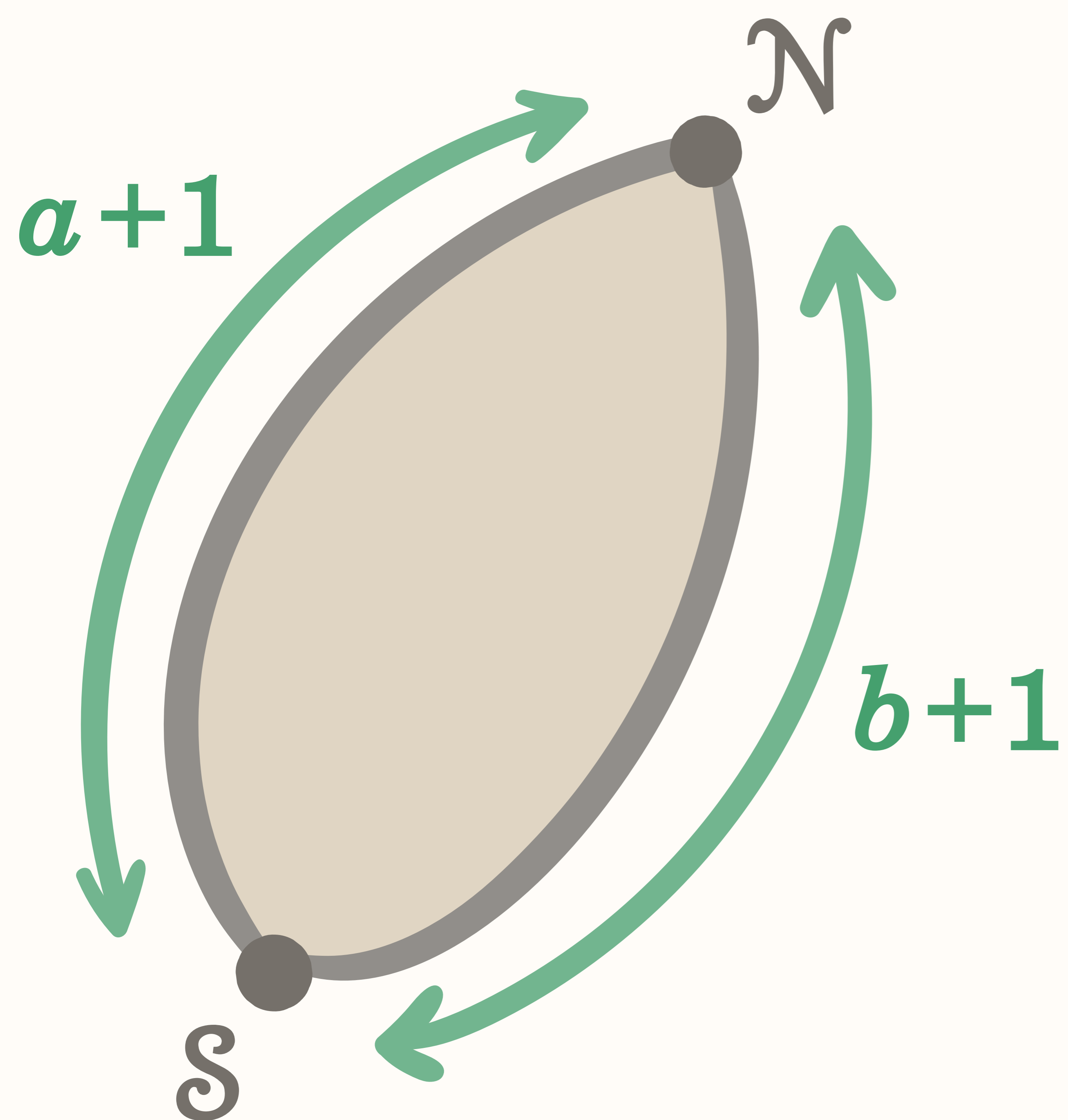


The KMSW bijection

Plane bipolar orientations



tandem walks in the quarter plane

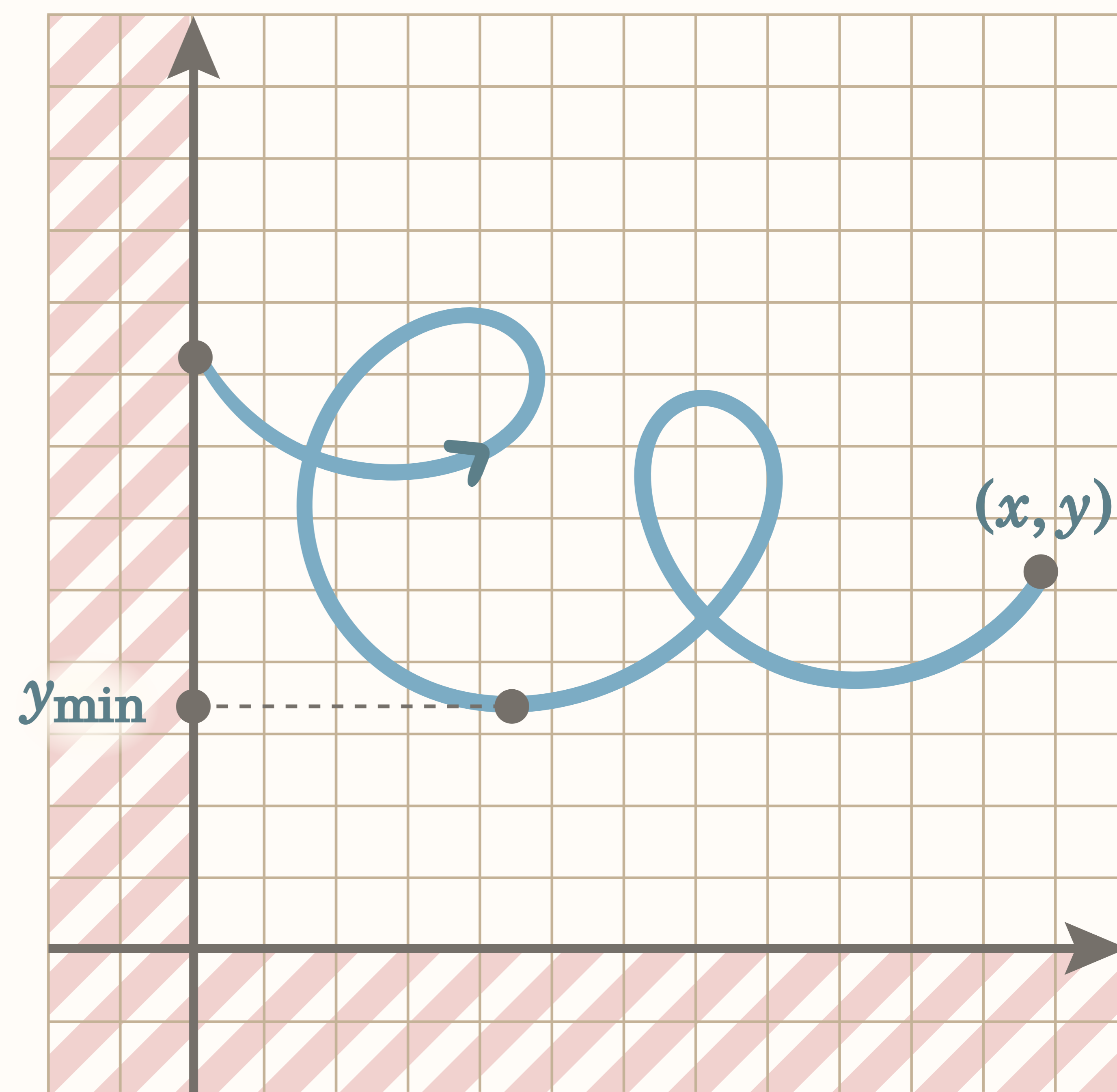
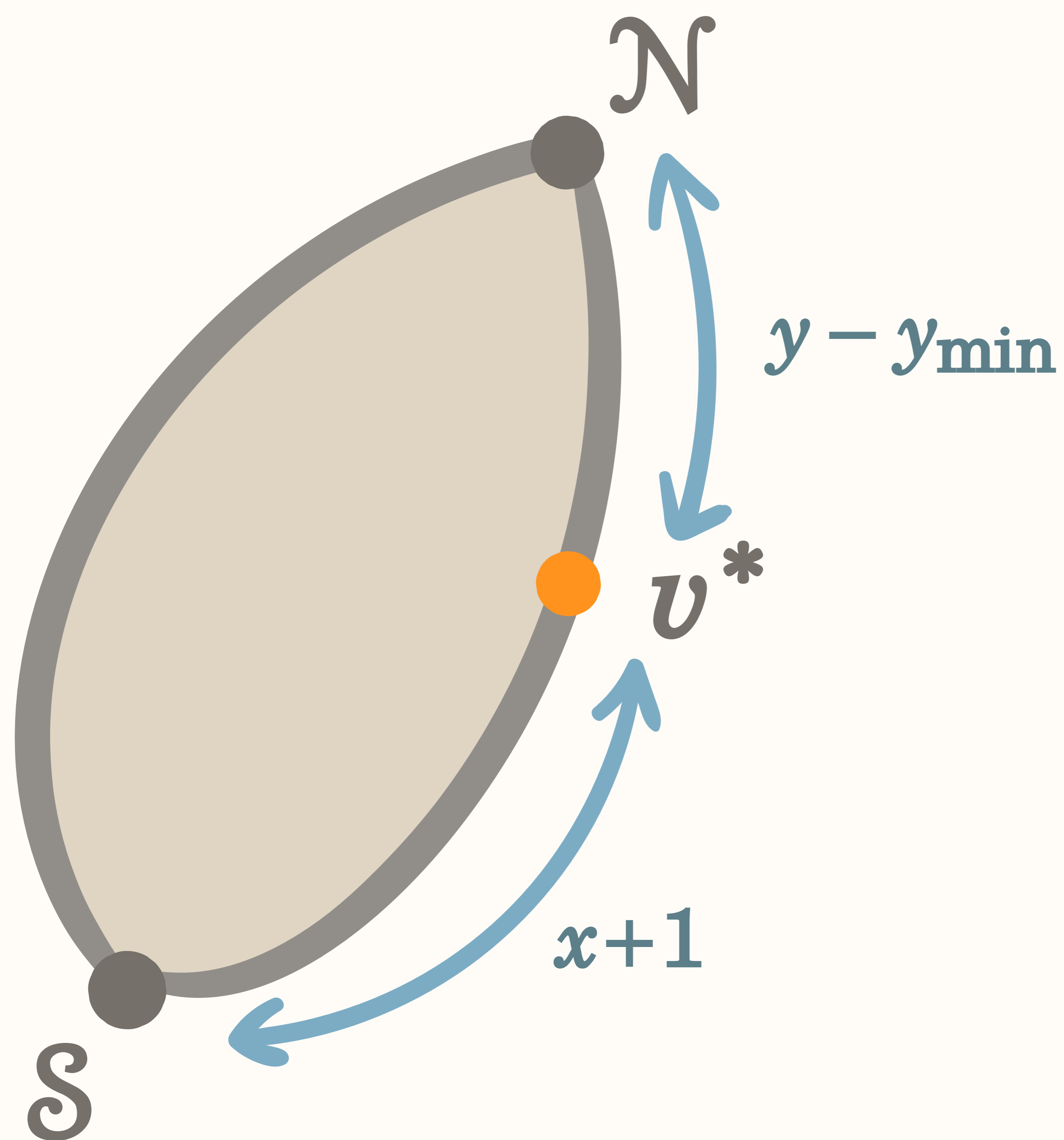


The KMSW bijection

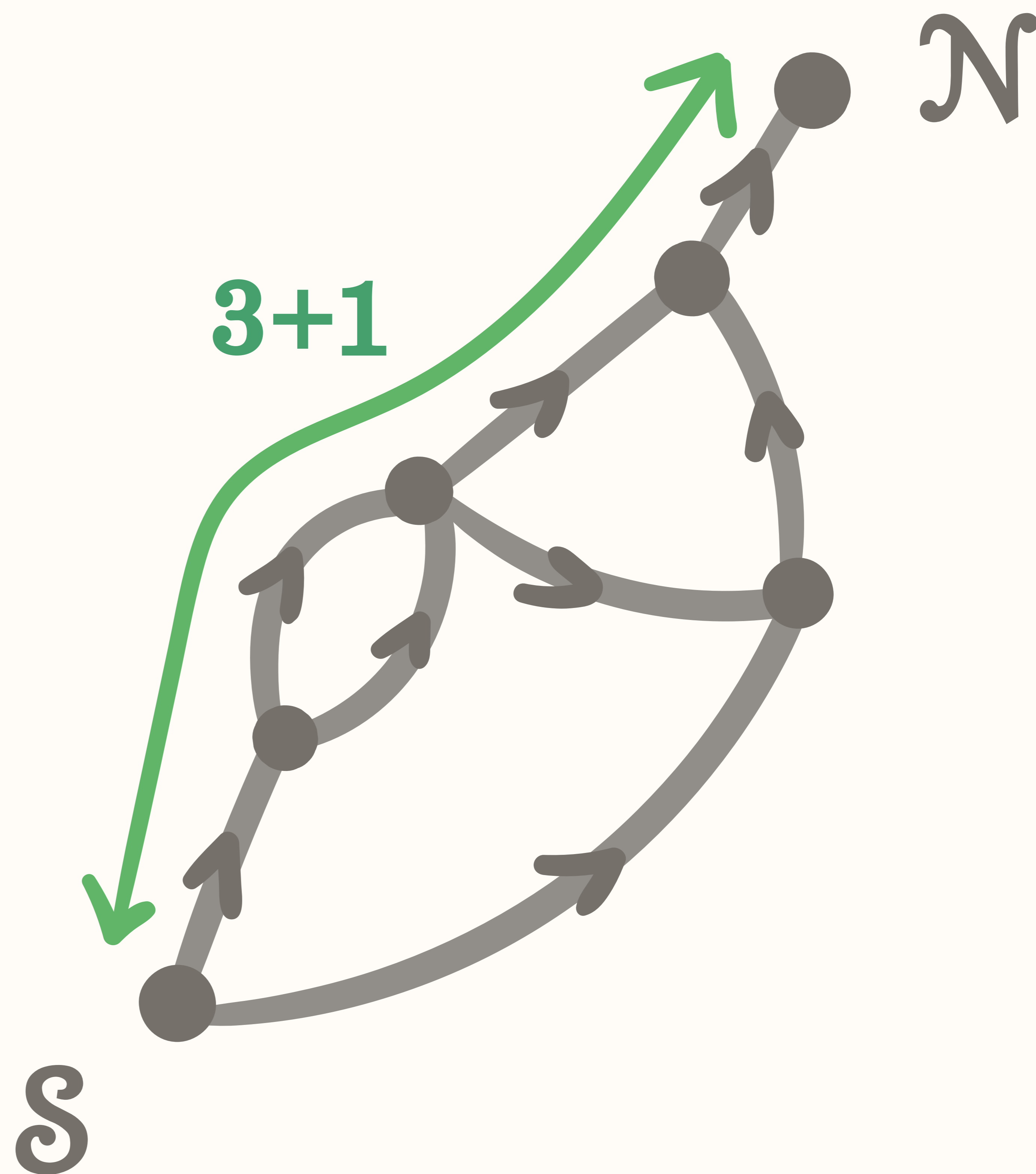
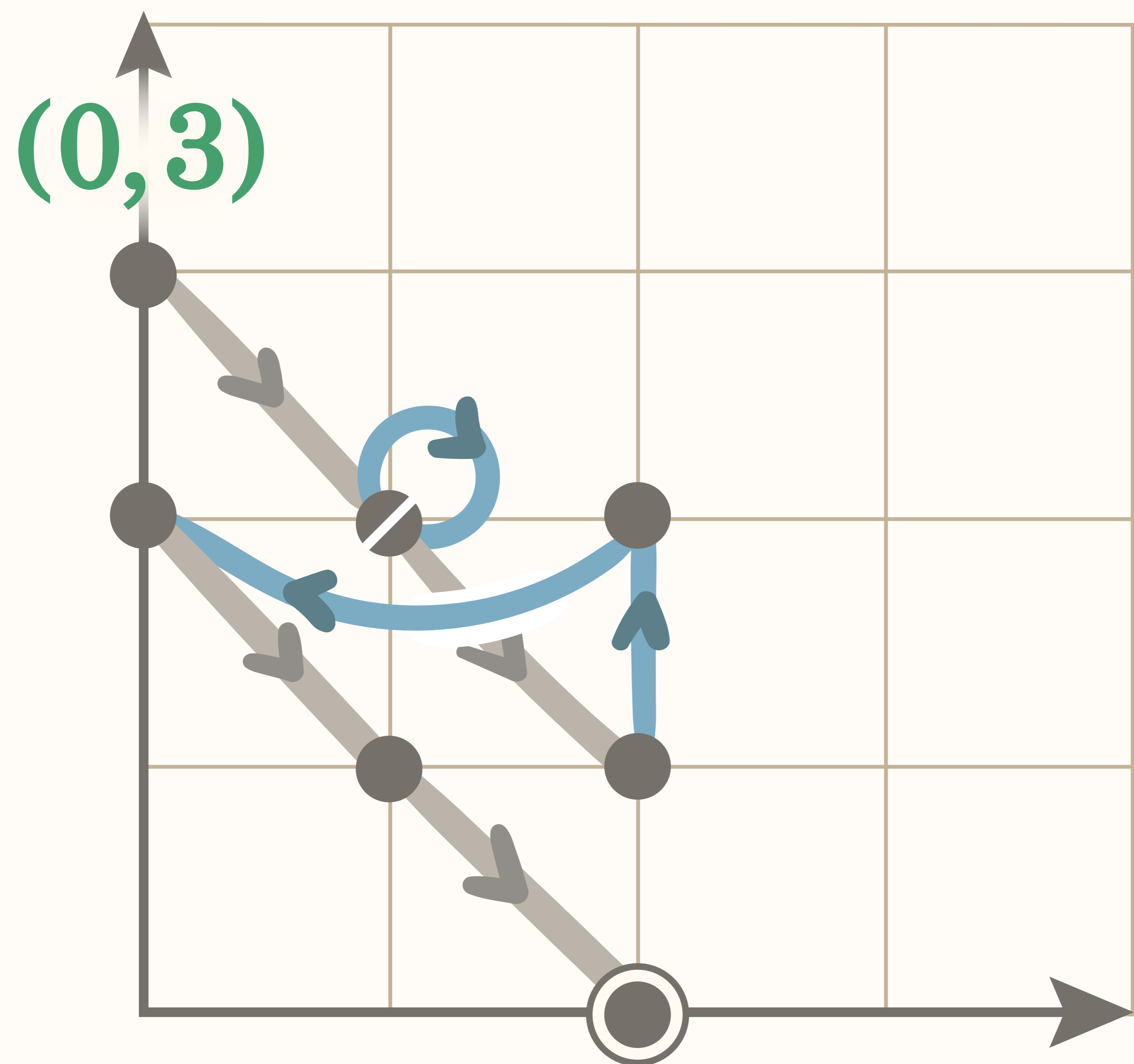
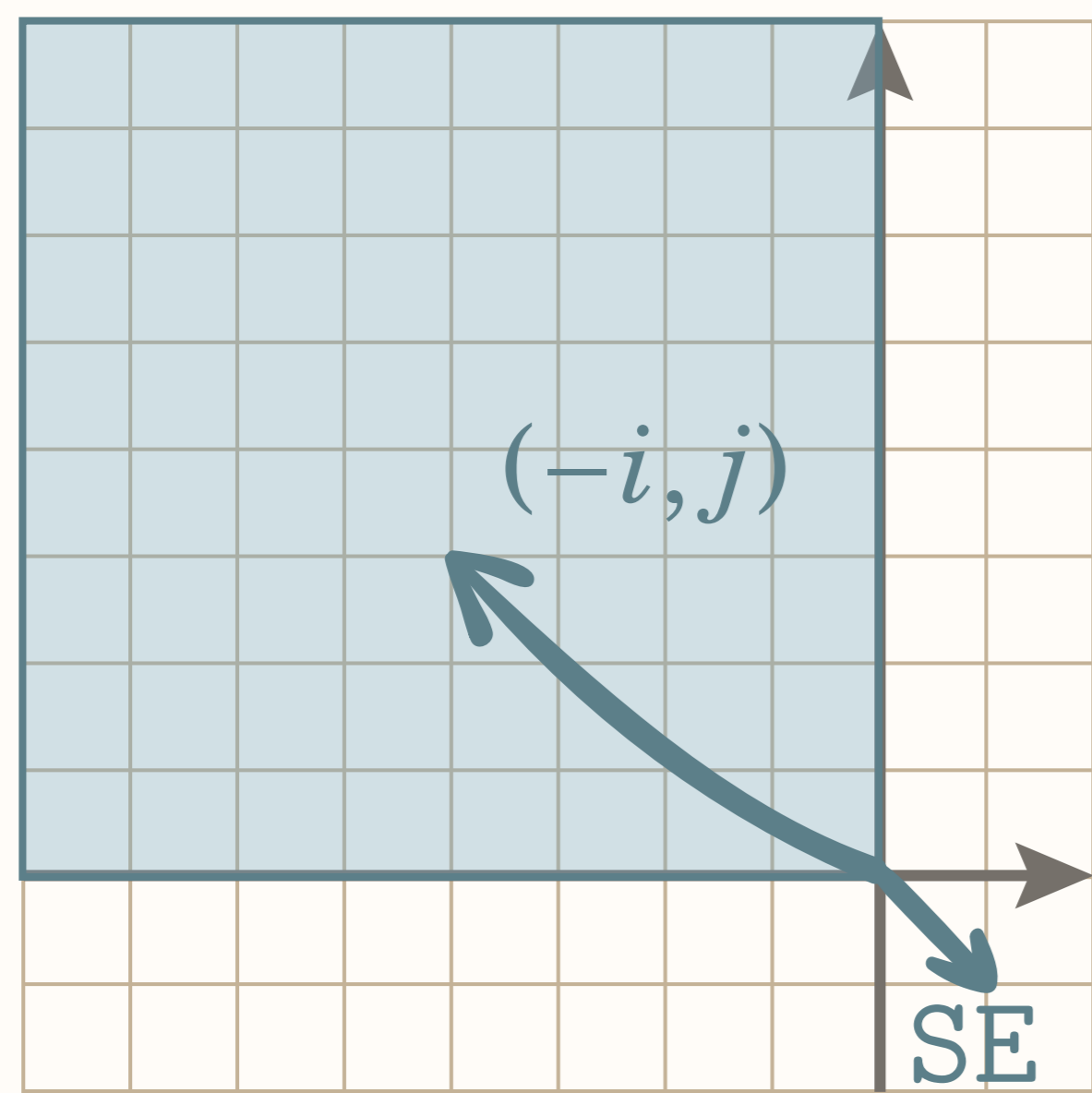
Plane bipolar orientations



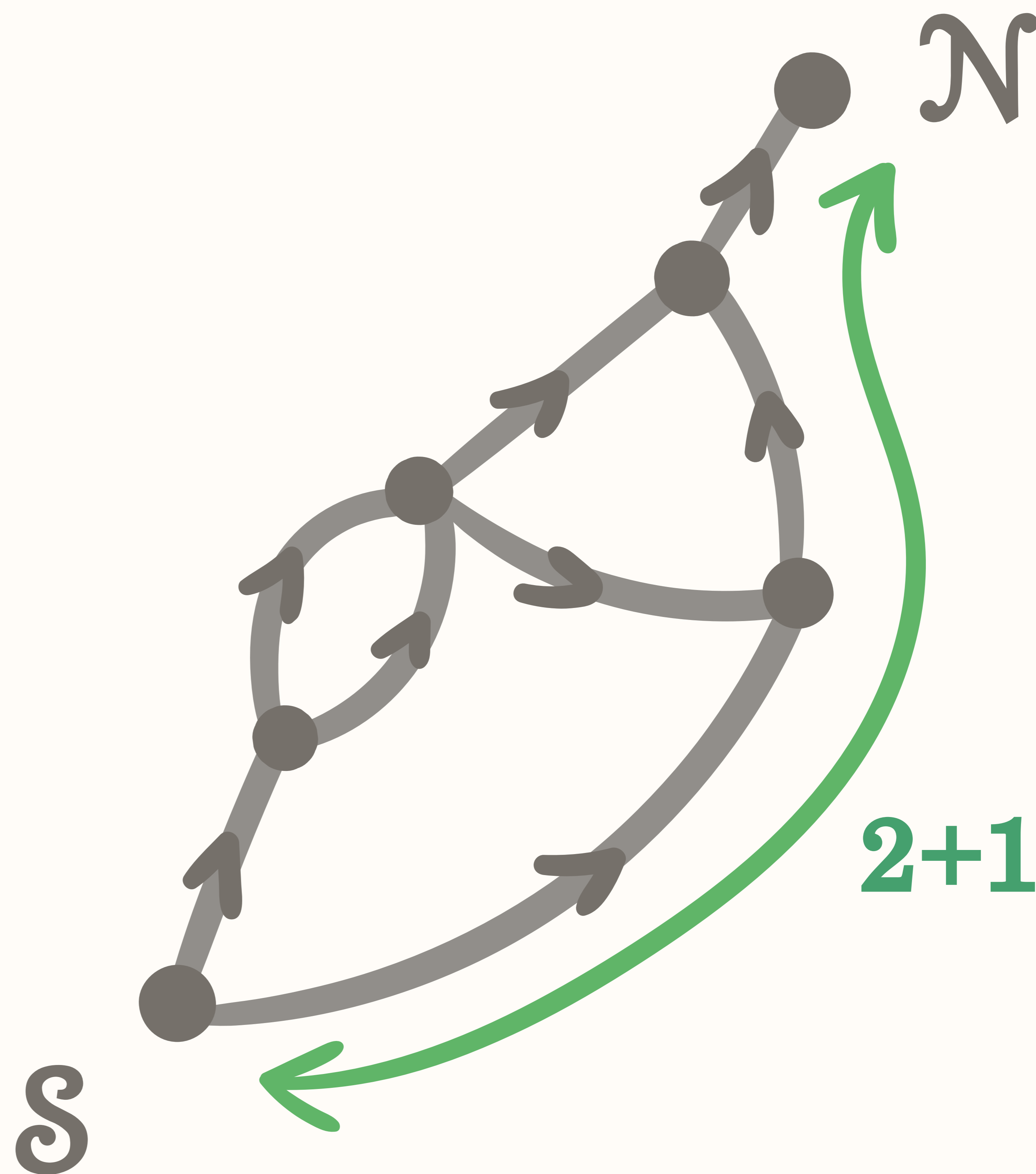
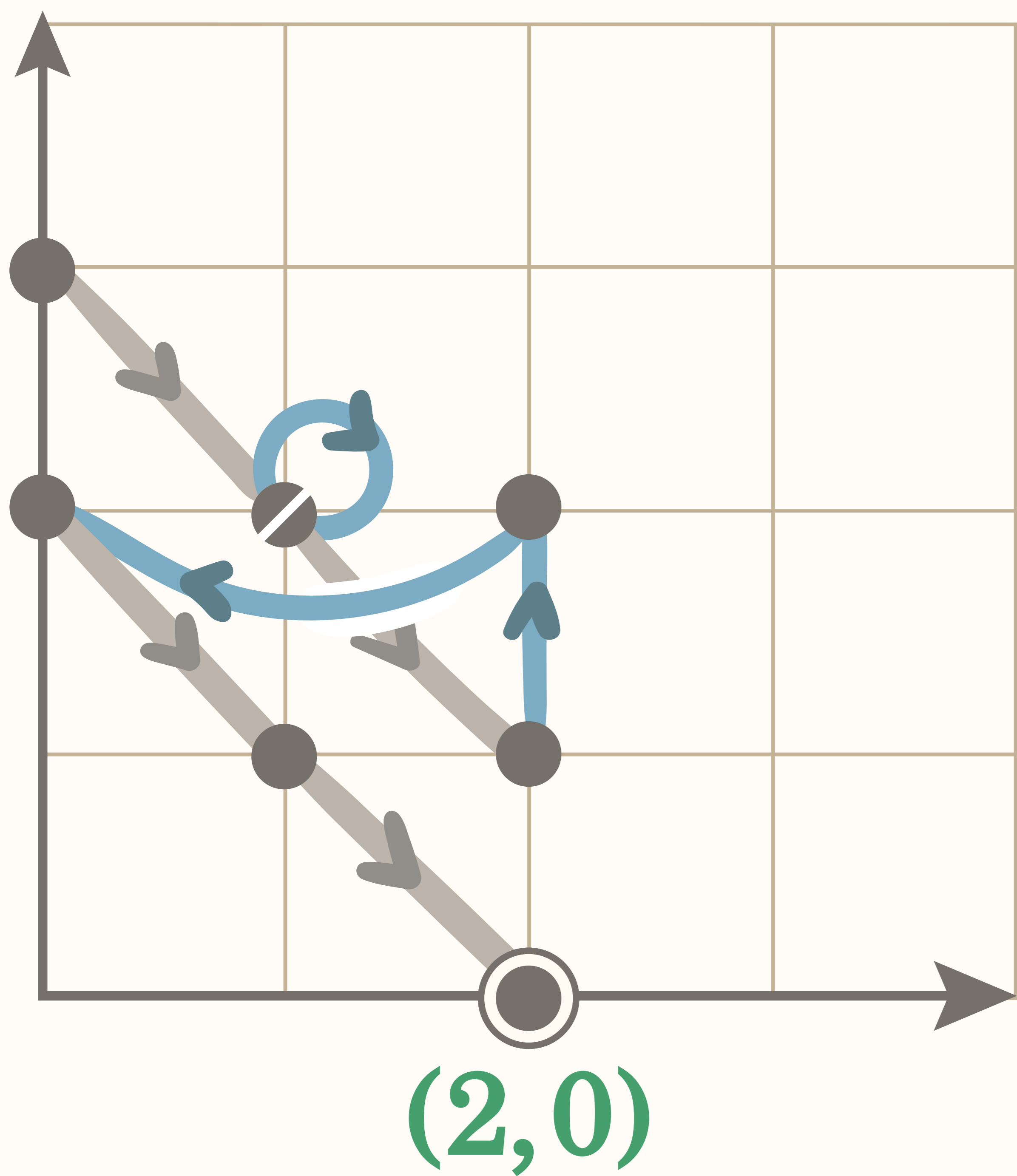
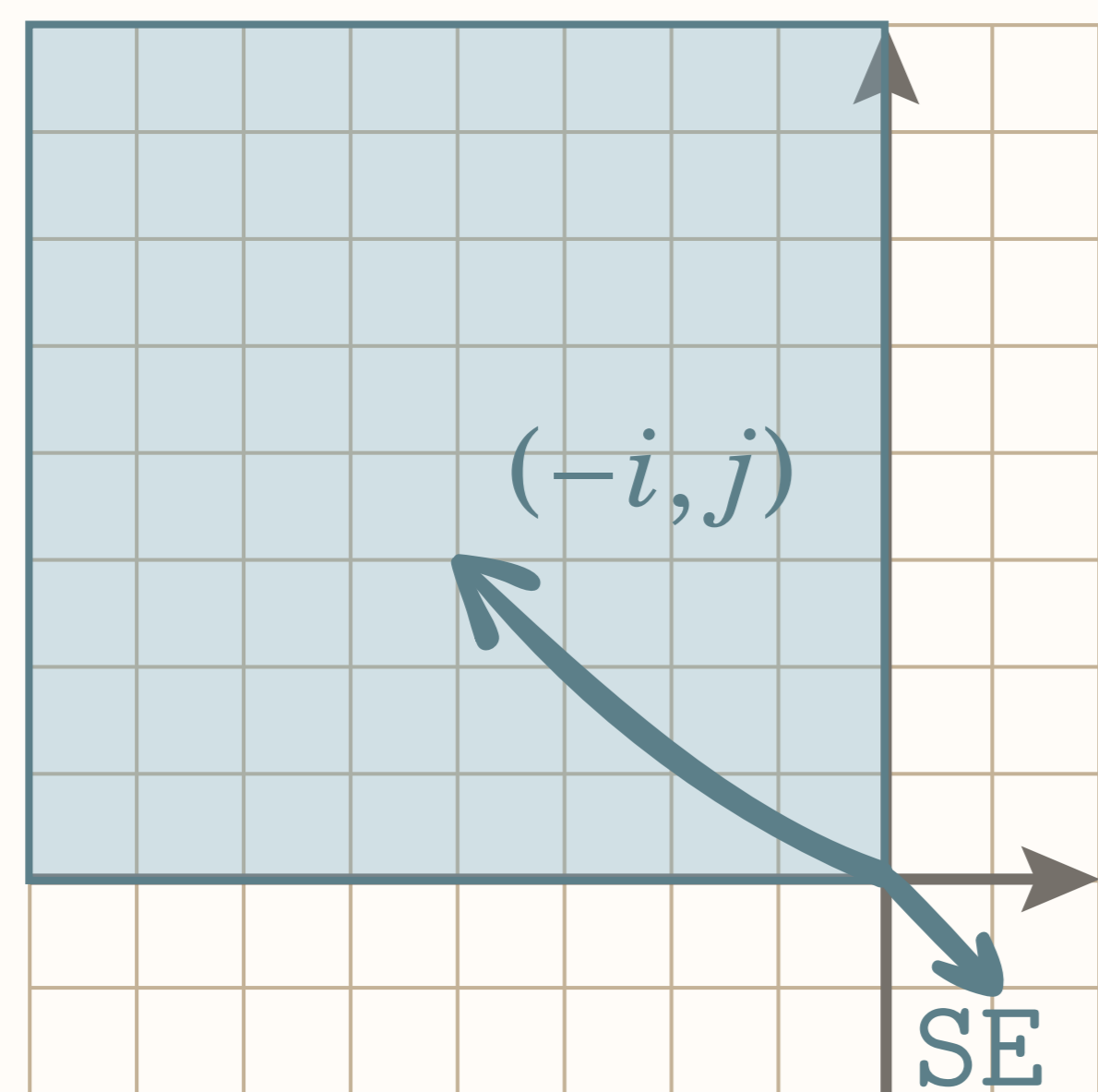
tandem walks in the quarter plane



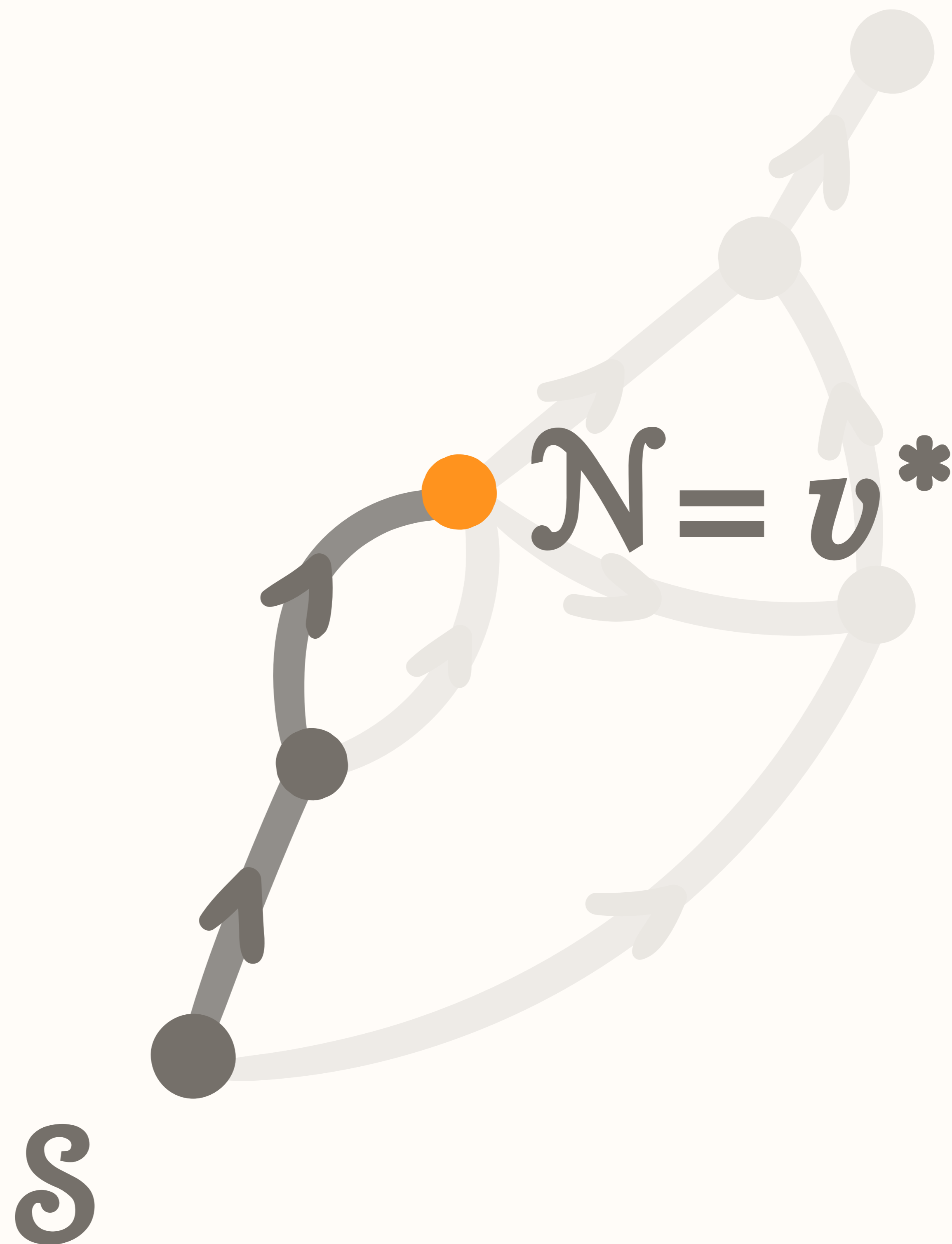
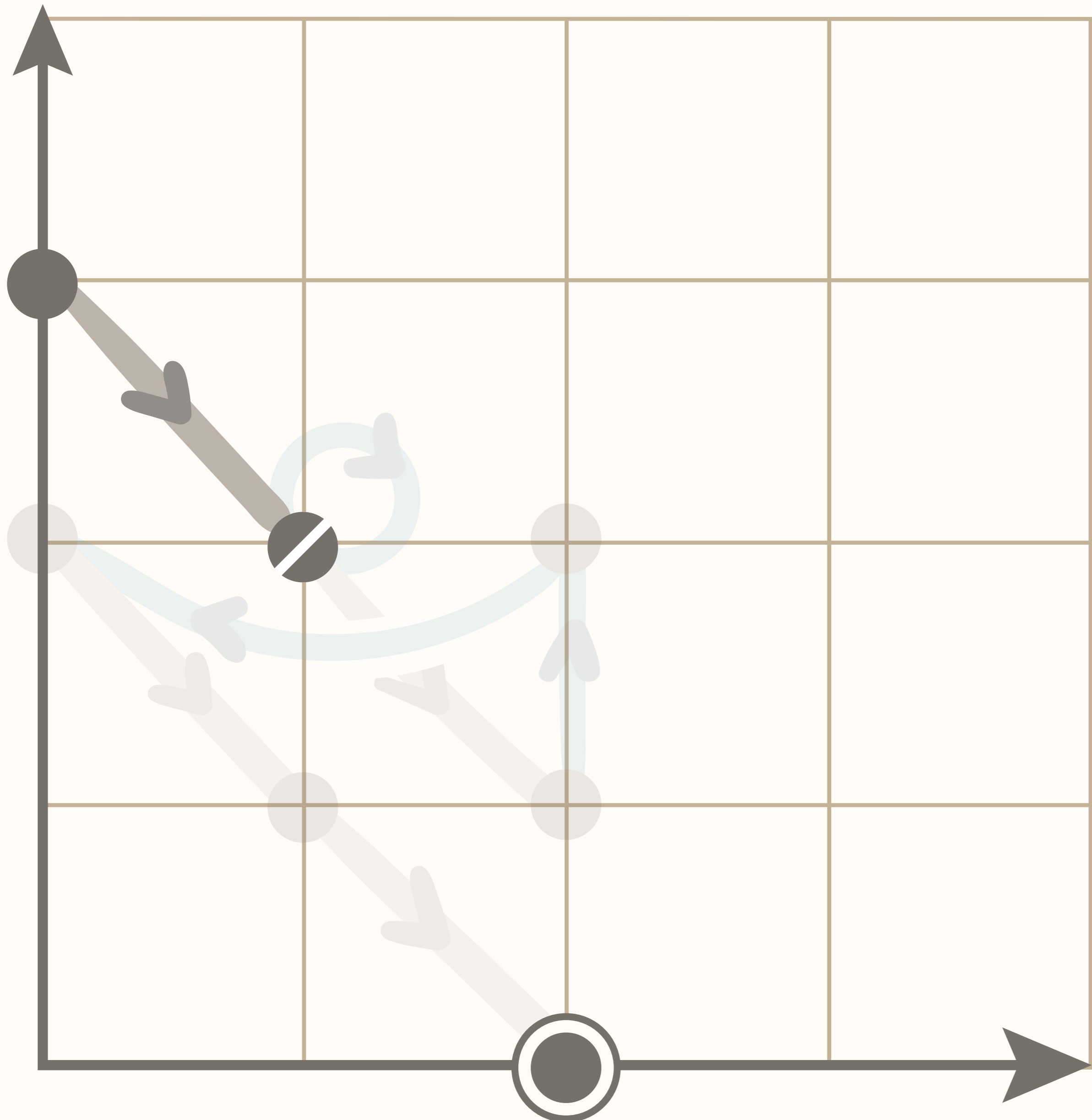
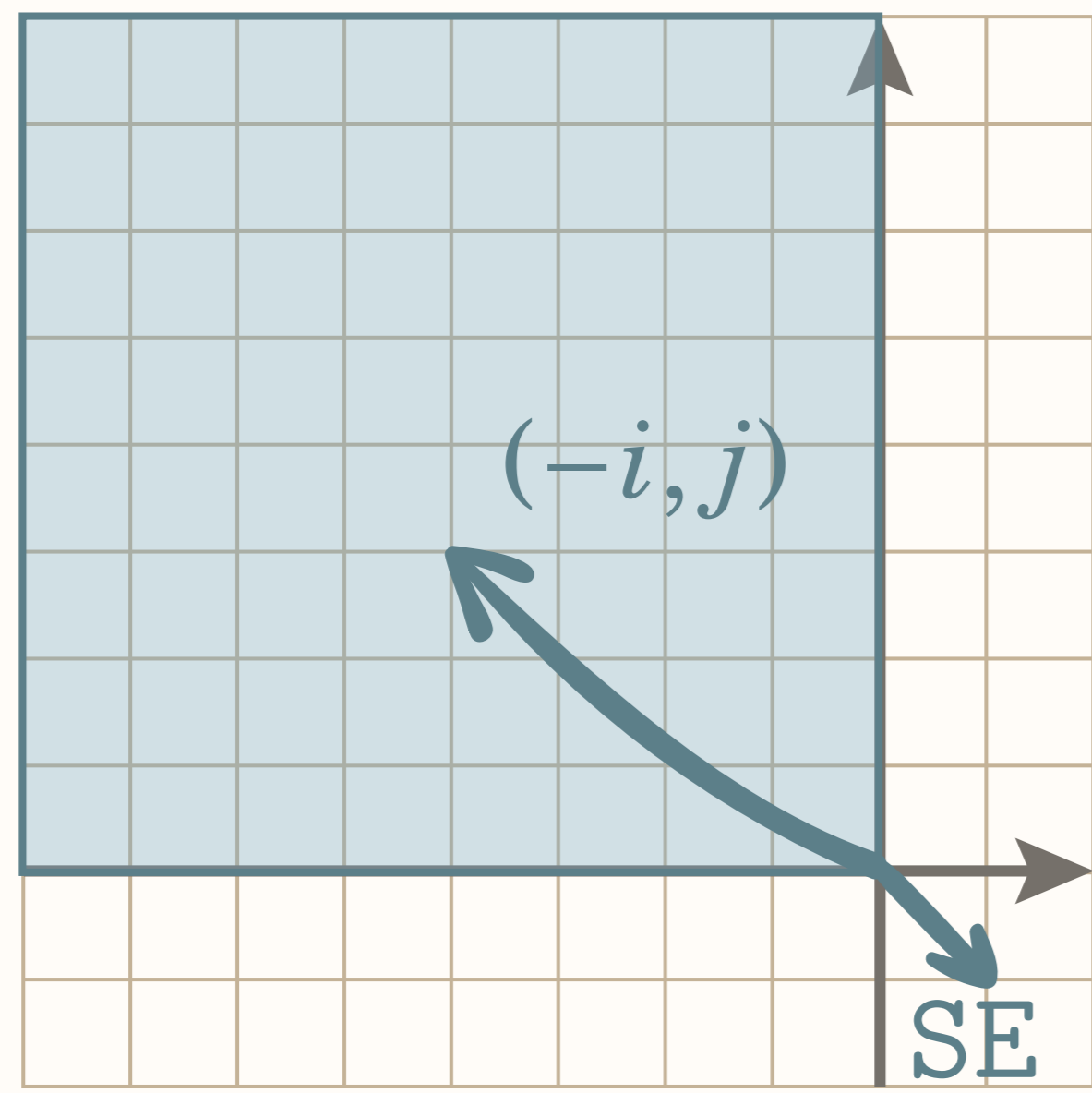
KMSW bijection example



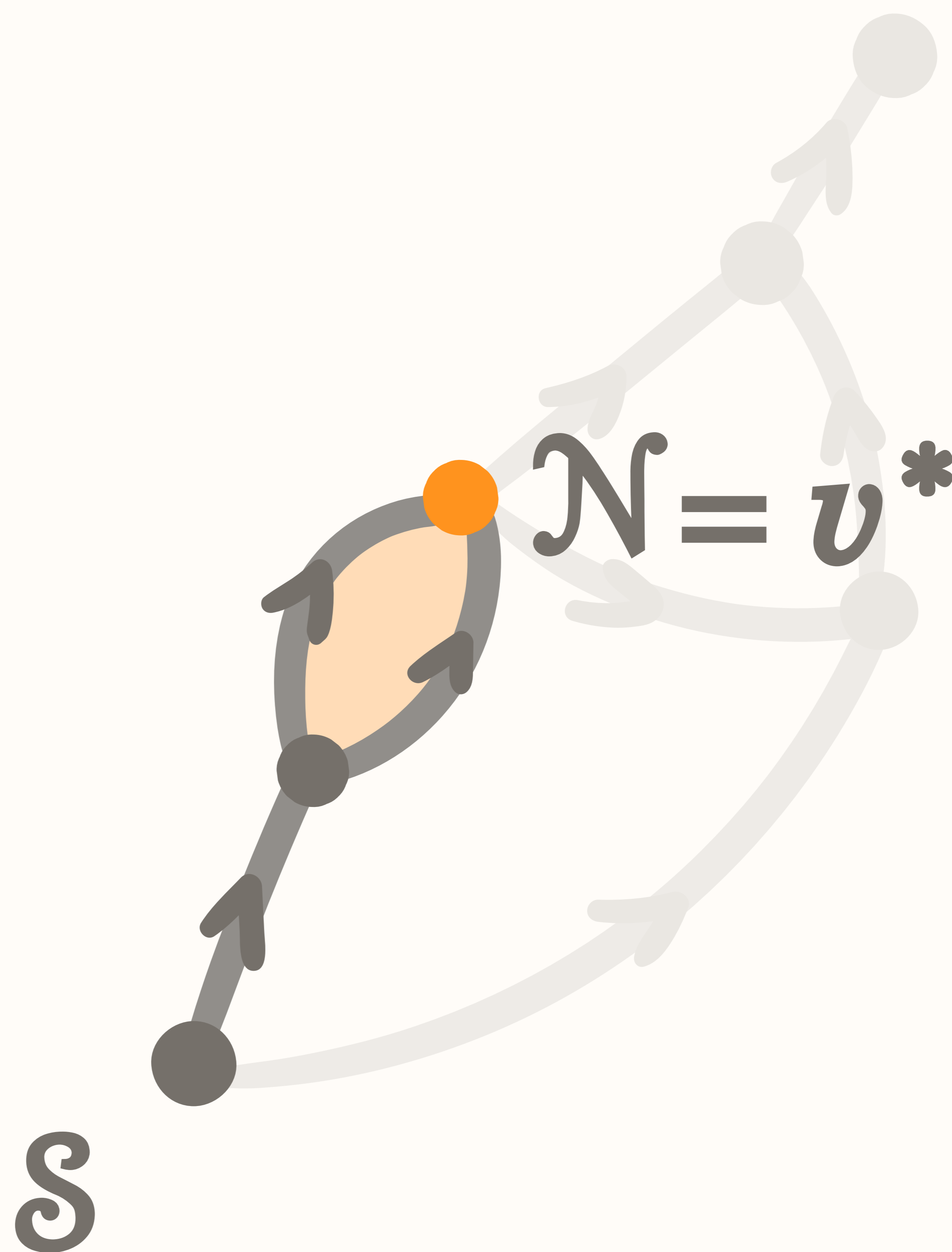
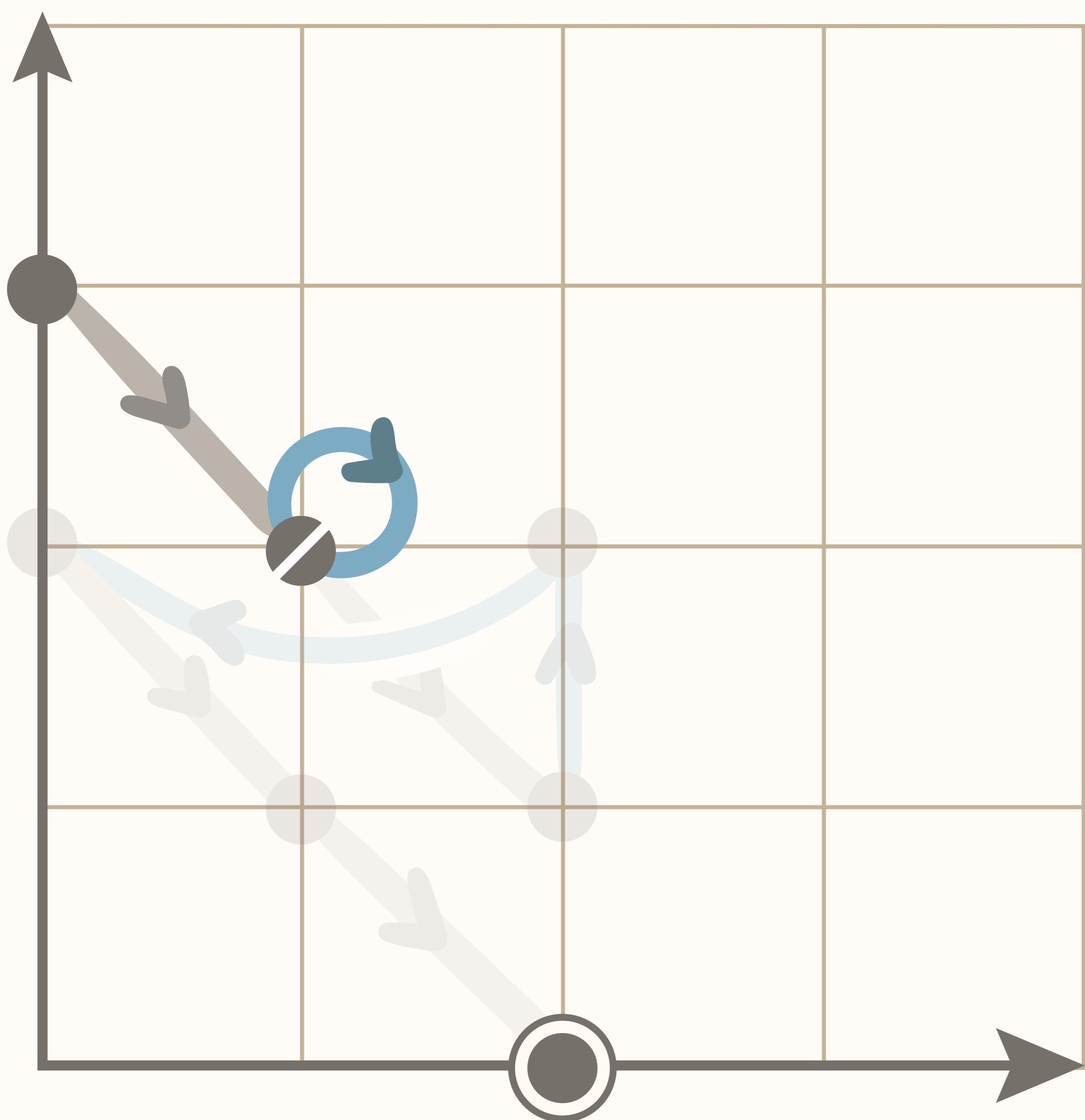
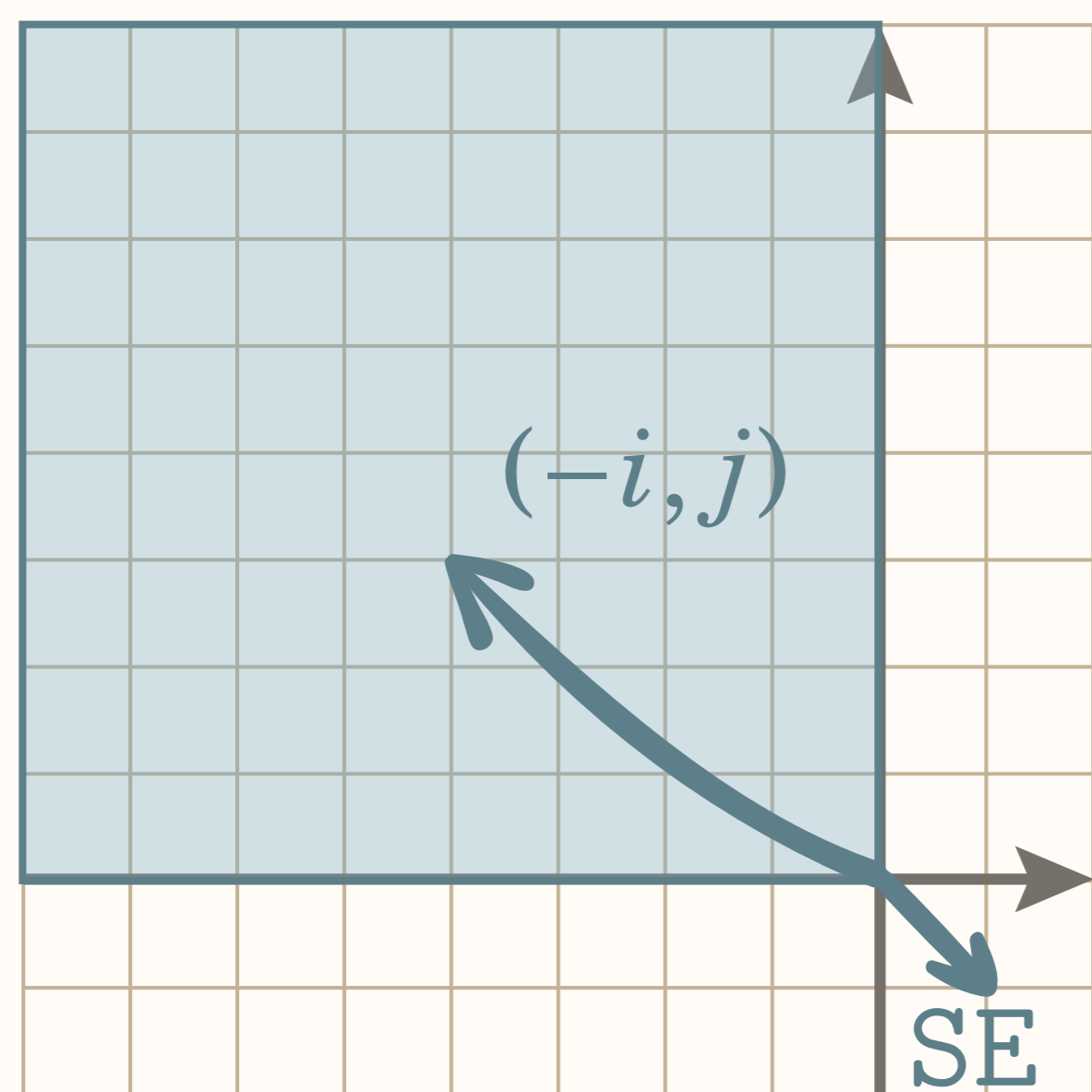
KMSW bijection example



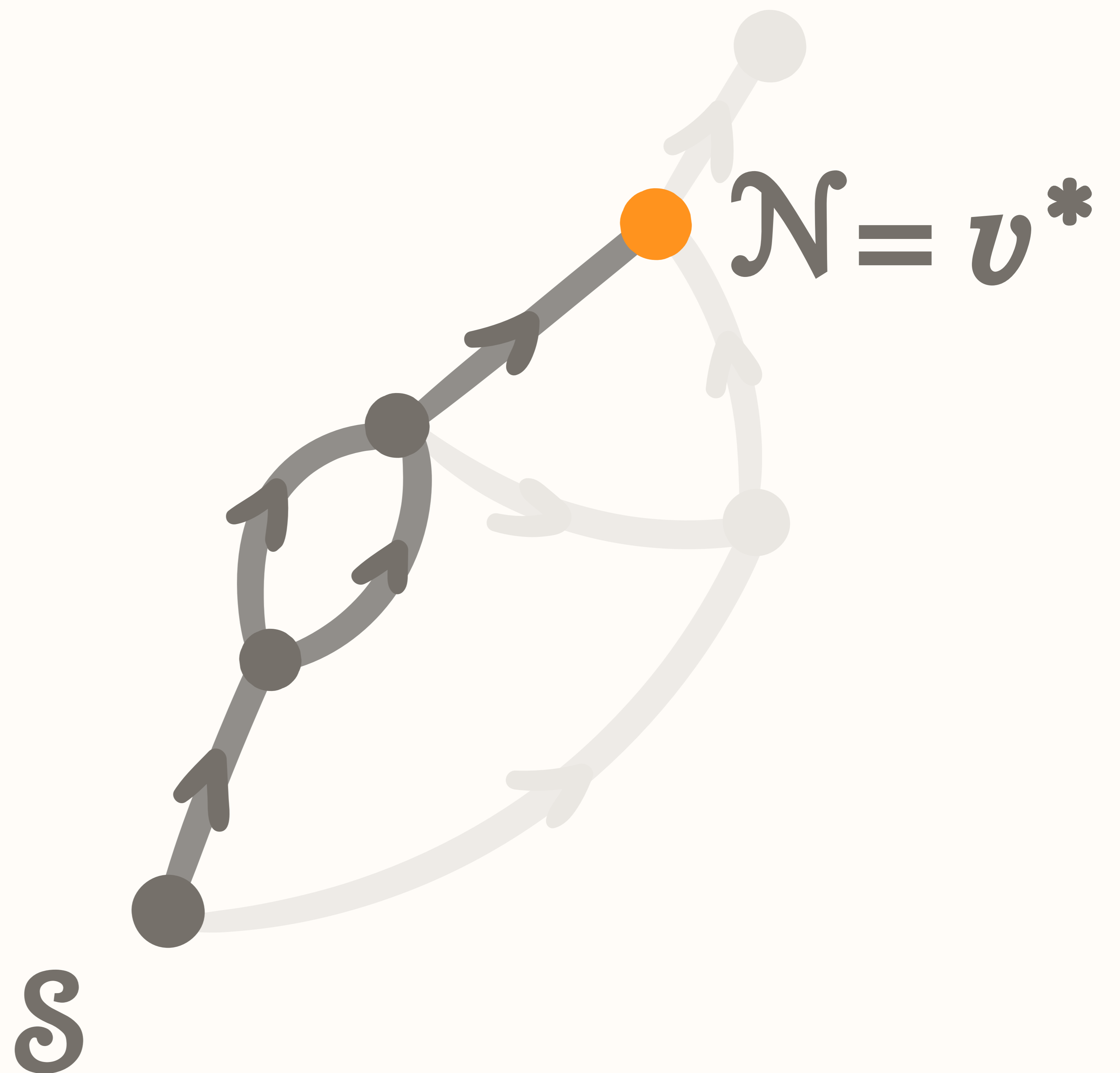
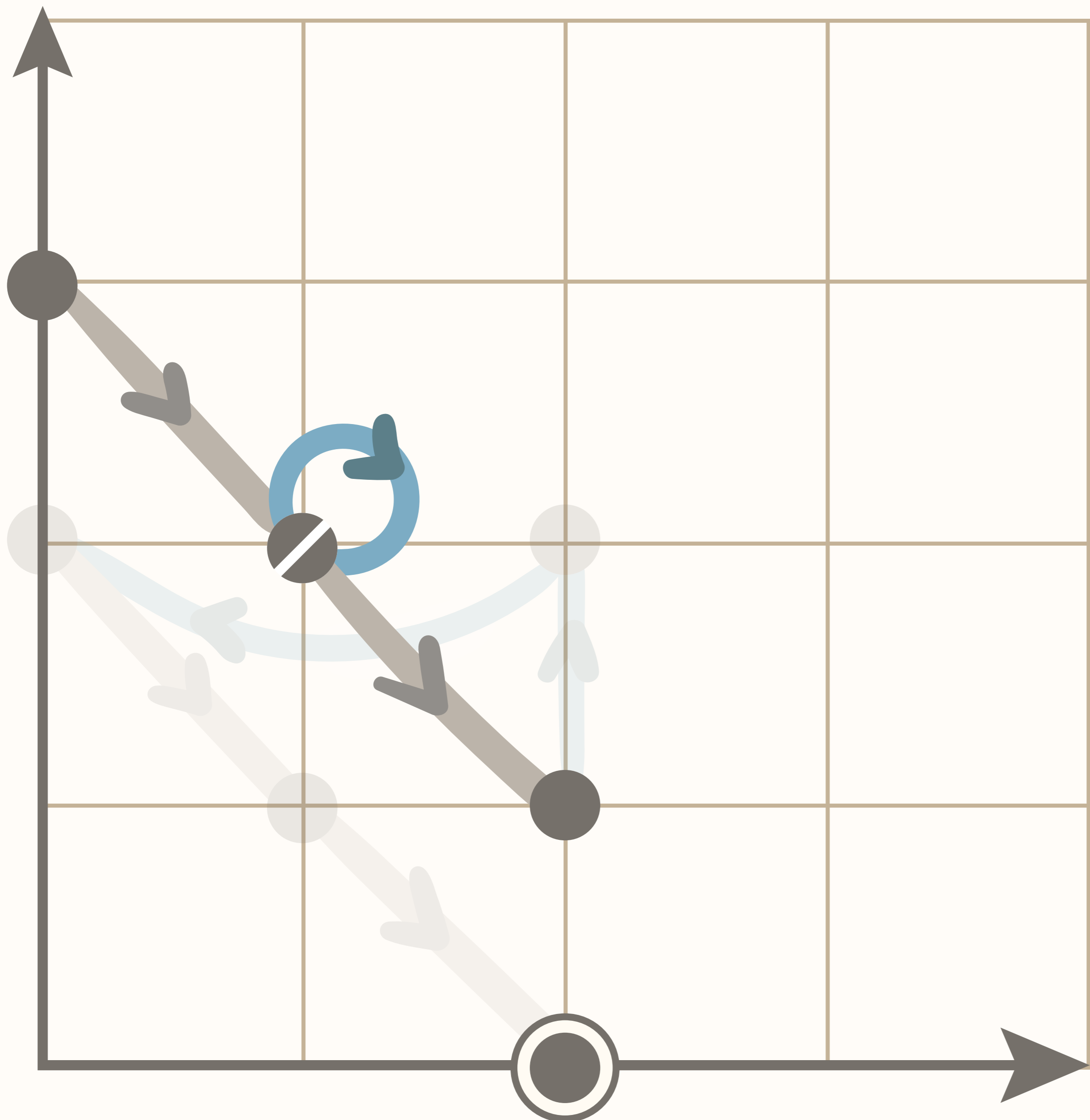
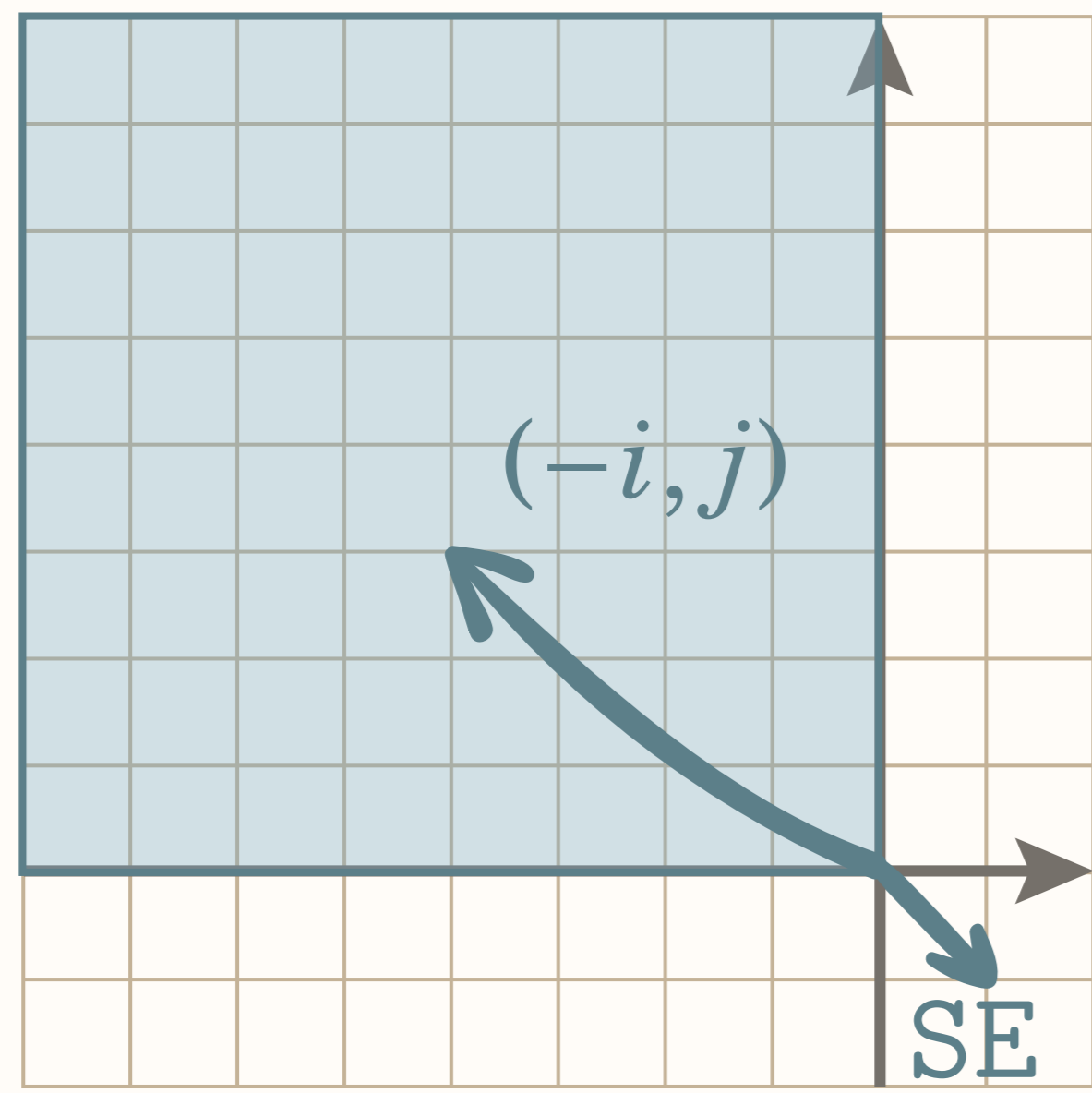
KMSW bijection example



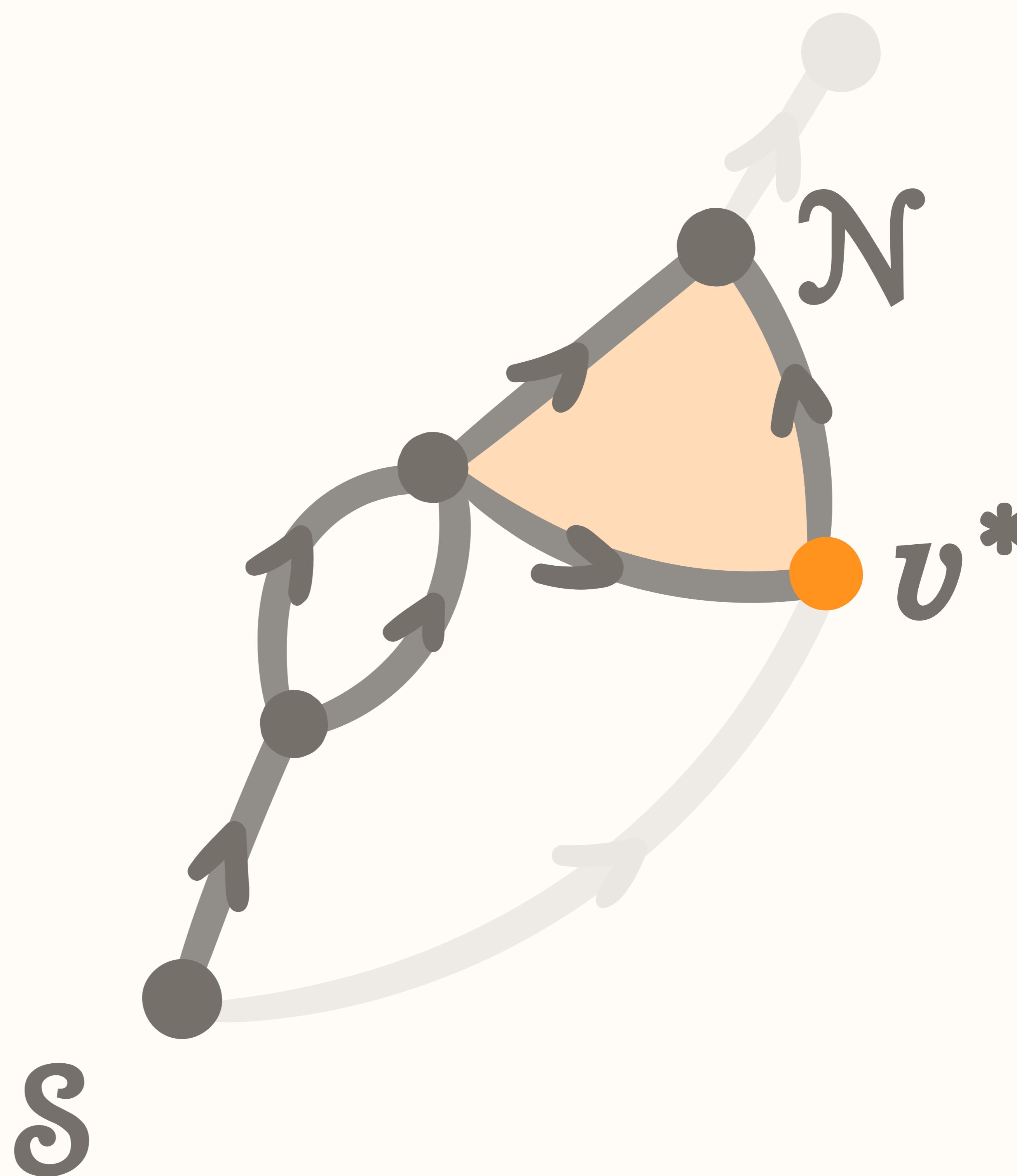
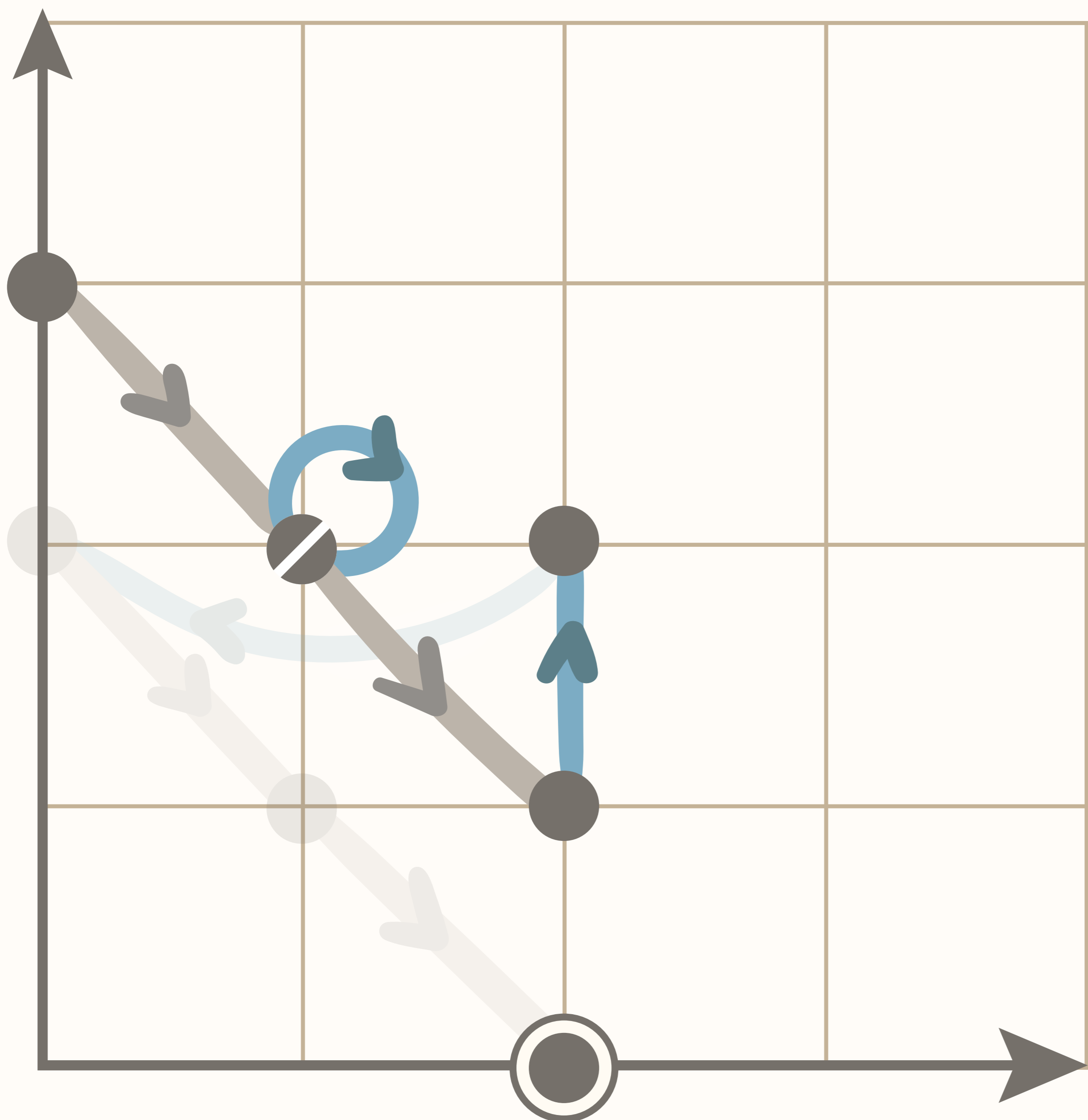
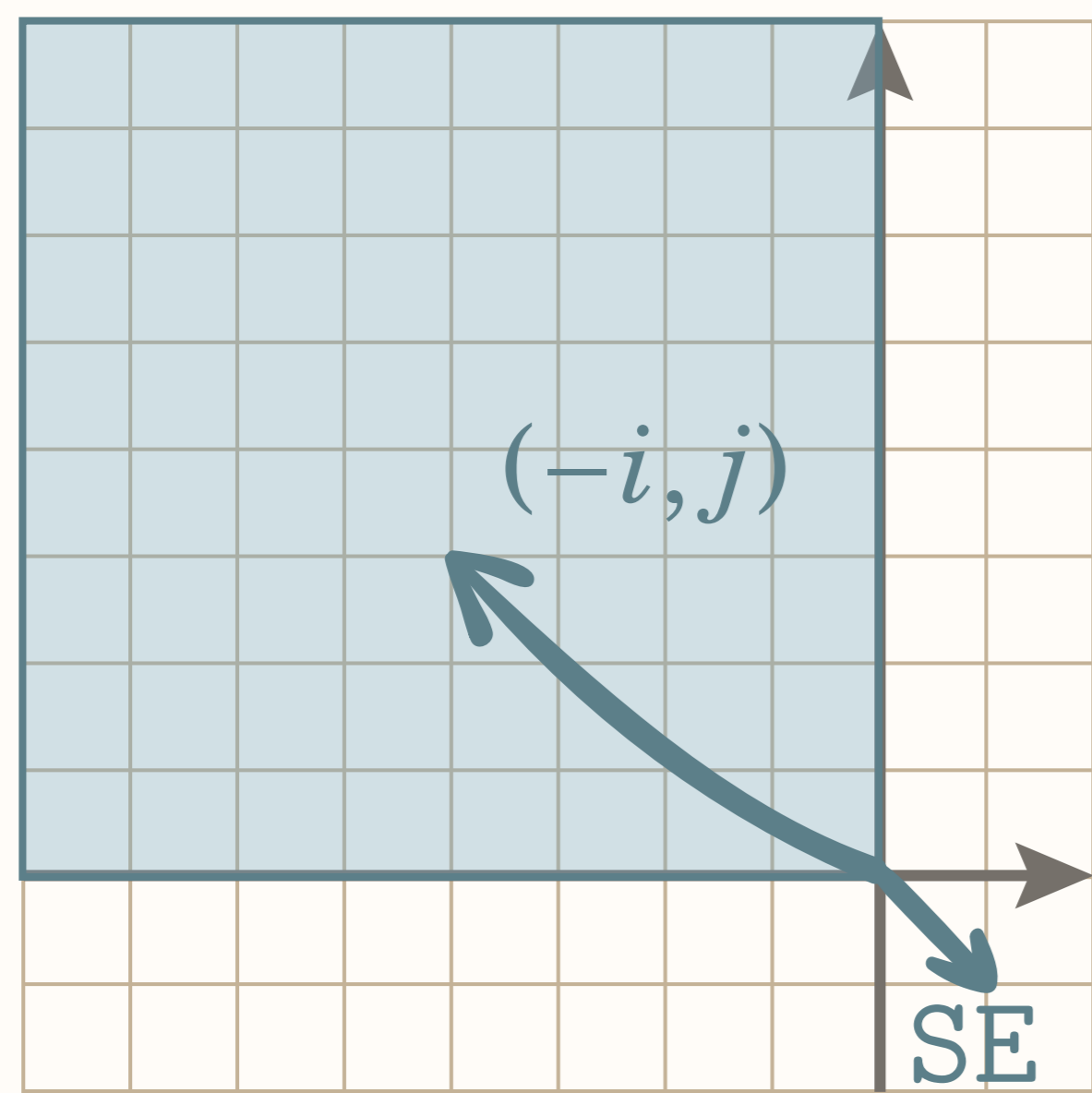
KMSW bijection example



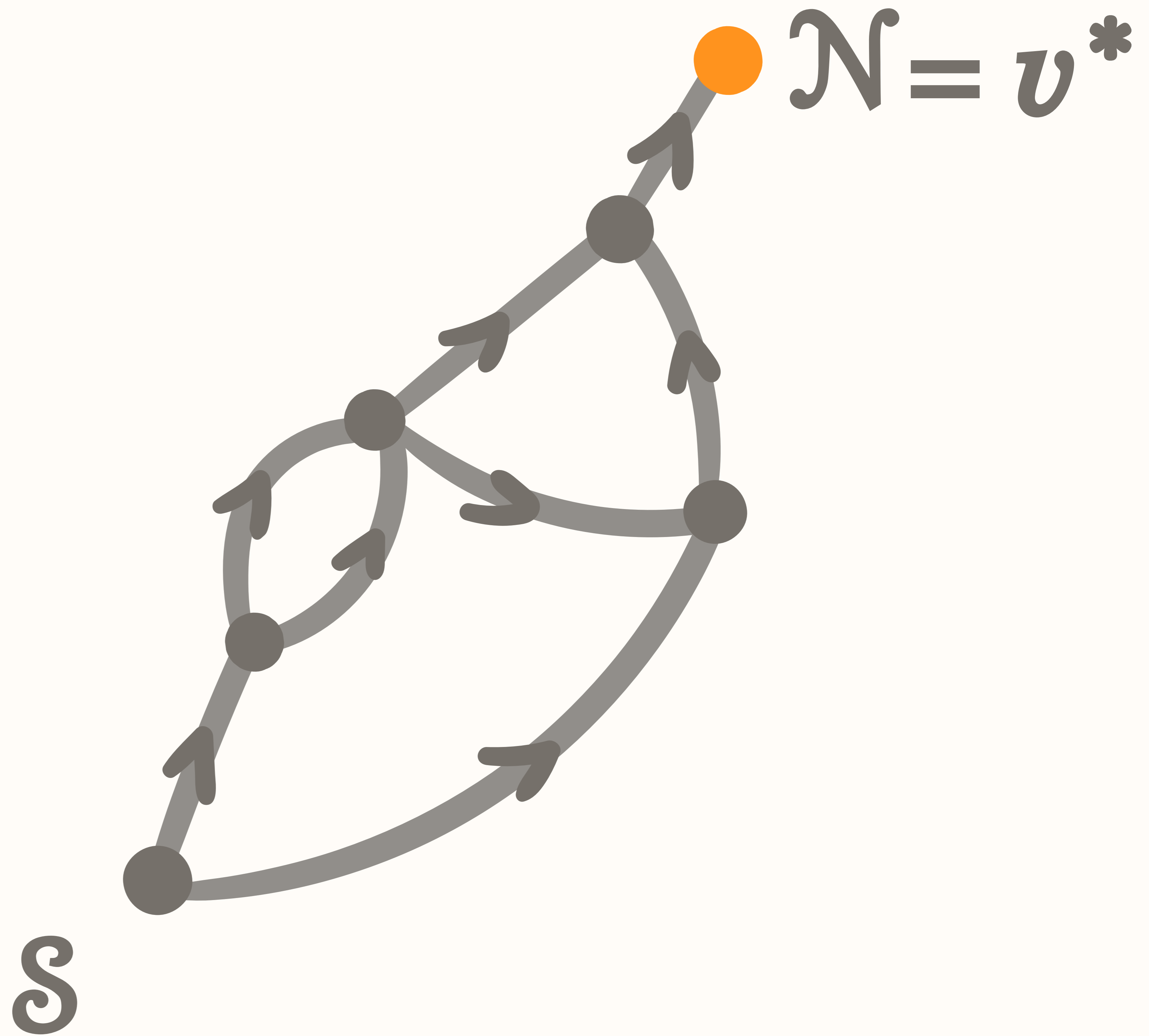
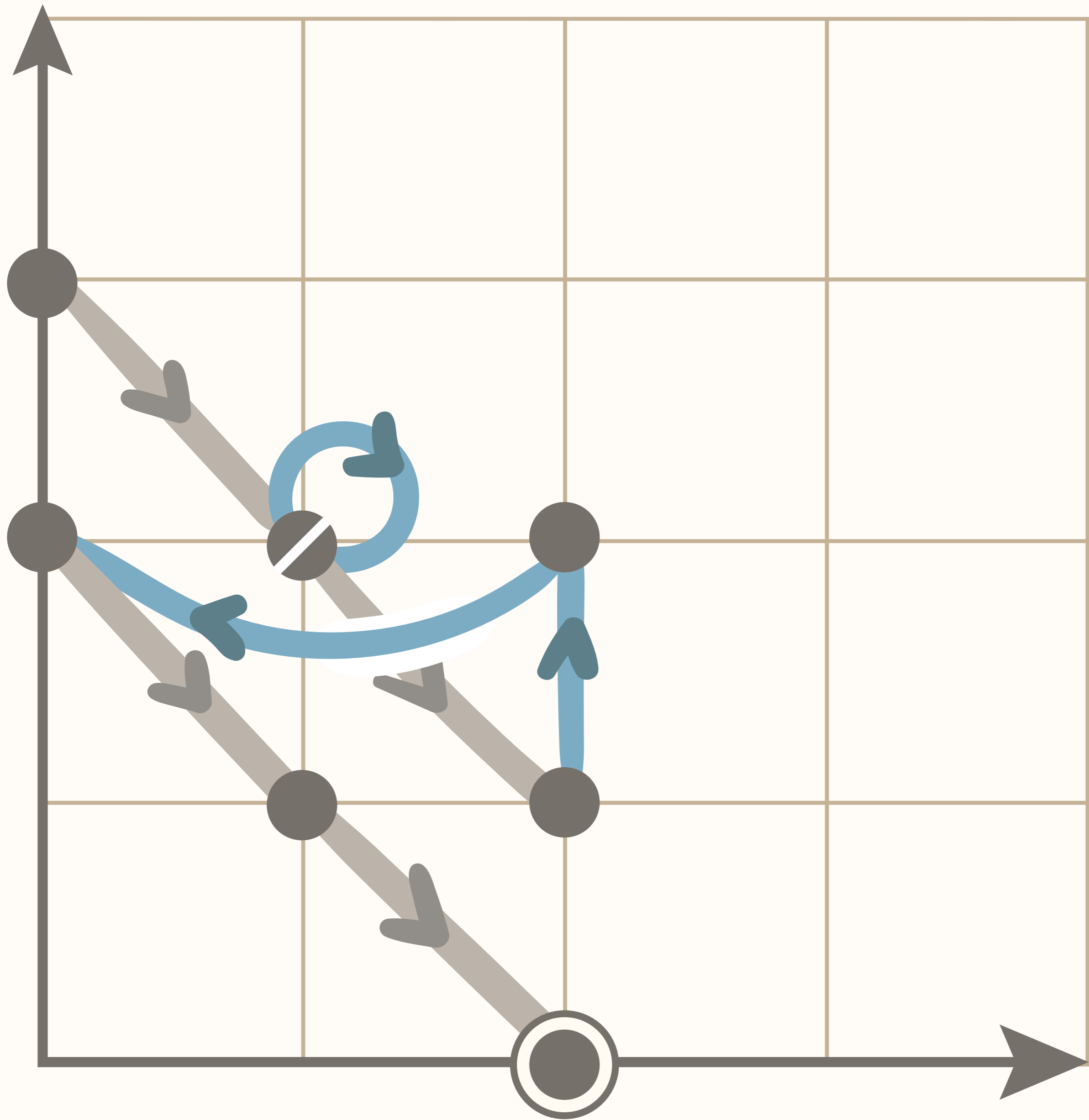
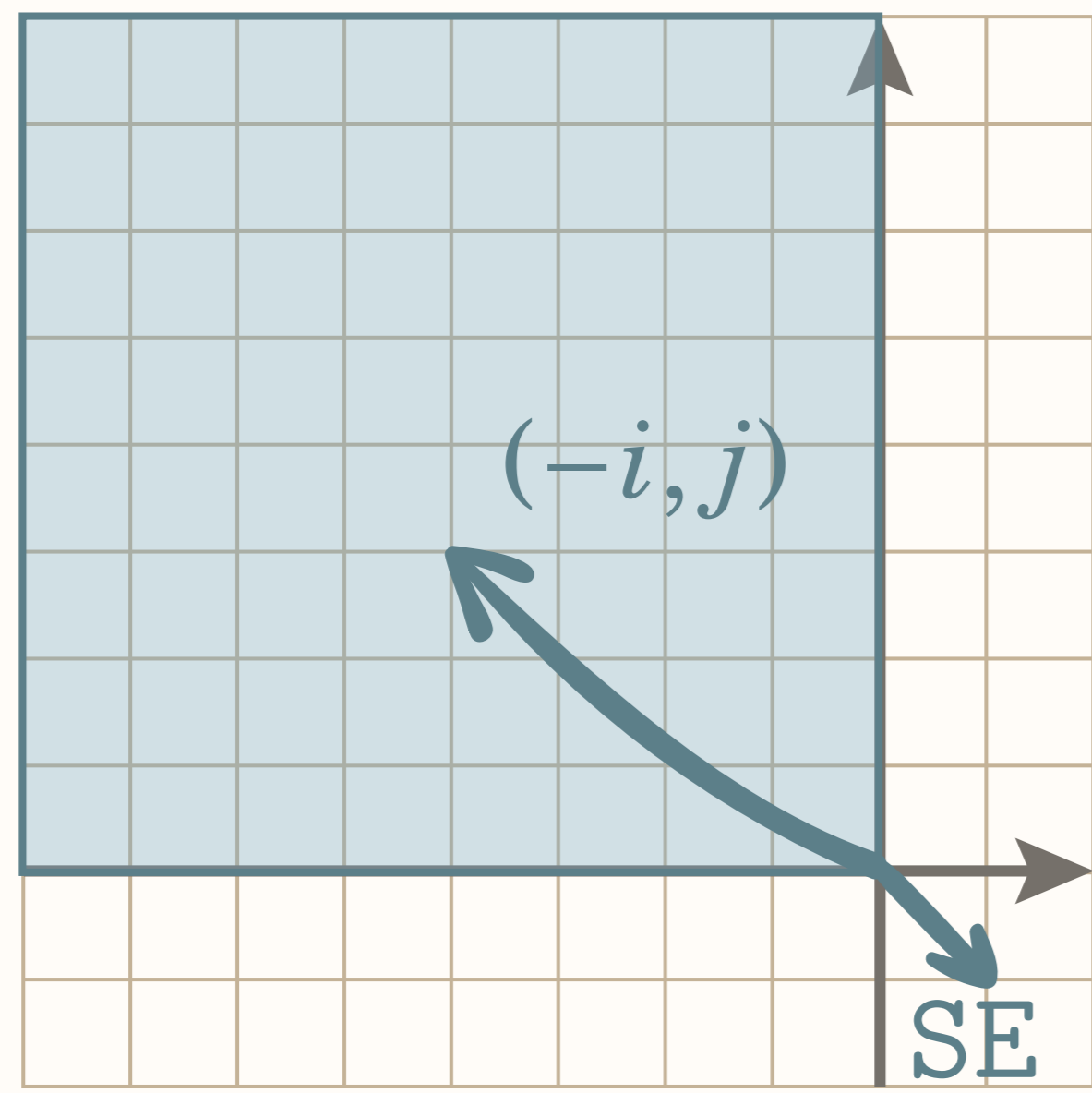
KMSW bijection example



KMSW bijection example



KMSW bijection example

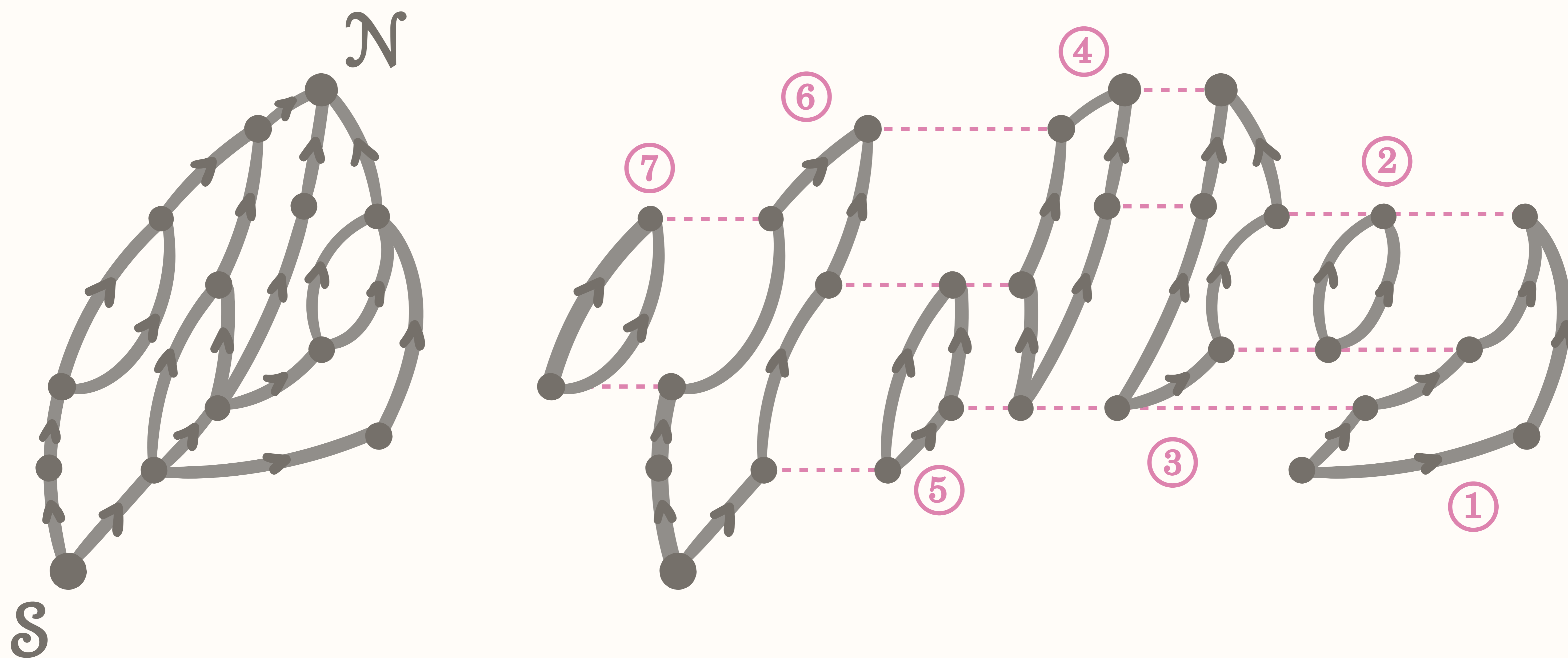


The KMSW bijection

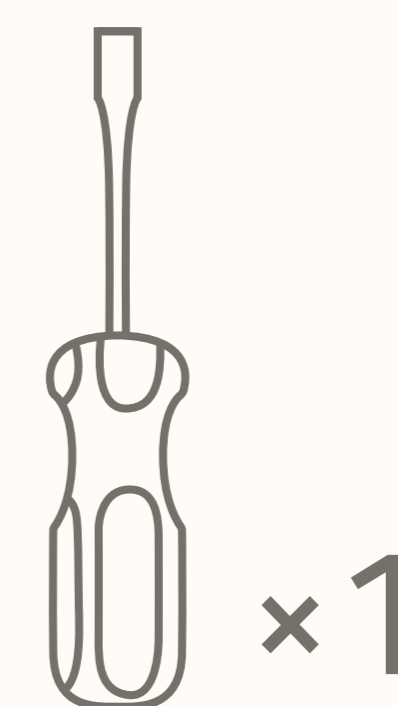
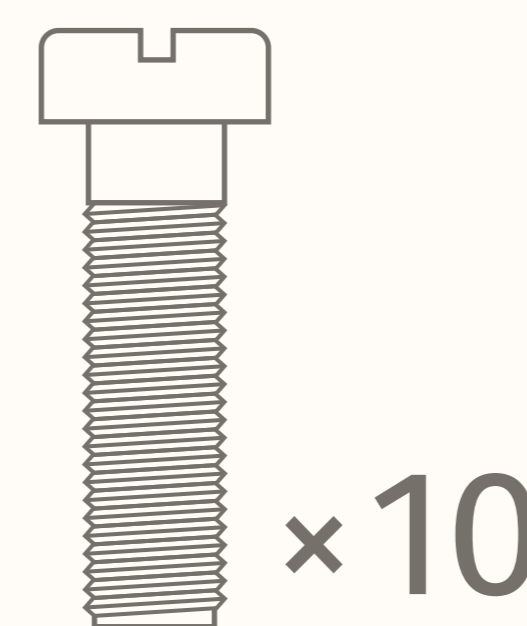
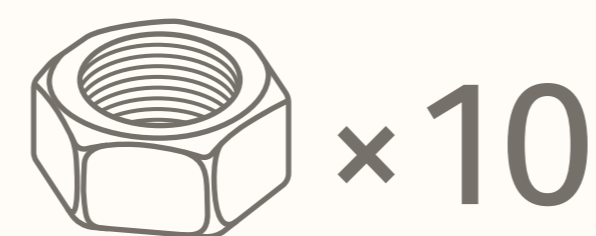
bipolar
orientations



tandem walks
in the quarter plane



→ Bipolar orientations on planar maps and SLE_{12} , R. Kenyon, J. Miller, S. Sheffield and D. Wilson (2015)

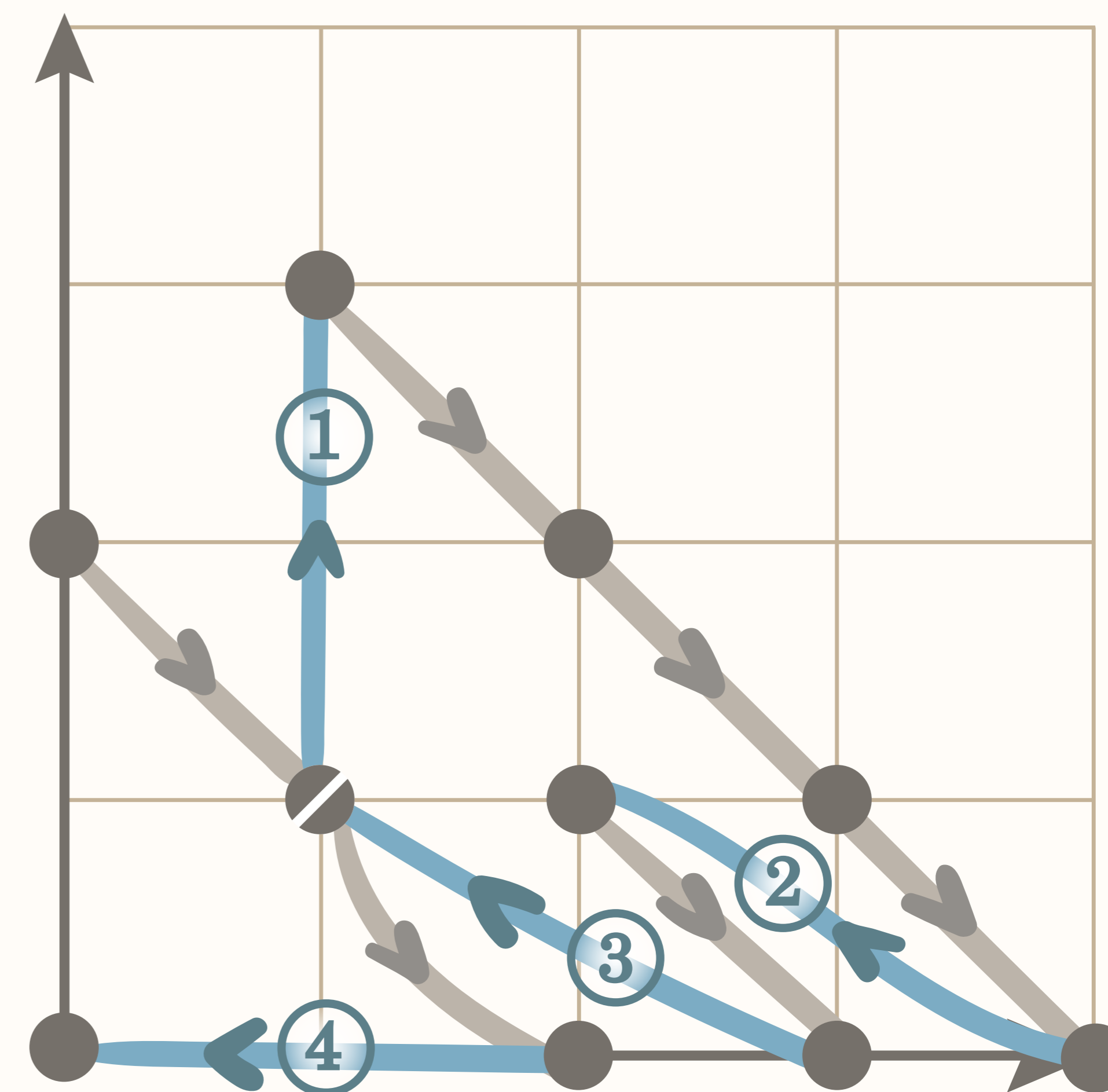
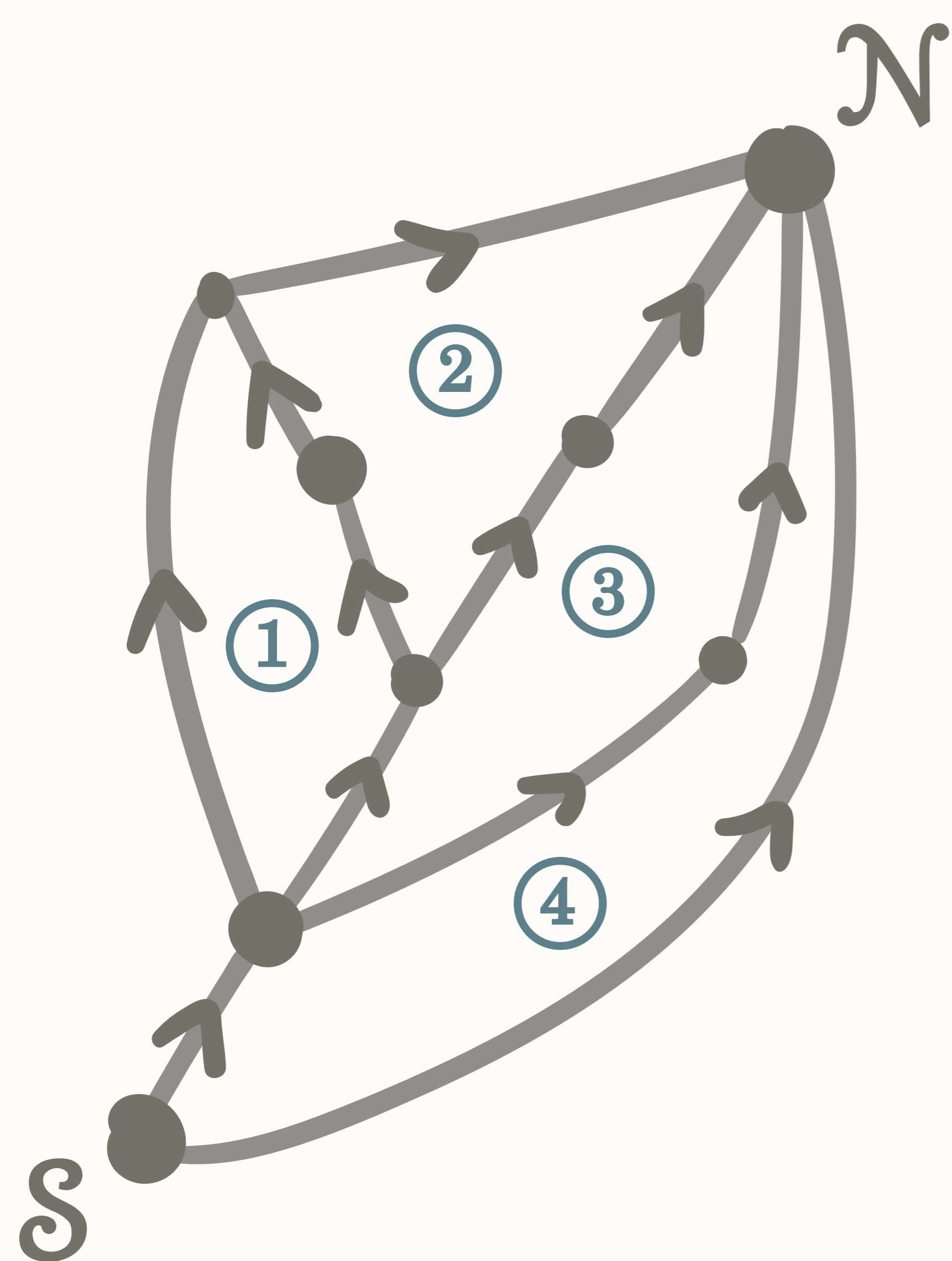


The KMSW bijection

*bipolar
orientations*



*tandem walks
in the quarter plane*



→ *Bipolar orientations on planar maps and SLE_{12} , R. Kenyon, J. Miller, S. Sheffield and D. Wilson (2015)*

Summary

Quadrant tandem walks

Asymptotics

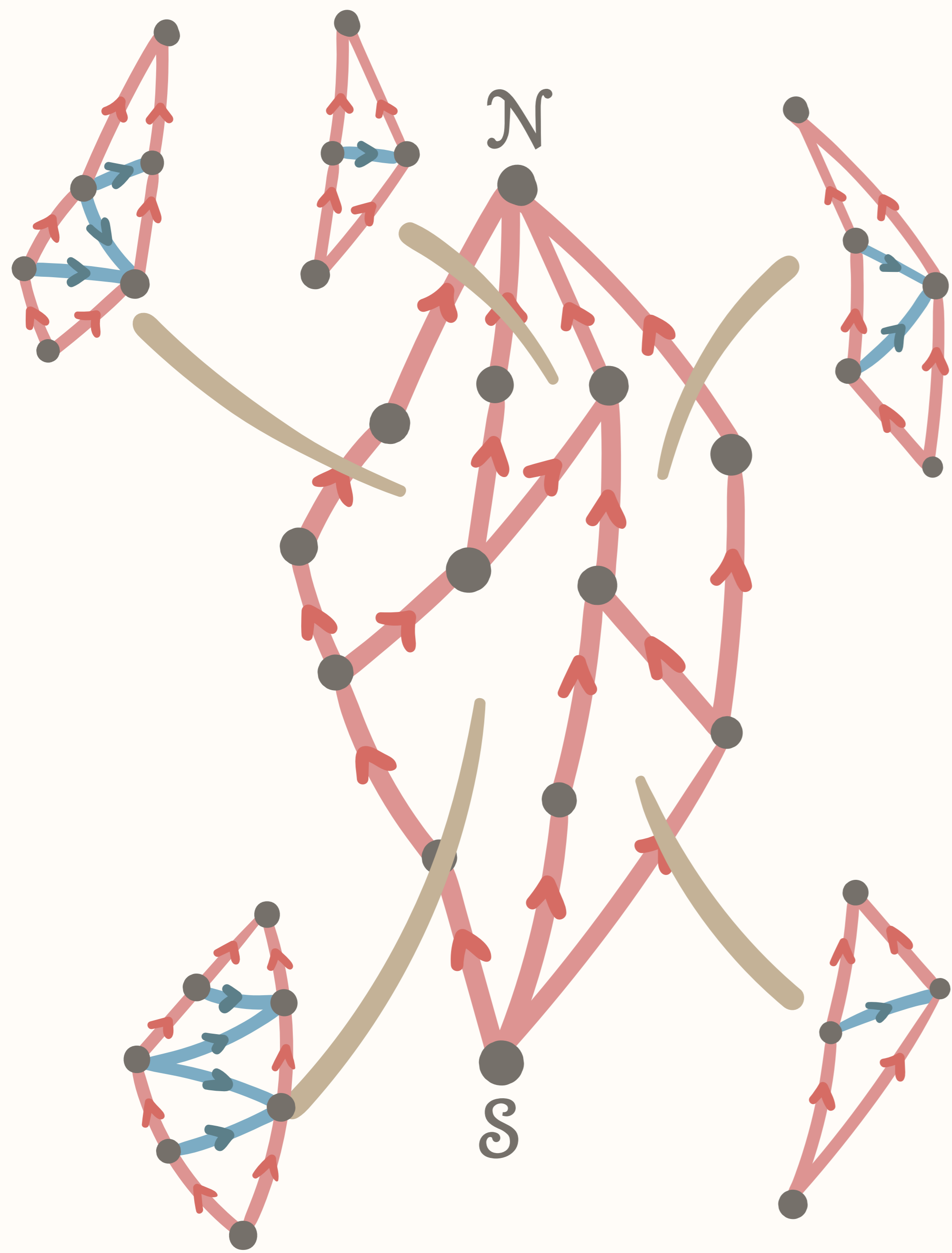
Small steps

Decorated bipolar orientations

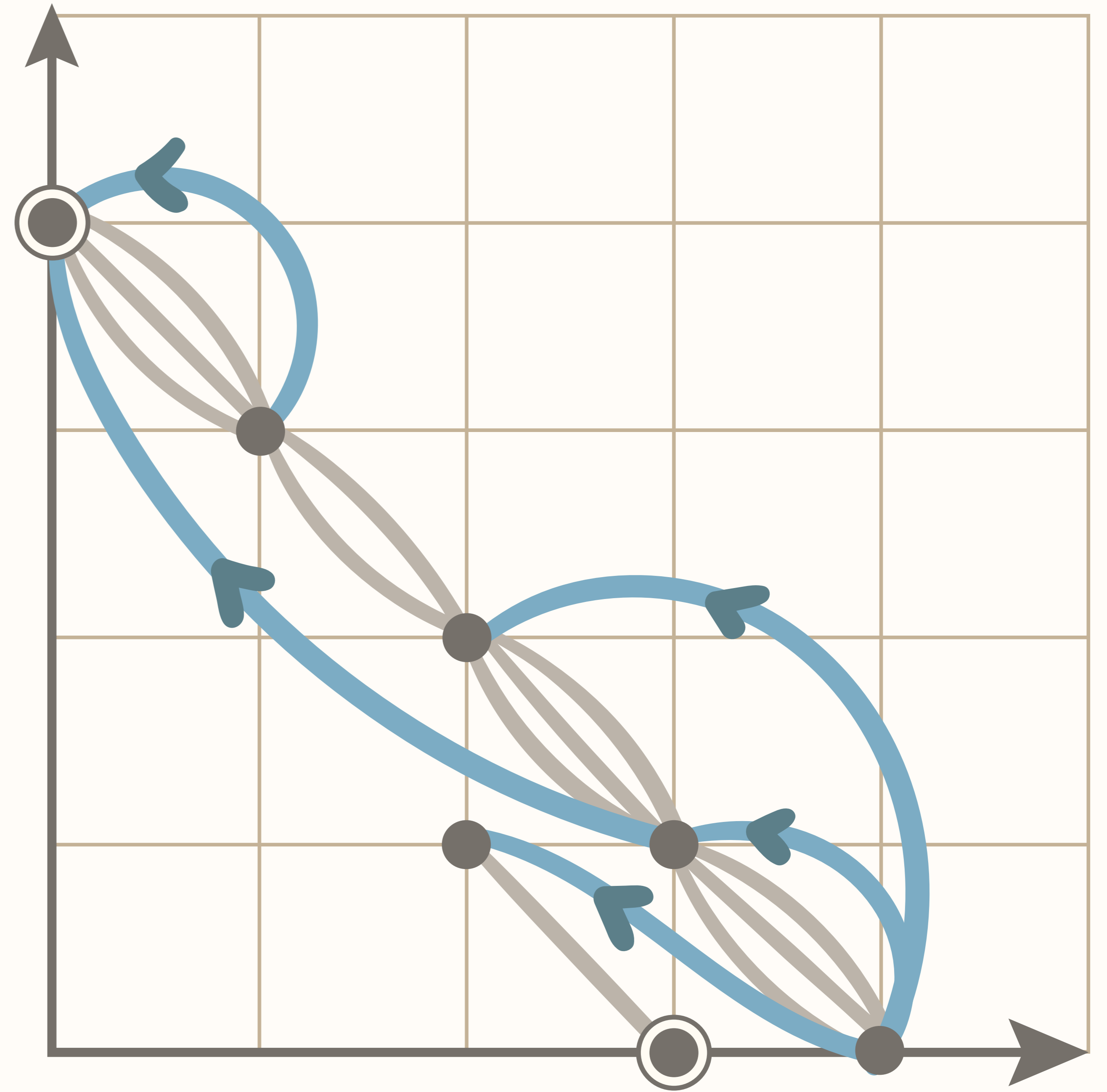
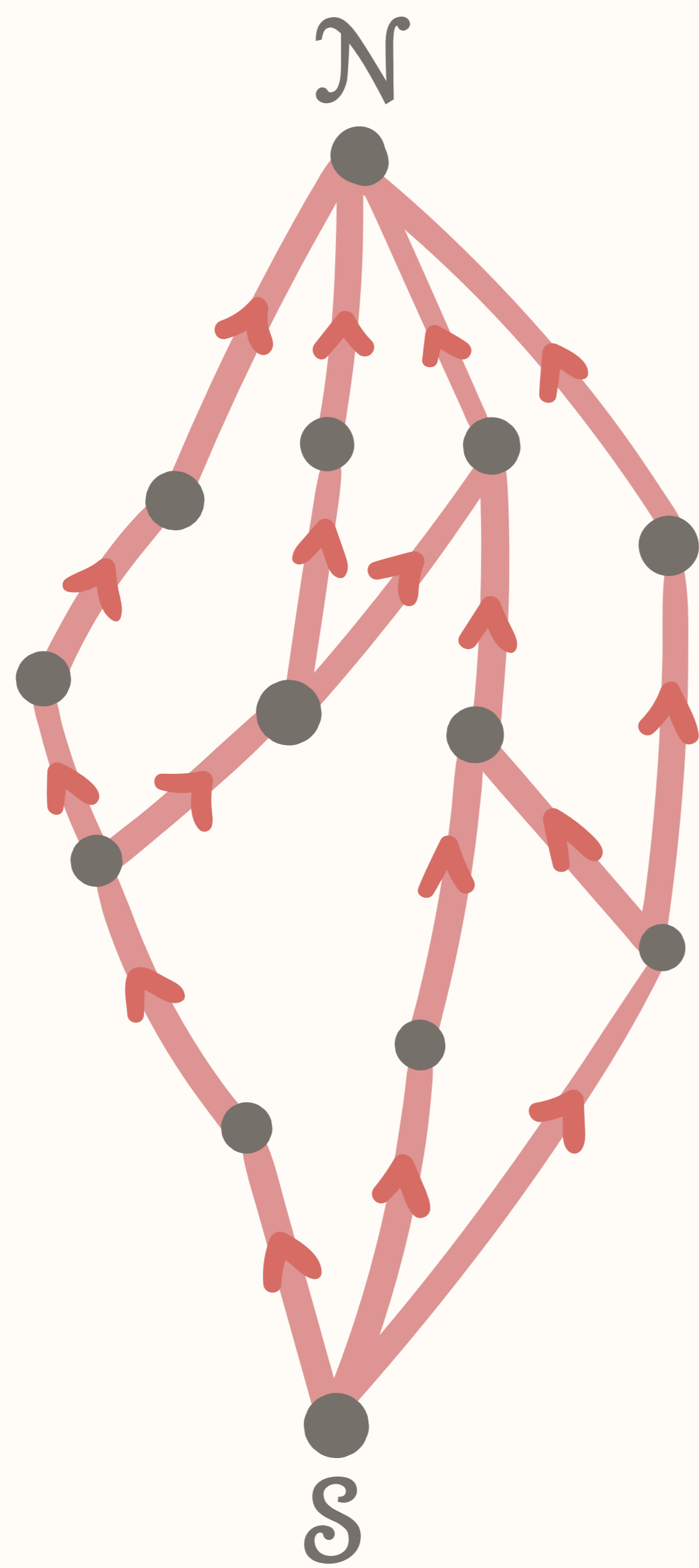
a. The KMSW bijection

b.  Application to transversal structures

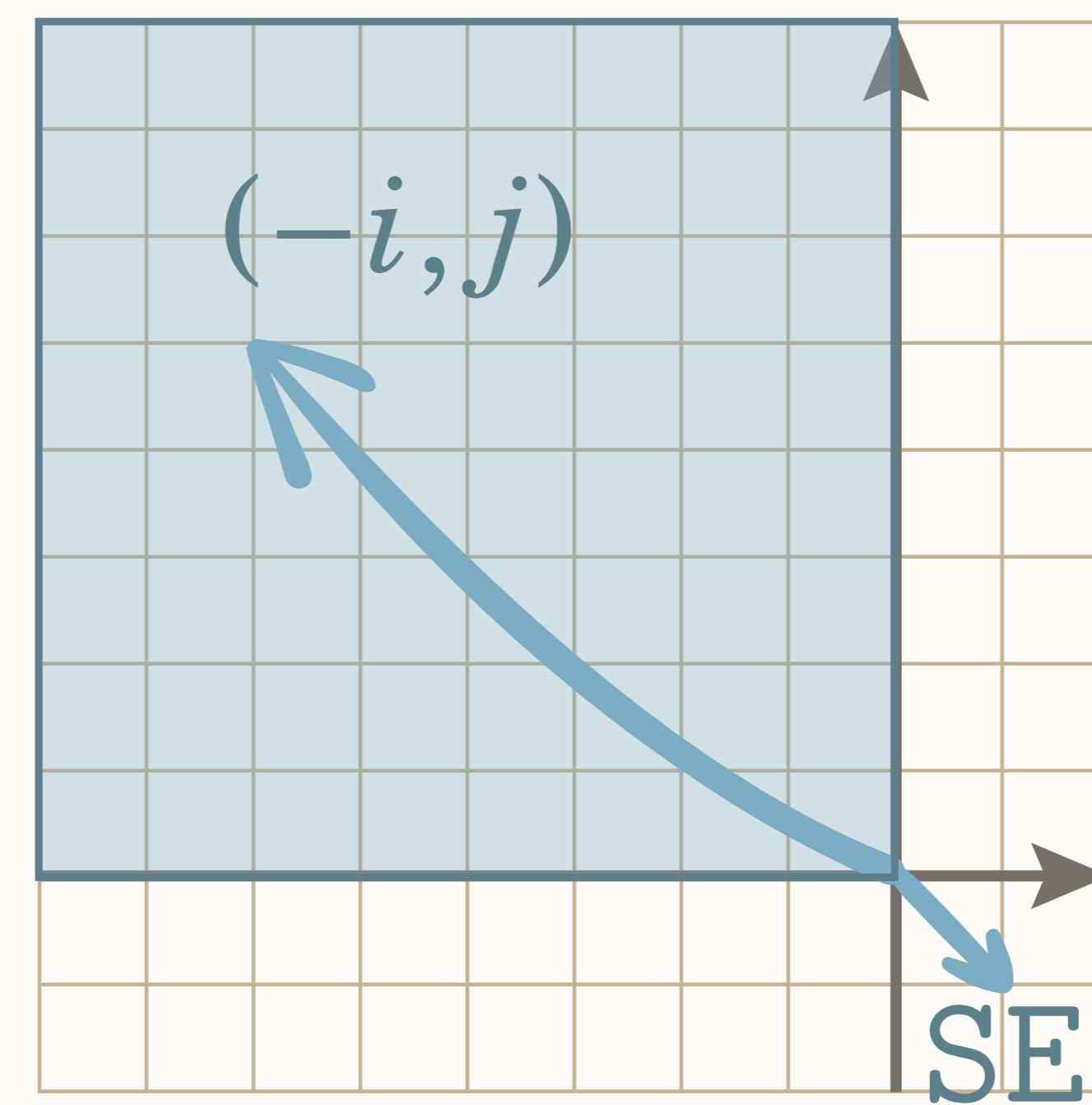
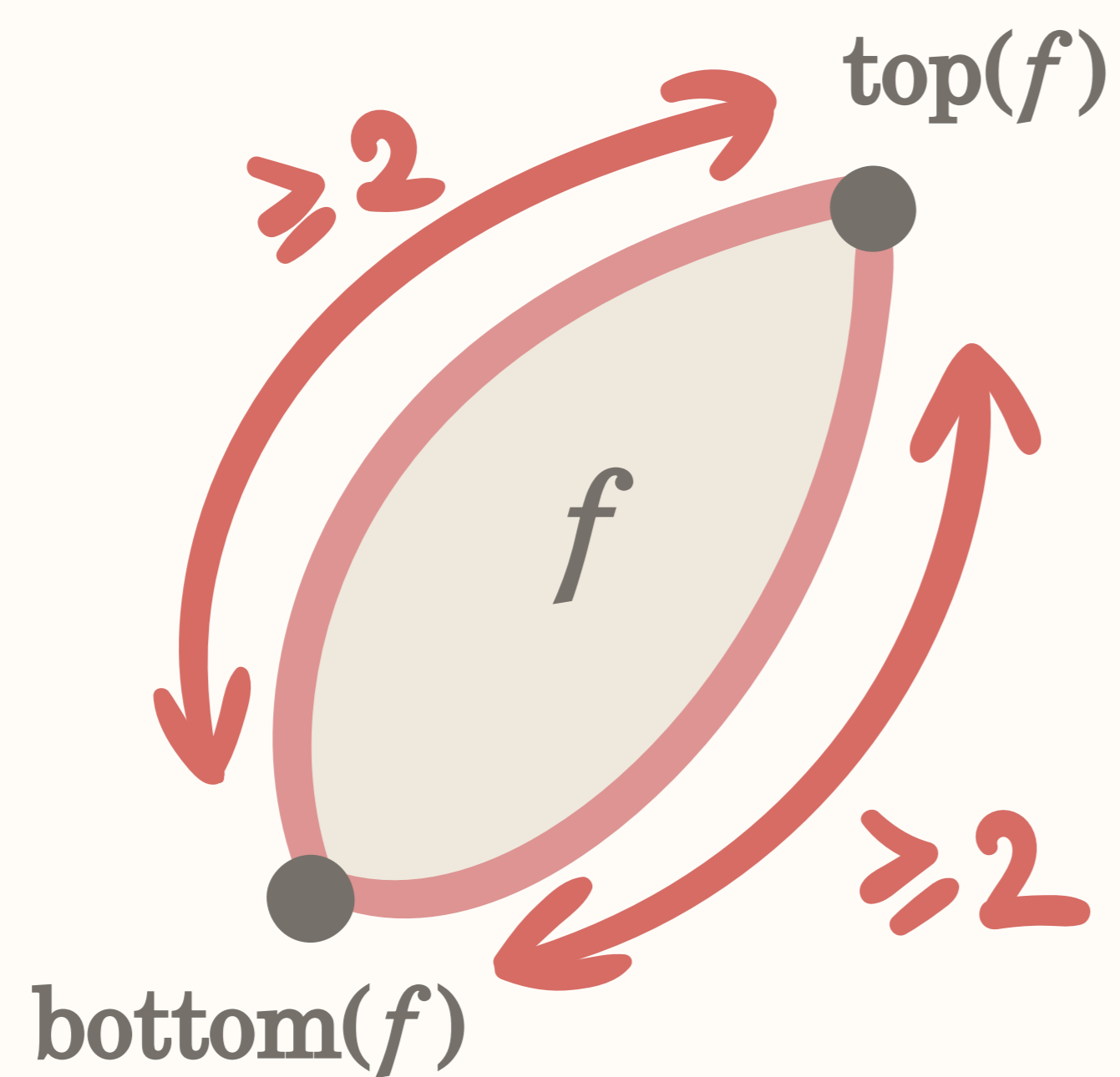
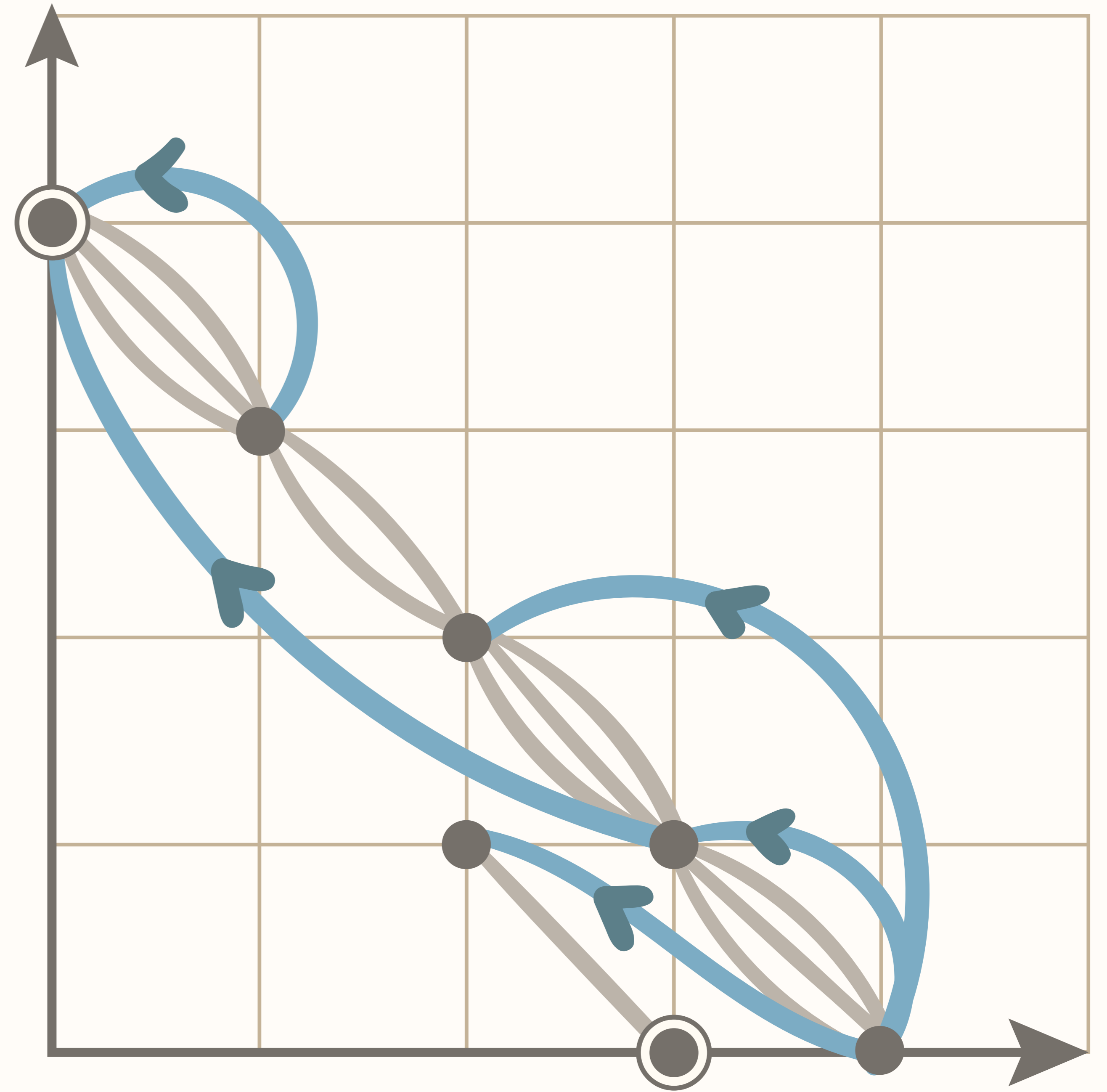
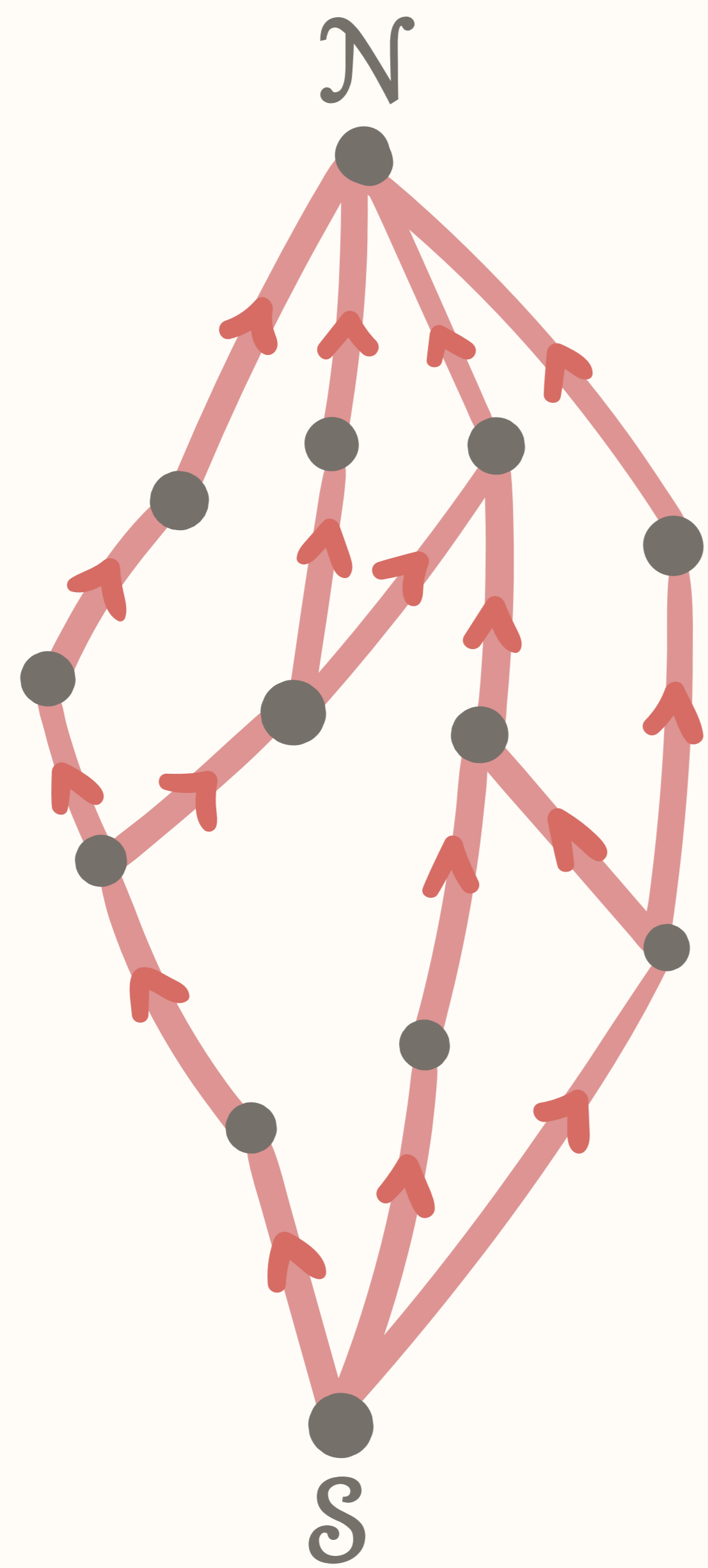
Application to transversal structures



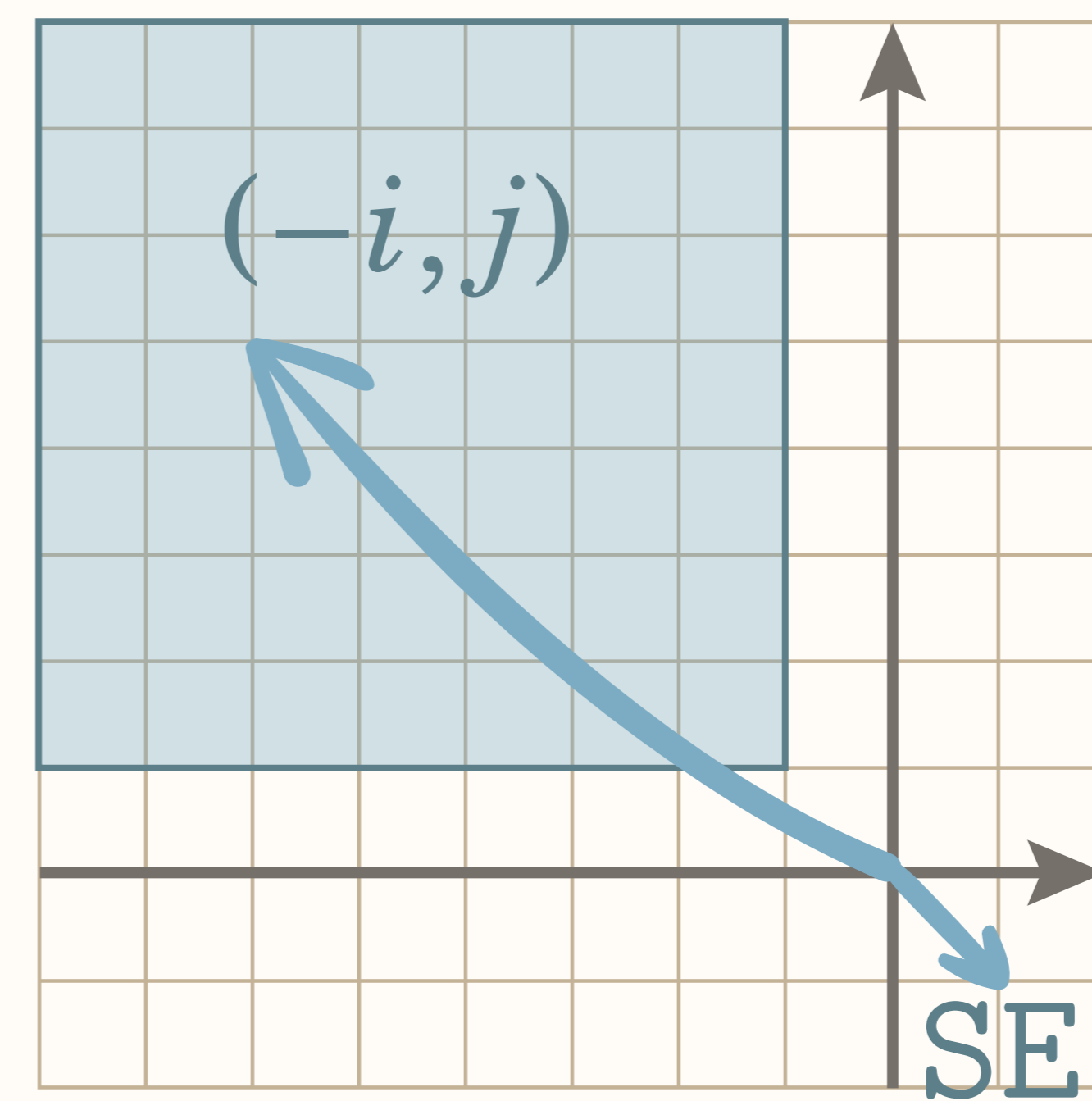
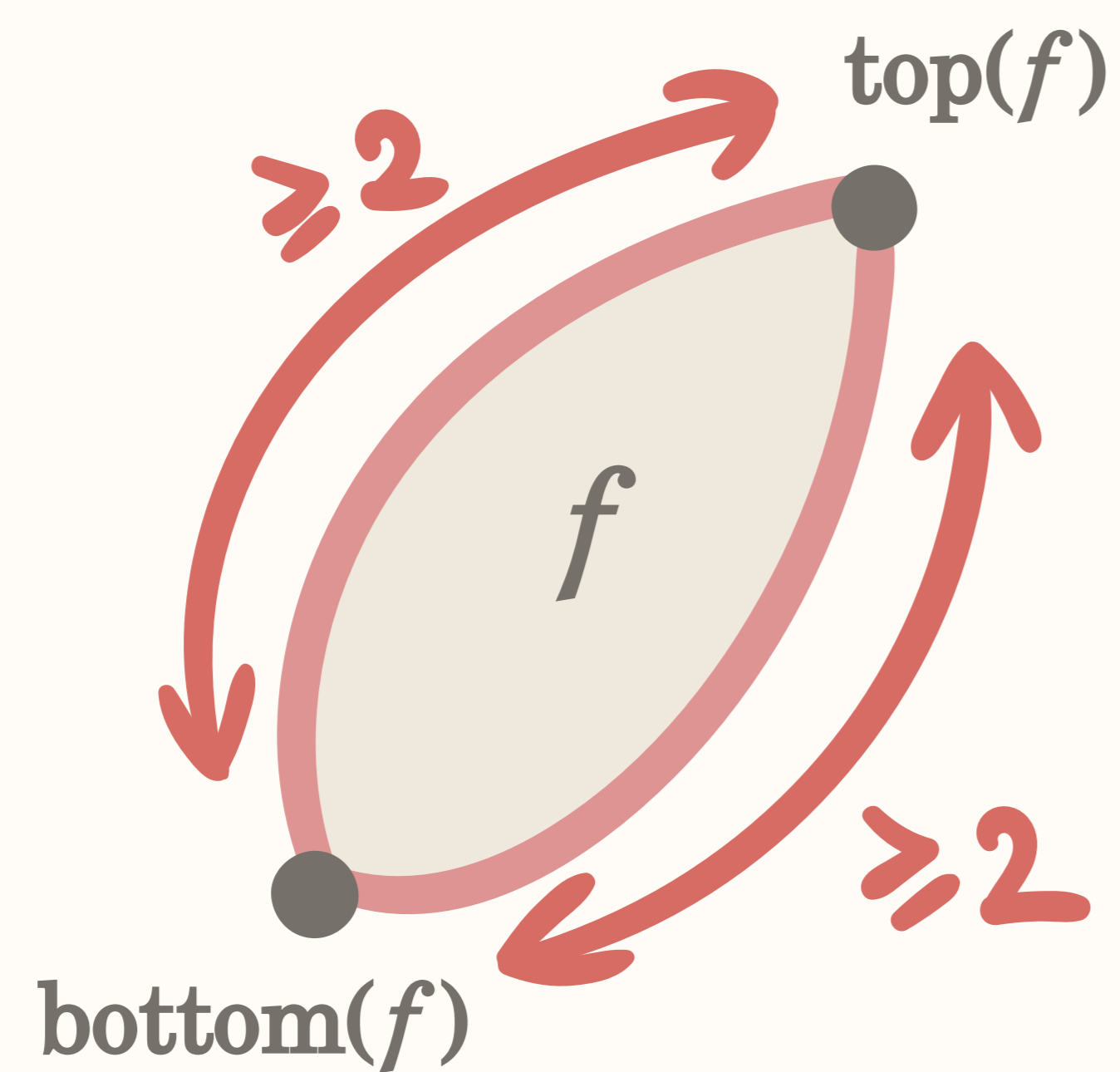
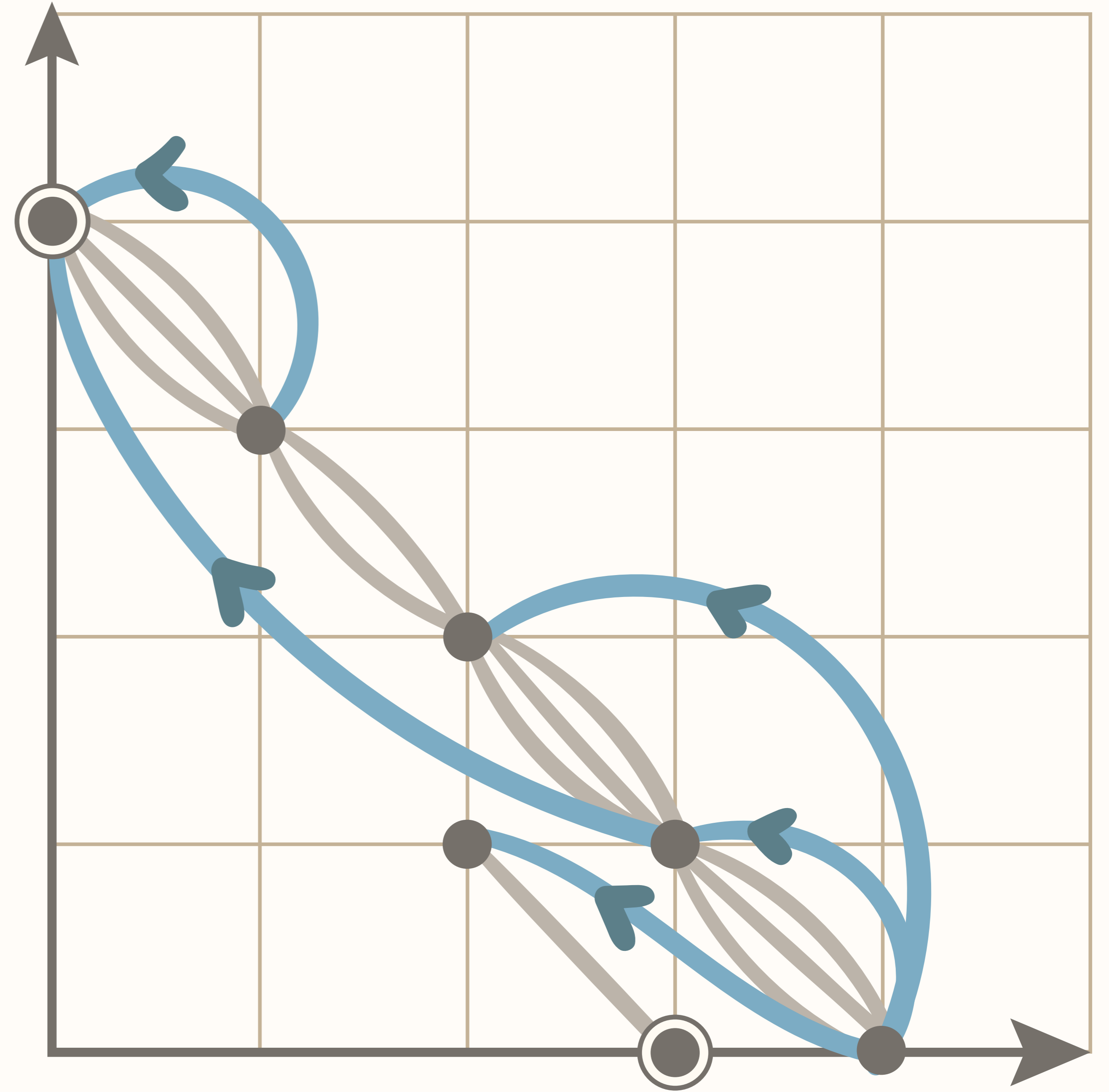
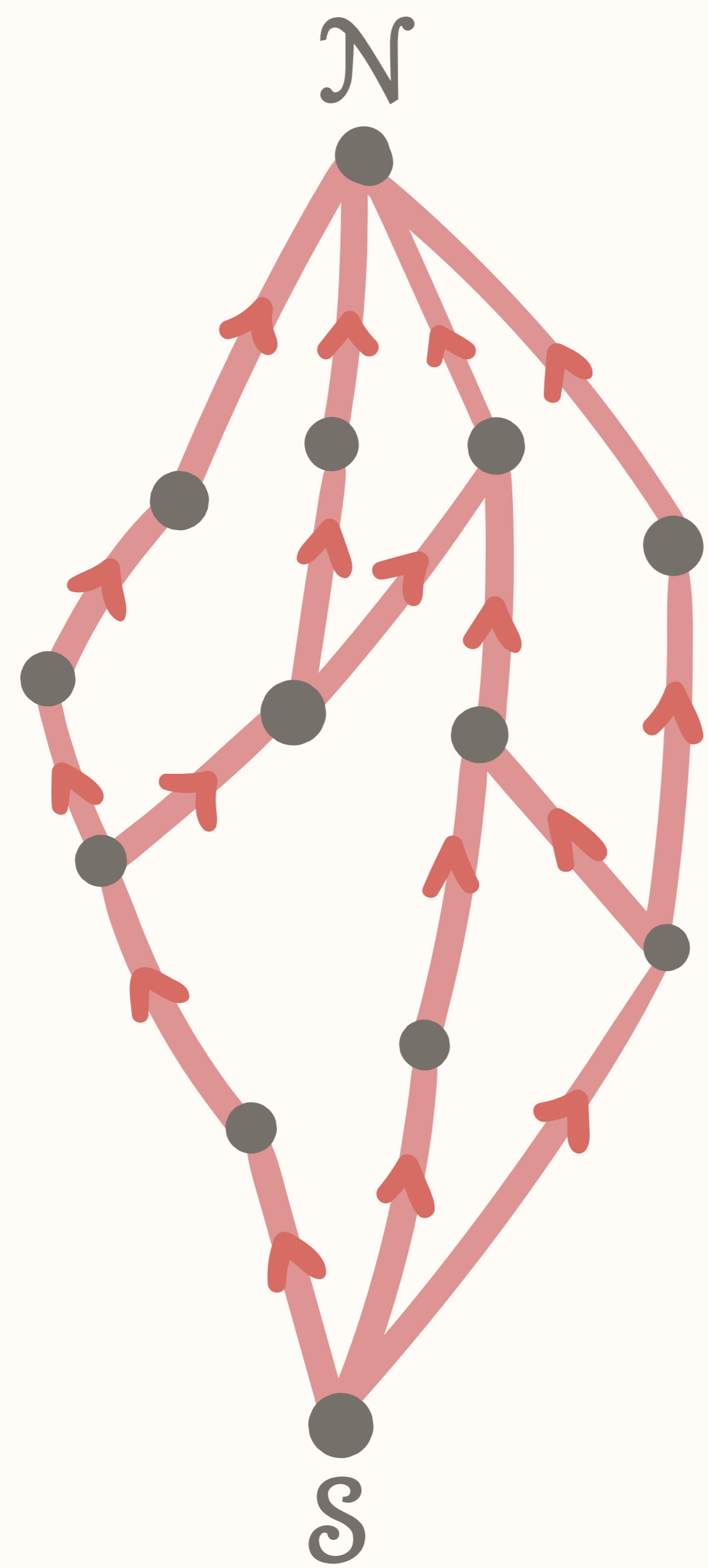
Application to transversal structures



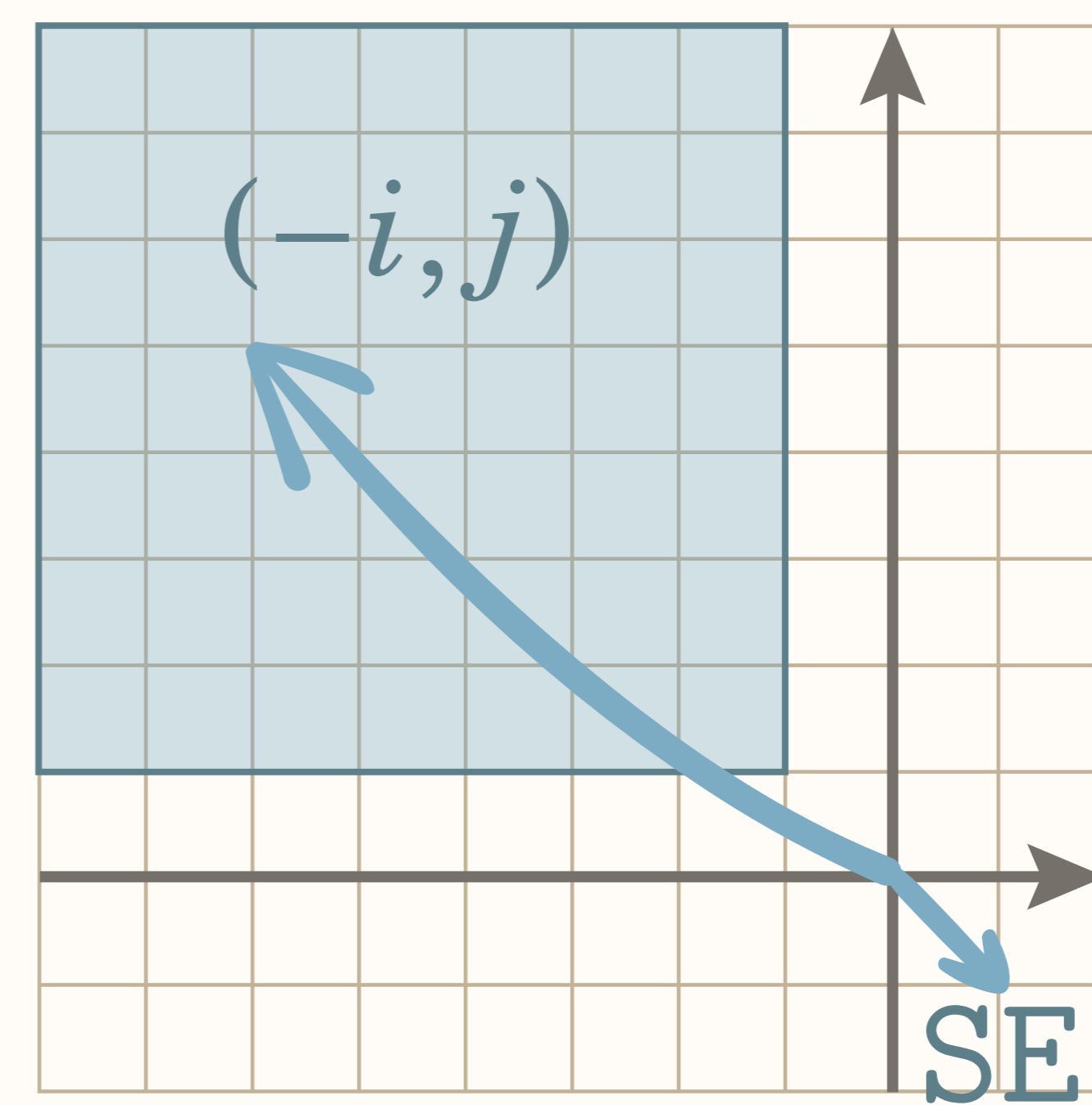
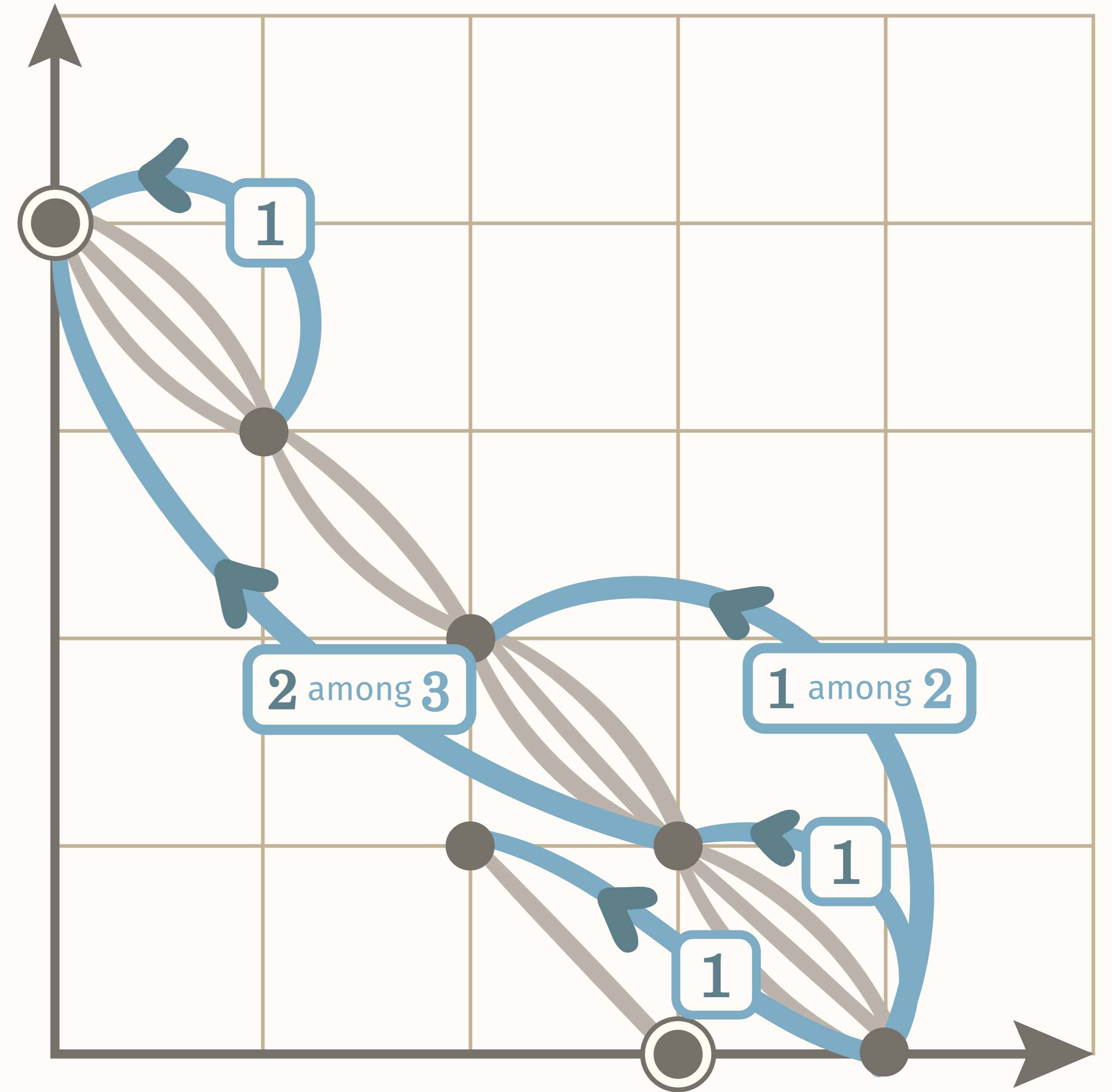
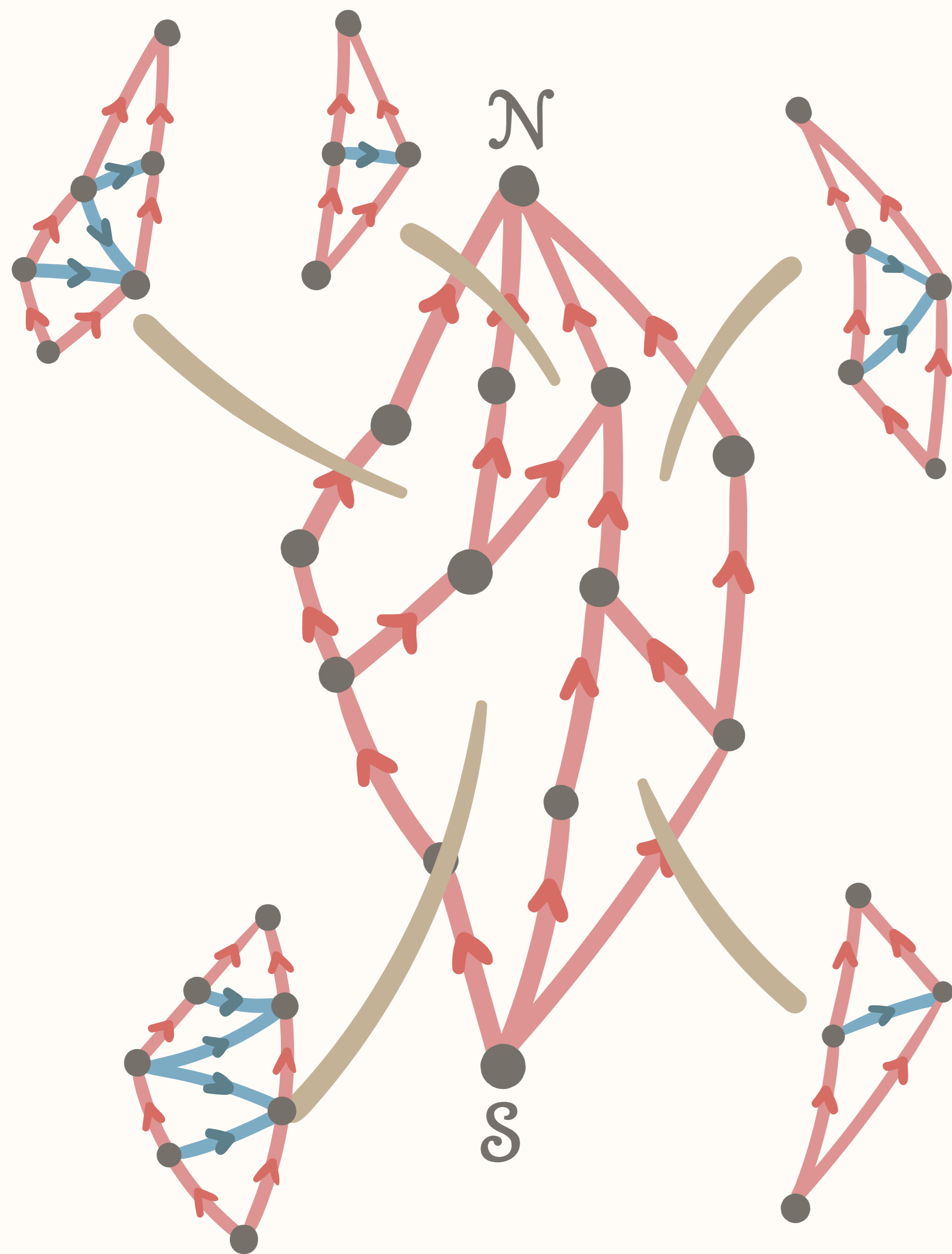
Application to transversal structures



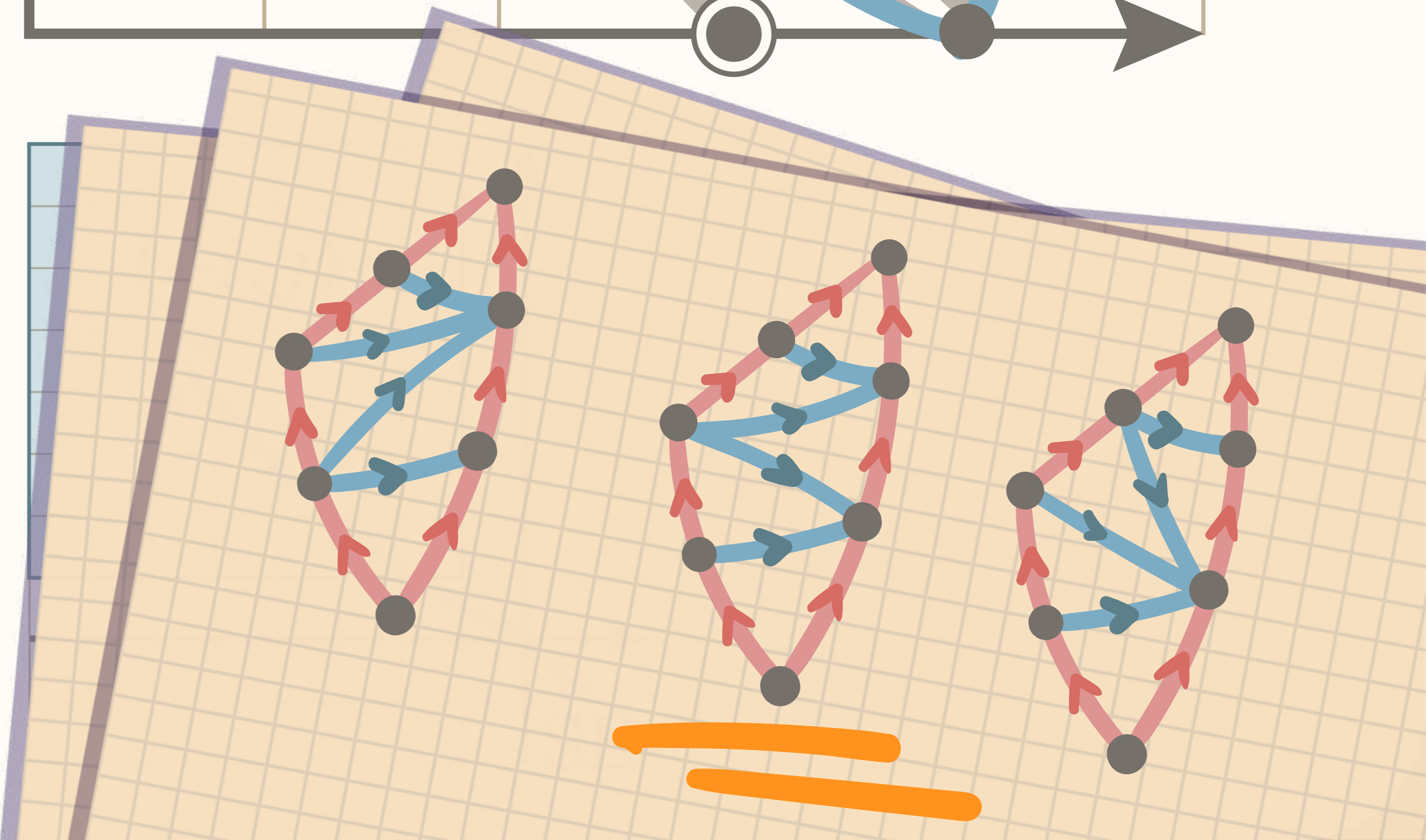
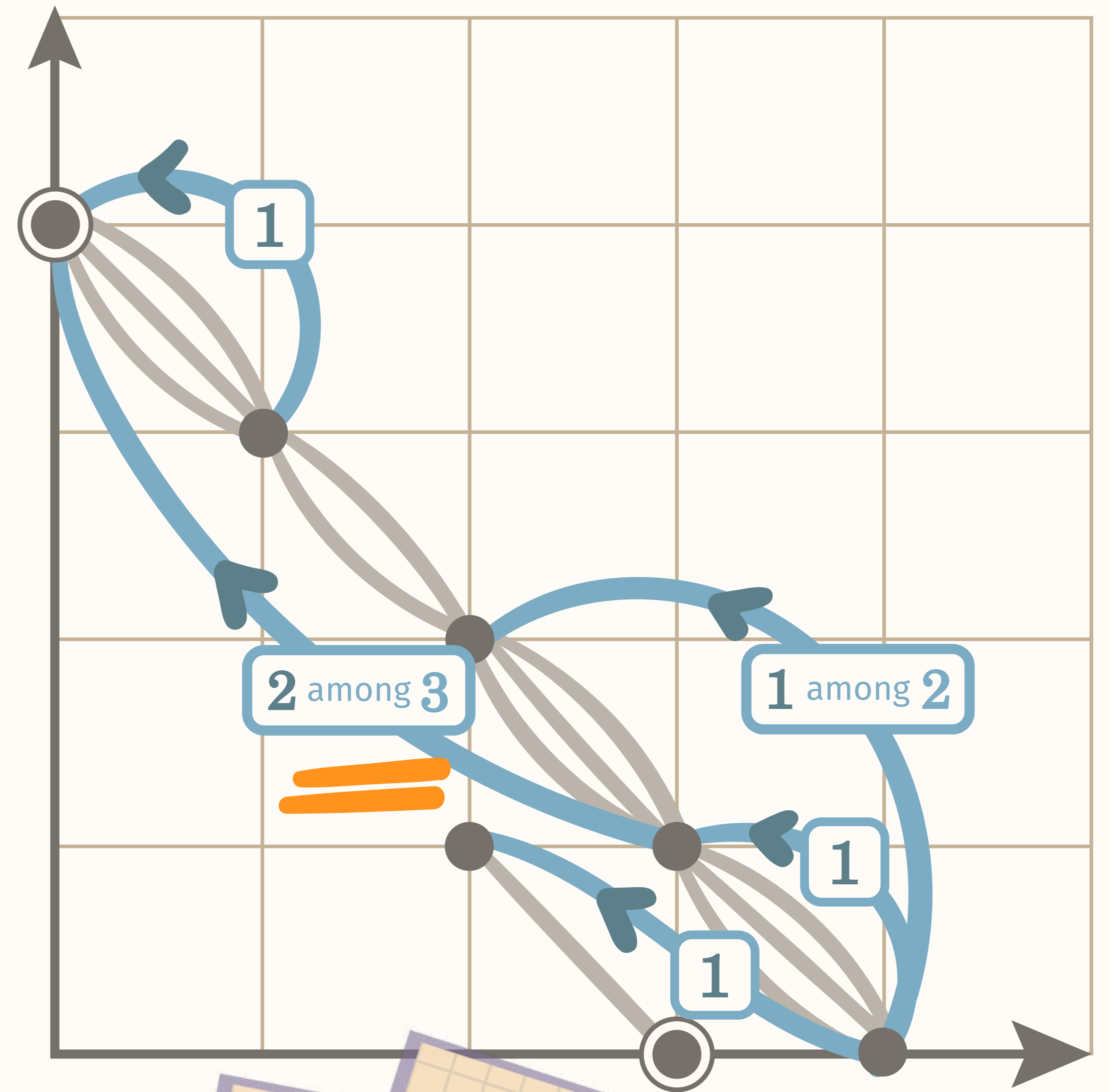
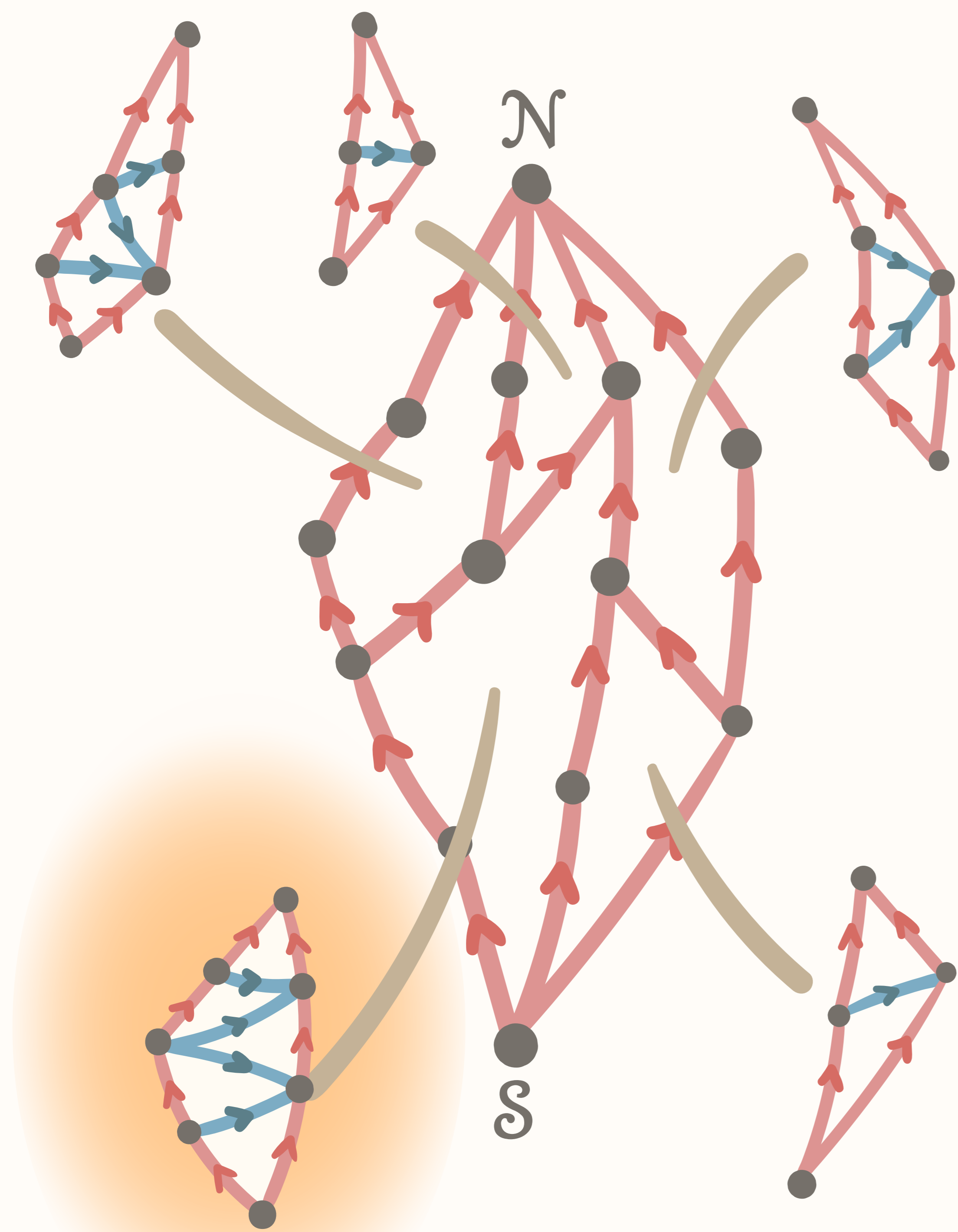
Application to transversal structures



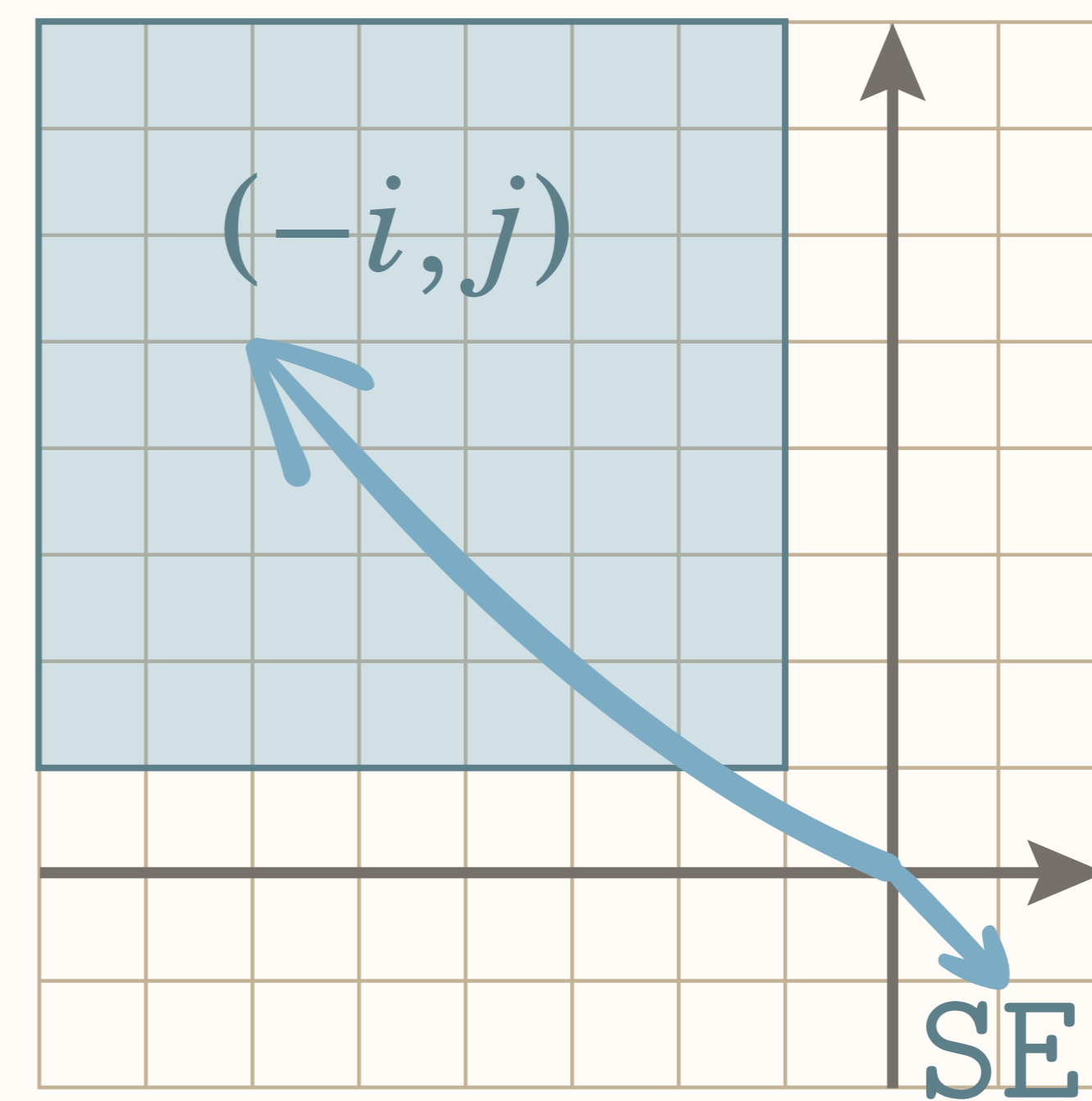
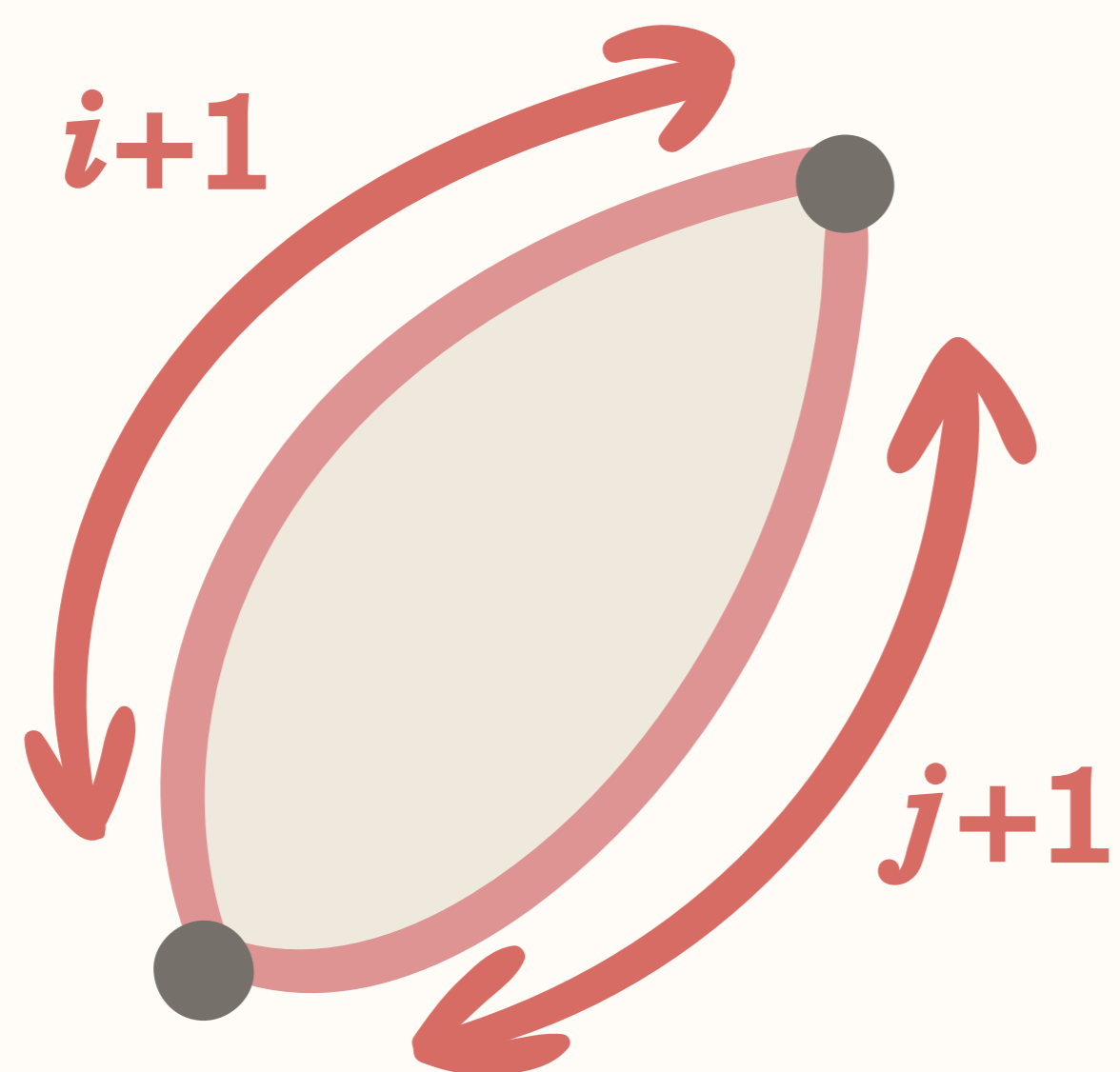
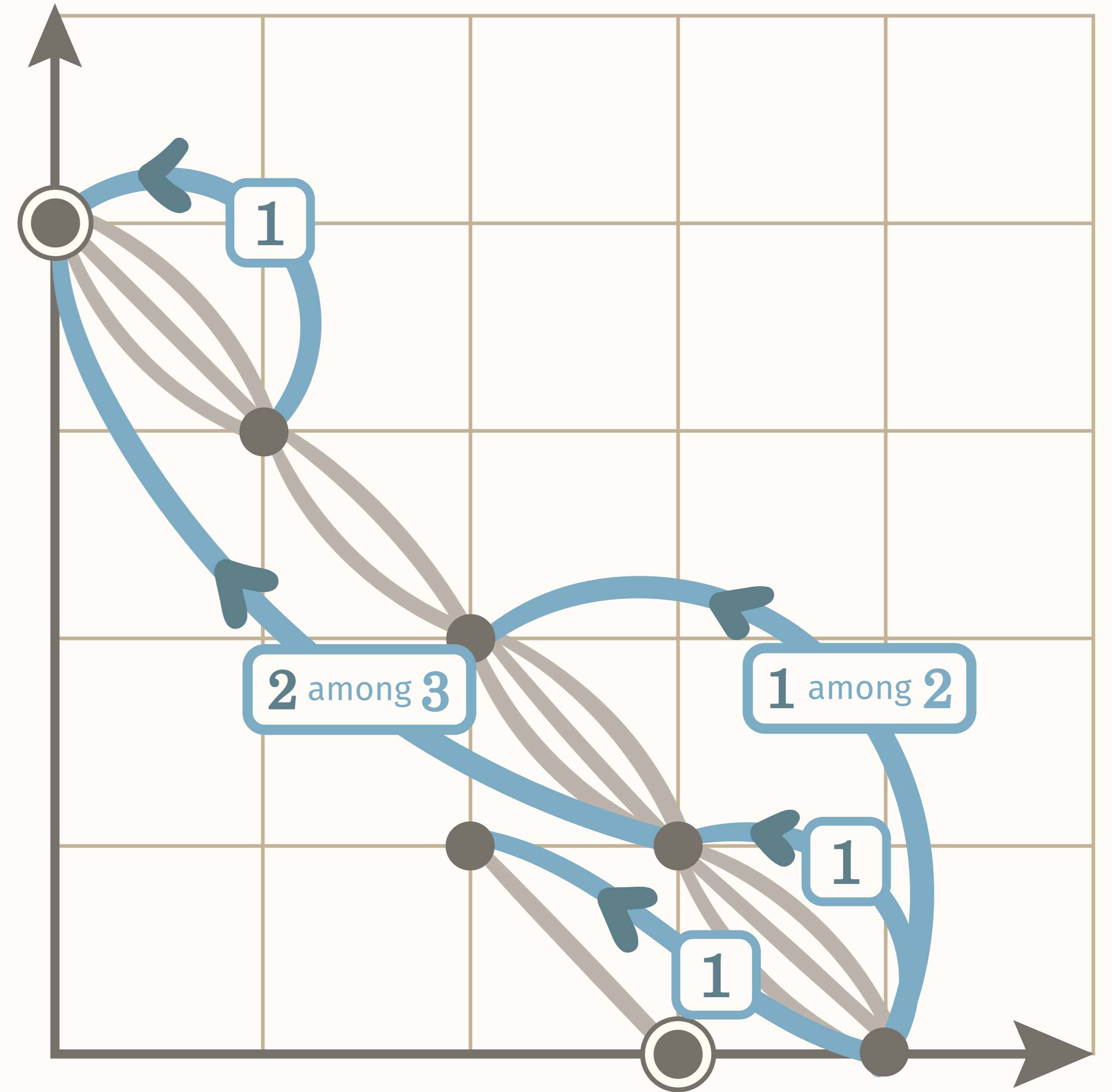
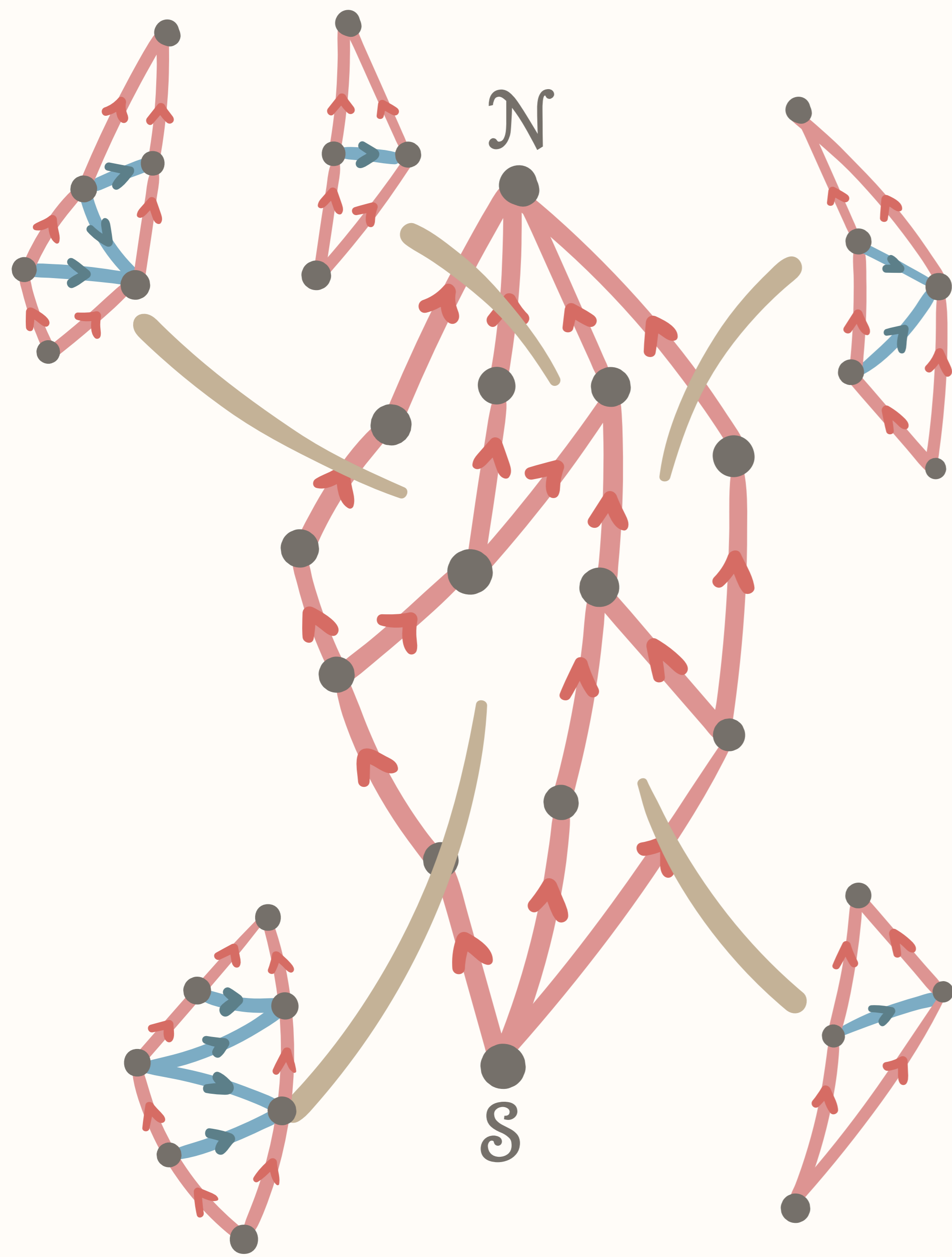
Application to transversal structures



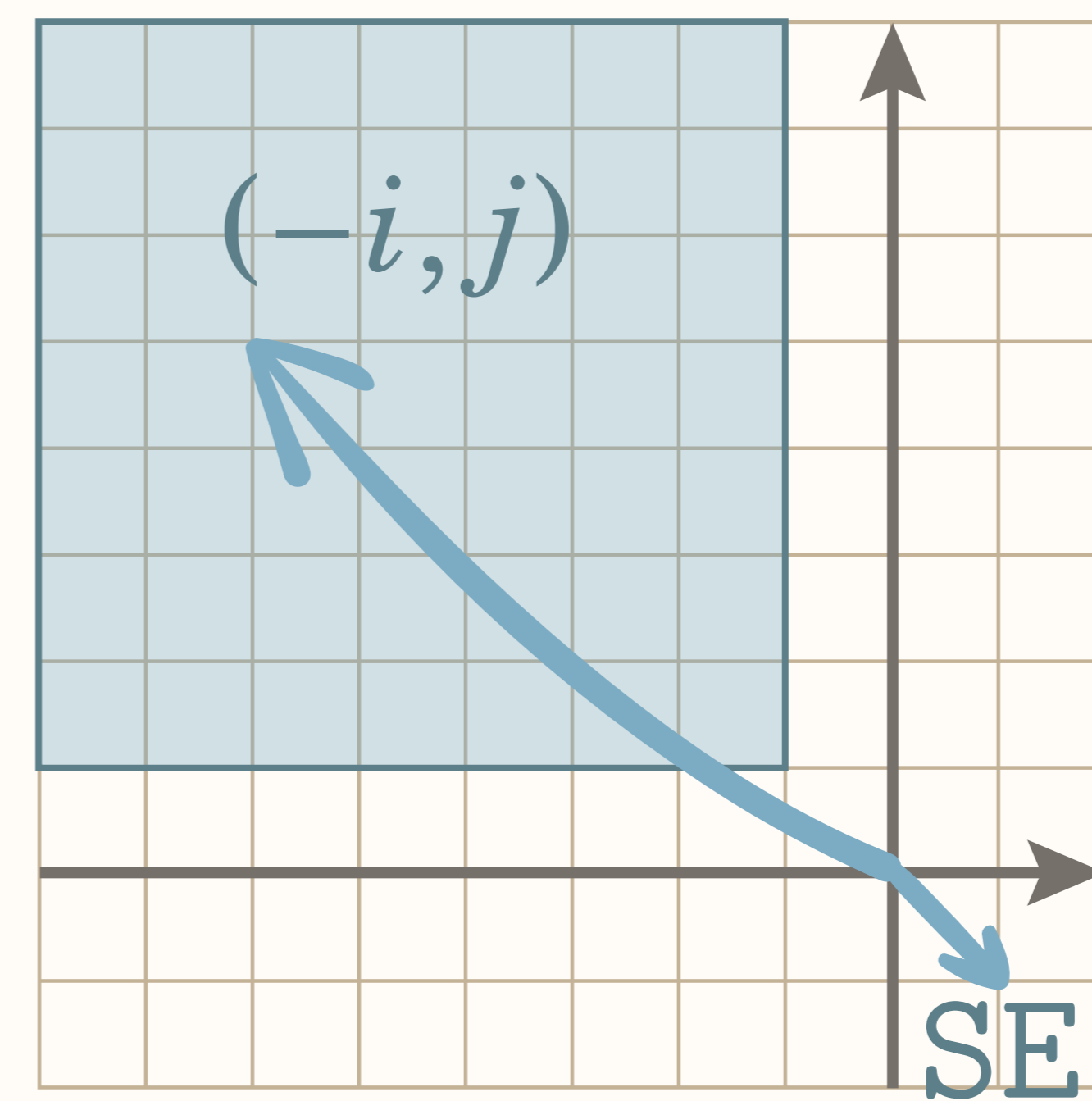
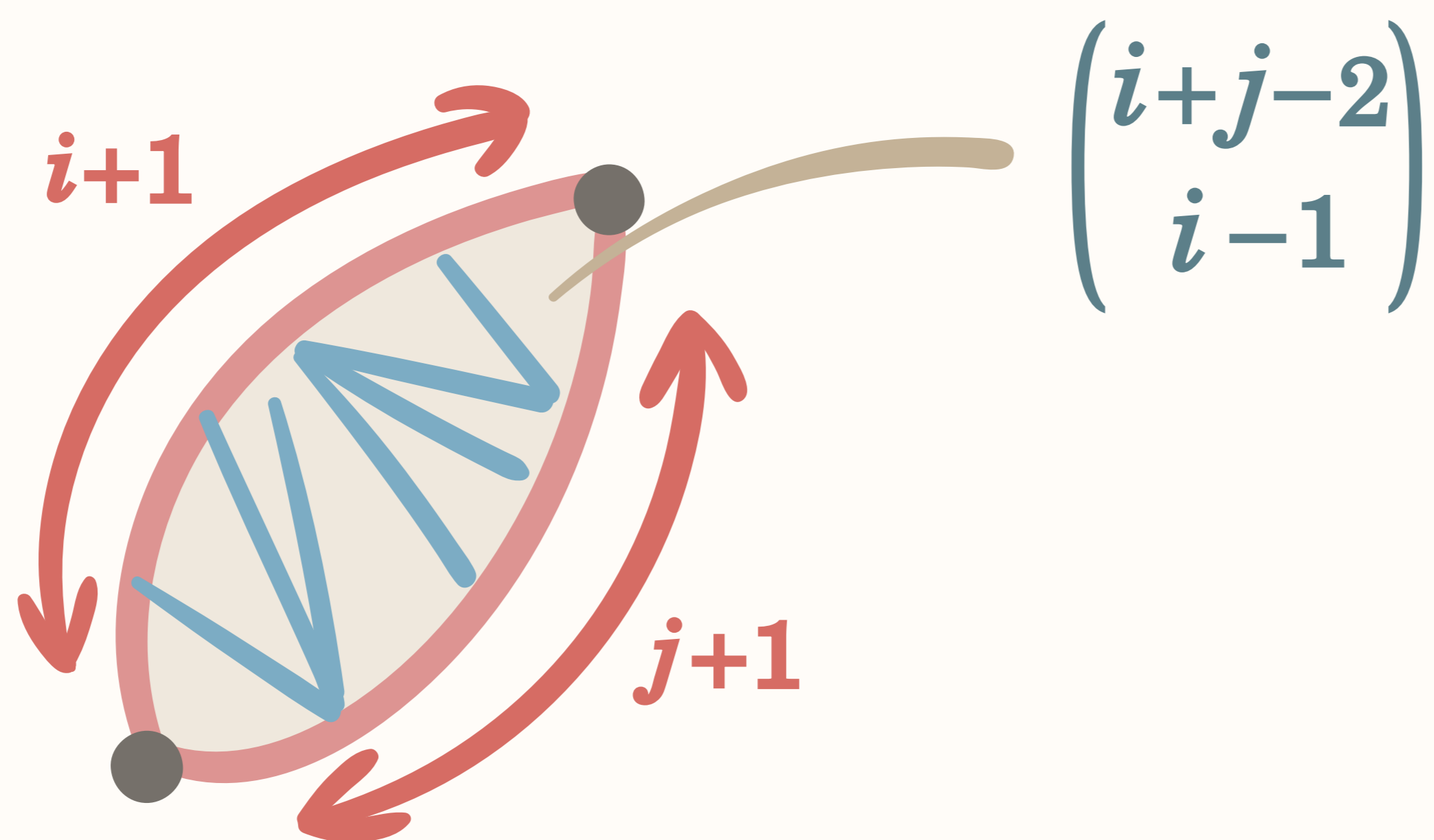
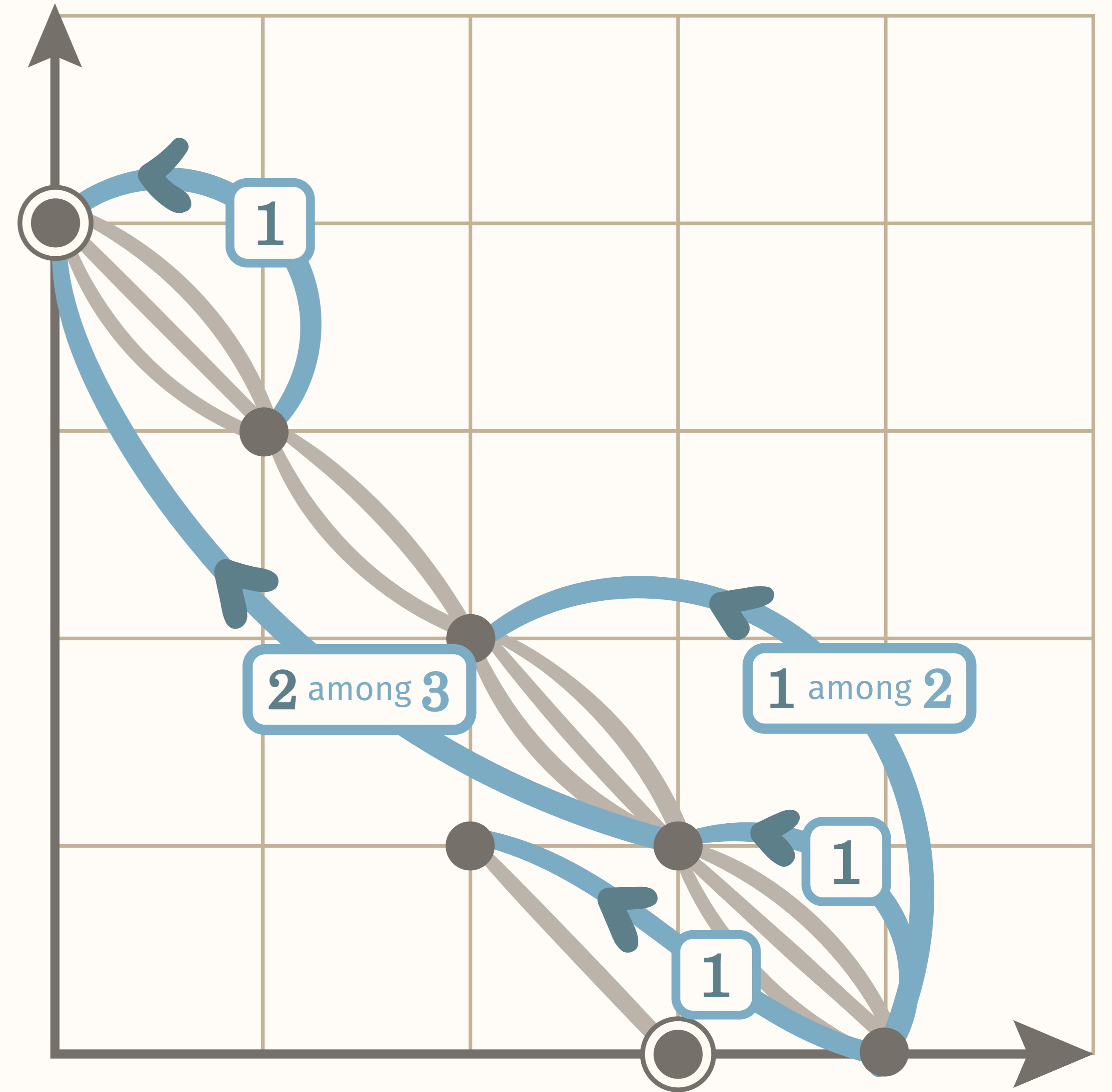
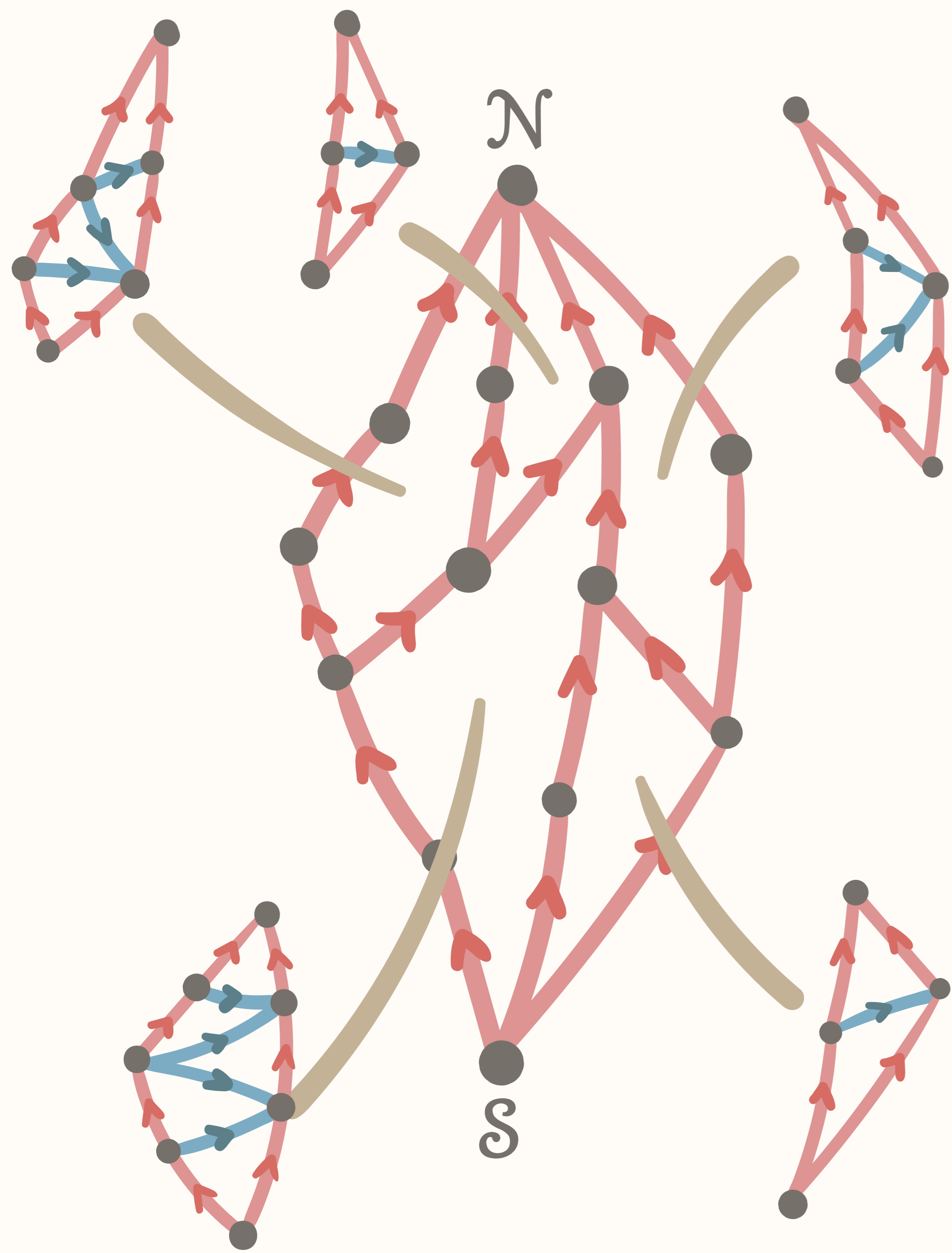
Application to transversal structures



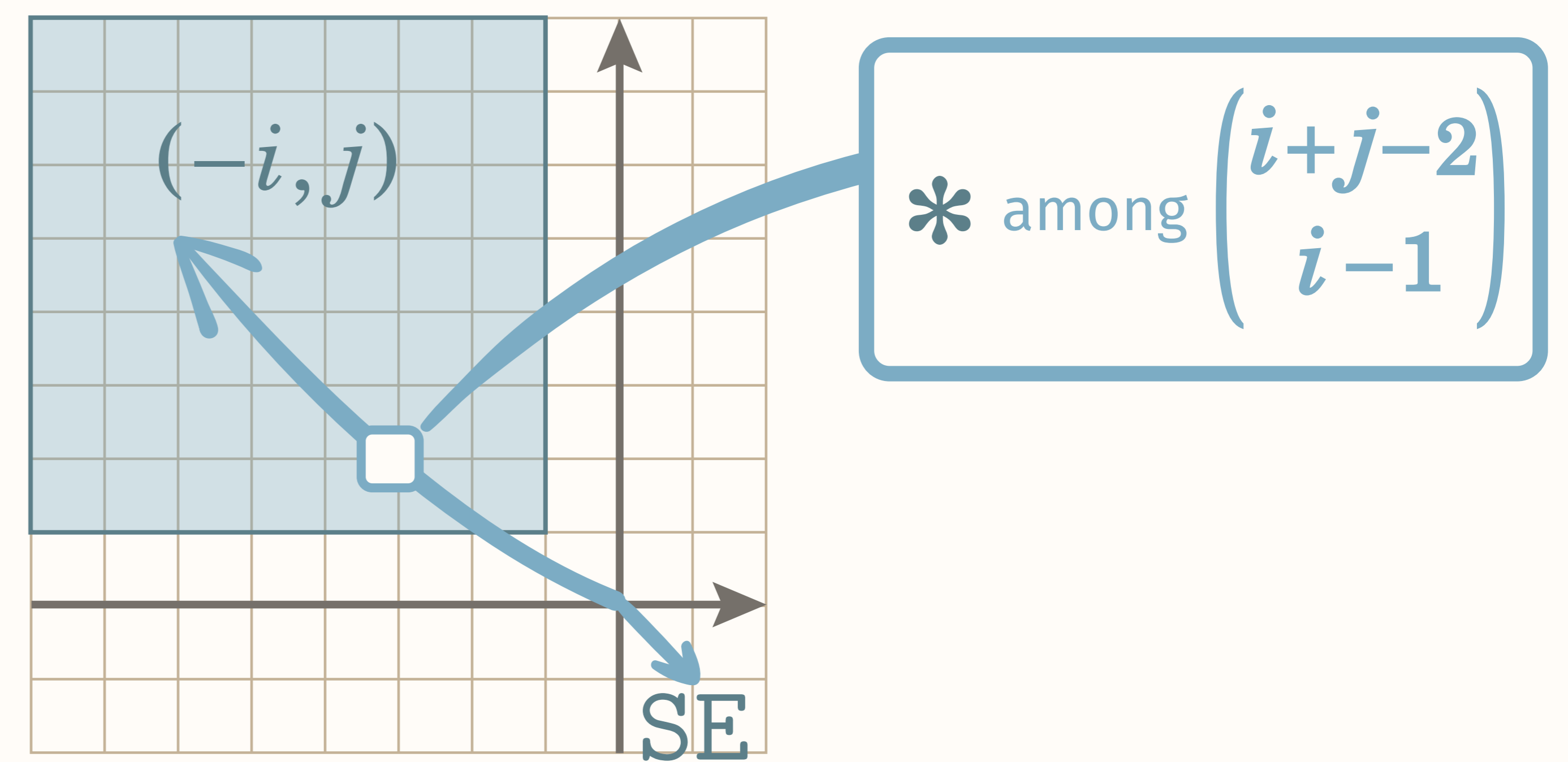
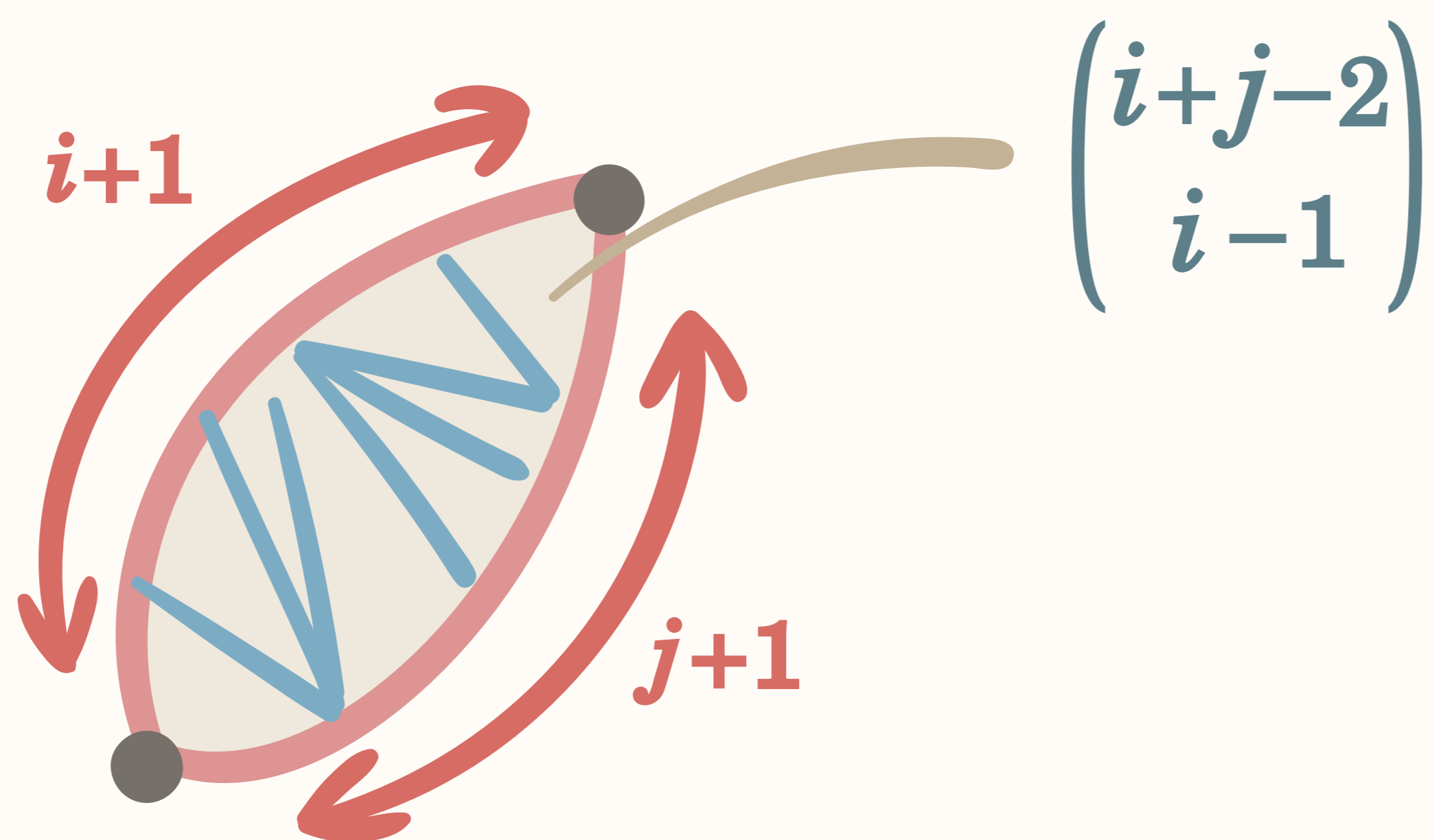
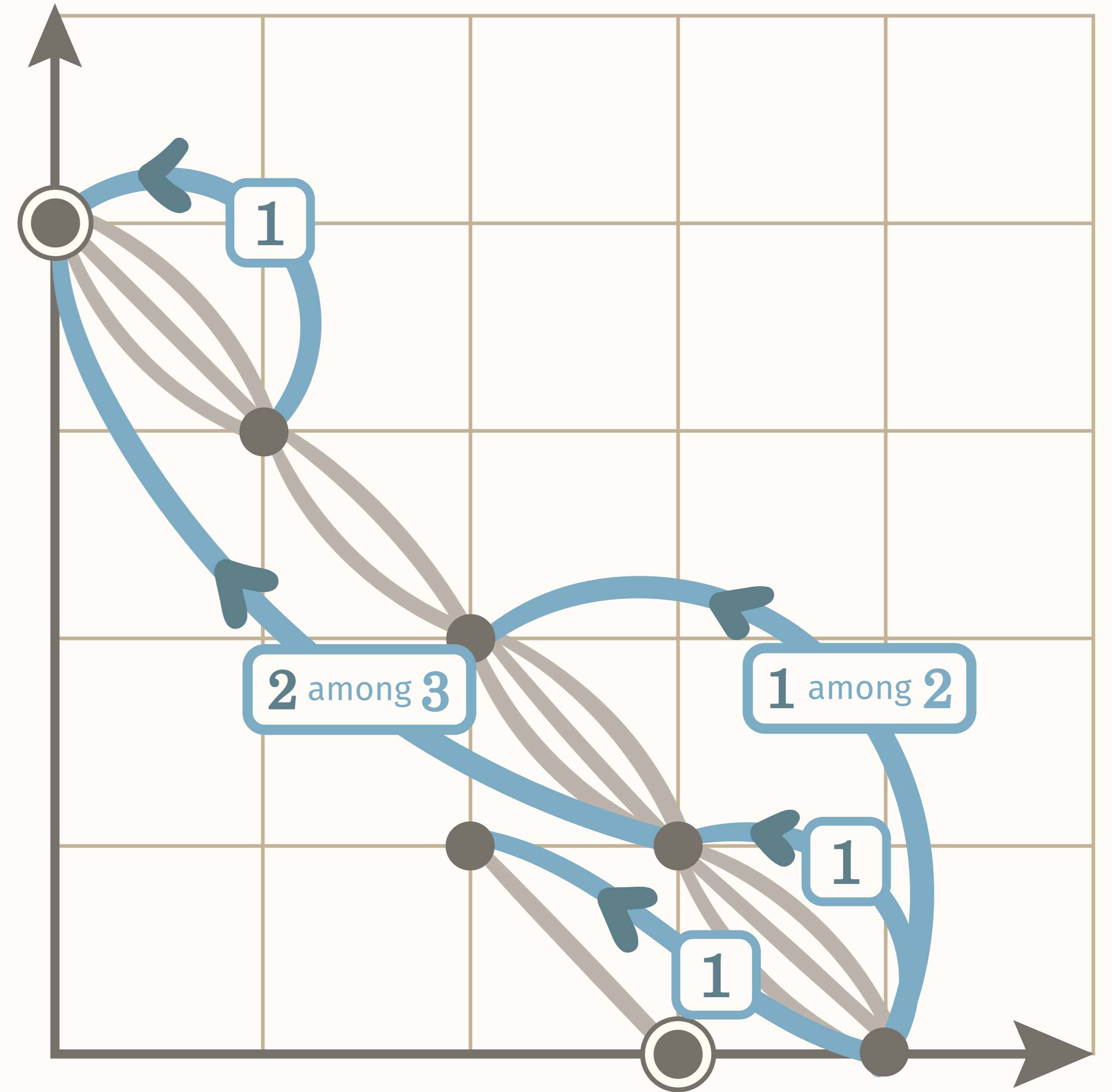
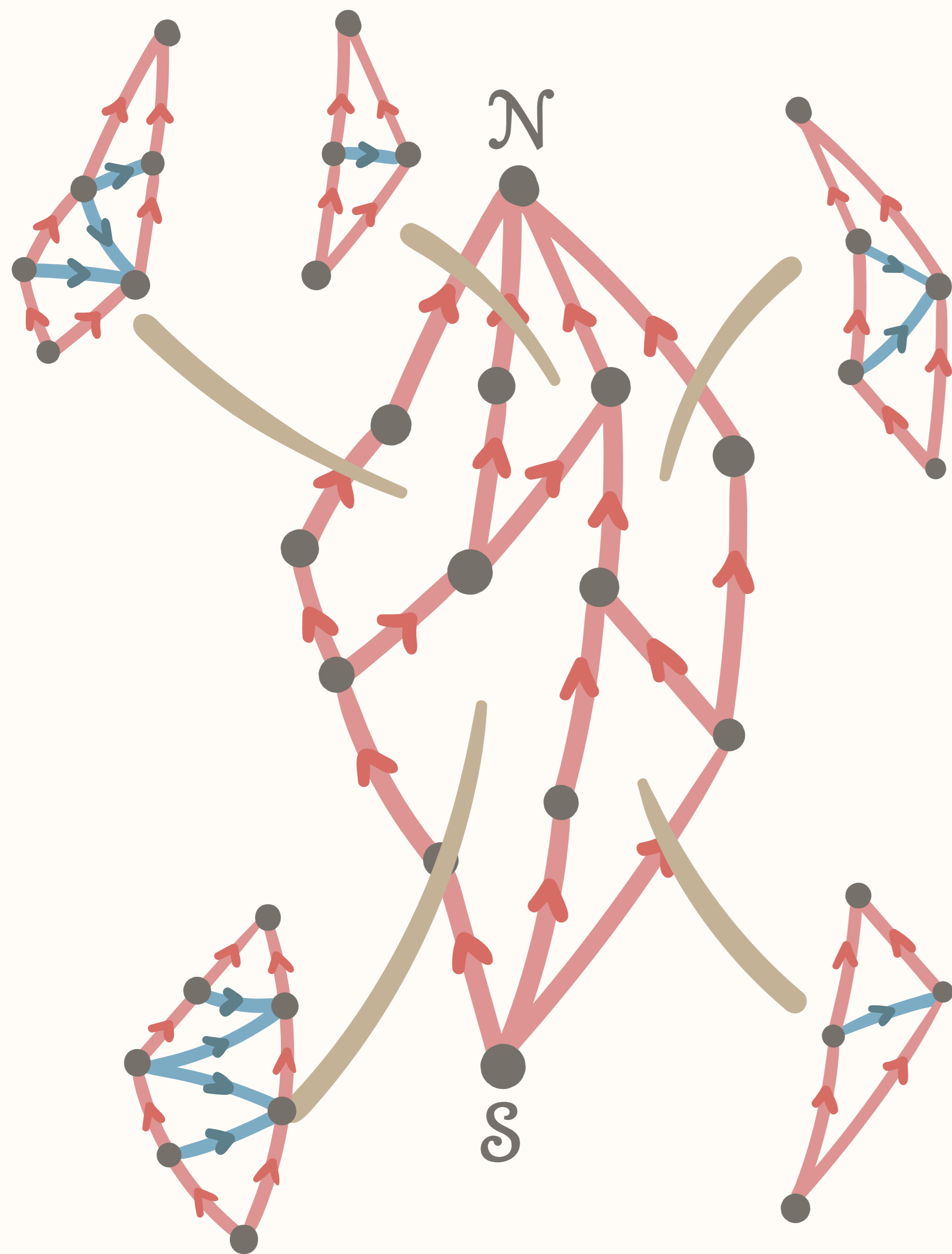
Application to transversal structures



Application to transversal structures

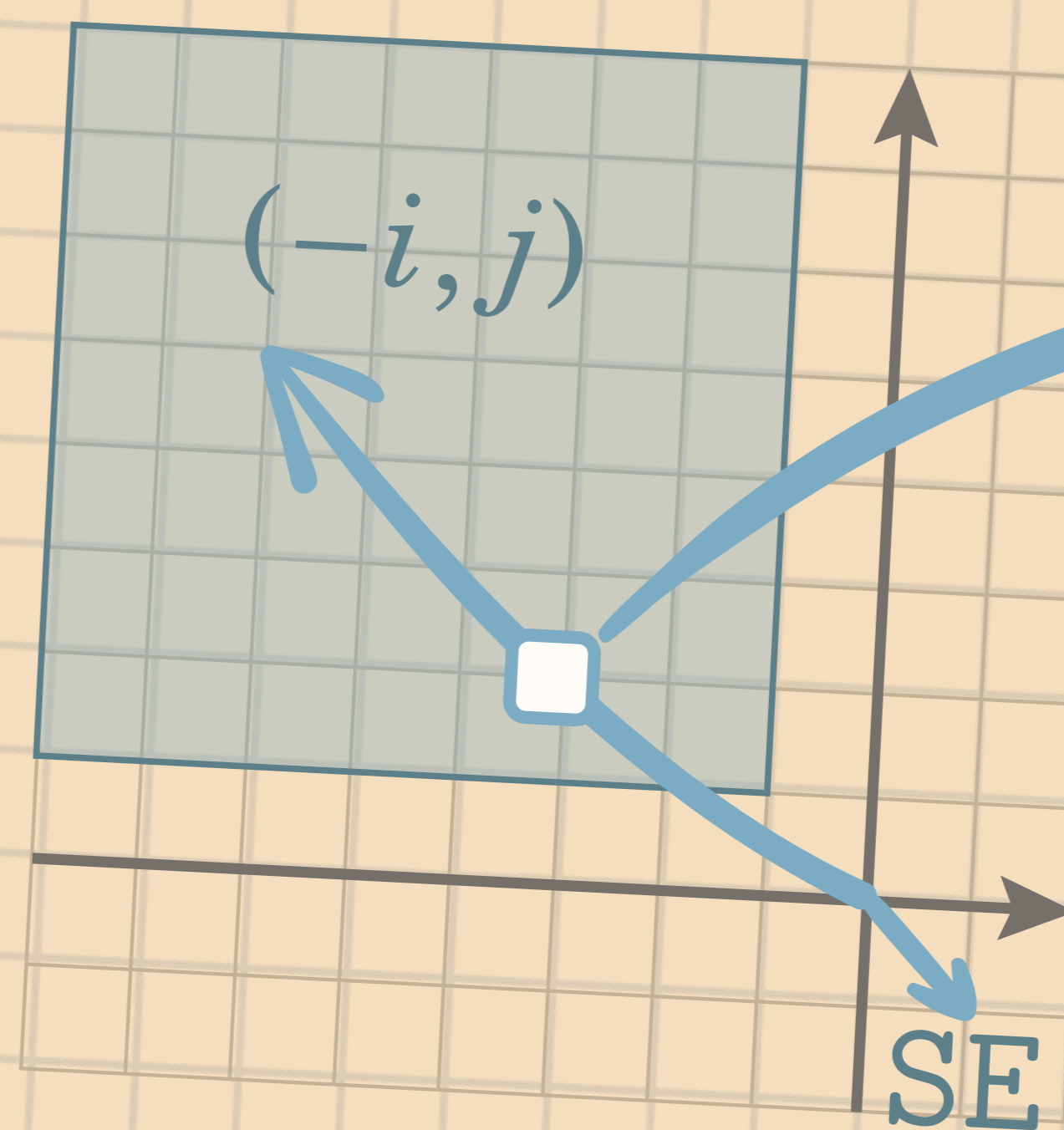


Application to transversal structures



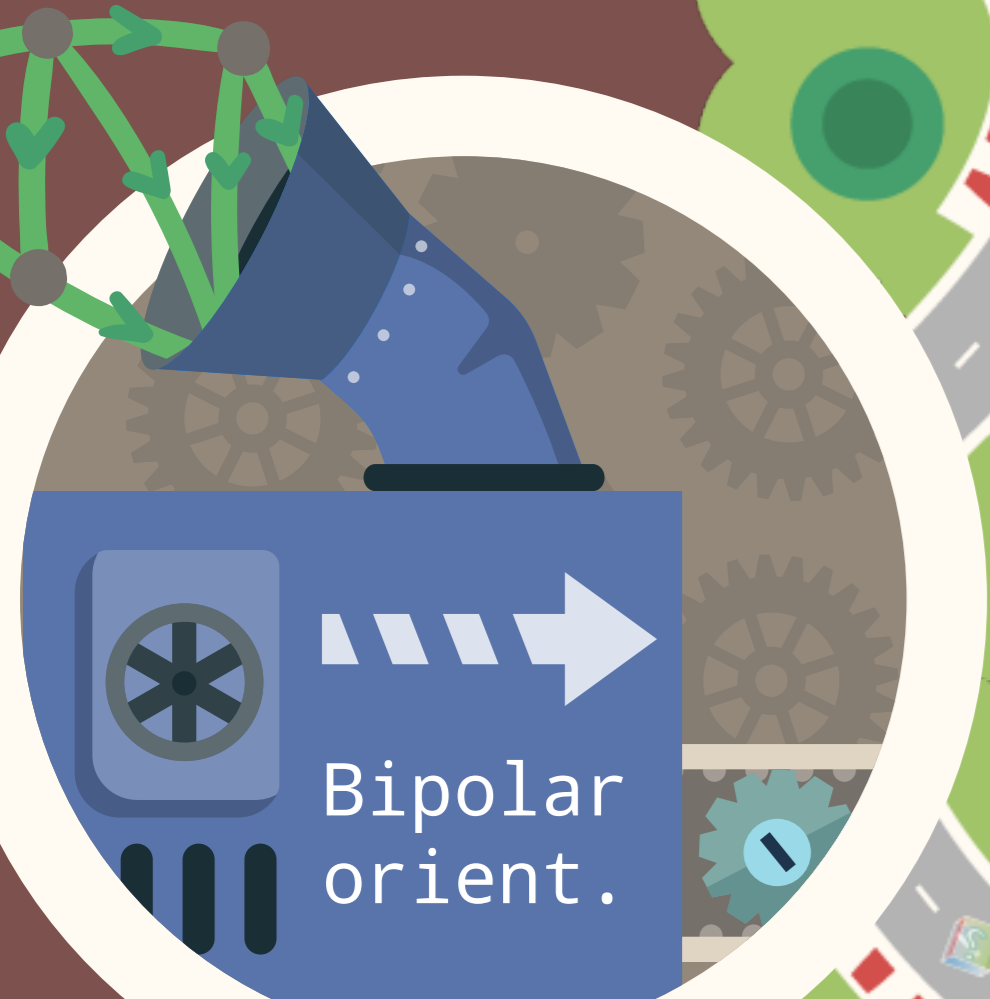
Summary

Quadrant tandem walks



* among $\binom{i+j-2}{i-1}$

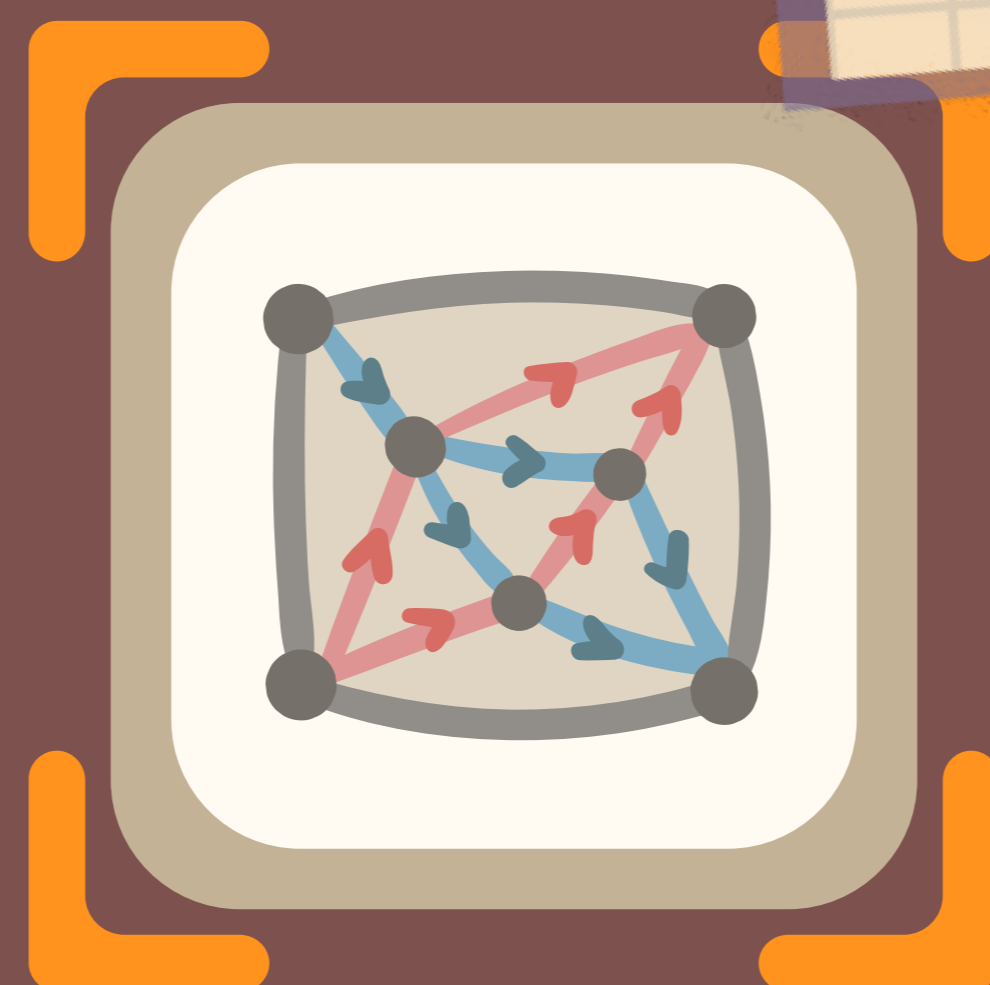
Decorated bipolar orientations



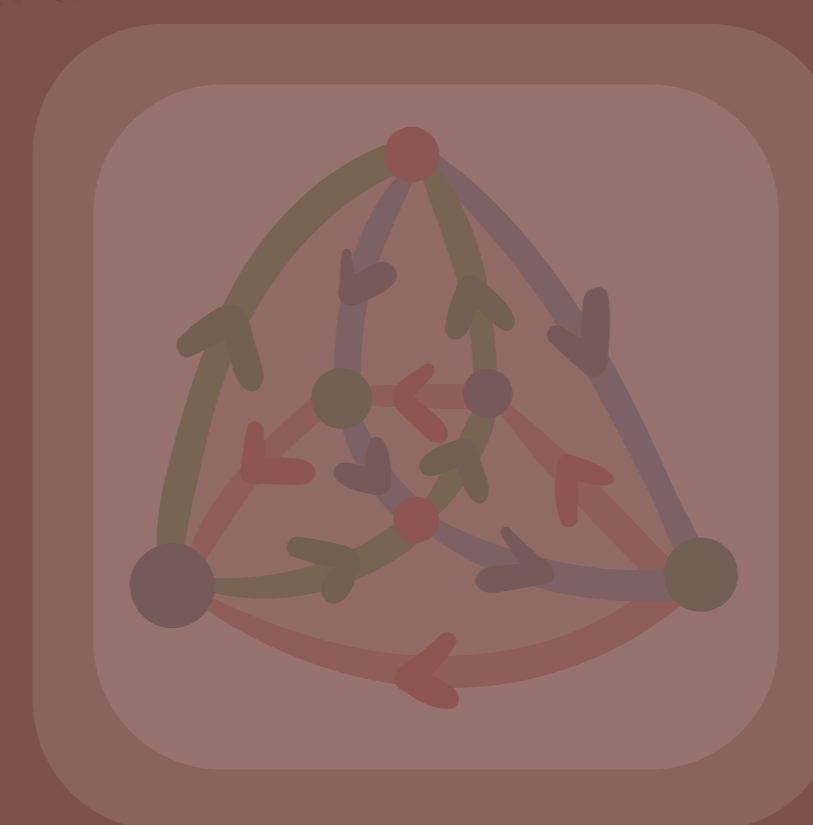
bipolar posets by vertices



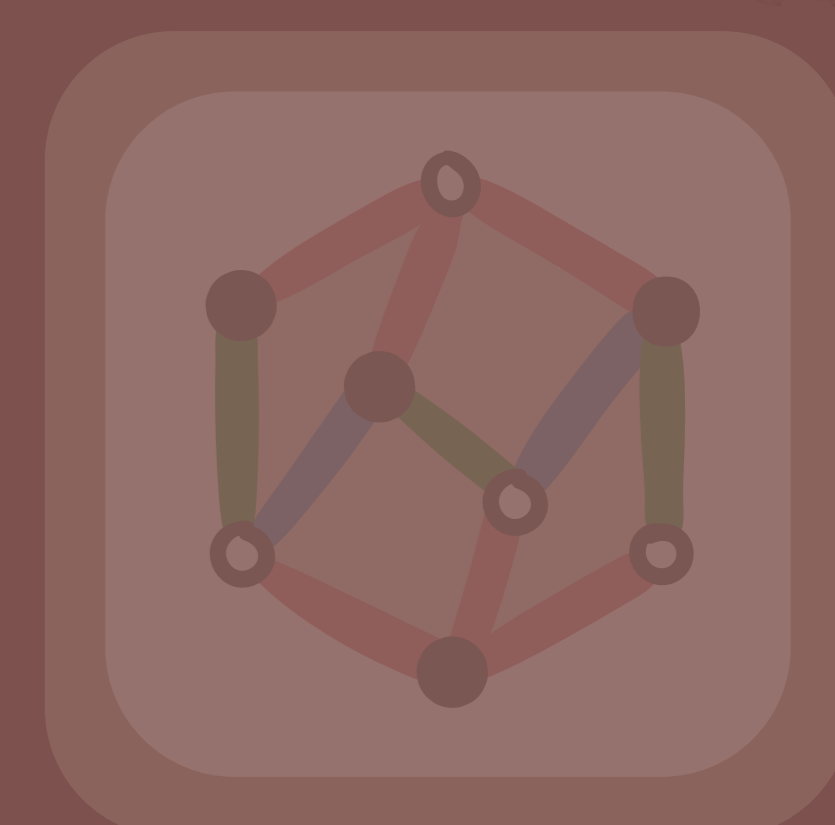
bipolar posets by edges



transversal structures



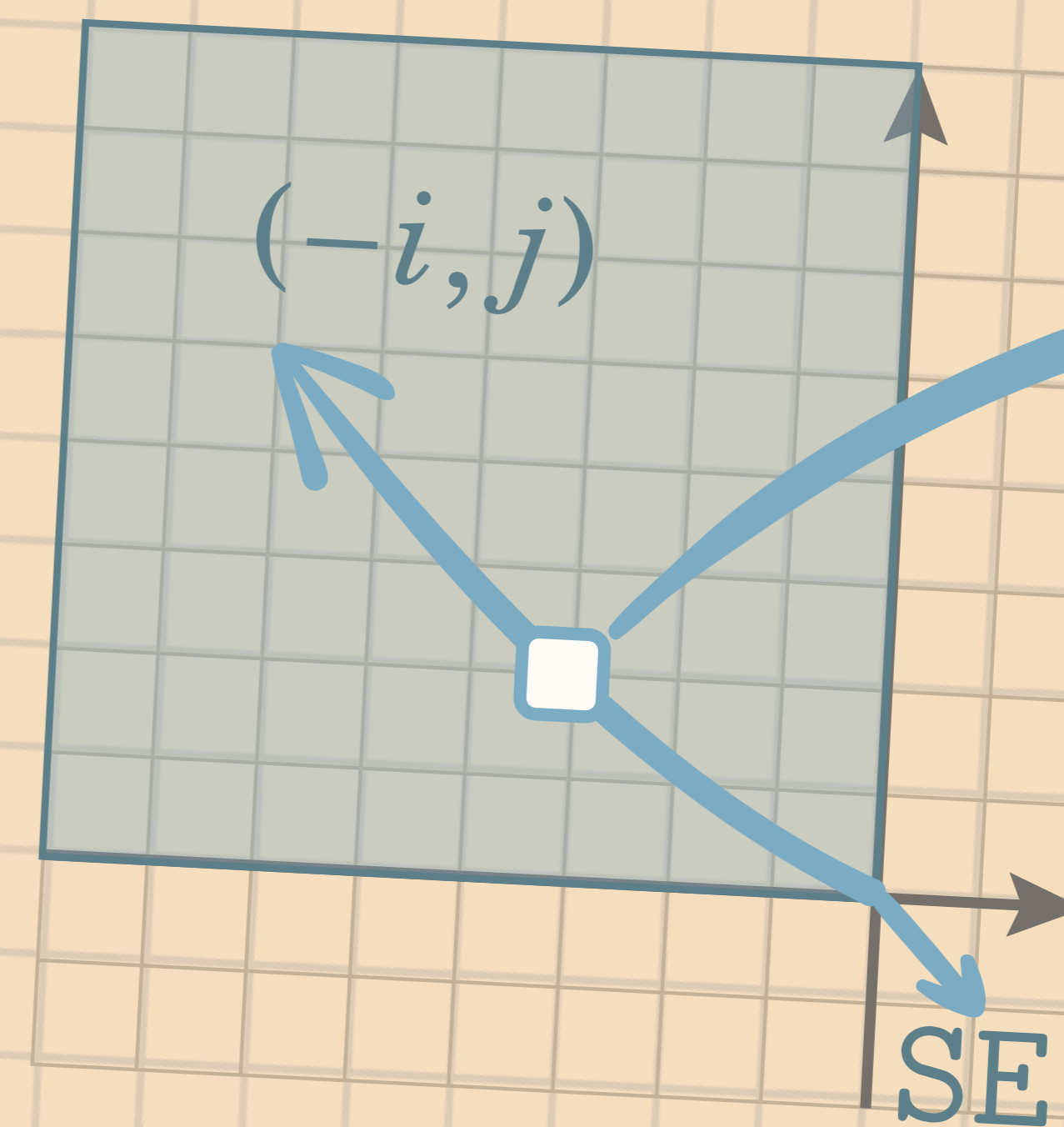
polyhedral orientations



Schnyder labelings

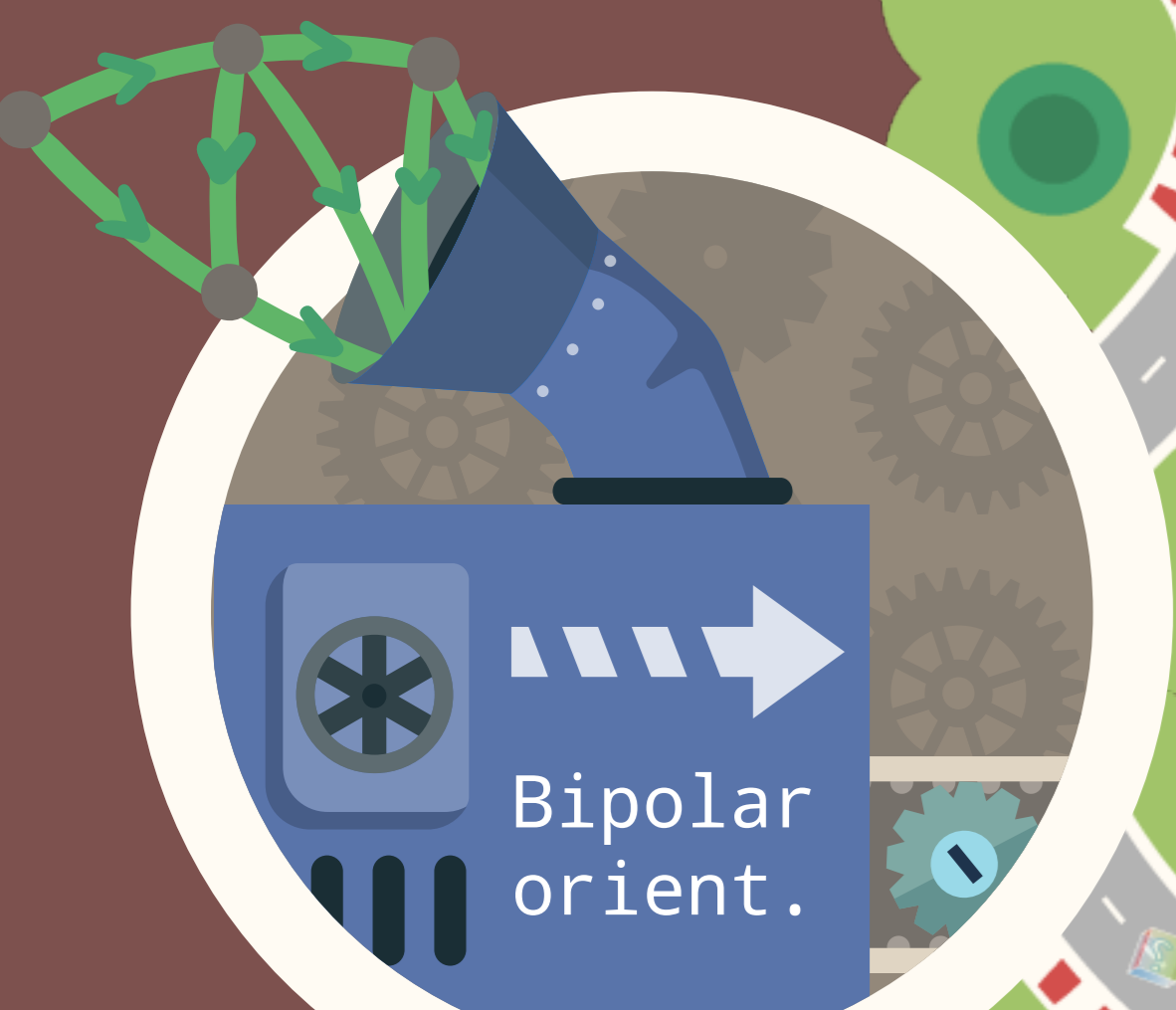
Summary

Quadrant tandem walks



* among $\binom{i+j}{i}$

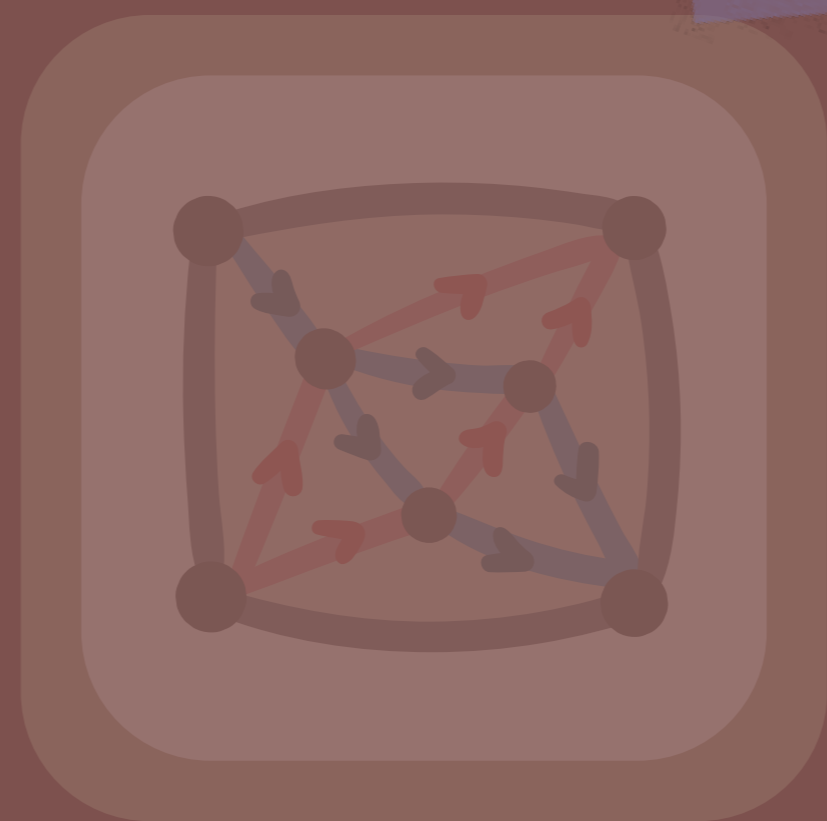
Decorated bipolar orientations



*bipolar posets
by vertices*



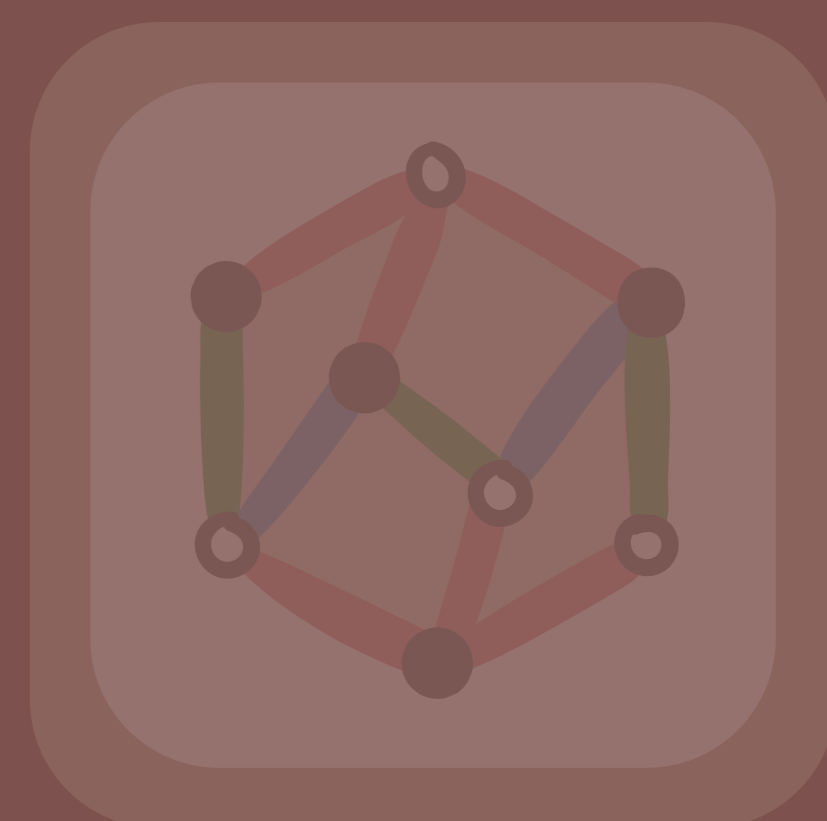
*bipolar posets
by edges*



*transversal
structures*



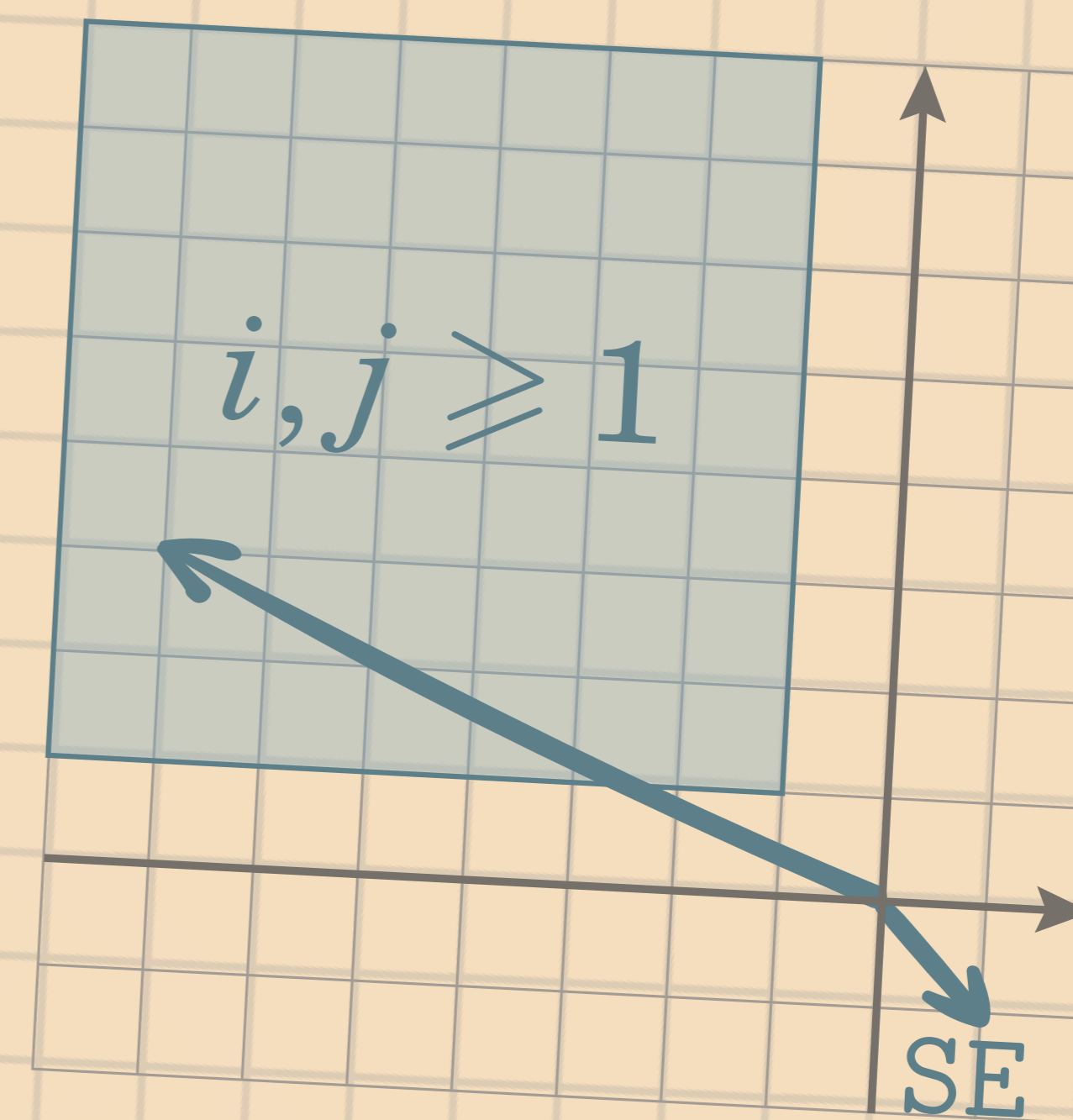
*polyhedral
orientations*



*Schnyder
labelings*

Summary

Quadrant
tandem walks



Bipolar
orient.

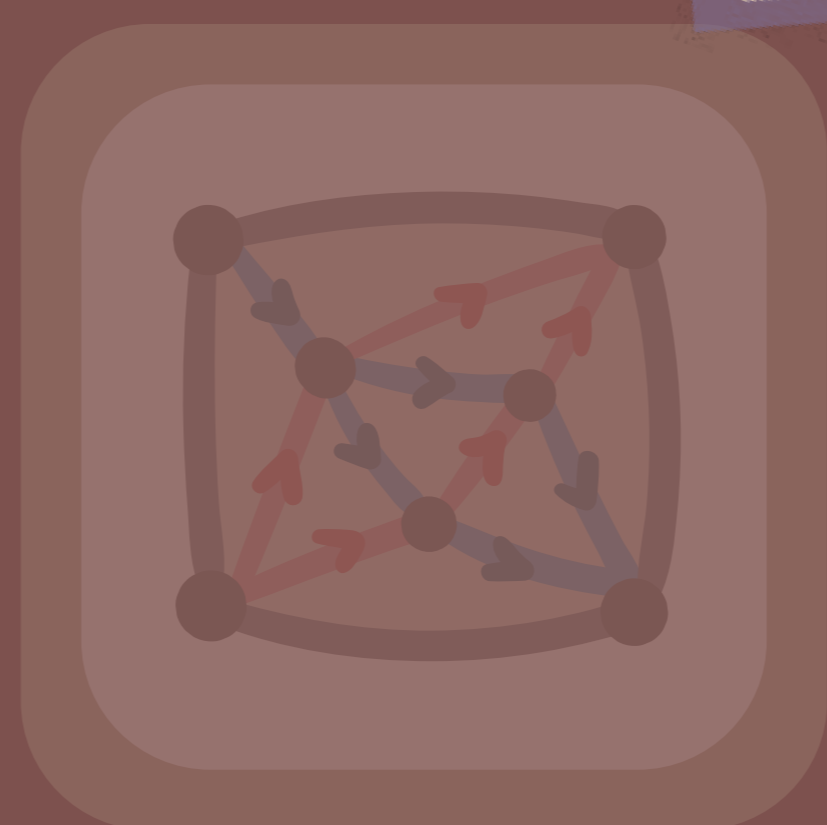
Decorated bipolar
orientations



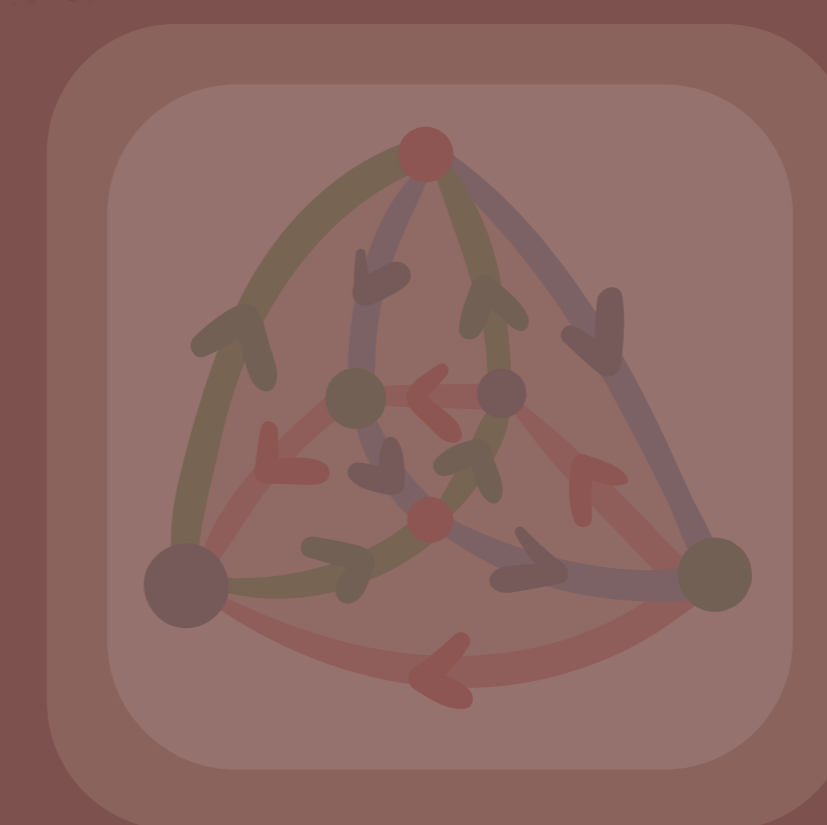
*bipolar posets
by vertices*



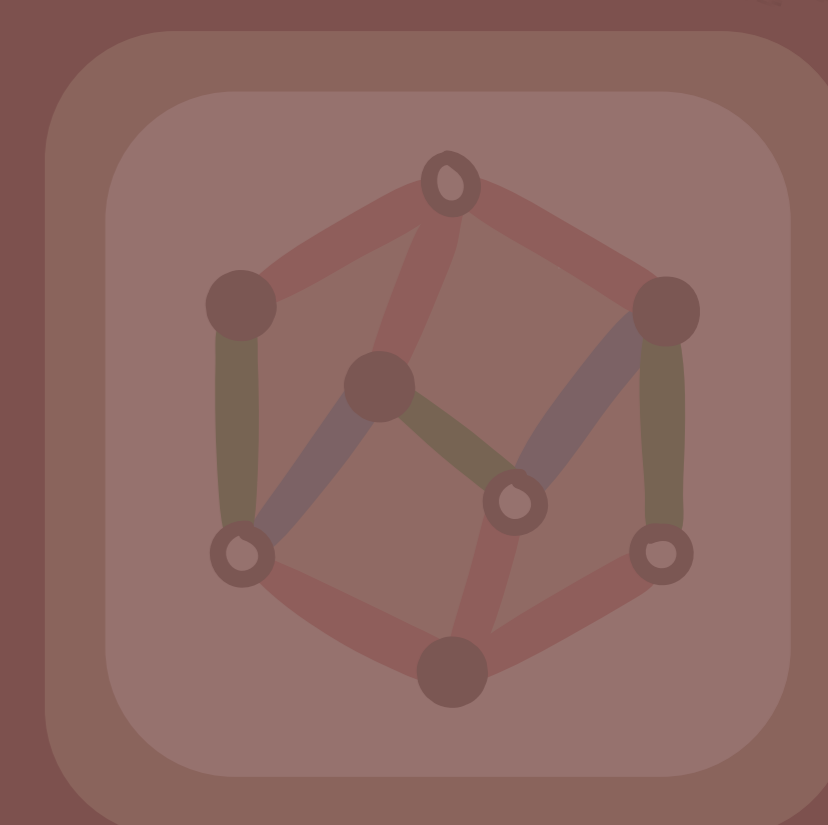
*bipolar posets
by edges*



*transversal
structures*



*polyhedral
orientations*



*Schnyder
labelings*

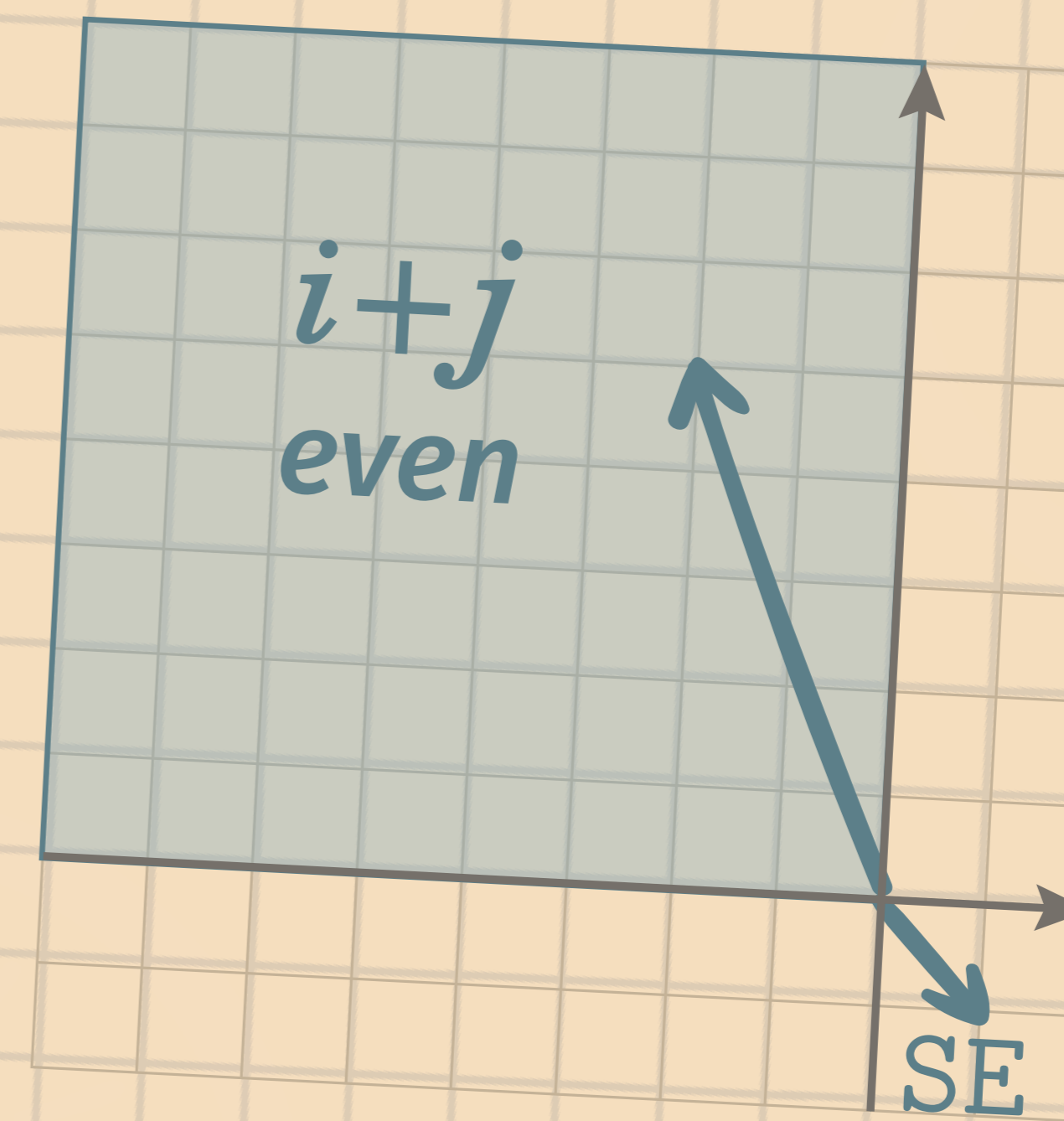
Summary

Quadrant
tandem walks



Bipolar
orient.

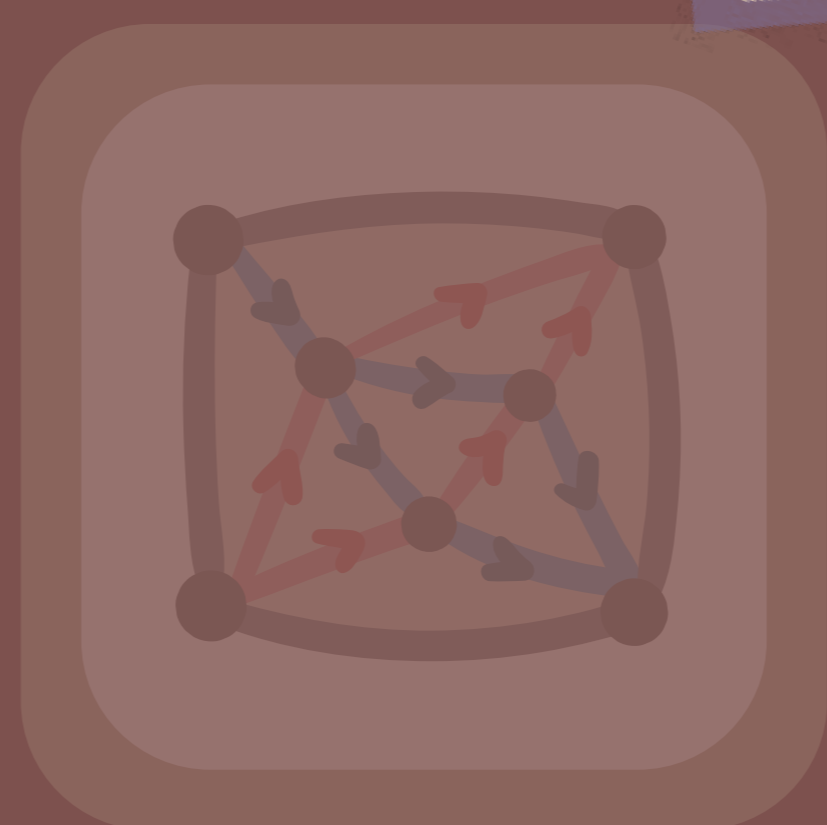
Decorated bipolar
orientations



*bipolar posets
by vertices*



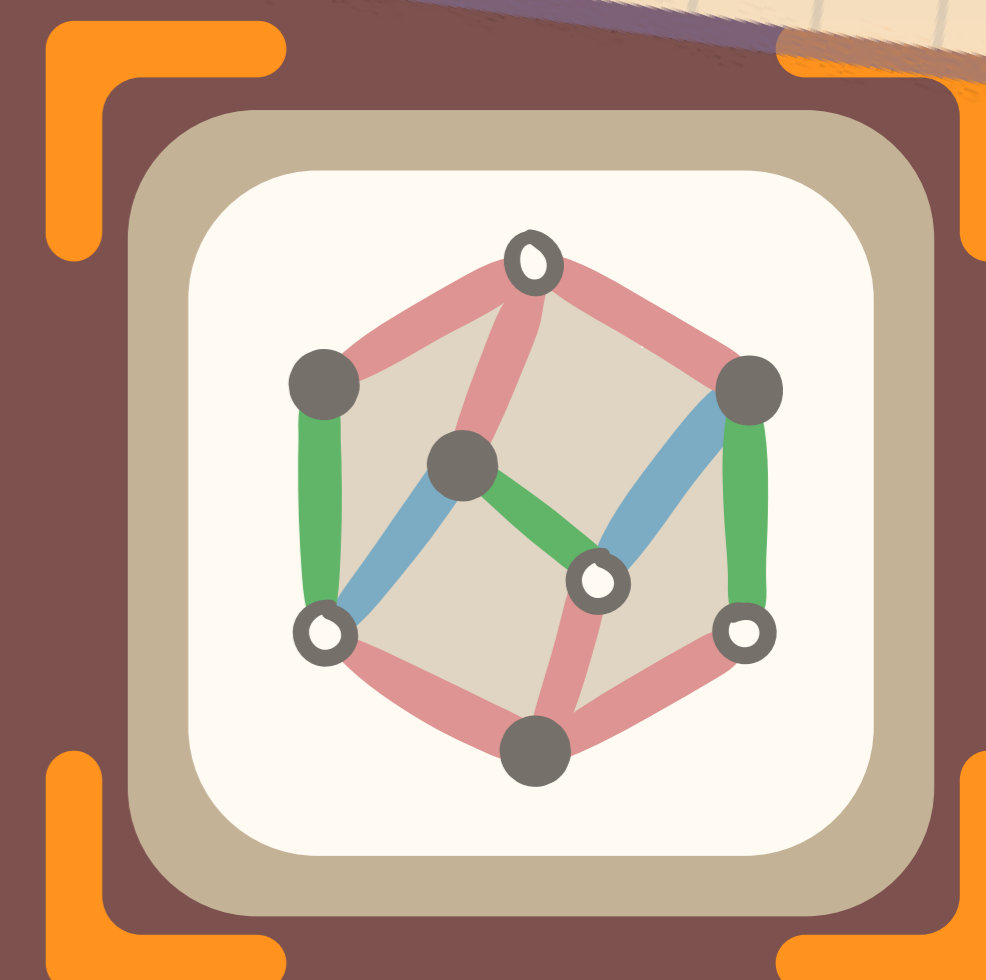
*bipolar posets
by edges*



*transversal
structures*



*polyhedral
orientations*

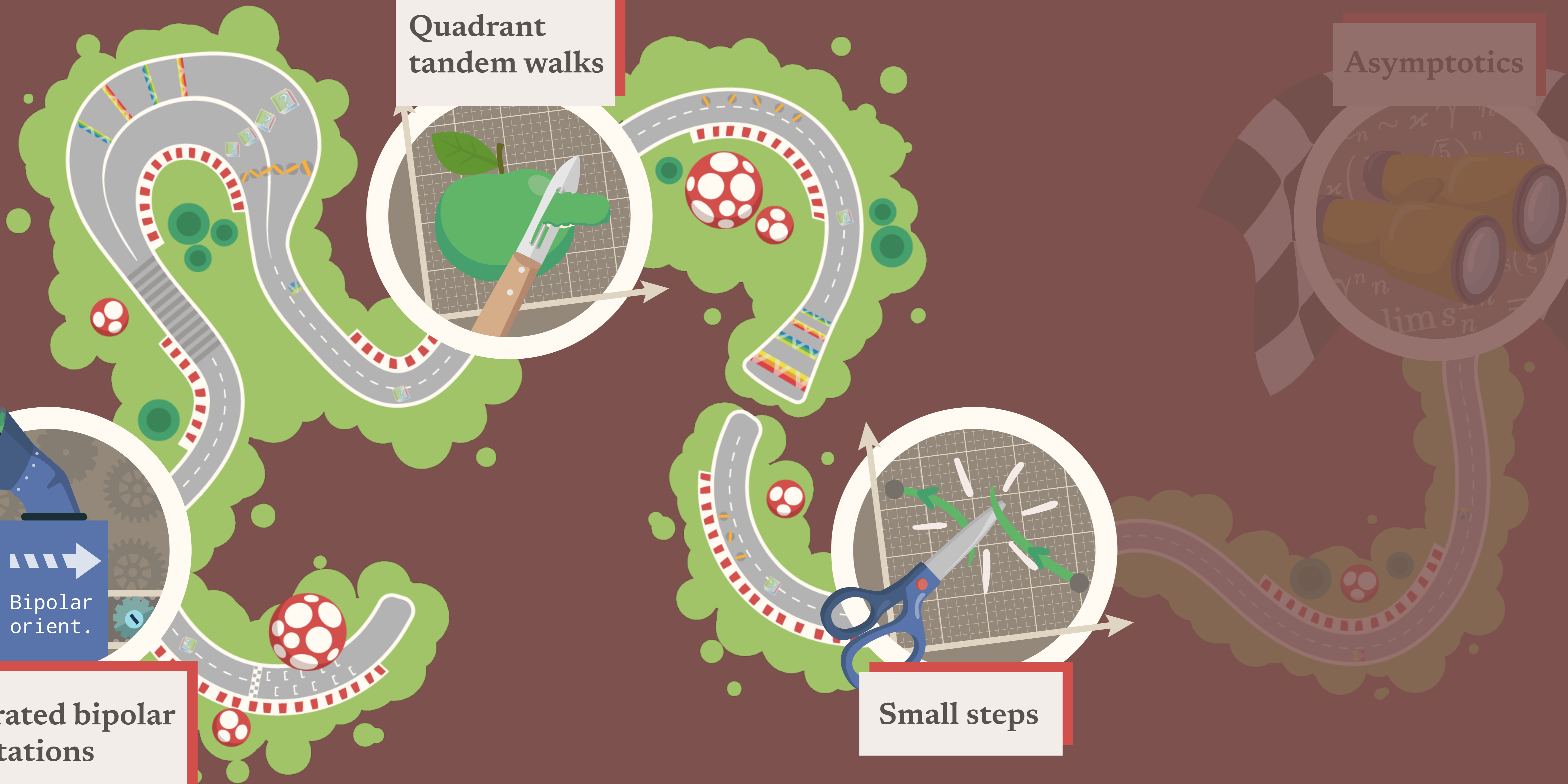


*Schnyder
labelings*

Summary

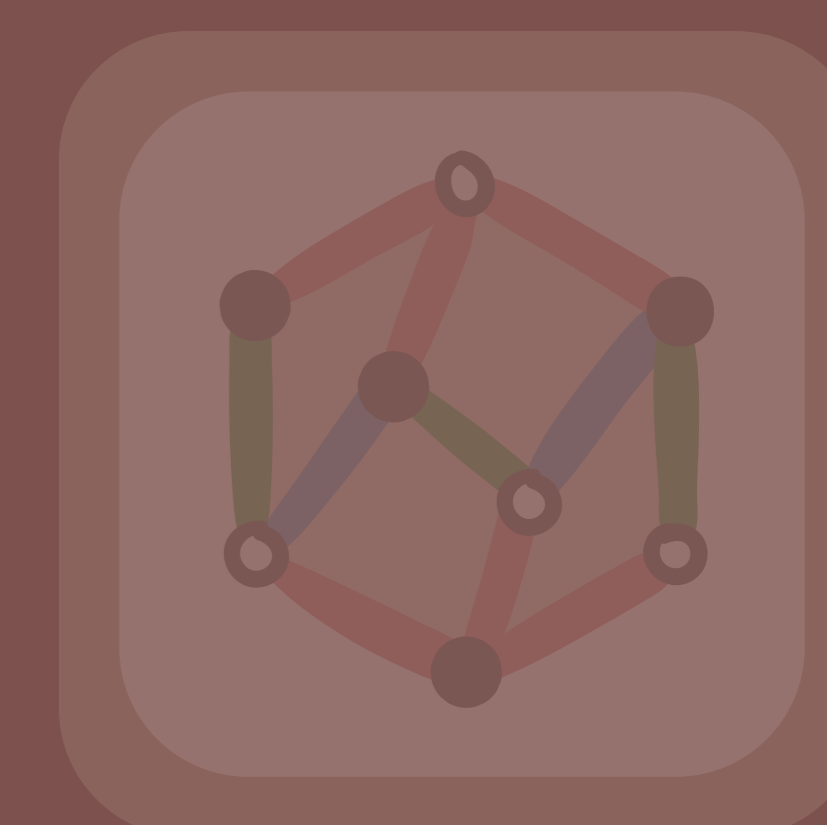
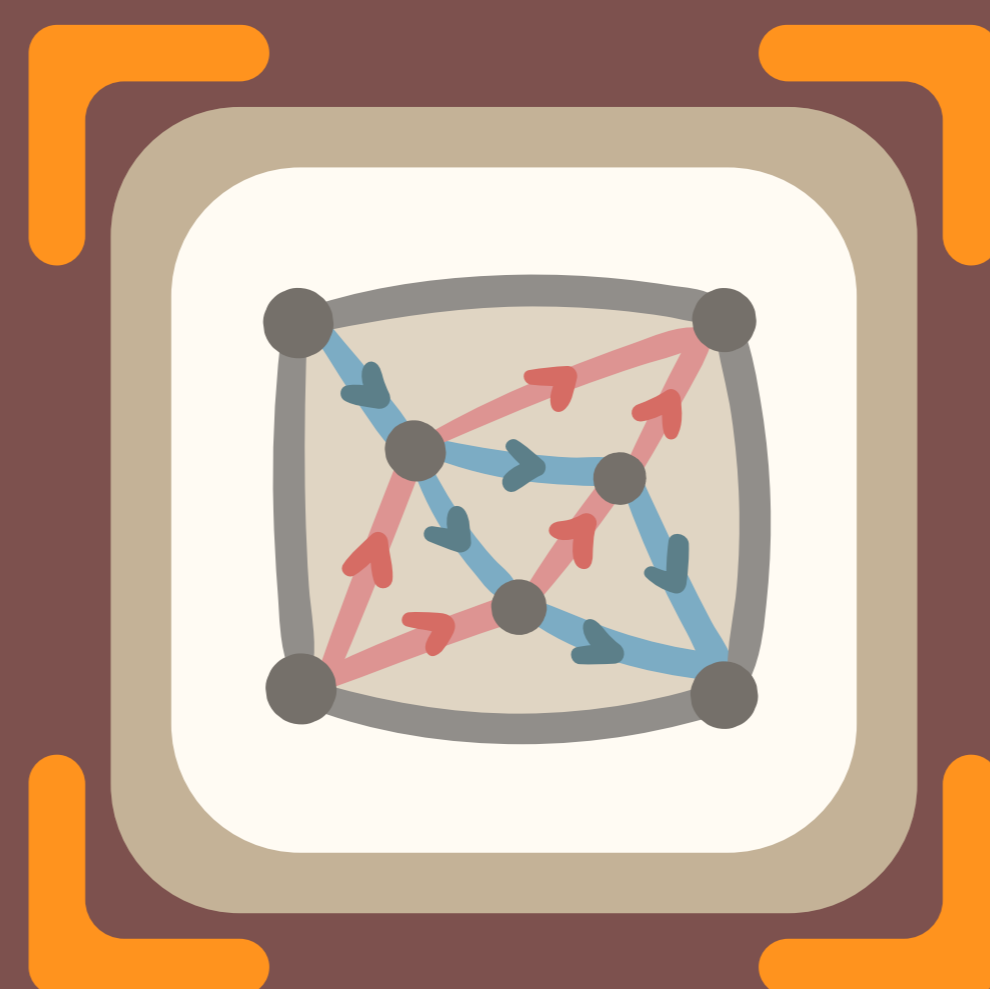
Quadrant
tandem walks

Asymptotics



Decorated bipolar
orientations

Small steps



*bipolar posets
by vertices*

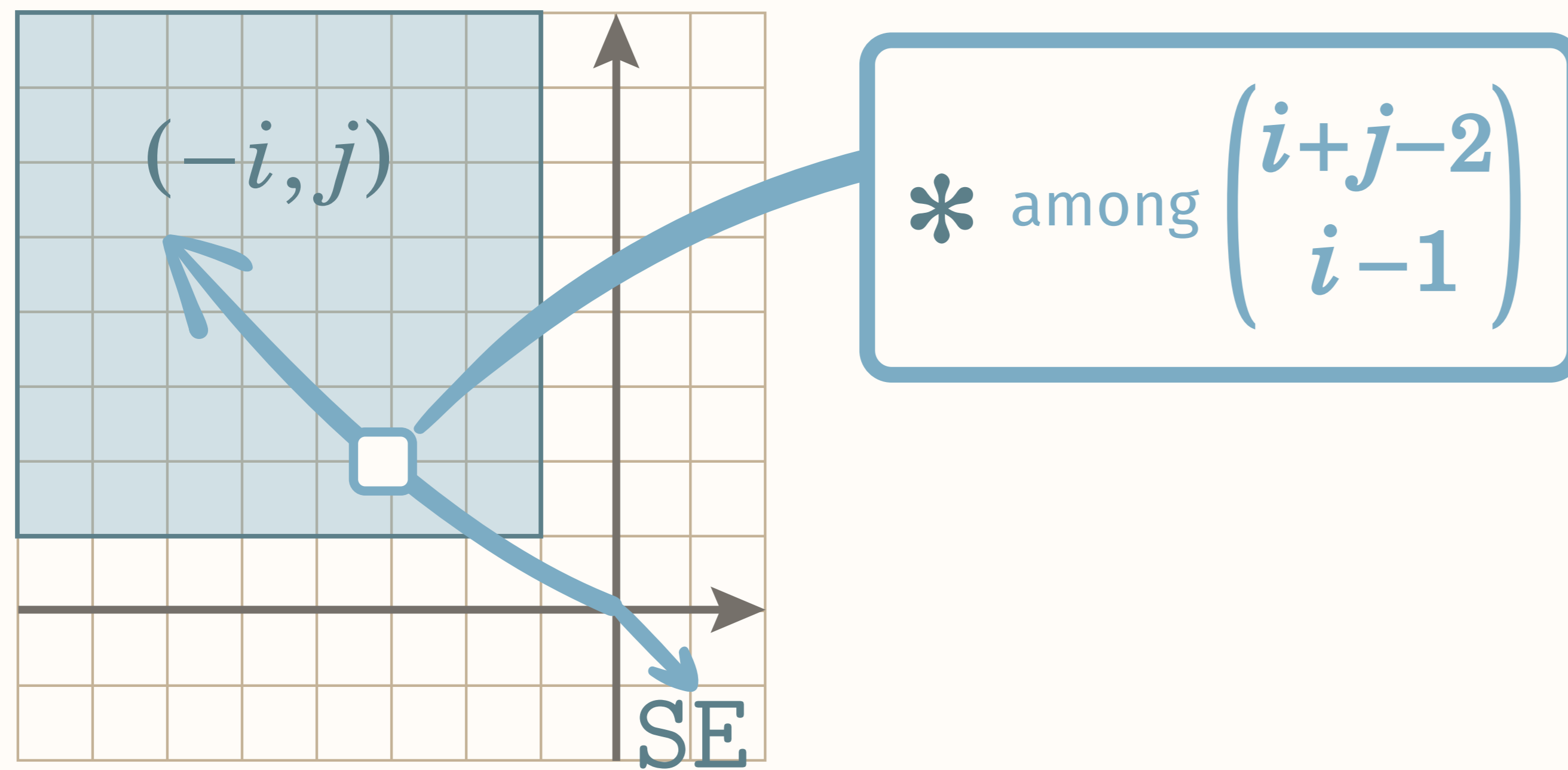
*bipolar posets
by edges*

*transversal
structures*

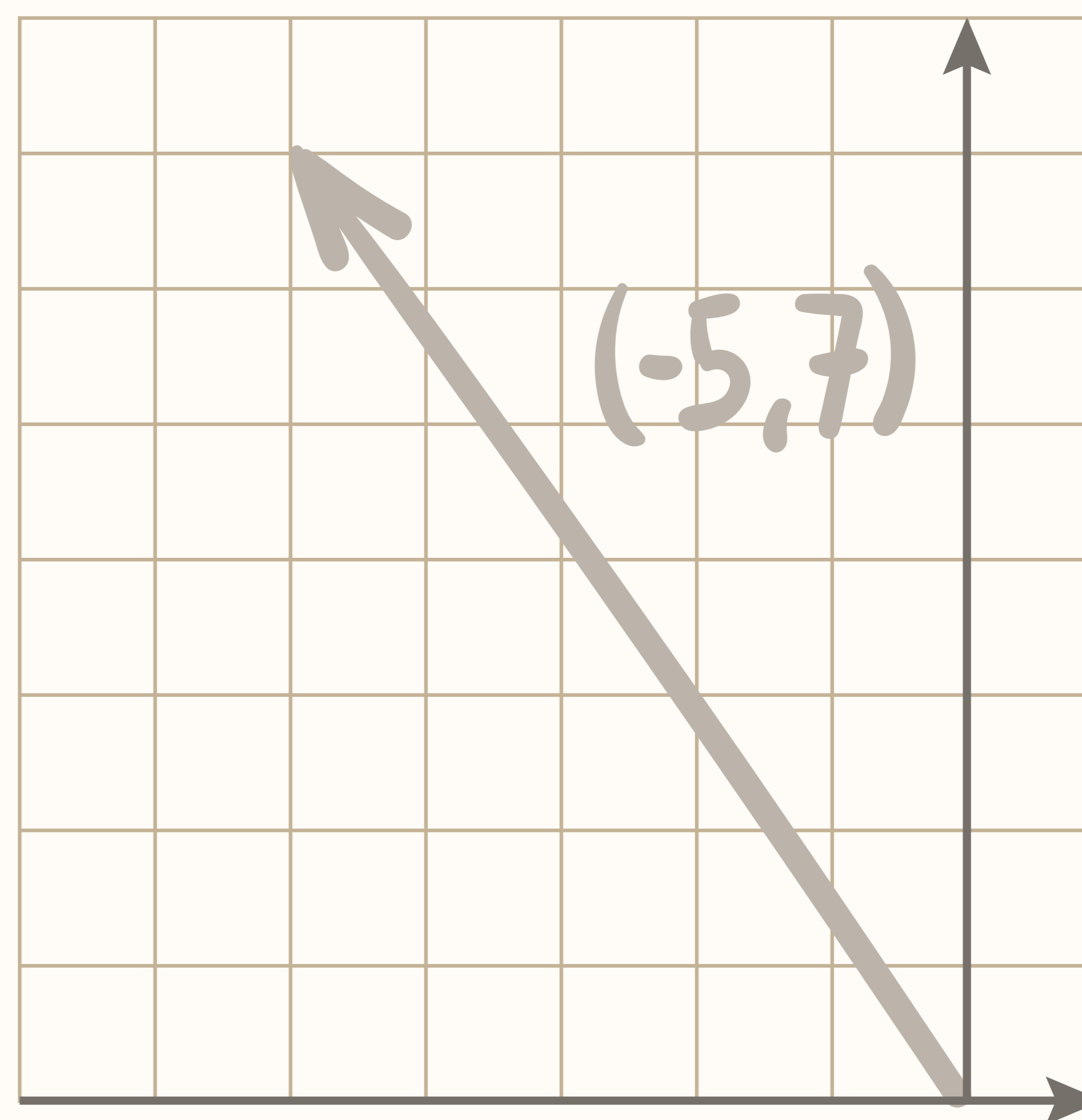
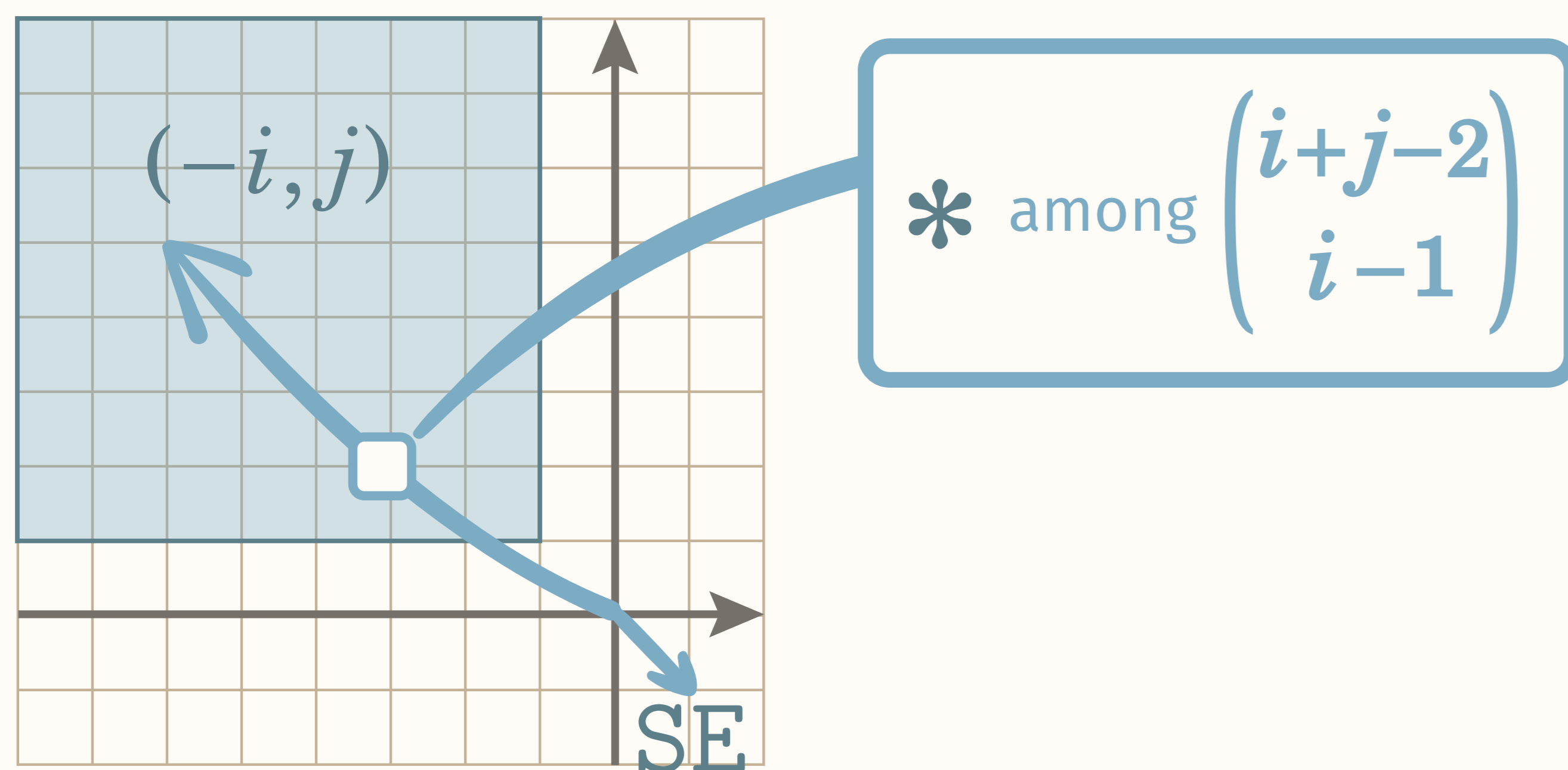
*polyhedral
orientations*

*Schnyder
labelings*

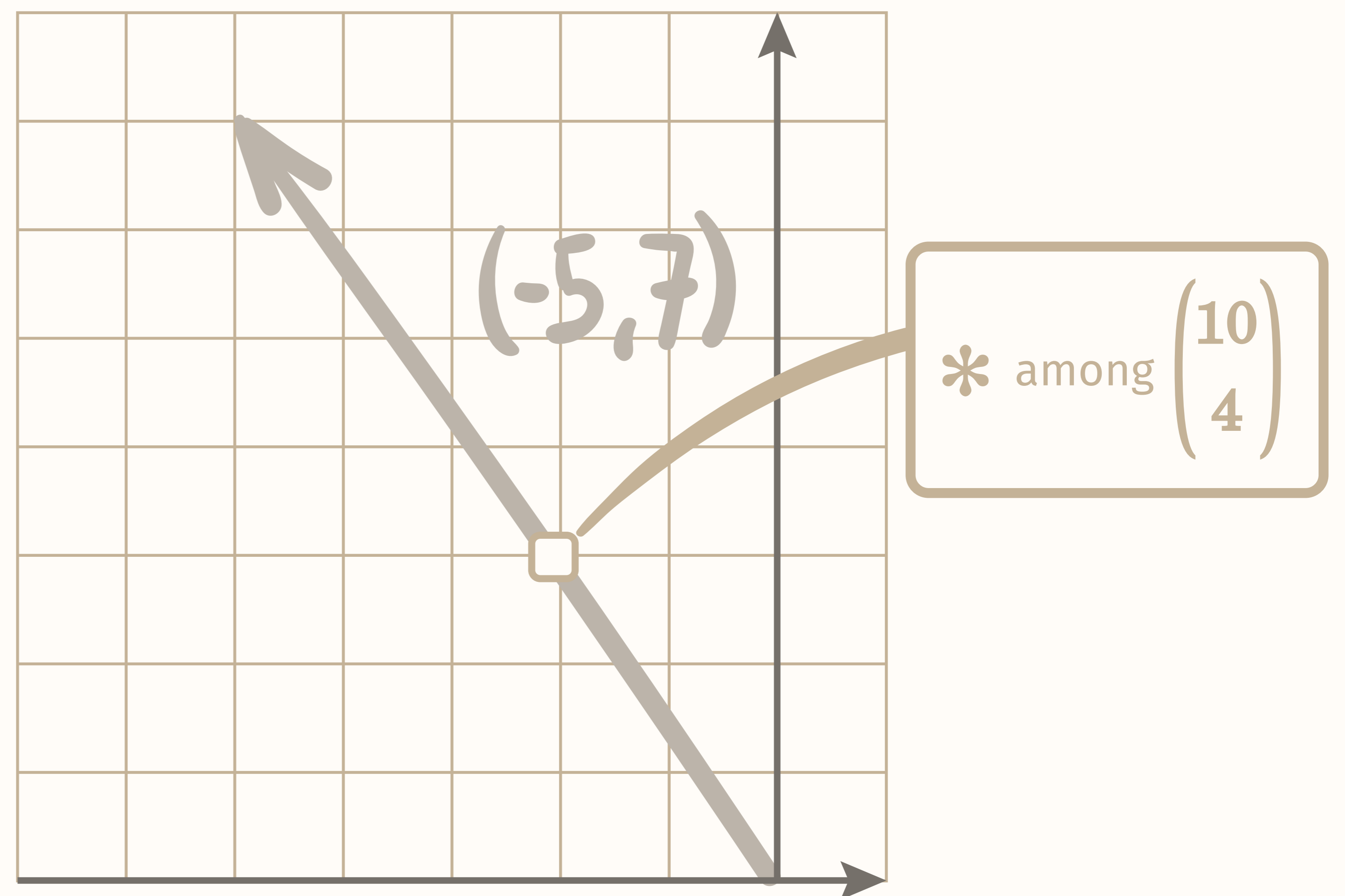
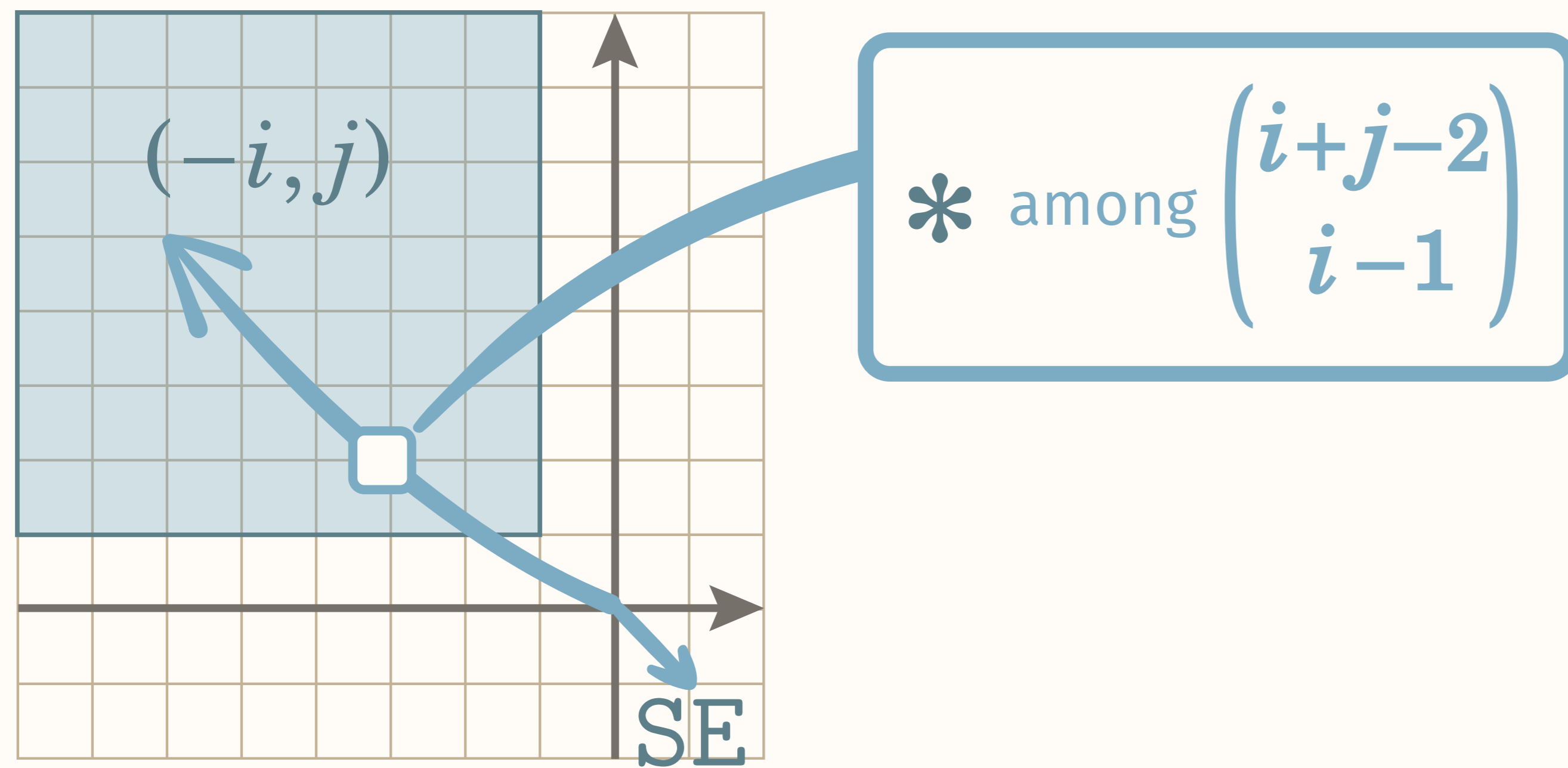
Breaking into small steps



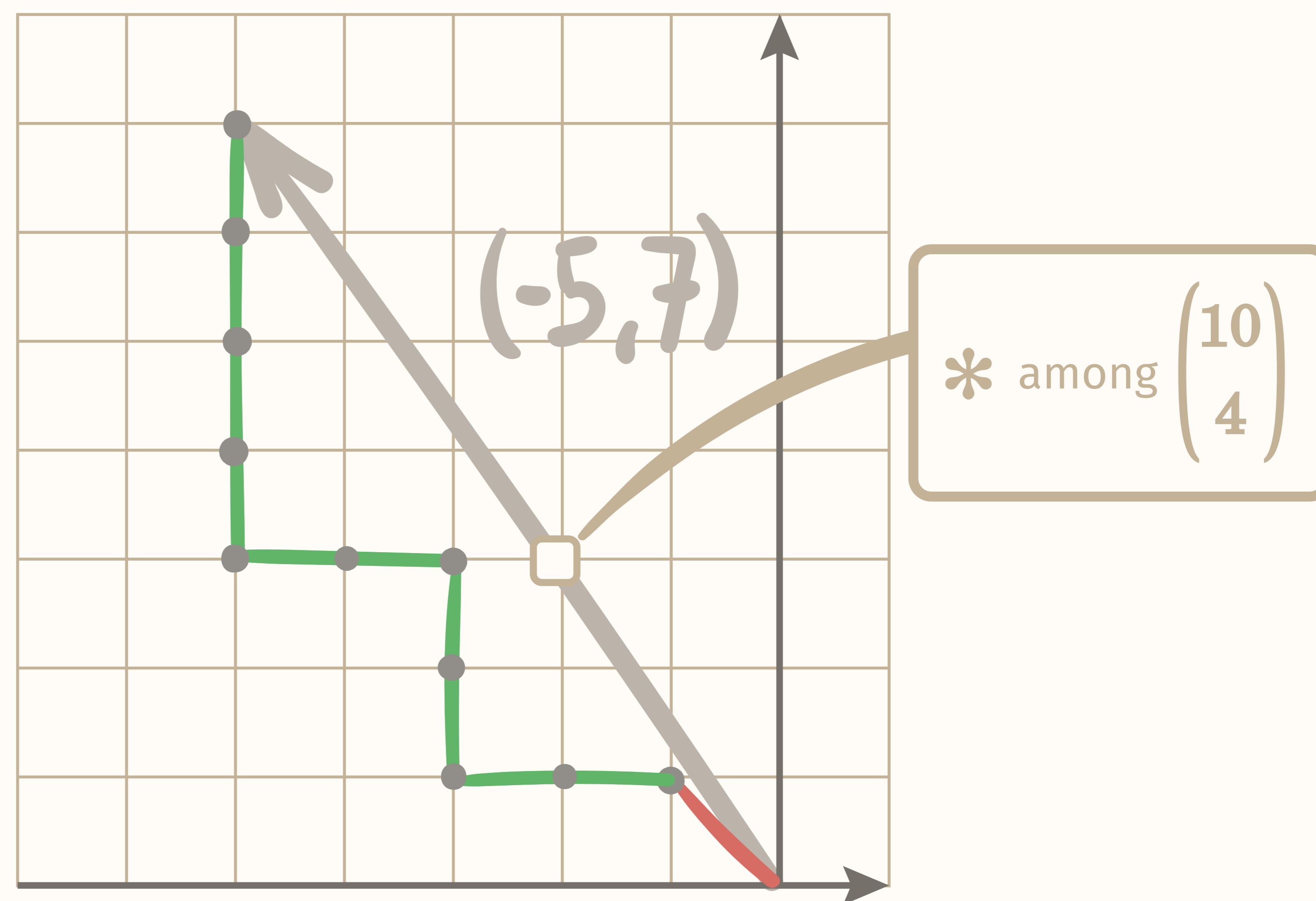
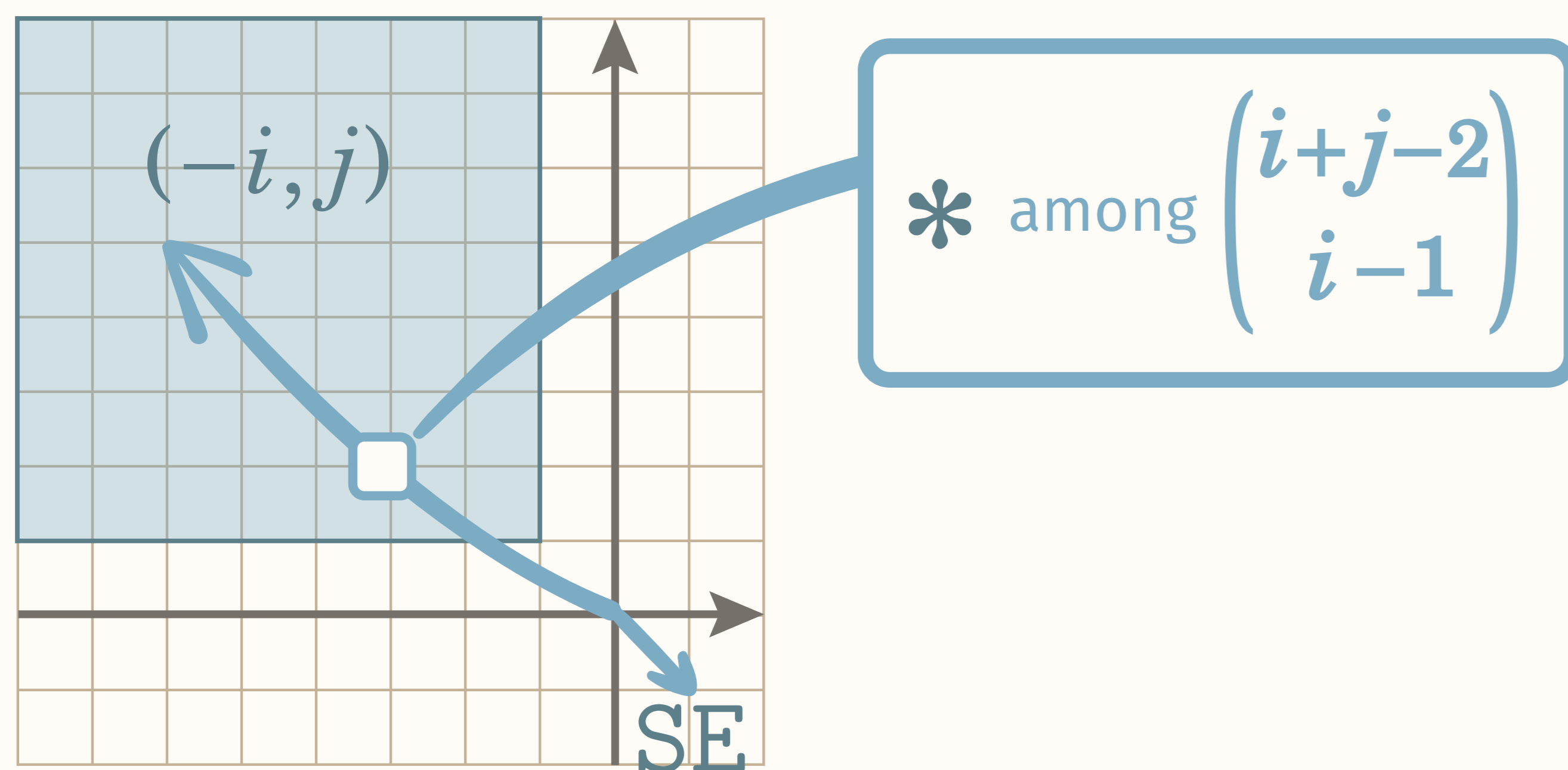
Breaking into small steps



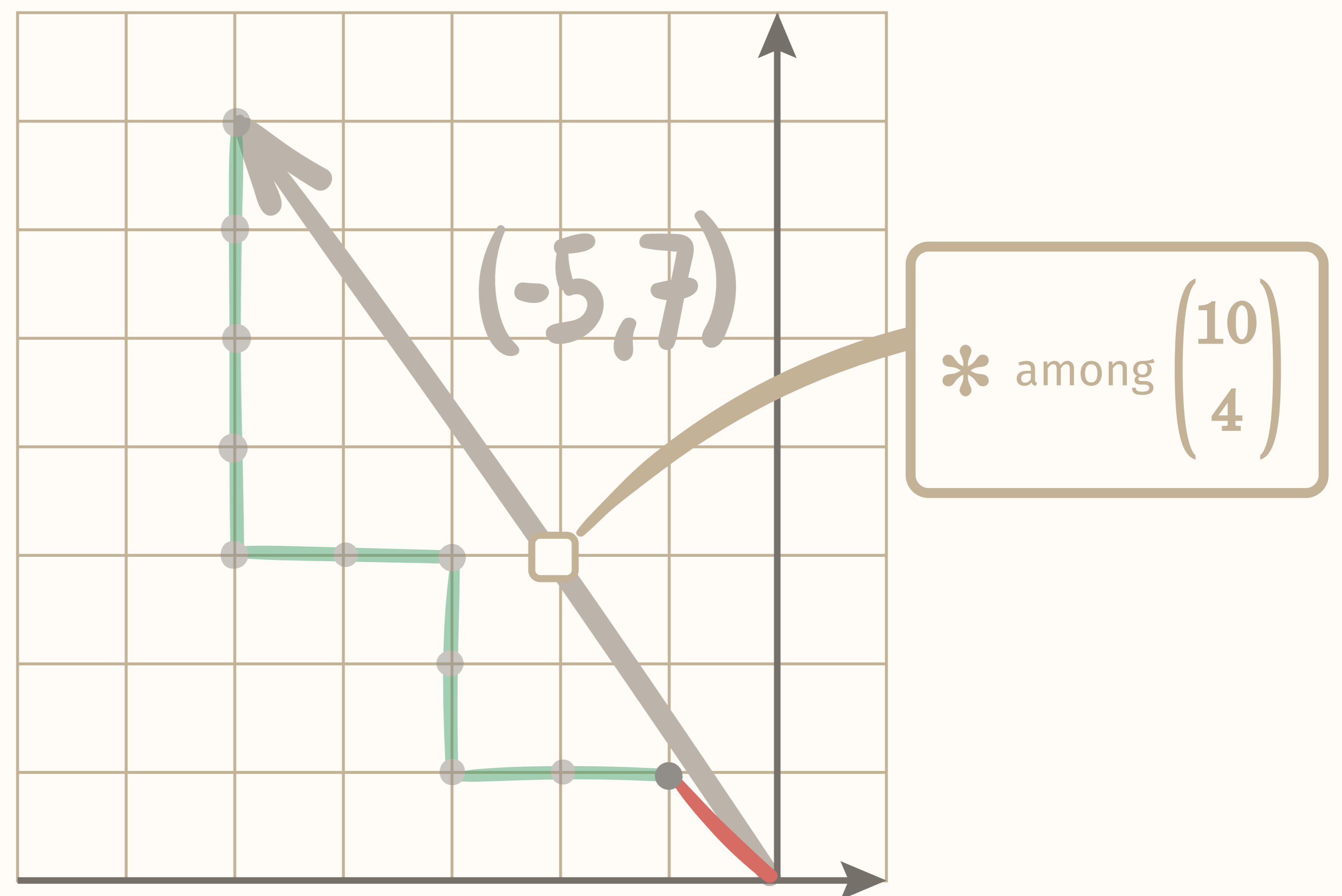
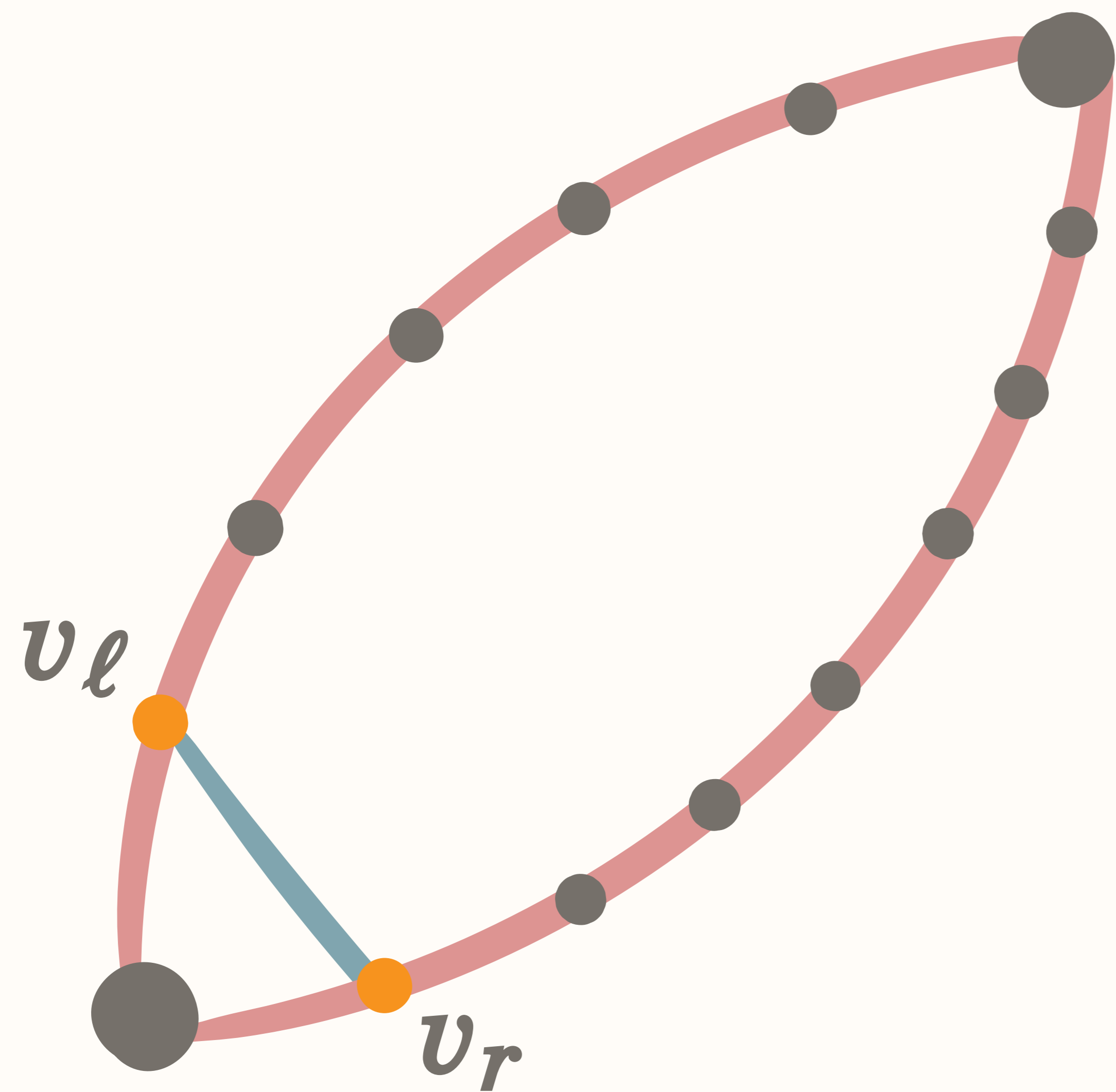
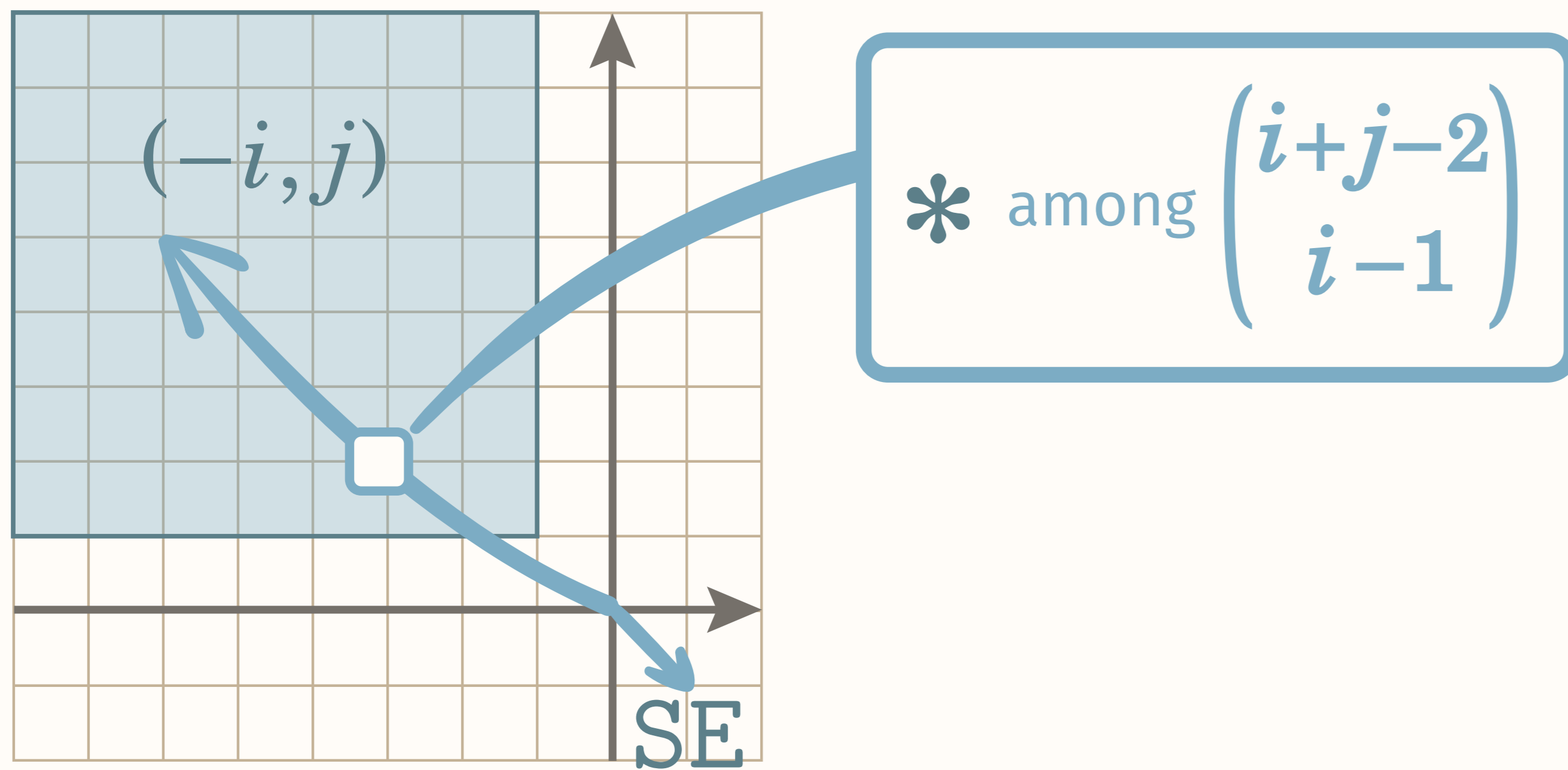
Breaking into small steps



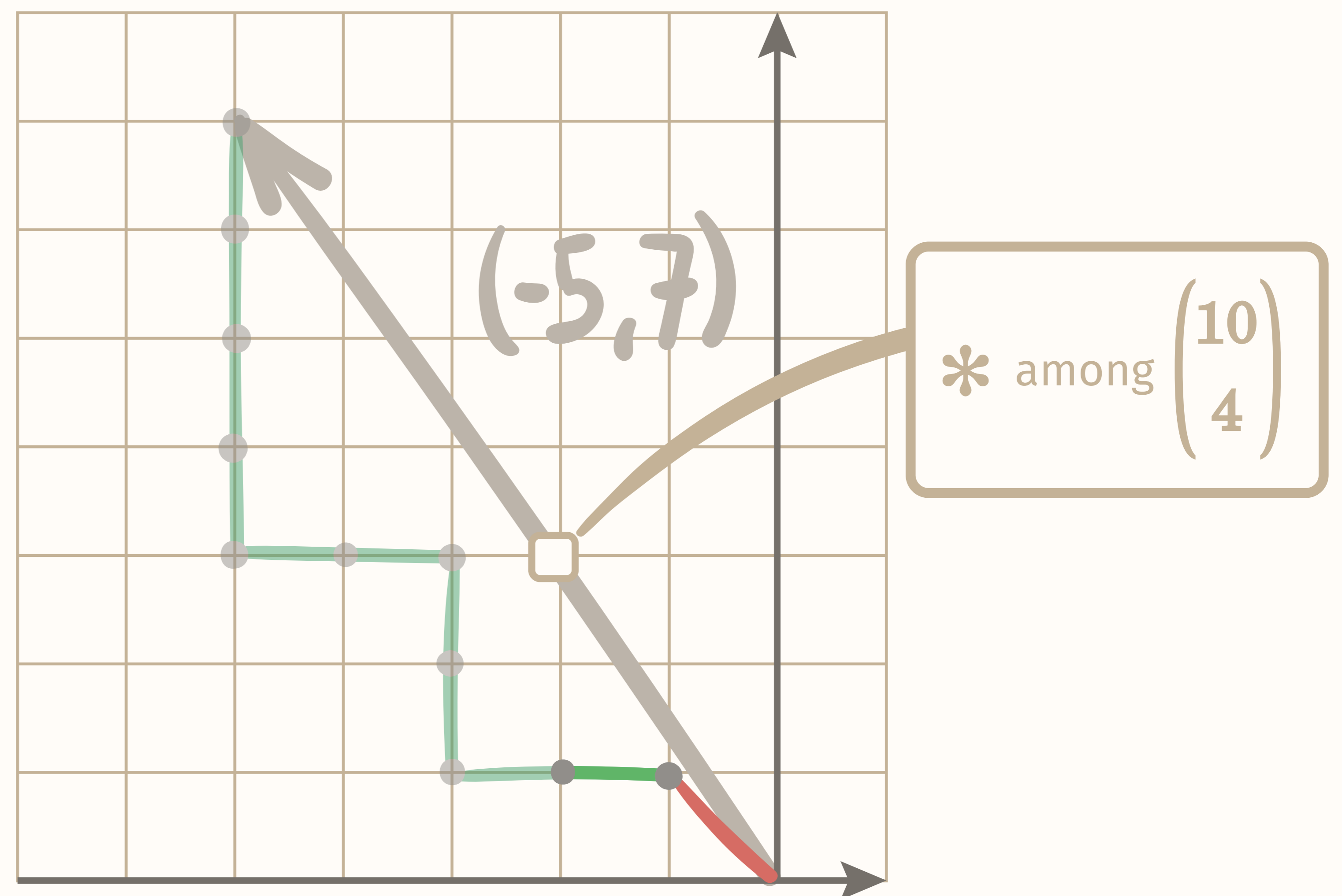
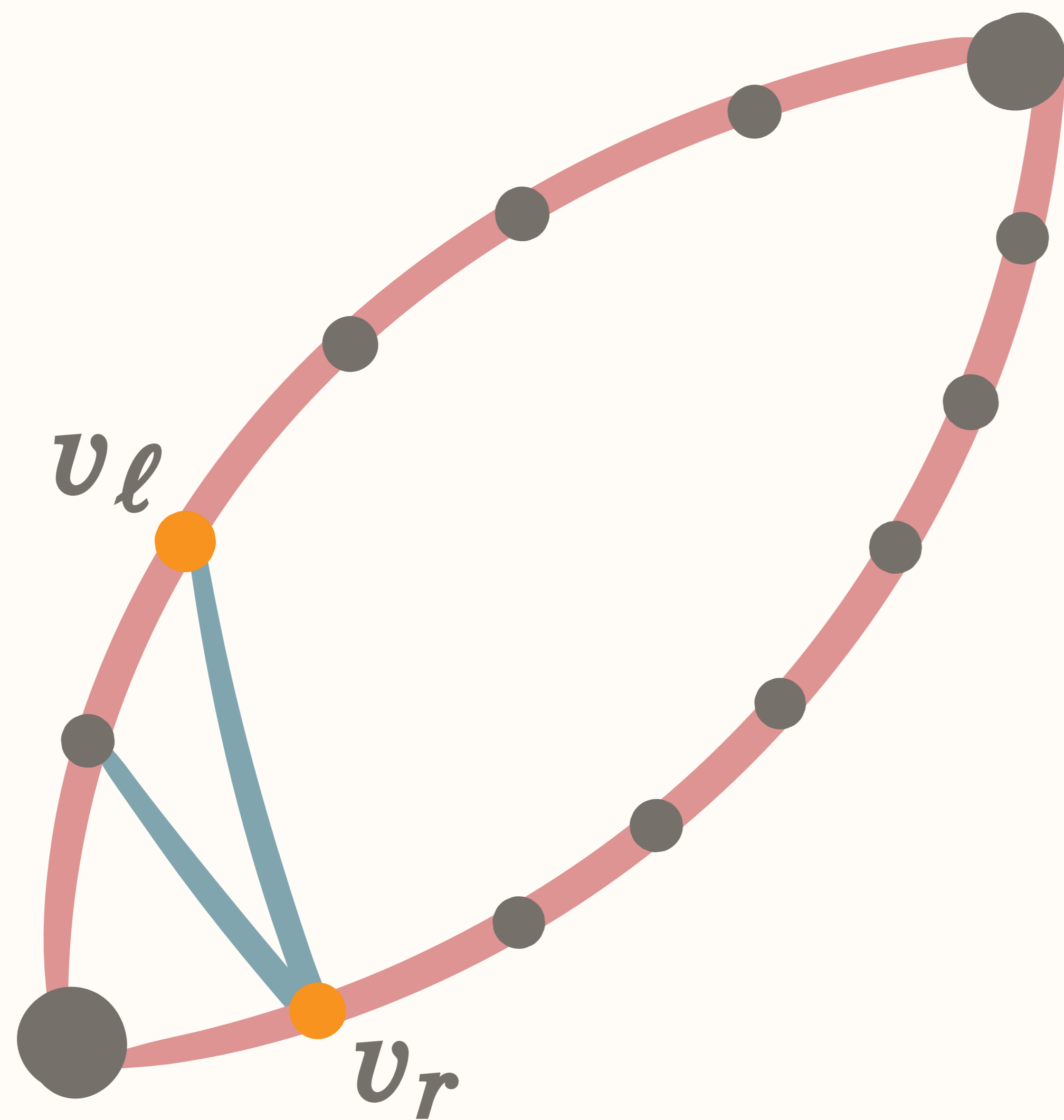
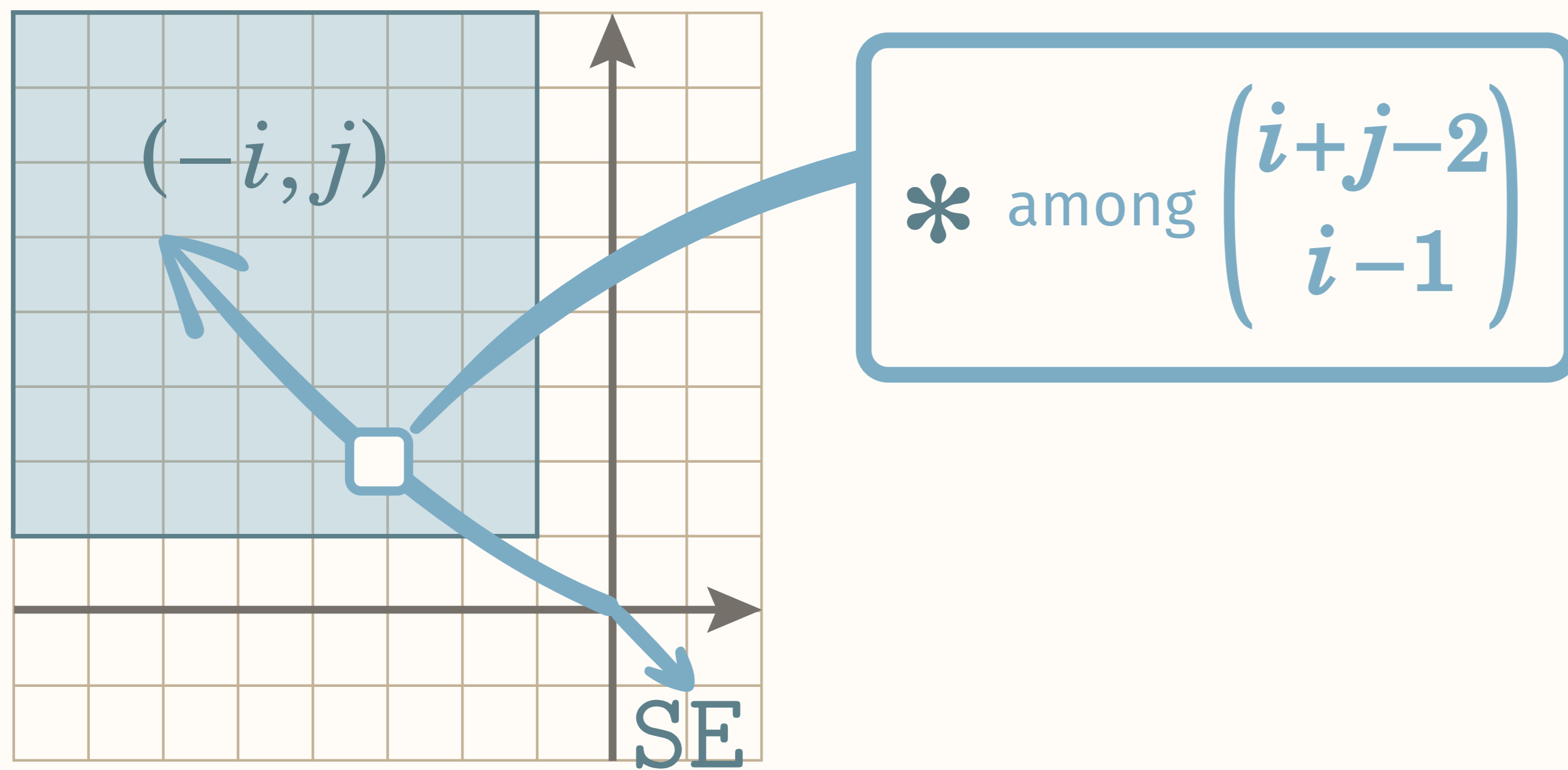
Breaking into small steps



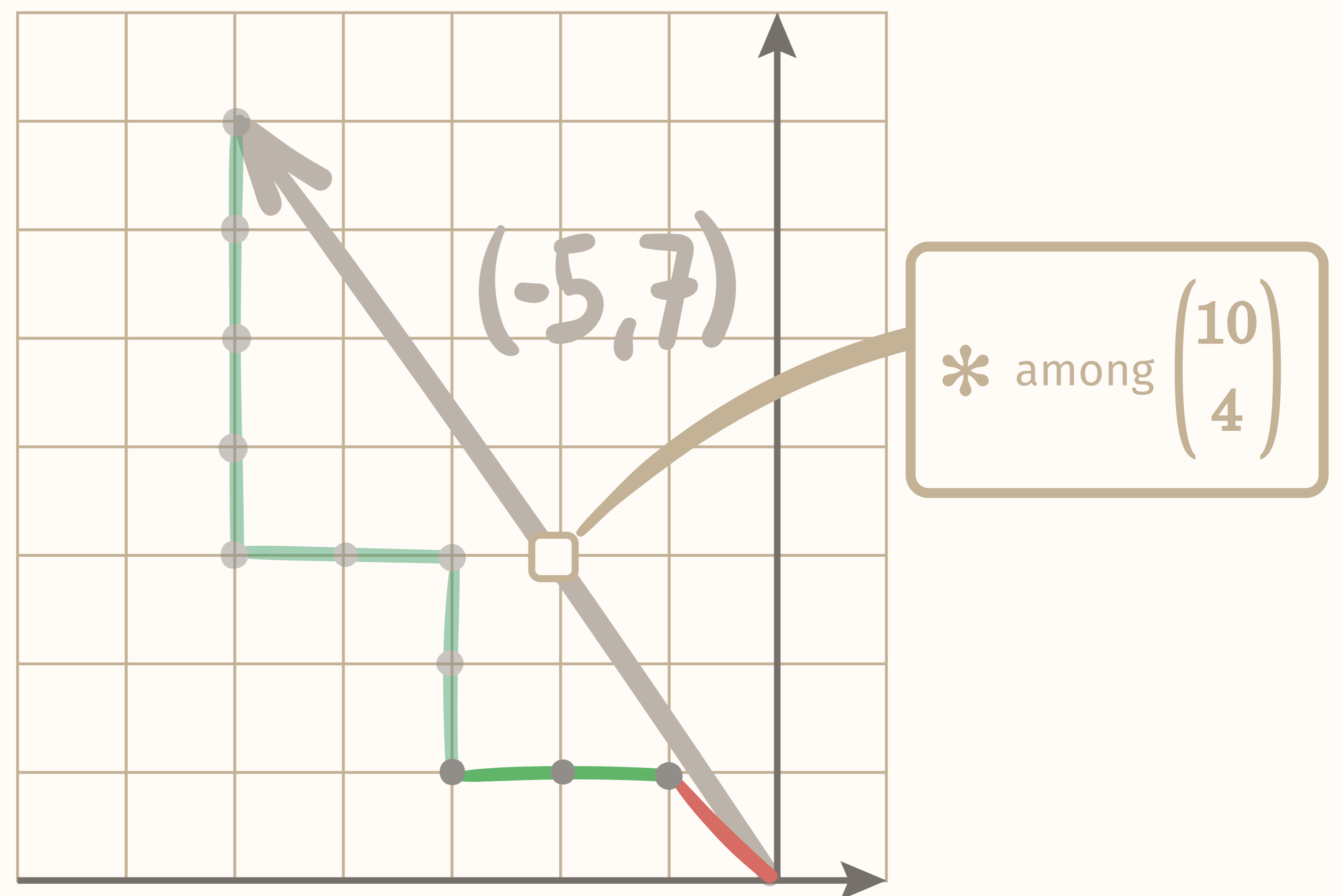
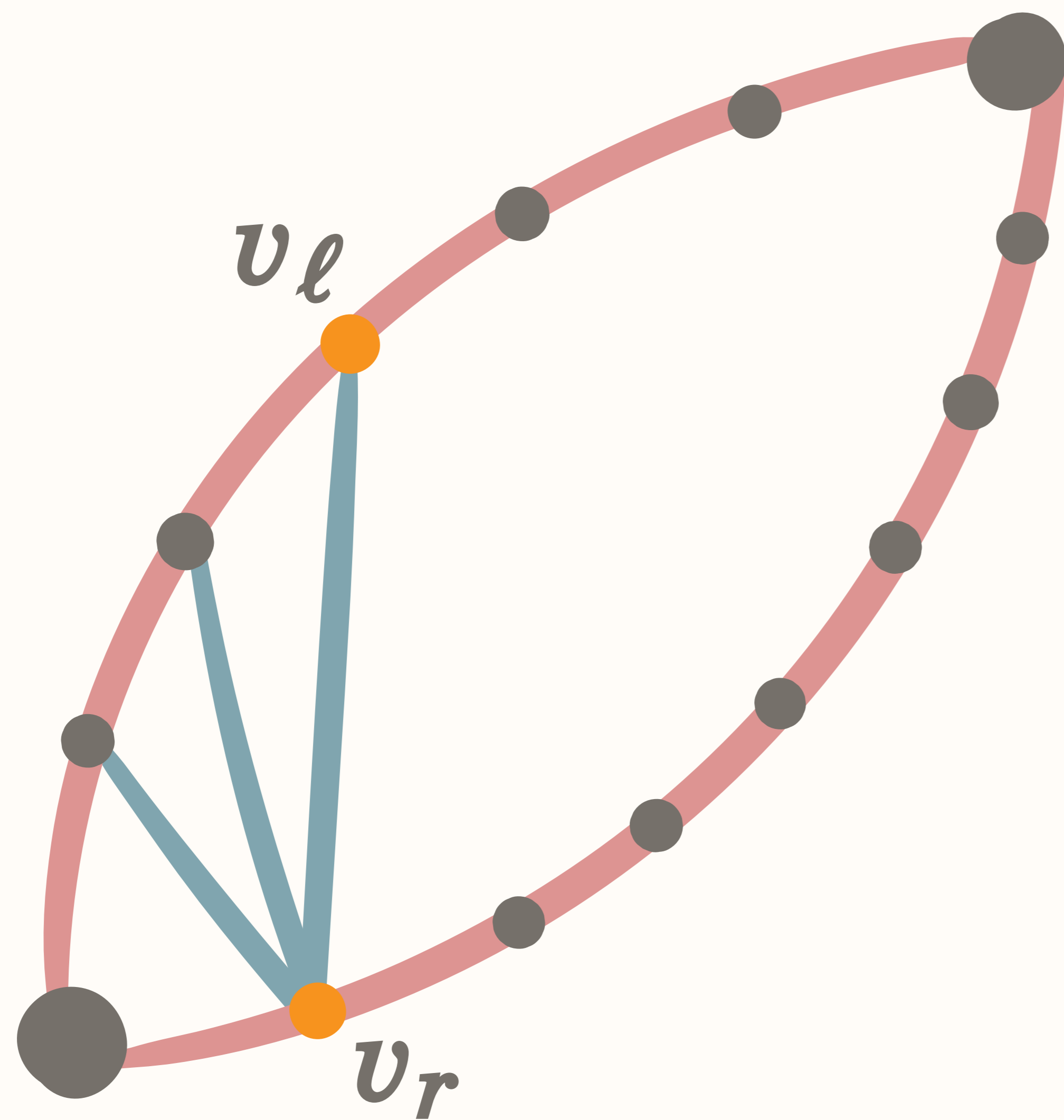
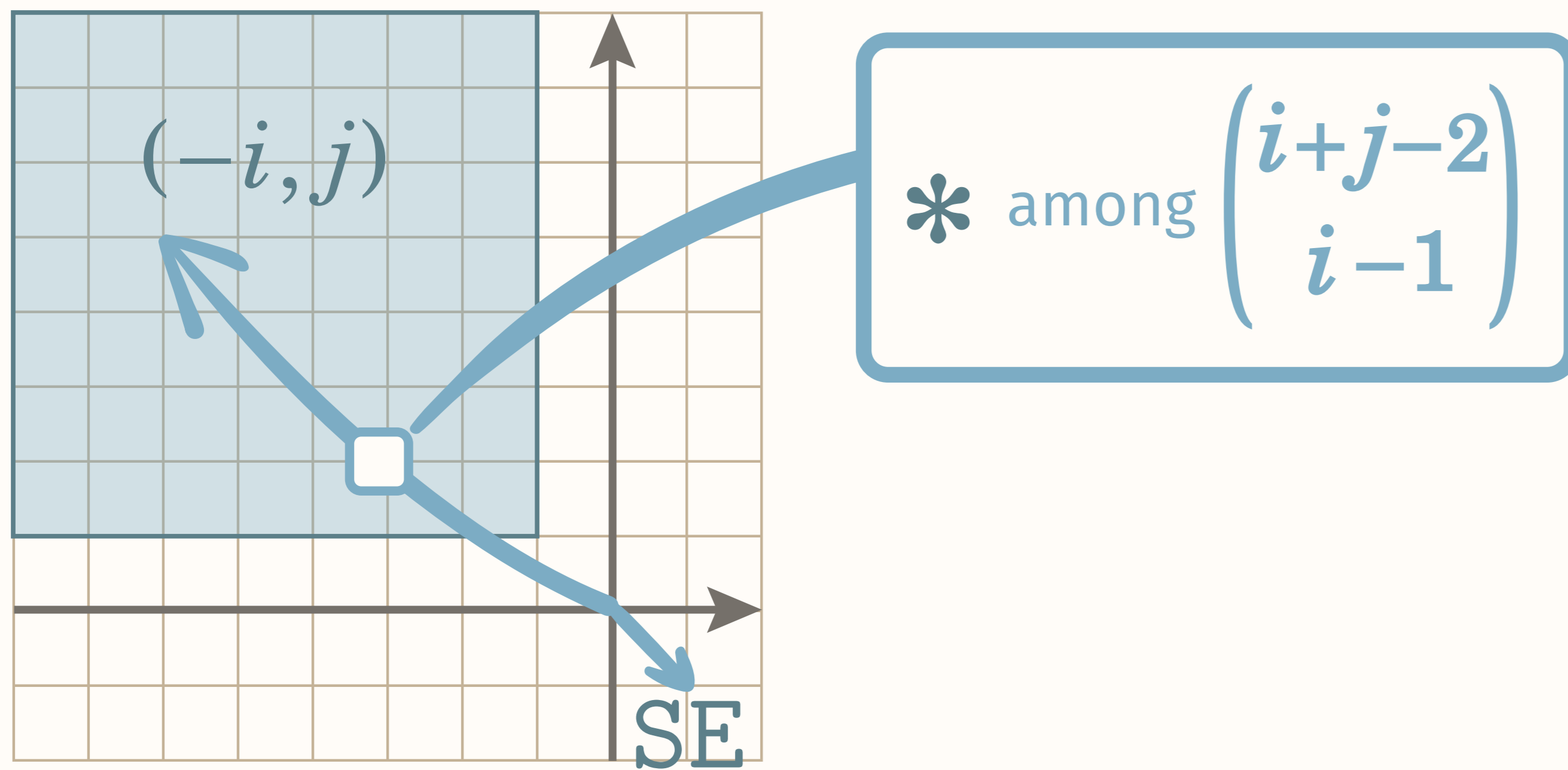
Breaking into small steps



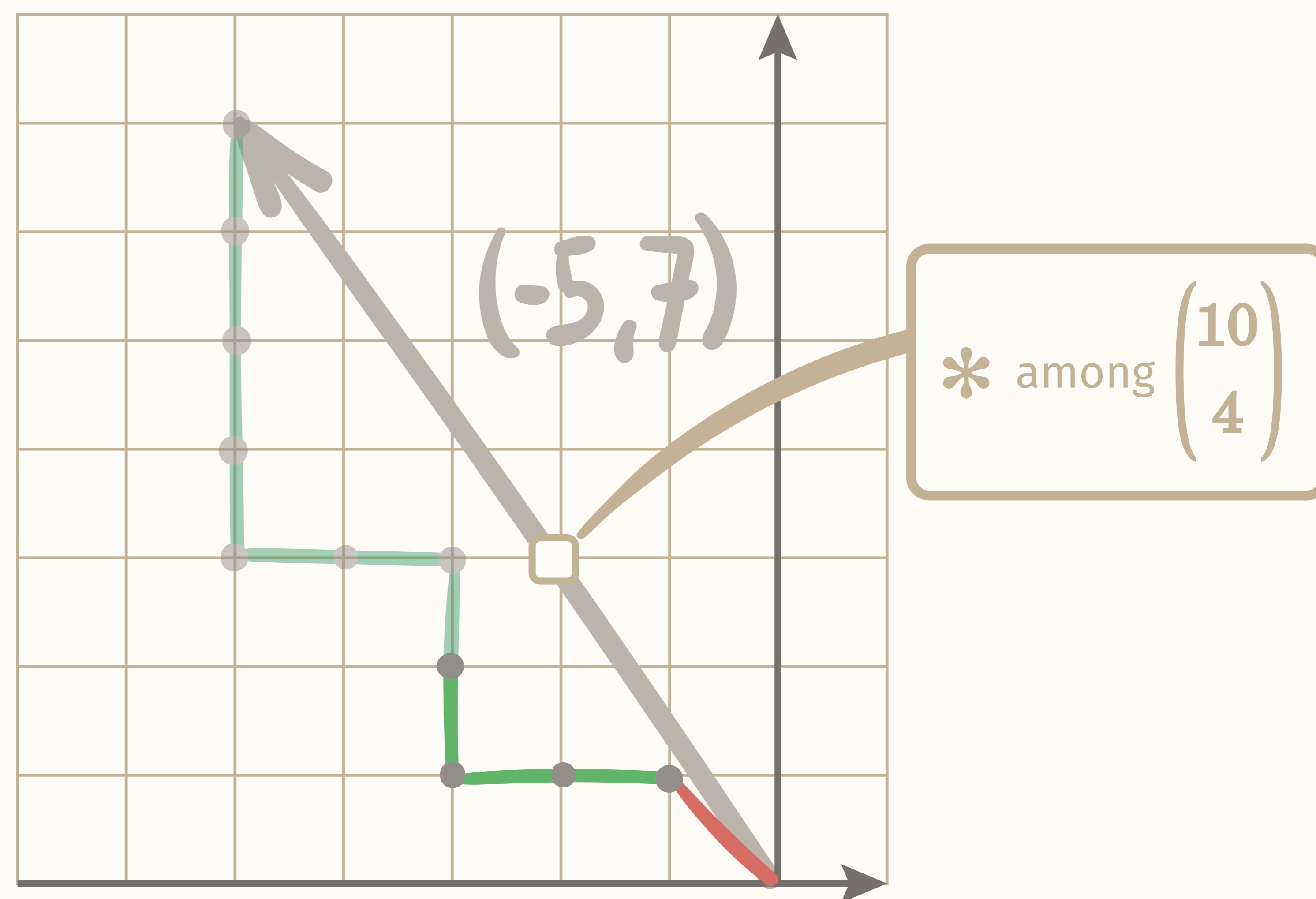
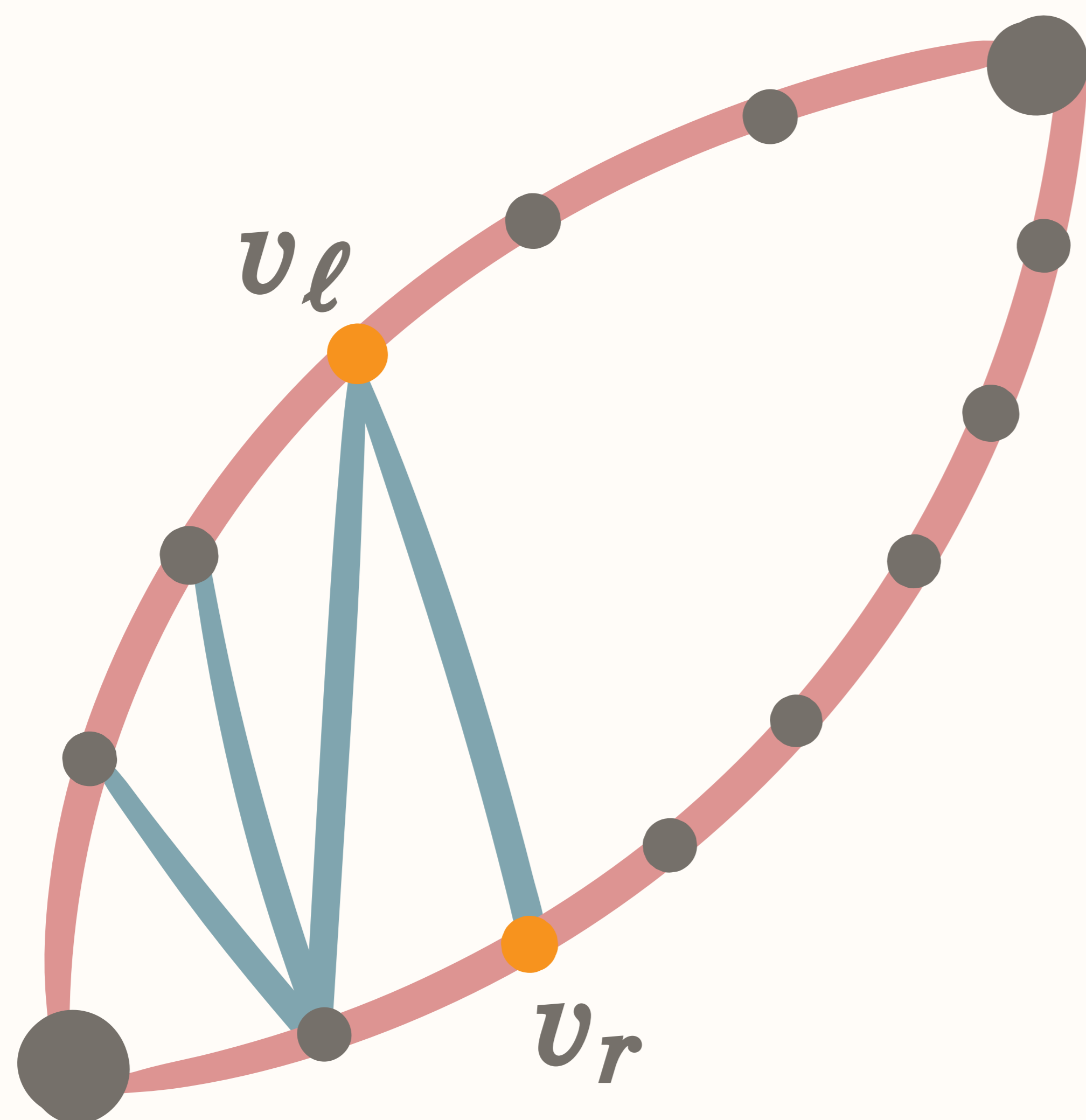
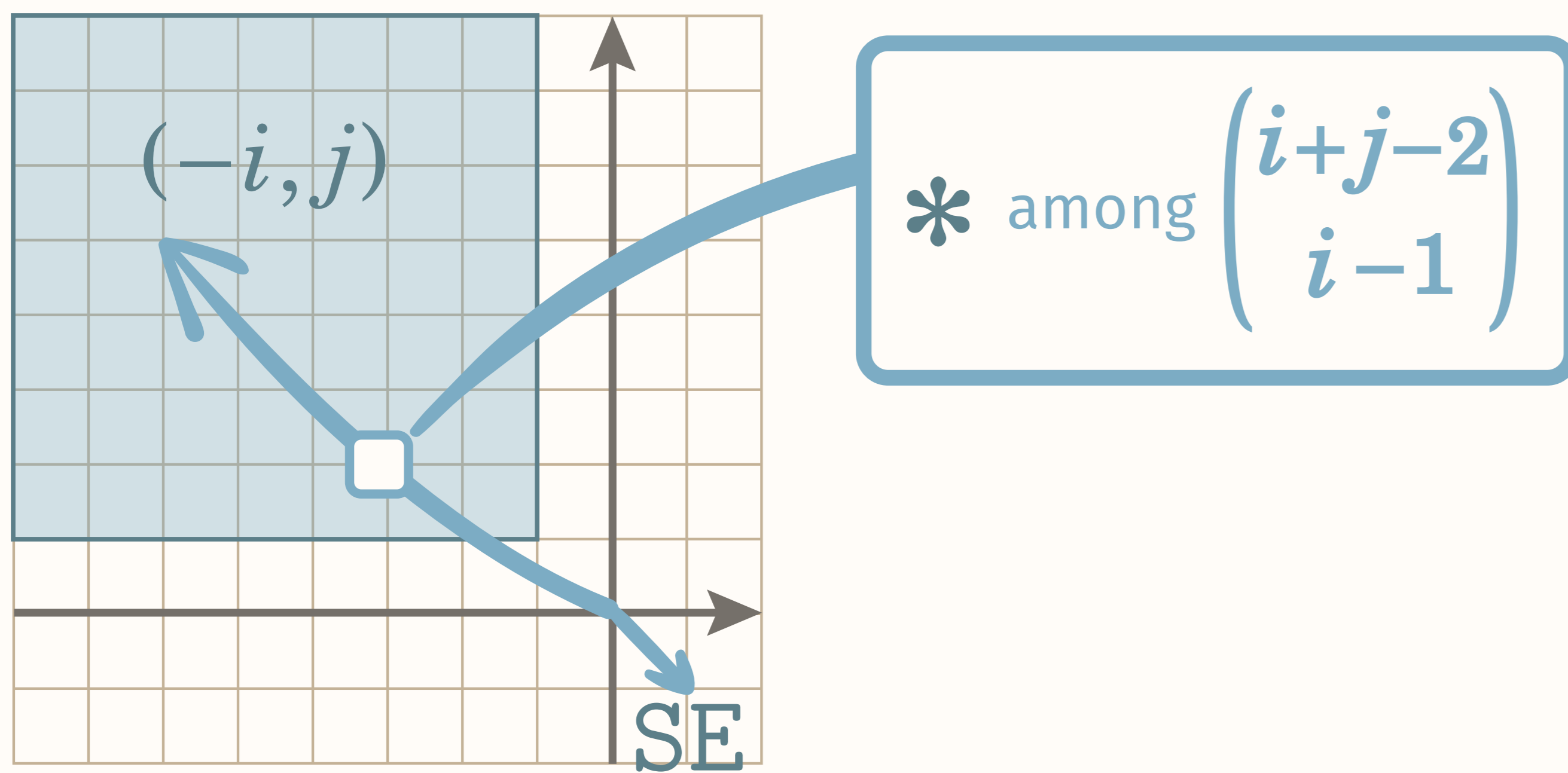
Breaking into small steps



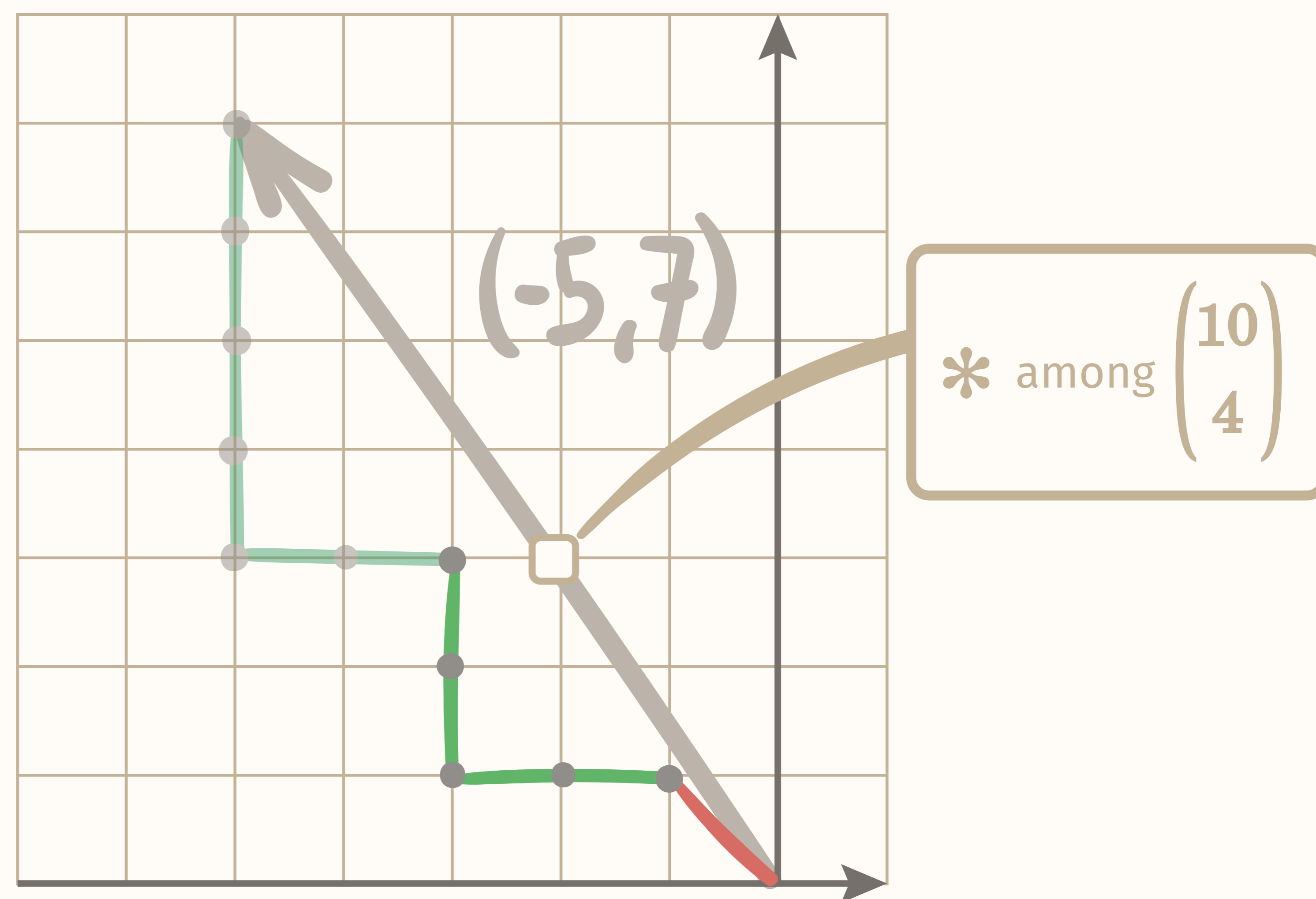
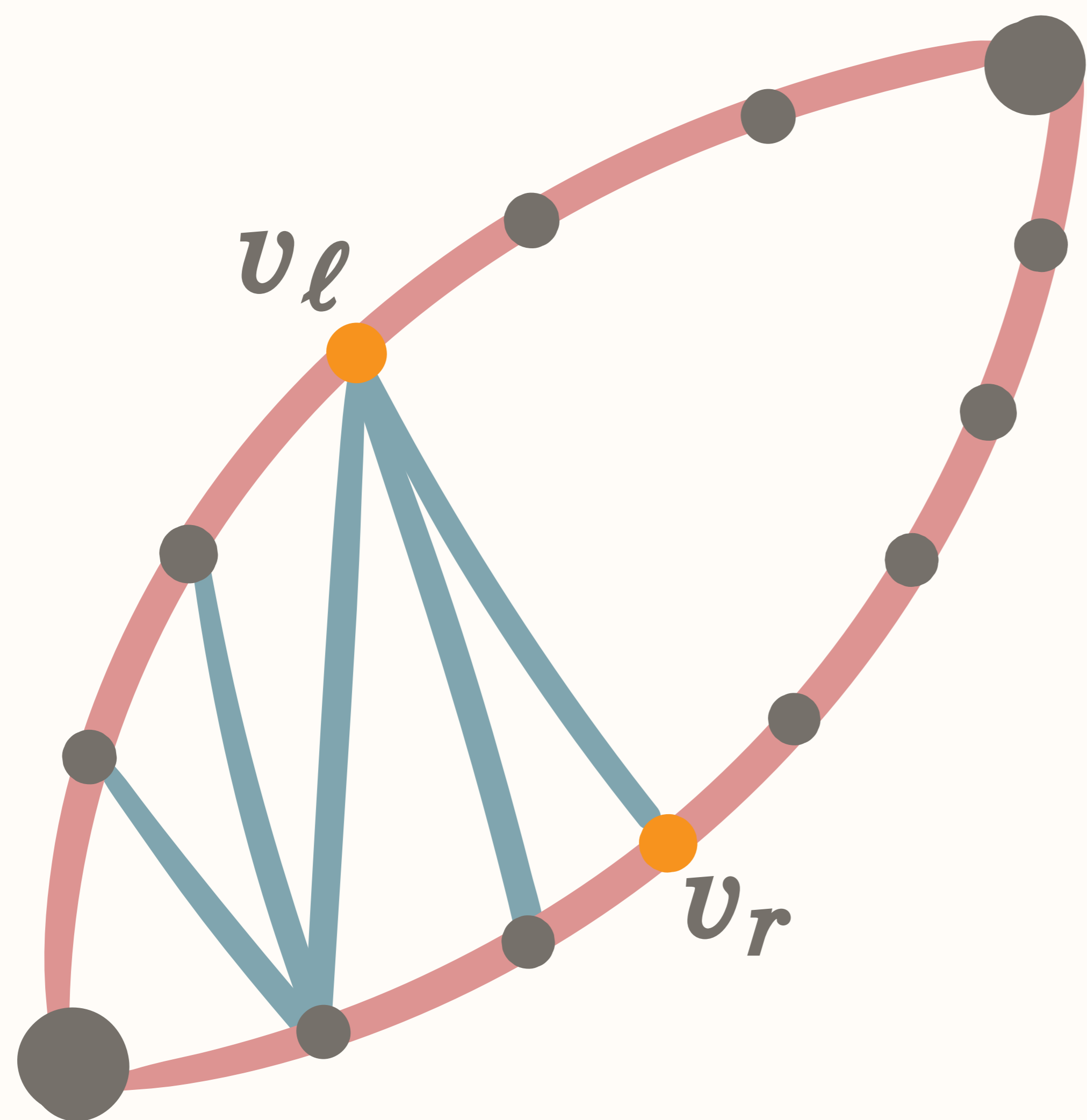
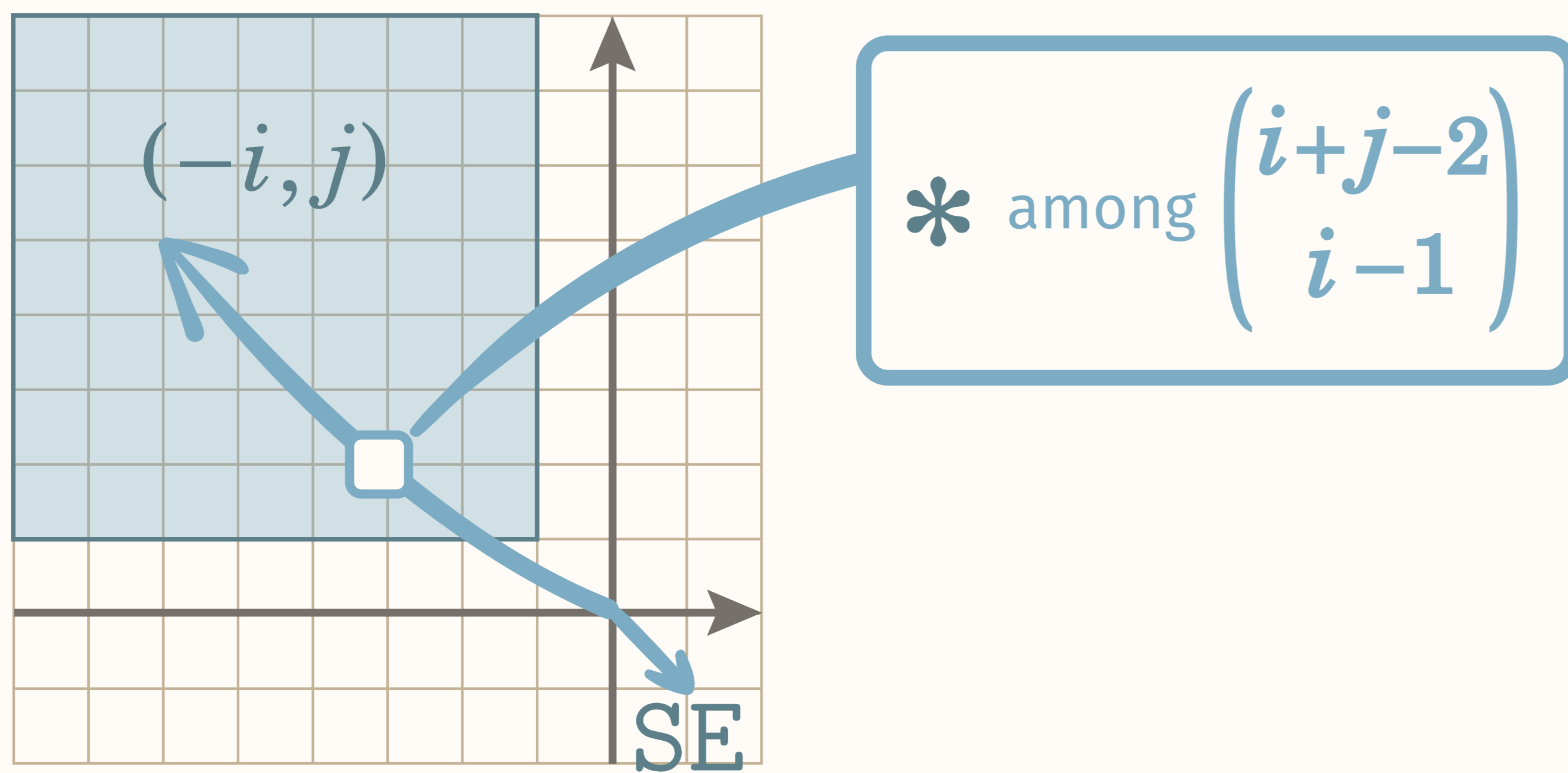
Breaking into small steps



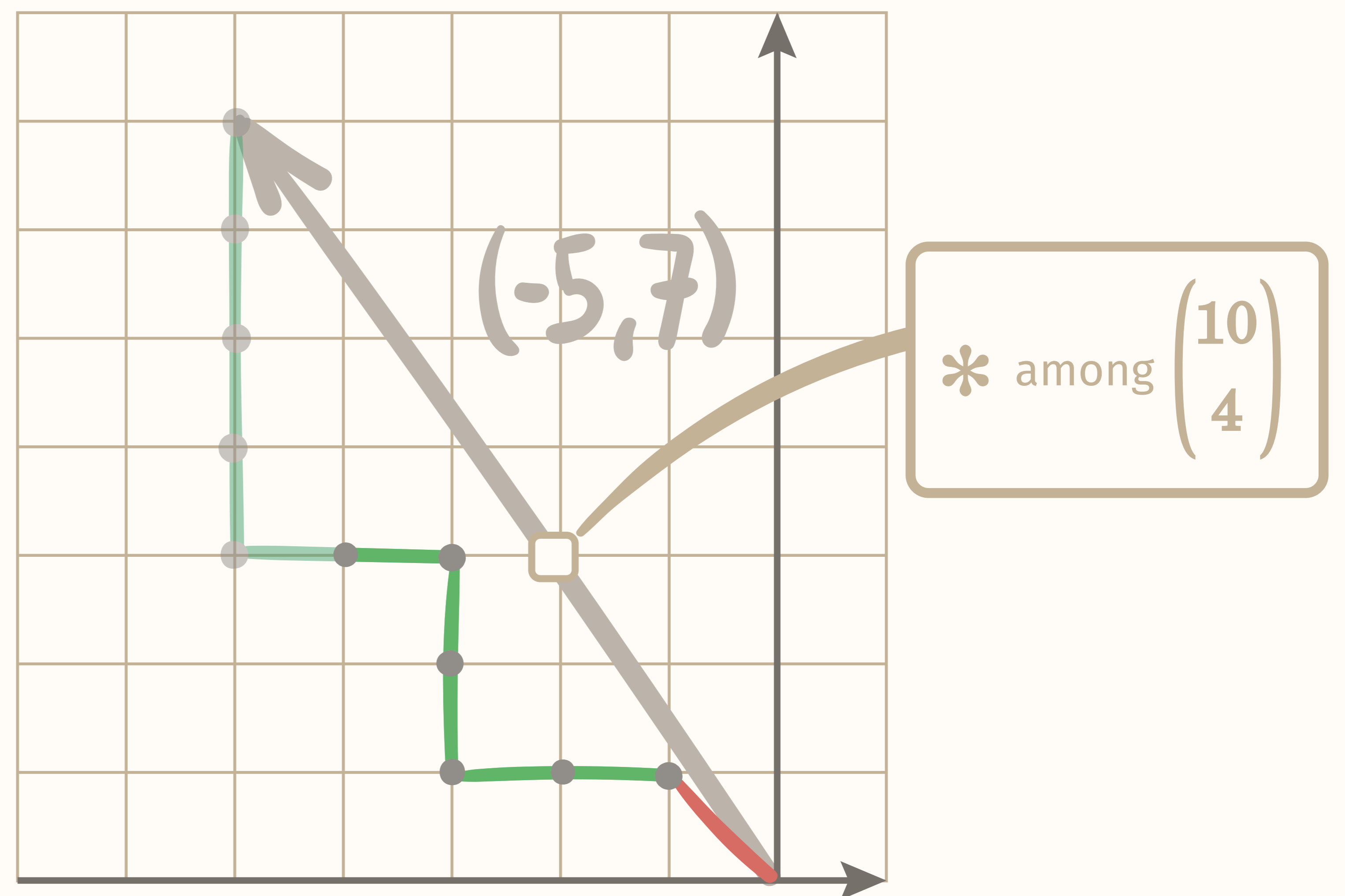
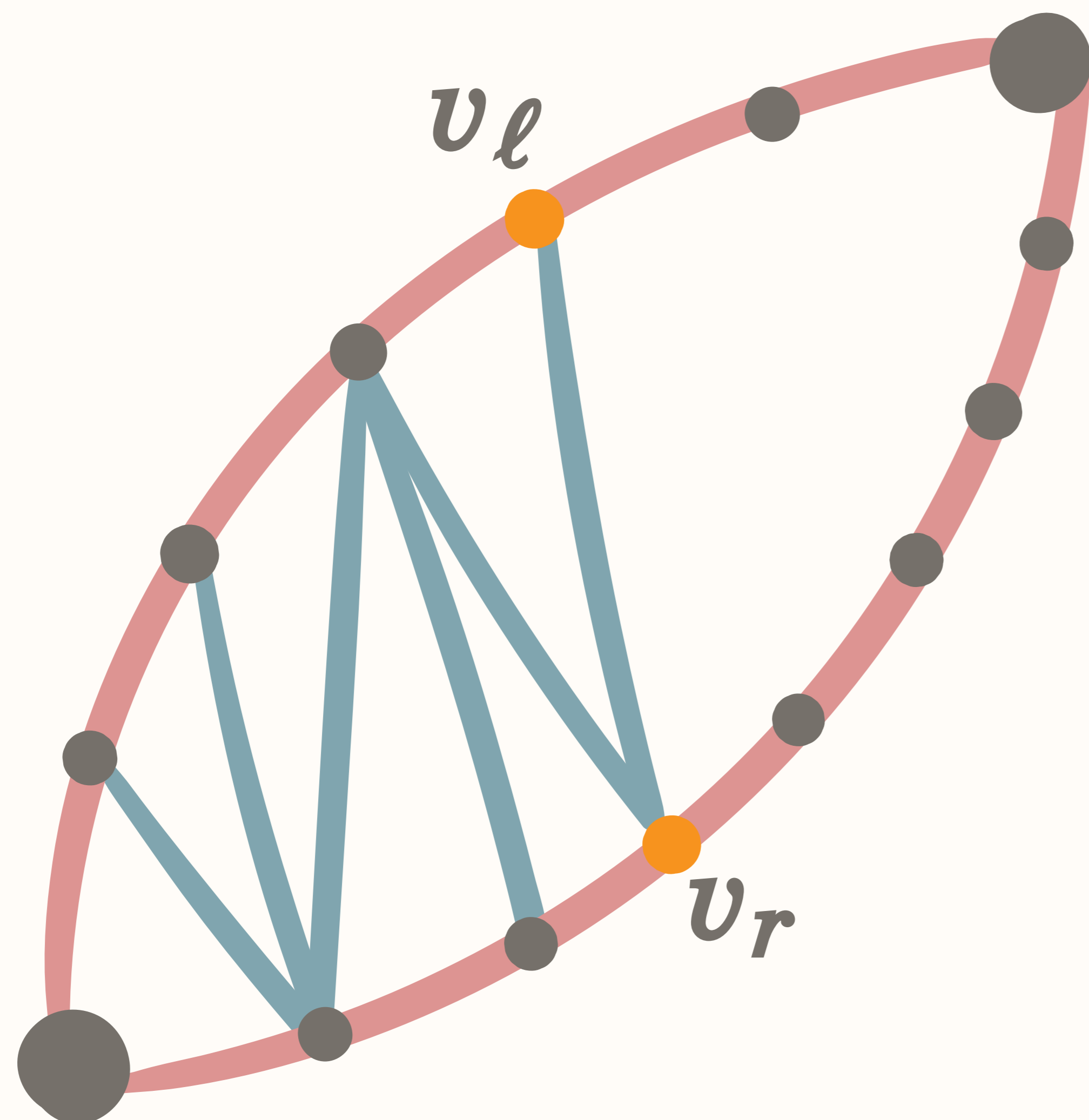
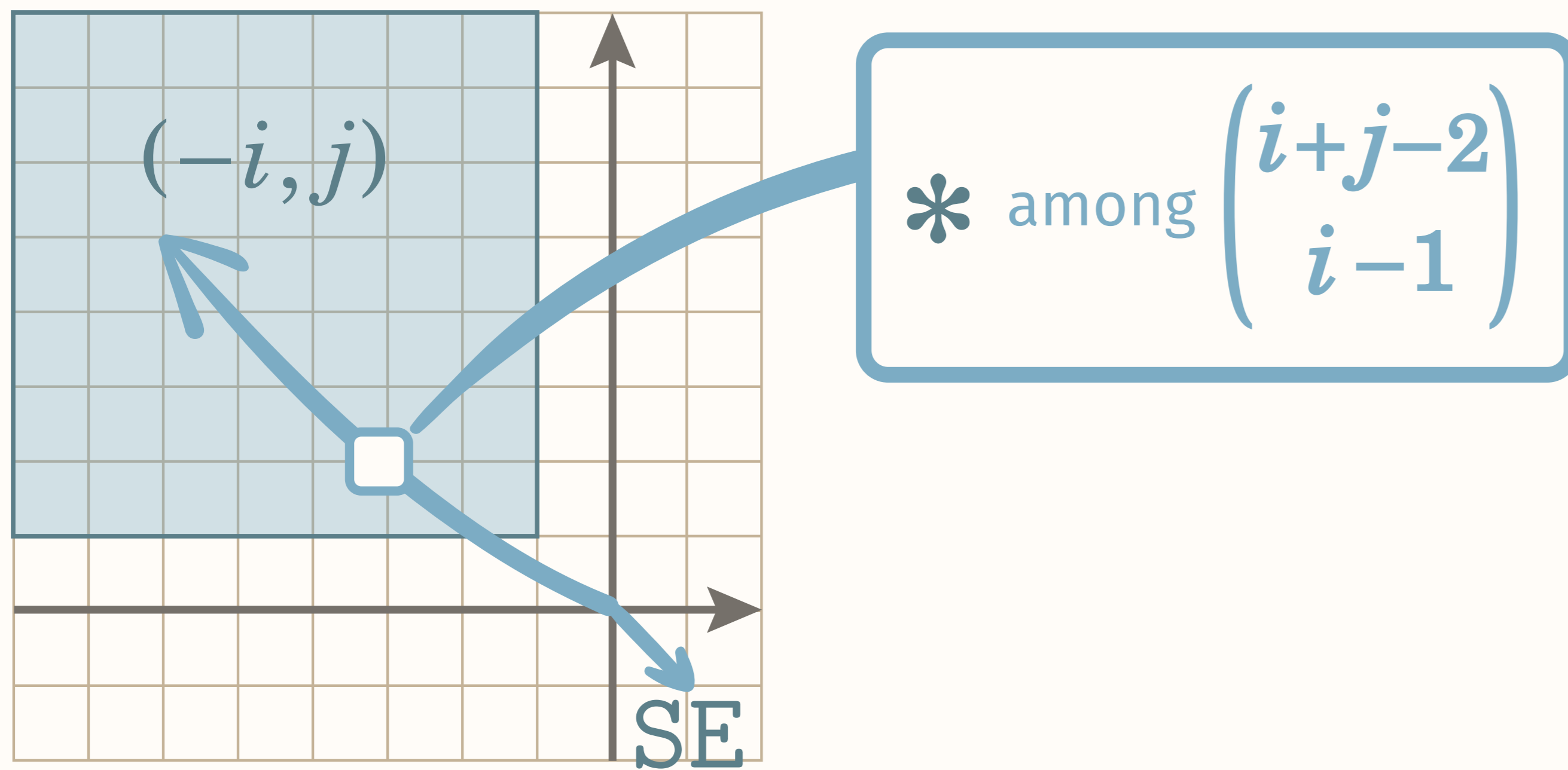
Breaking into small steps



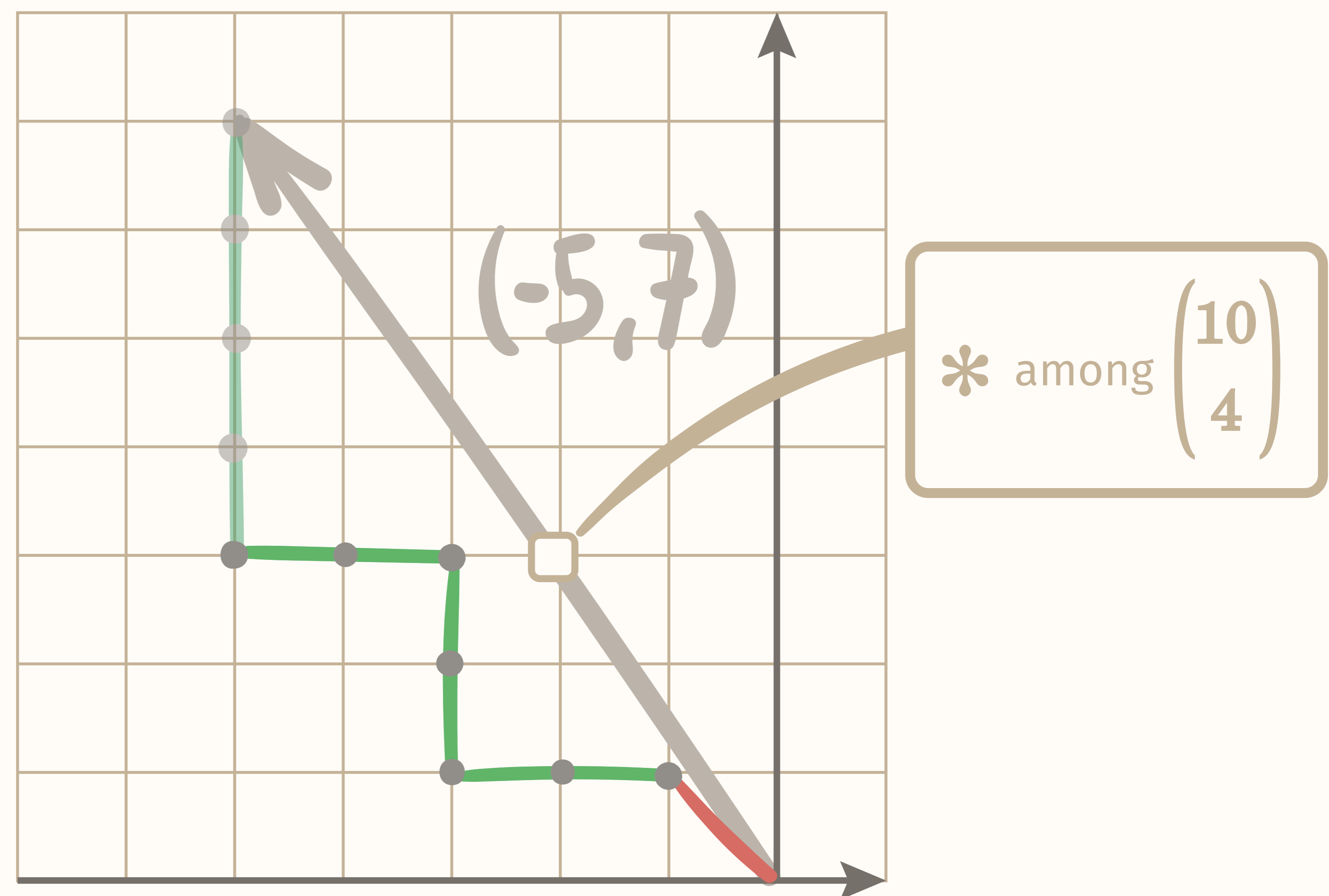
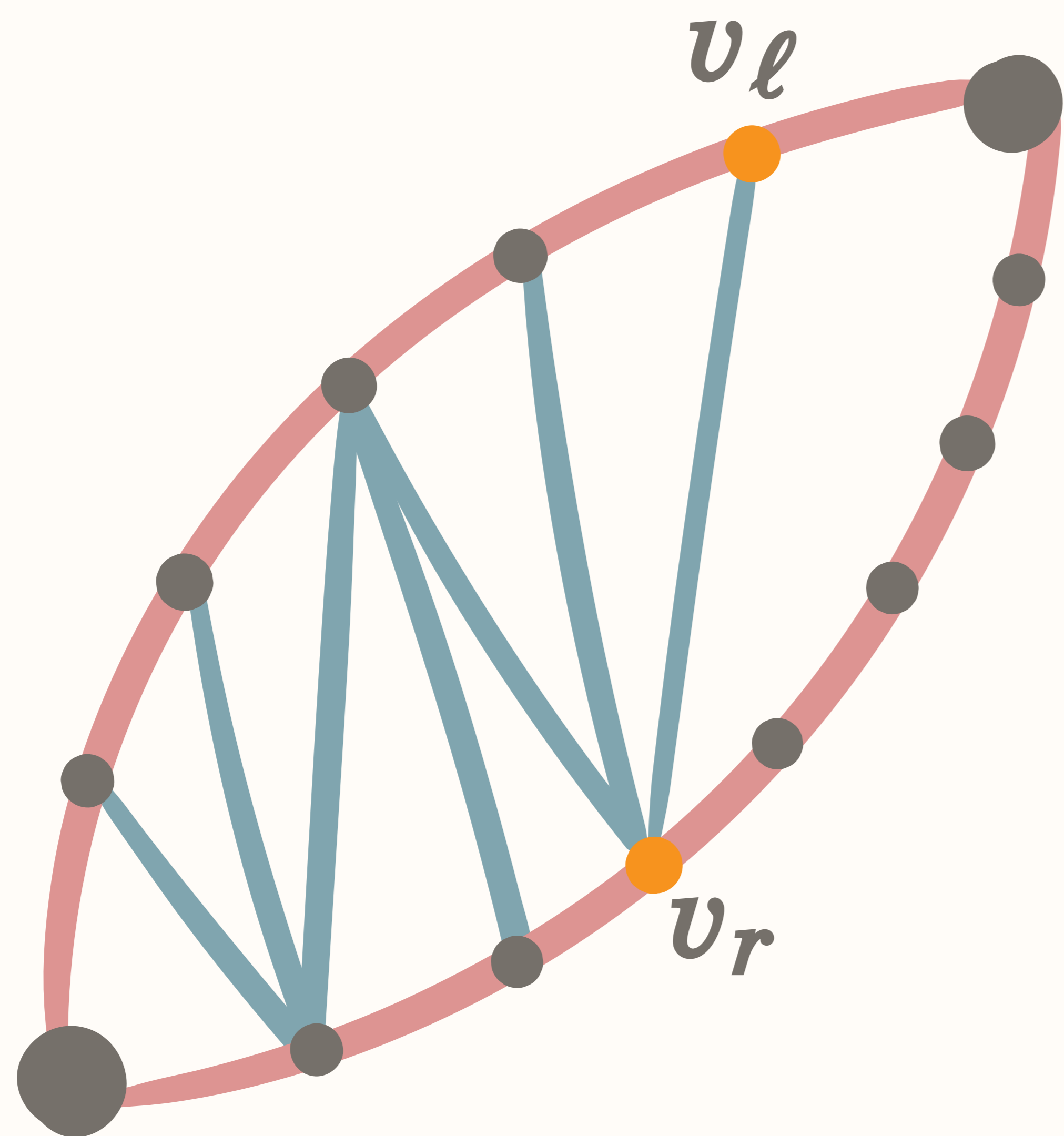
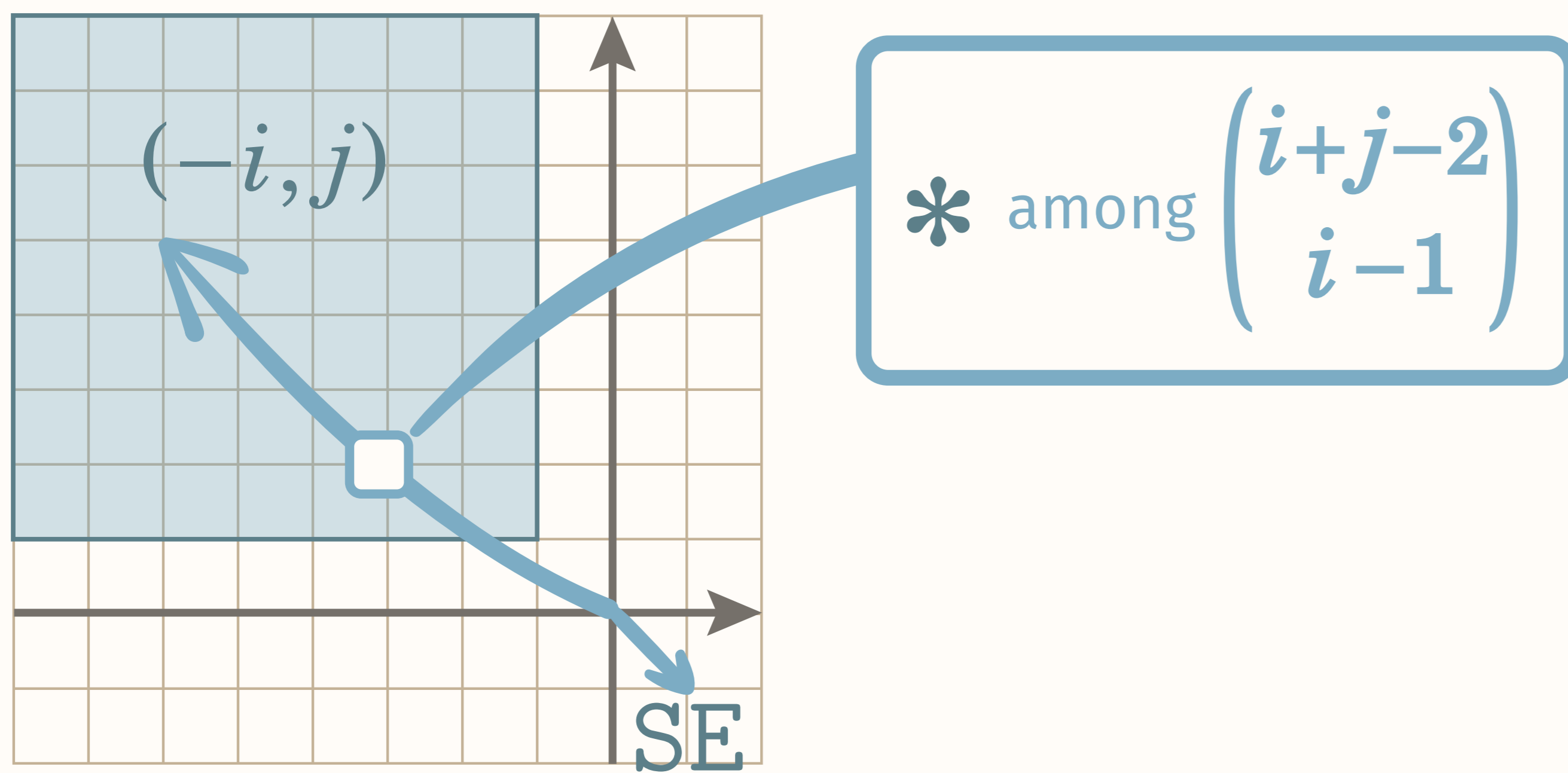
Breaking into small steps



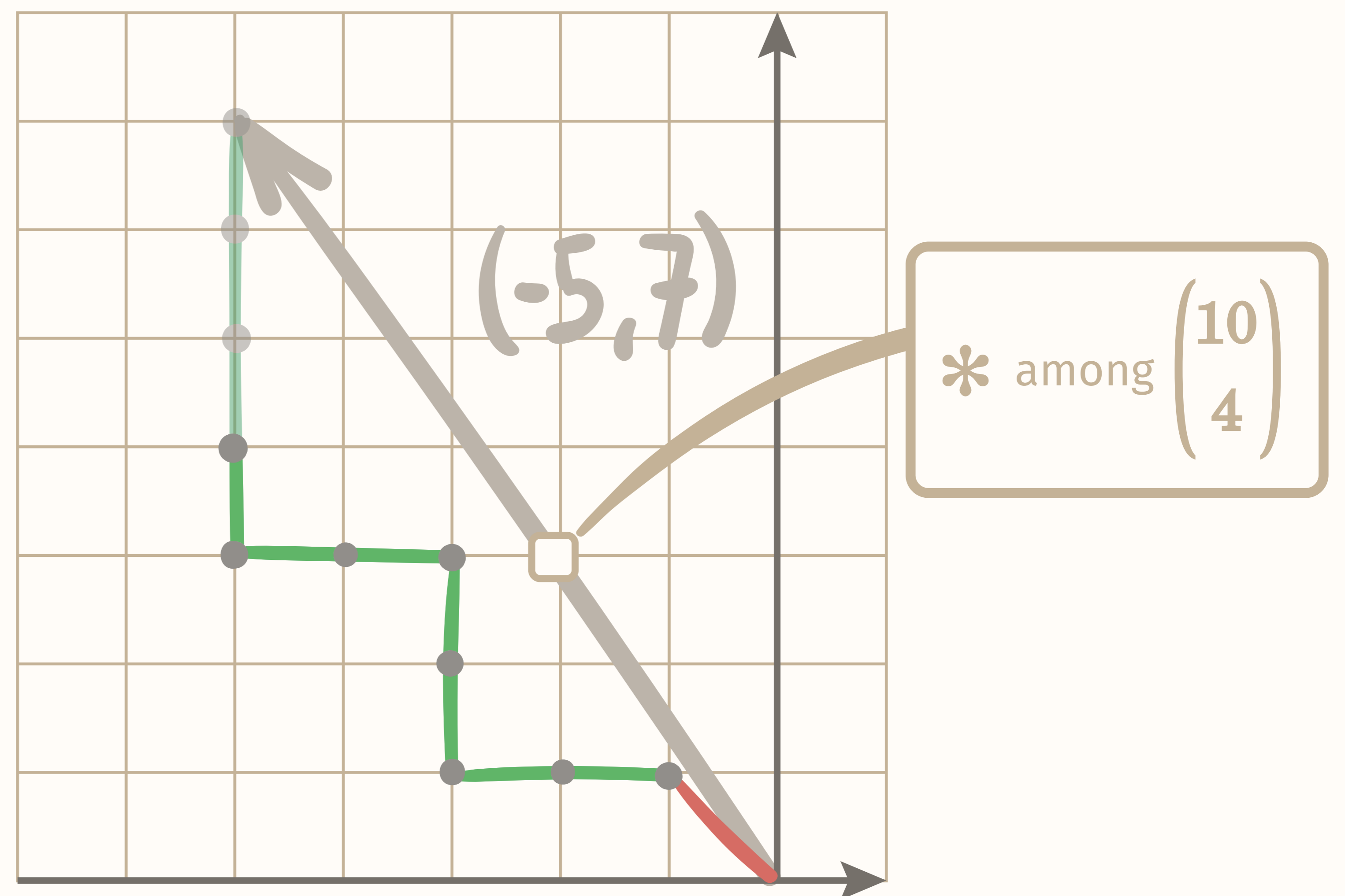
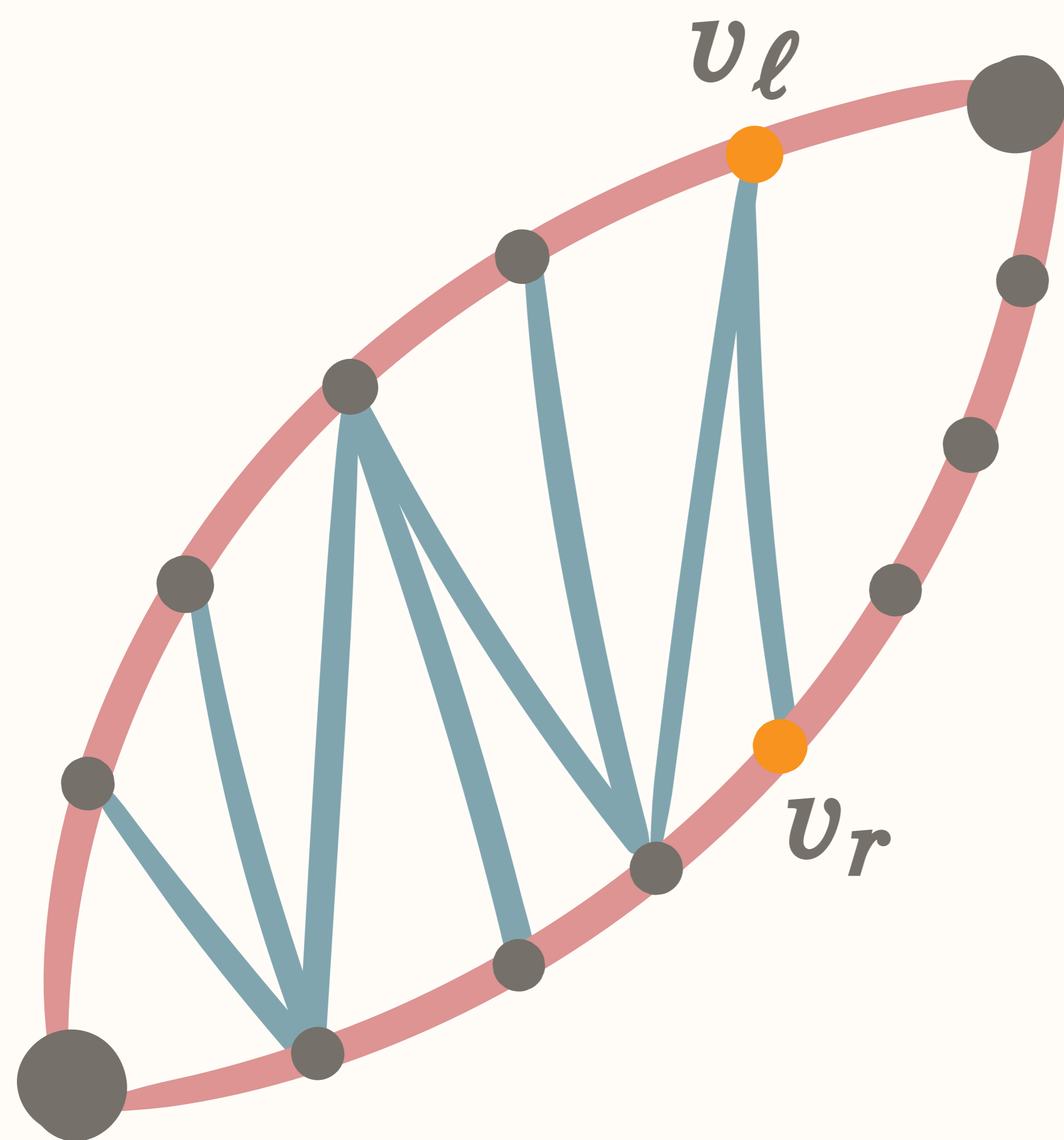
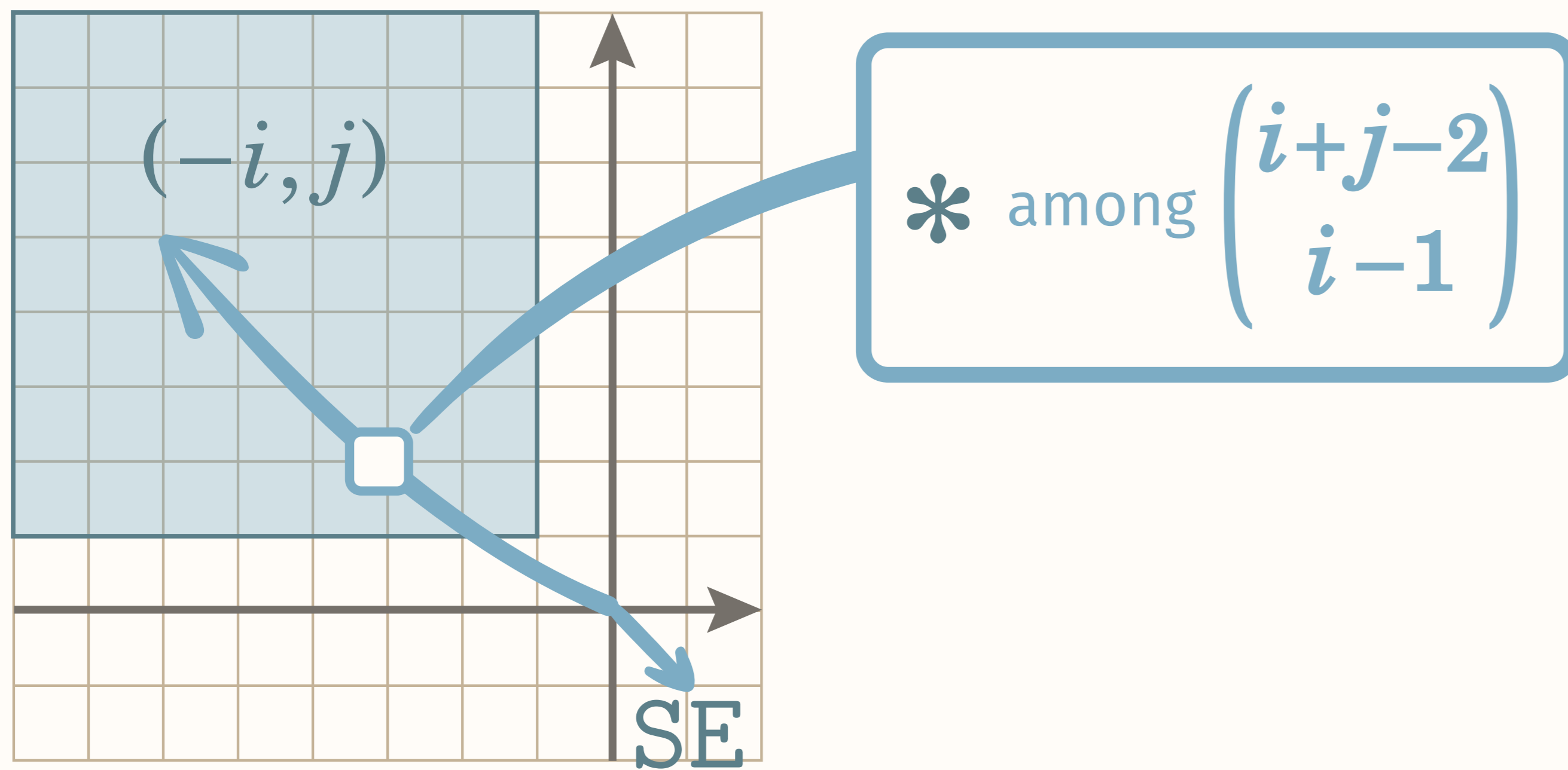
Breaking into small steps



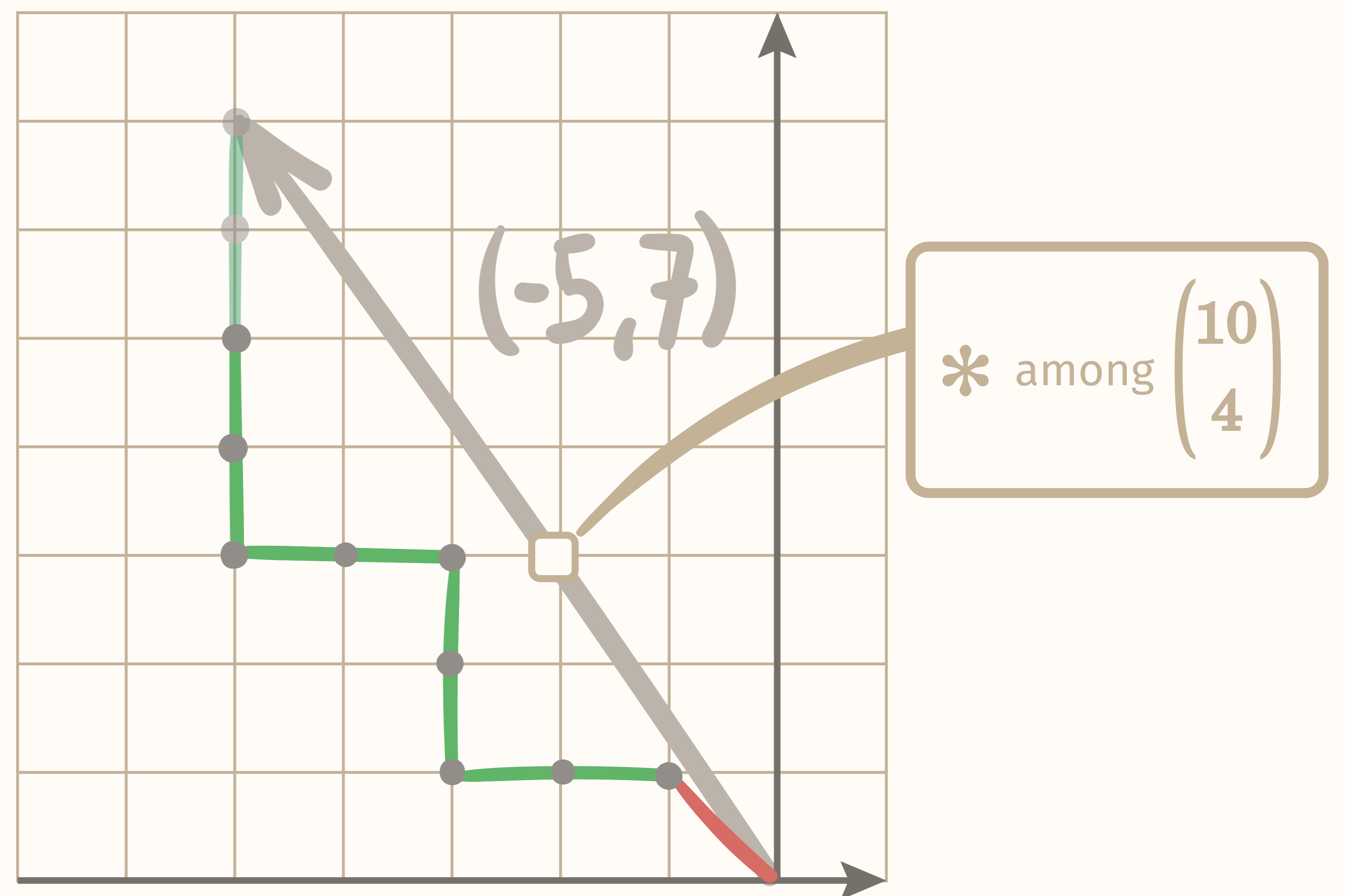
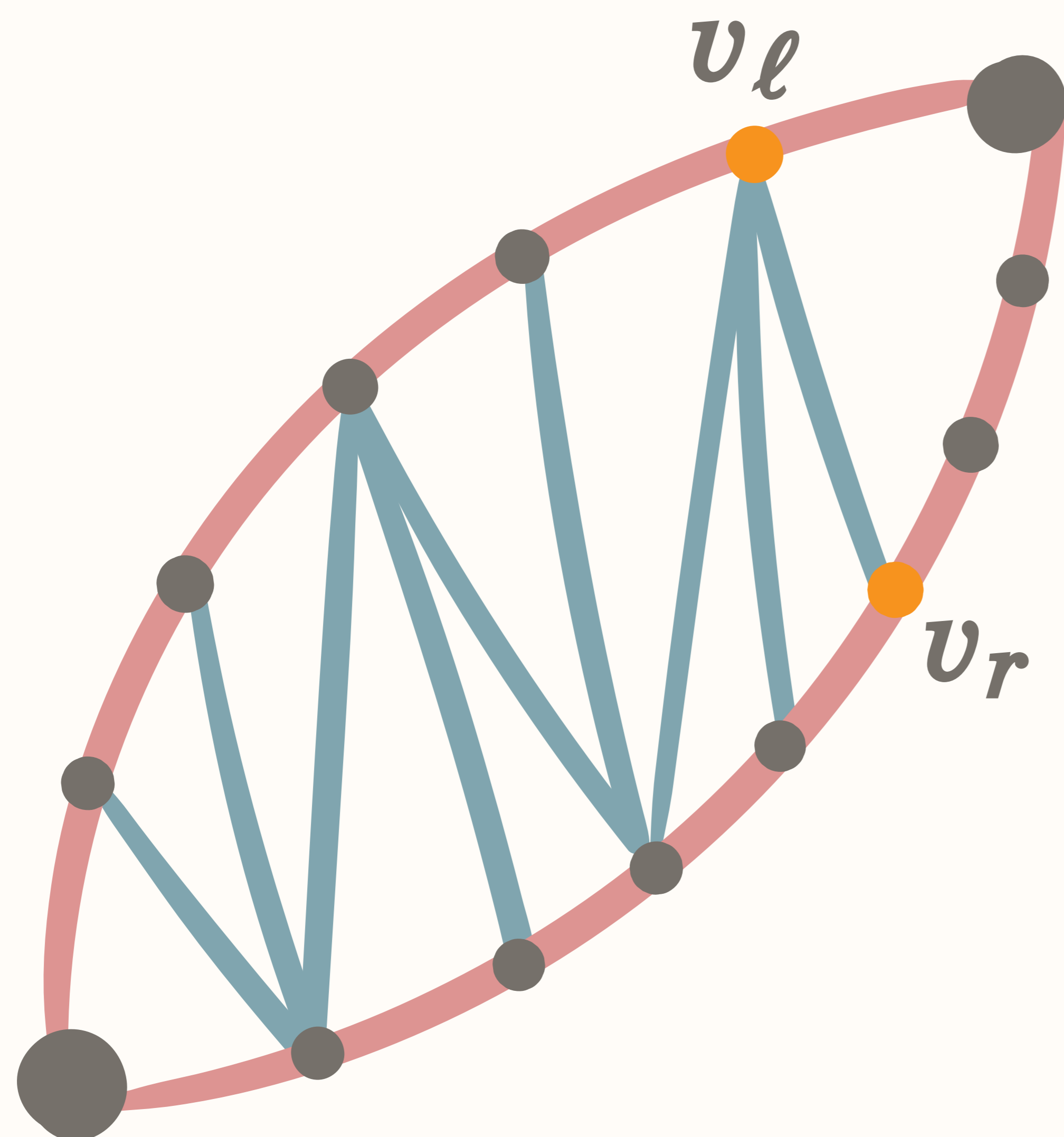
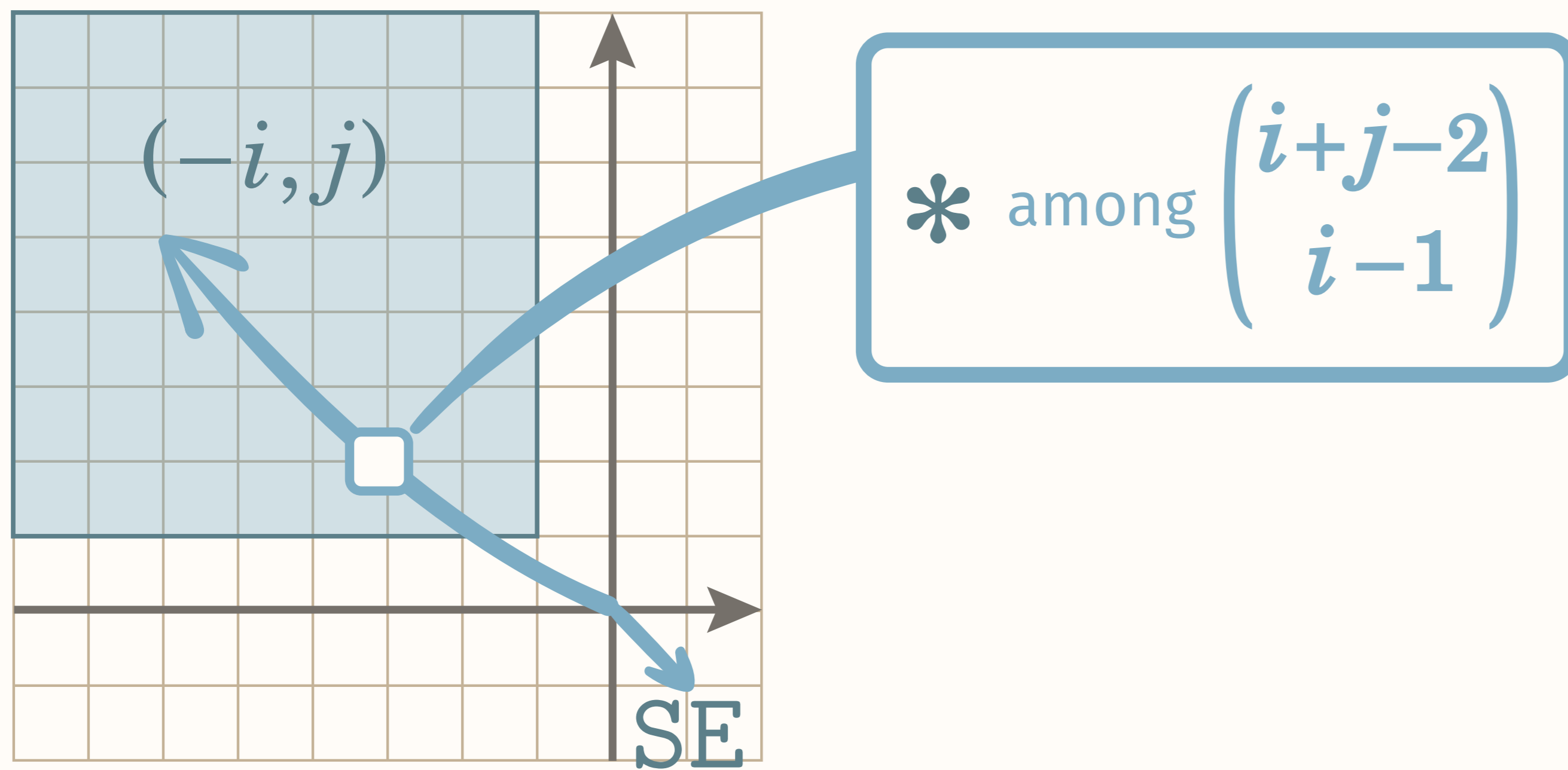
Breaking into small steps



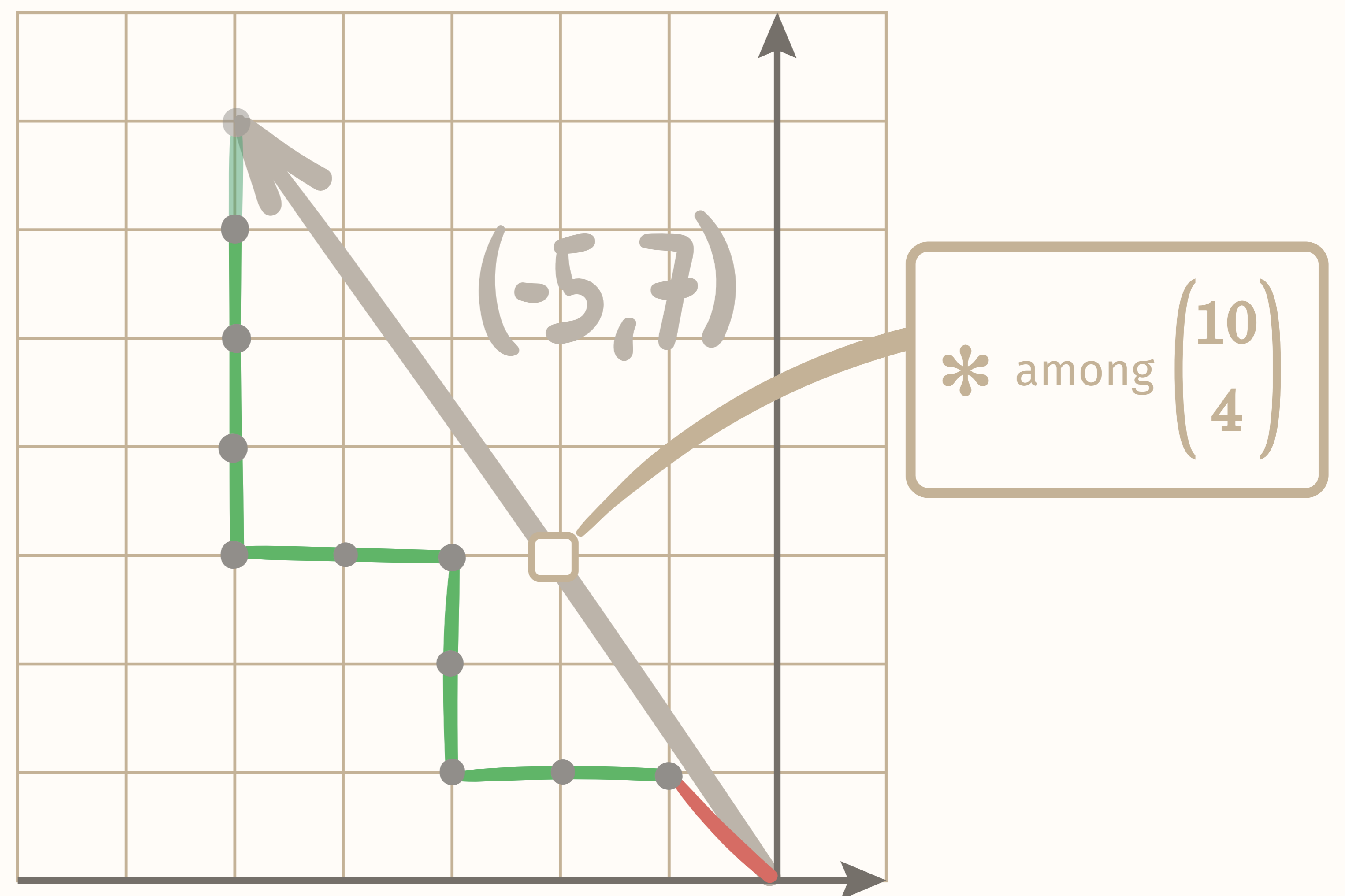
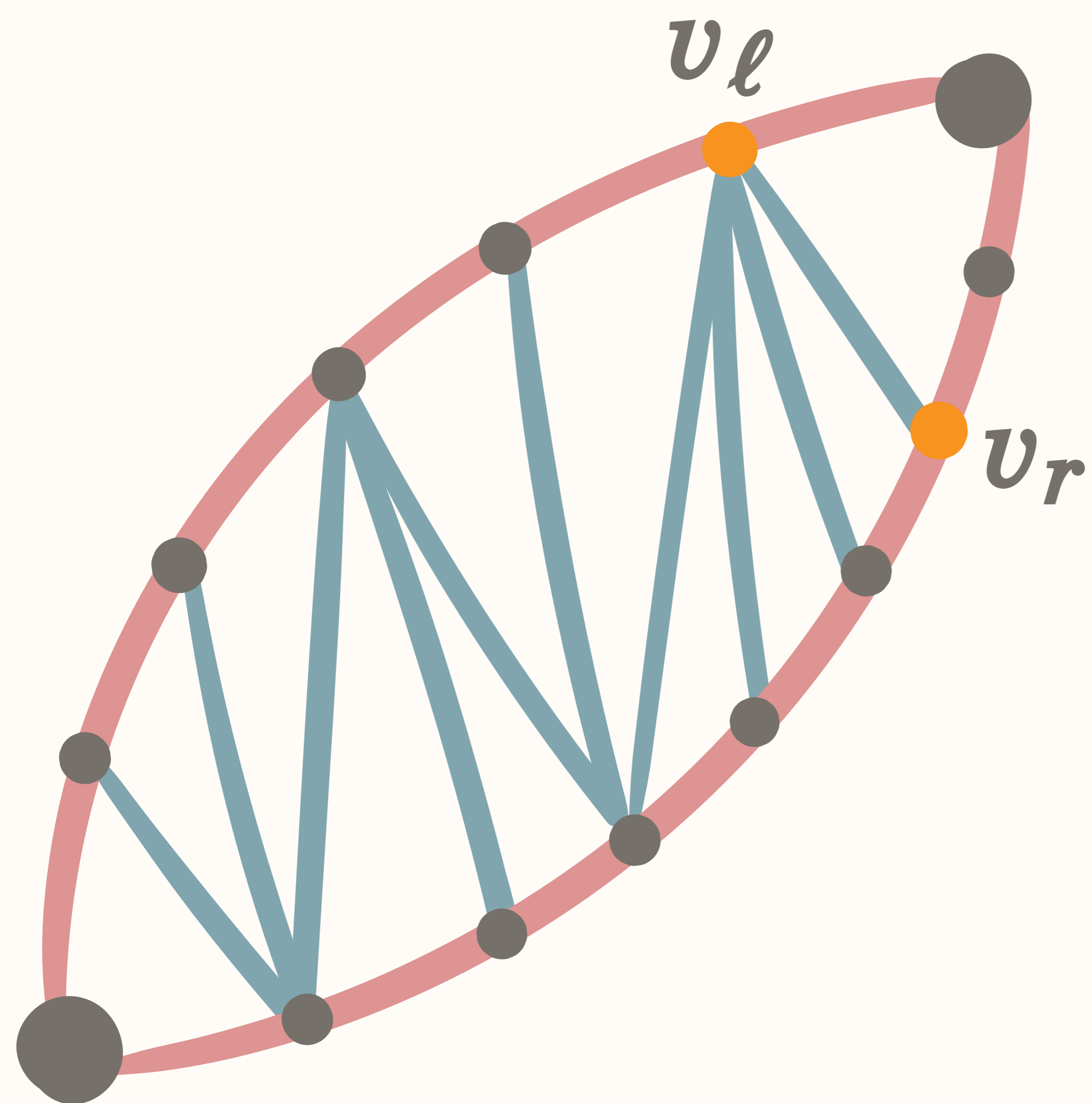
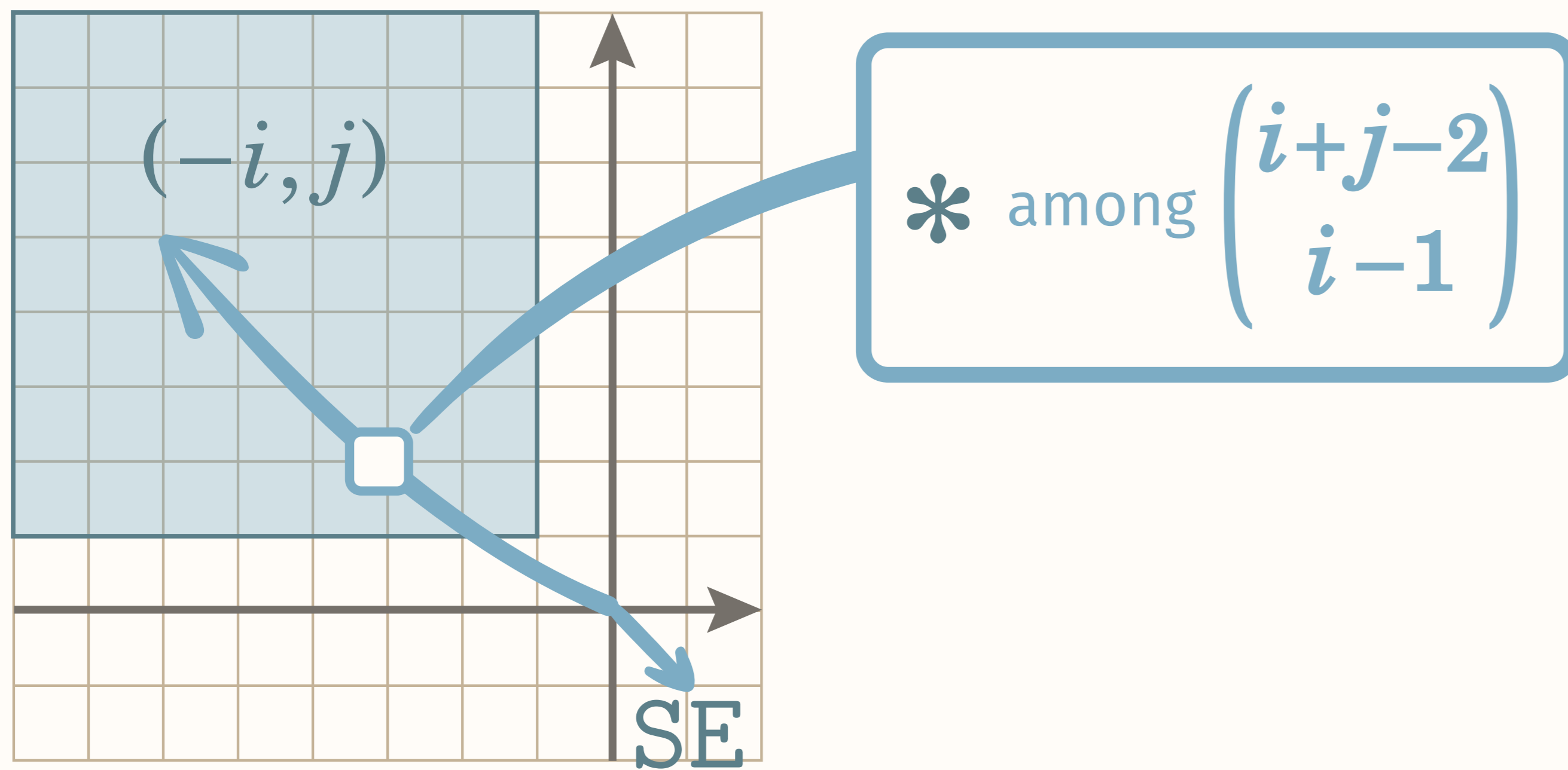
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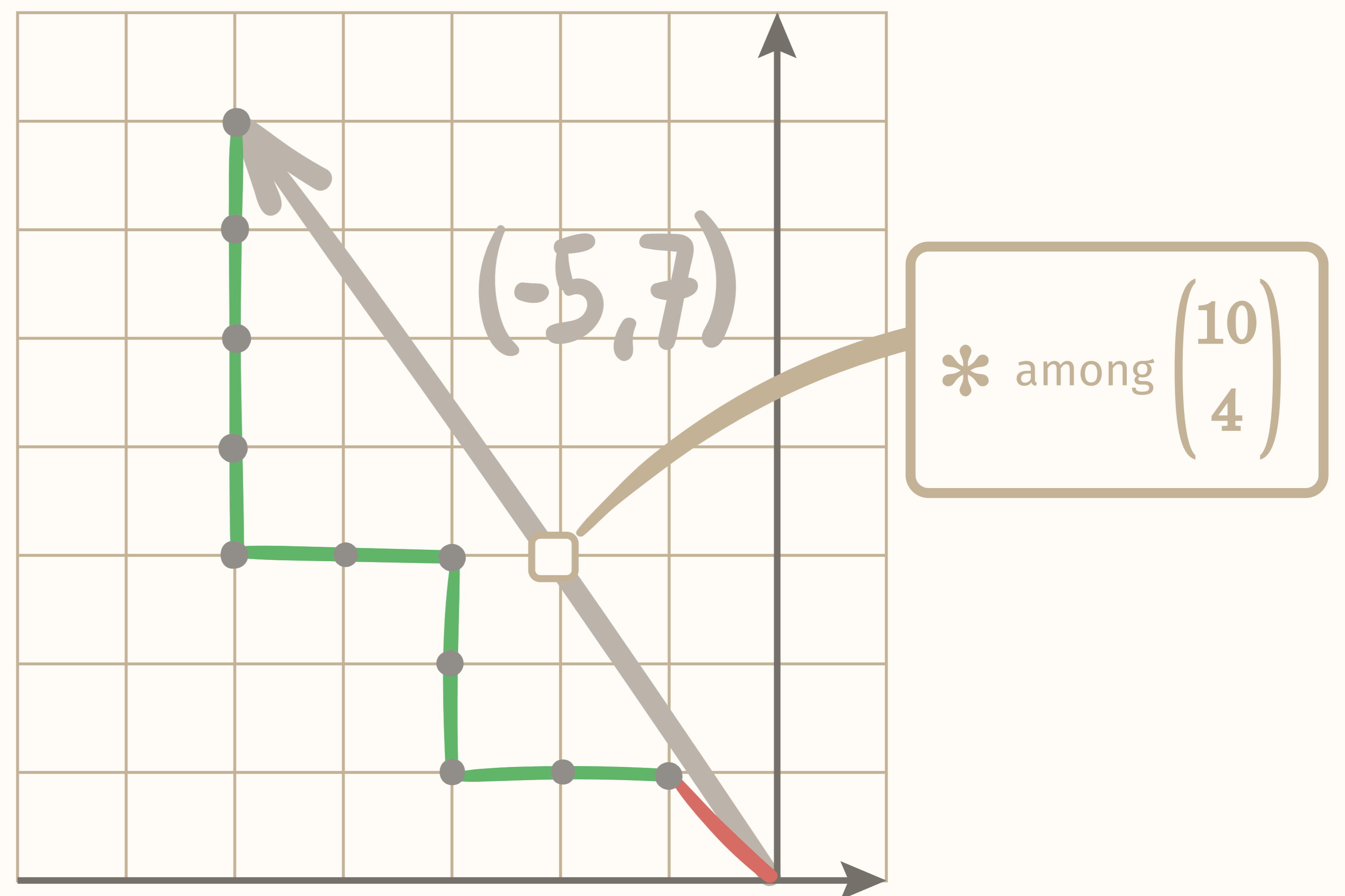
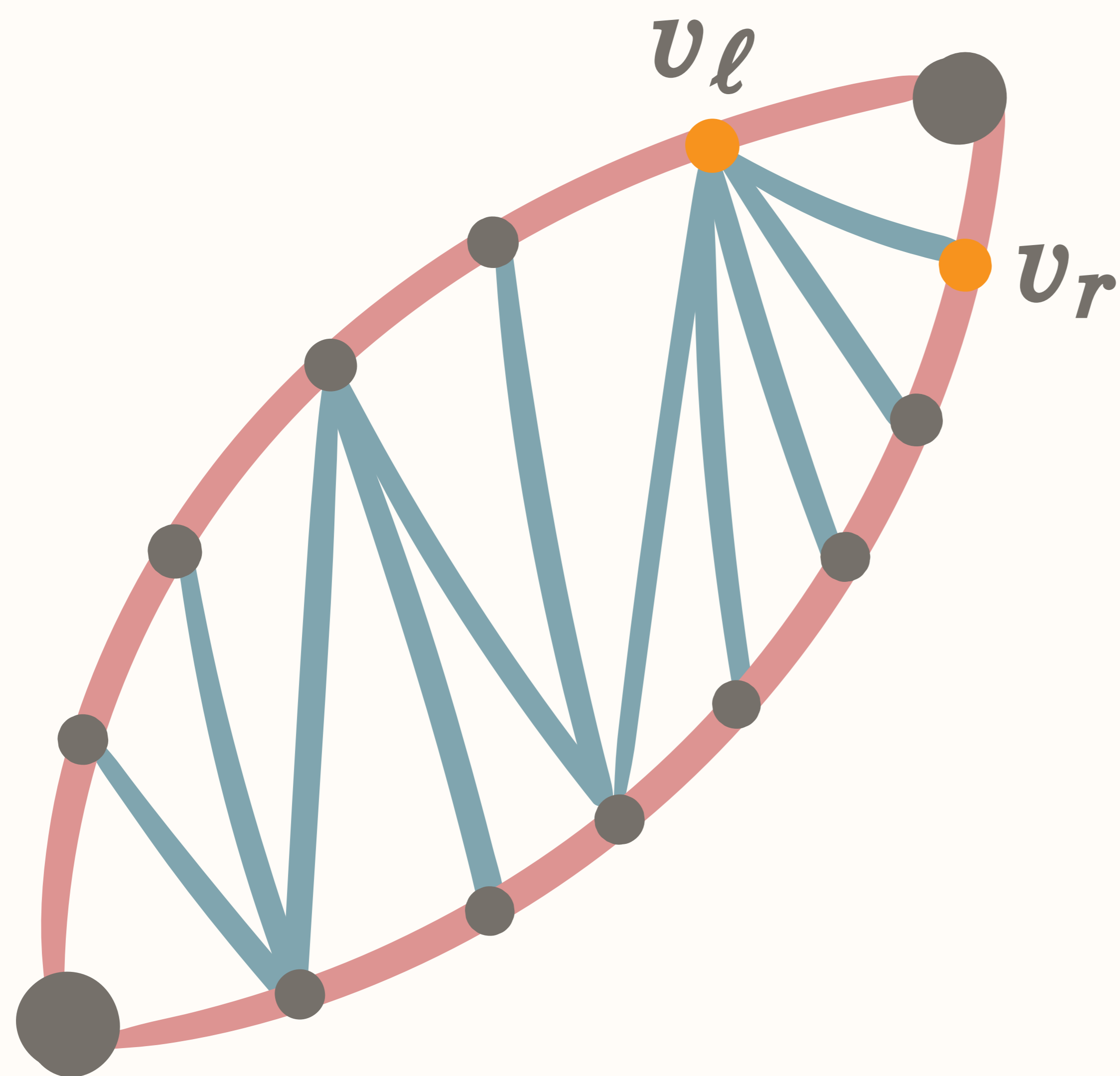
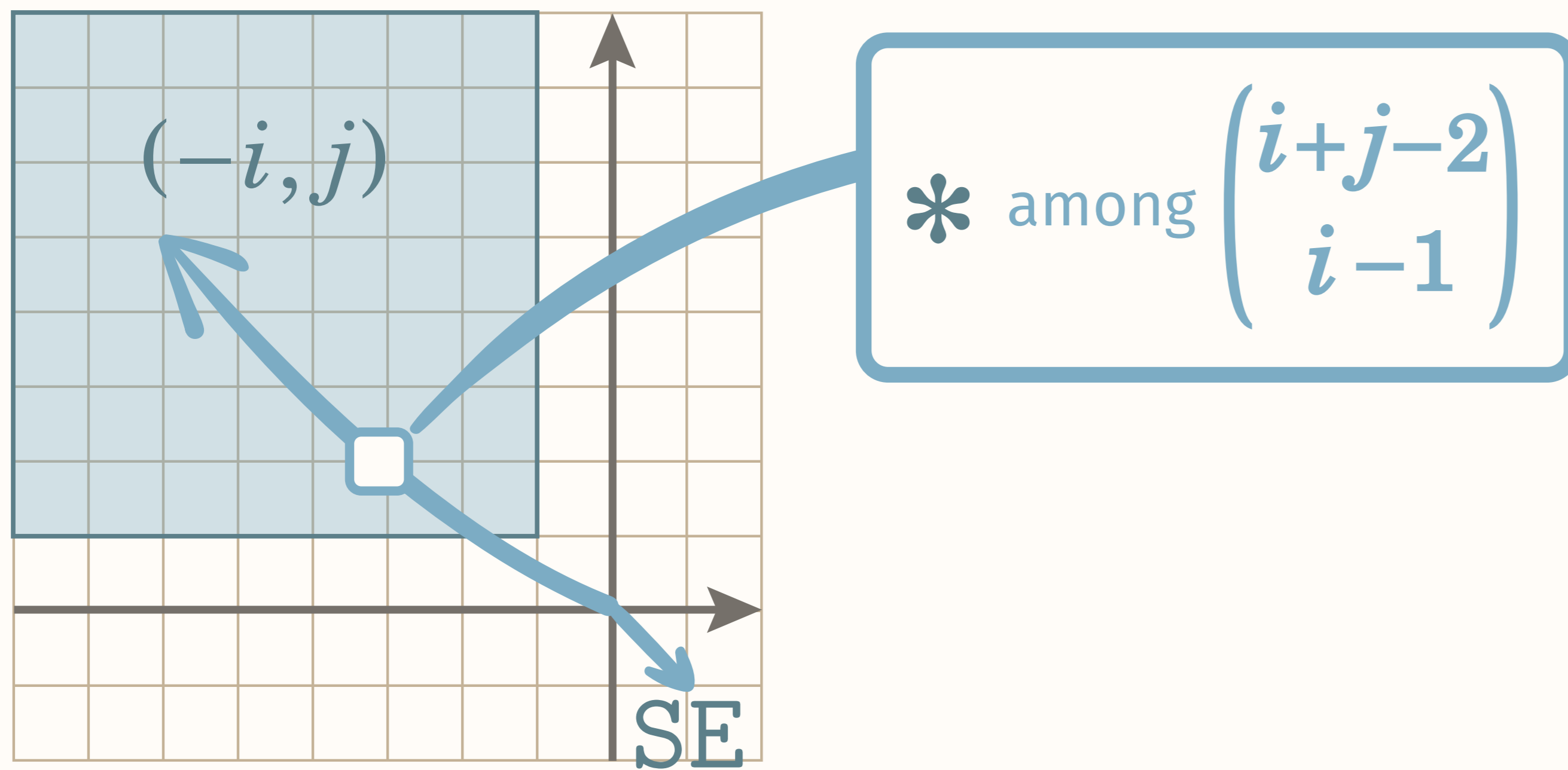
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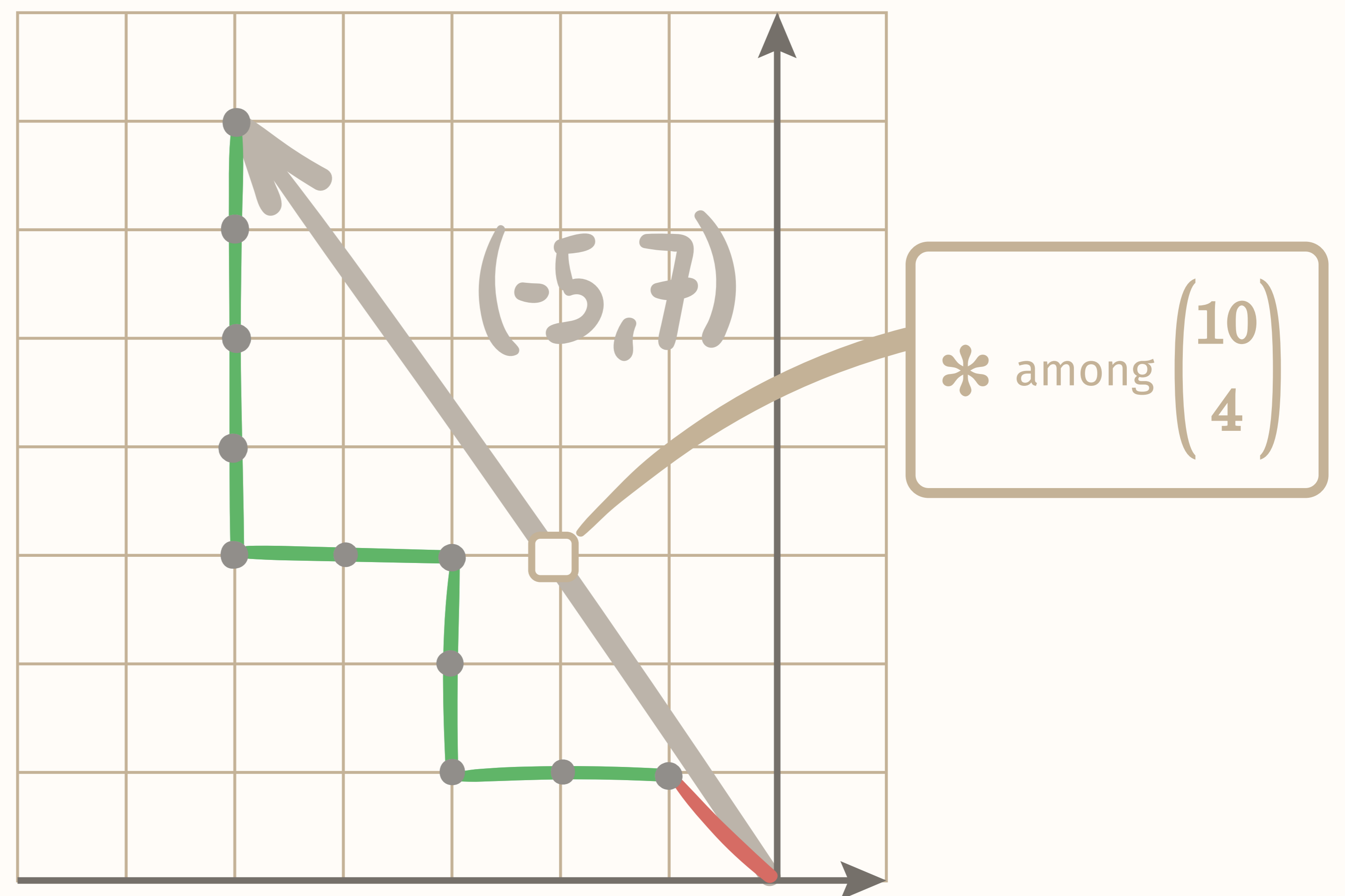
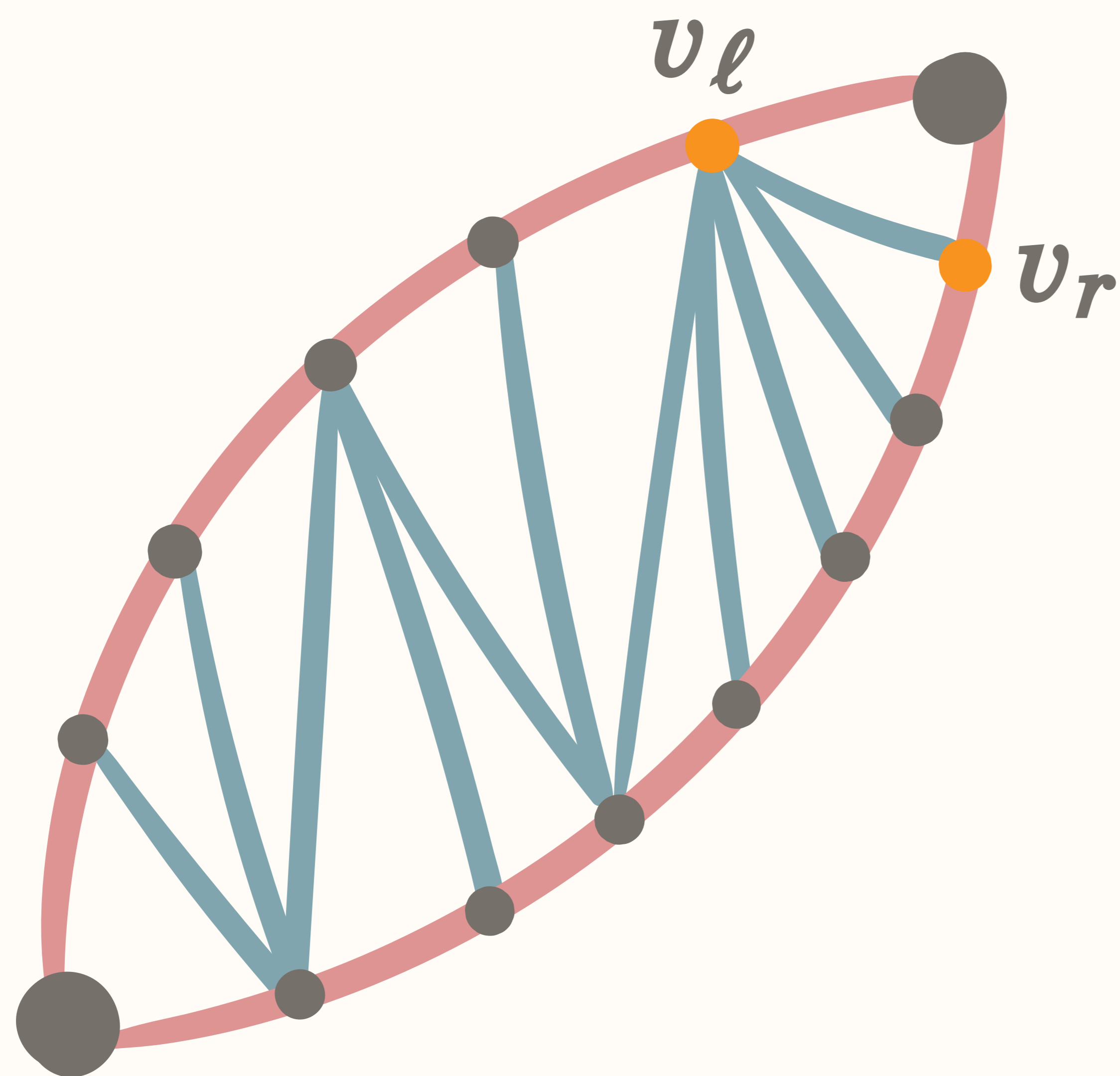
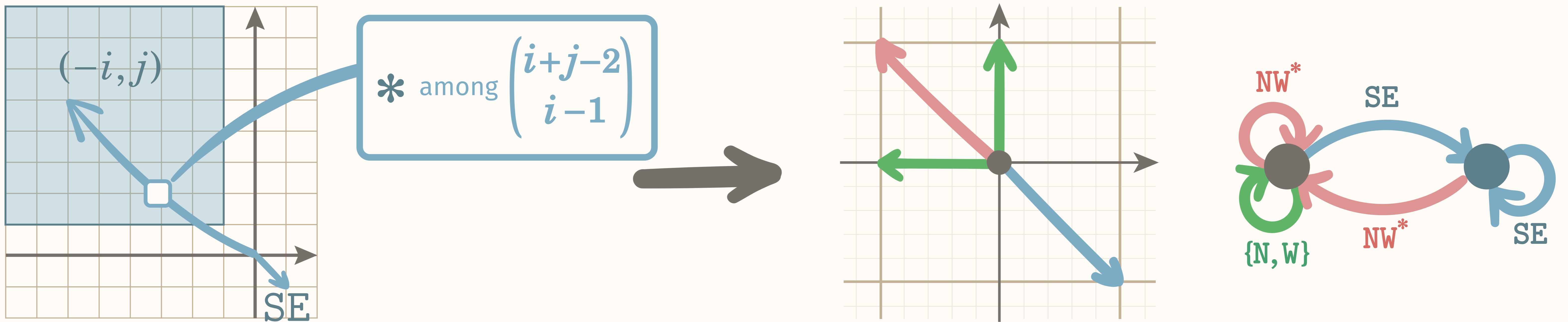
Breaking into small steps



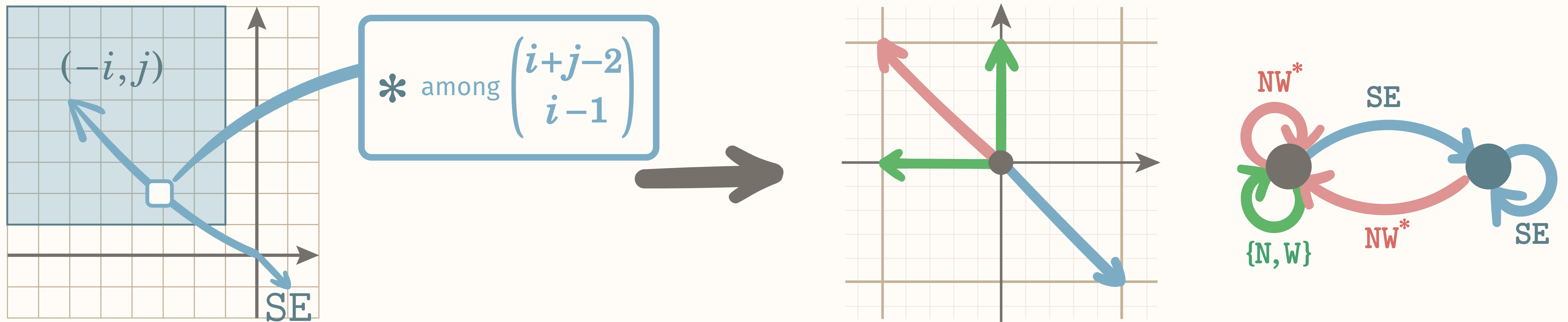
Breaking into small steps



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Breaking into small steps



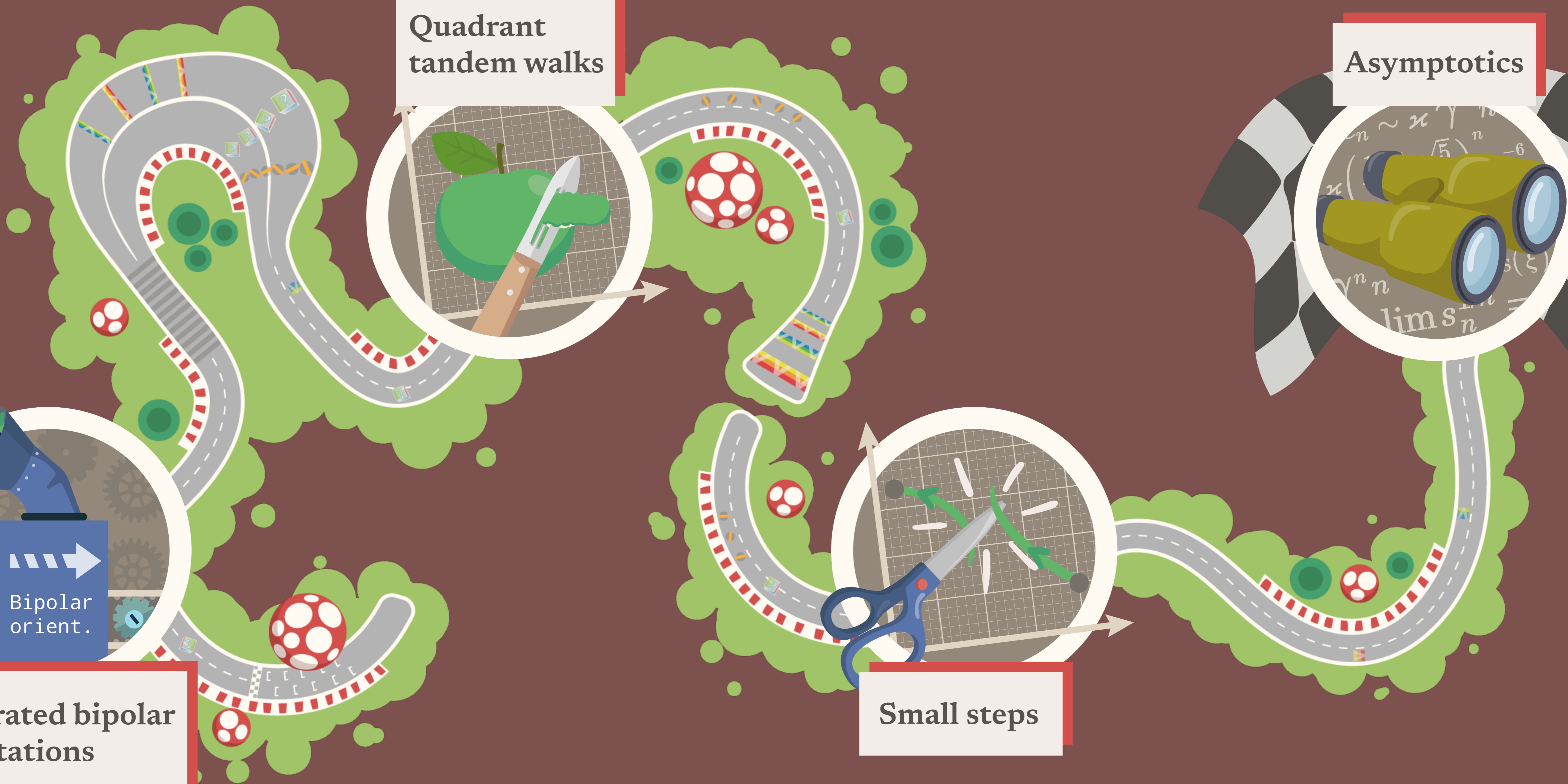
1, 1, 2, 6, 24, 116, 642, 3938, 26194, 186042, ...

⇒ *Counting rectangular drawings* T. Takahashi,
R. Fujimaki; Y. Inoue (2009)

Summary

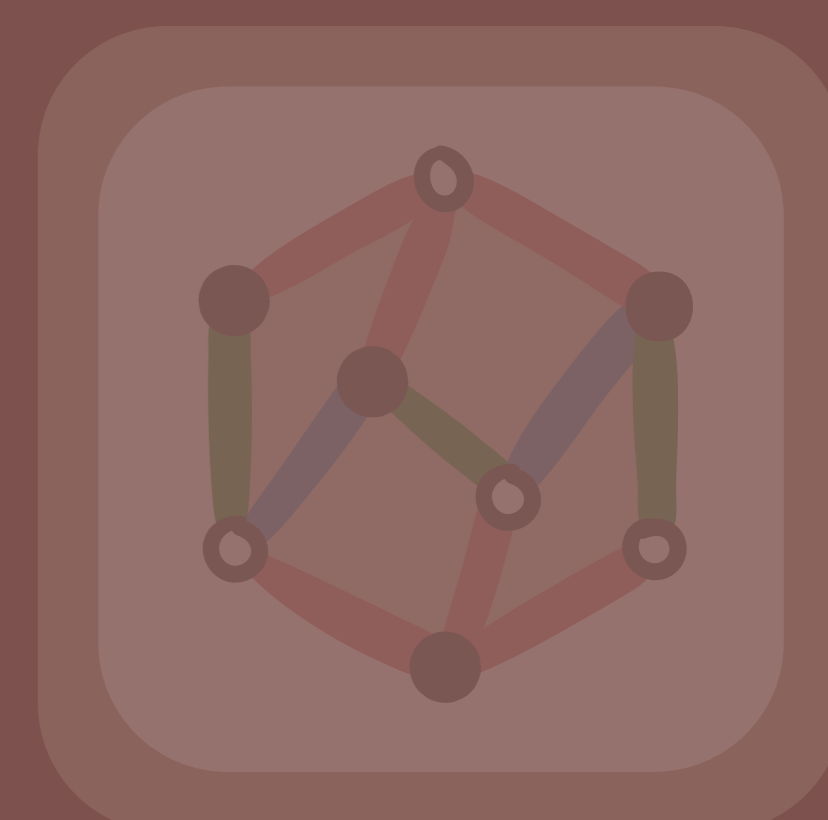
Quadrant
tandem walks

Asymptotics



Decorated bipolar
orientations

Small steps



*bipolar posets
by vertices*

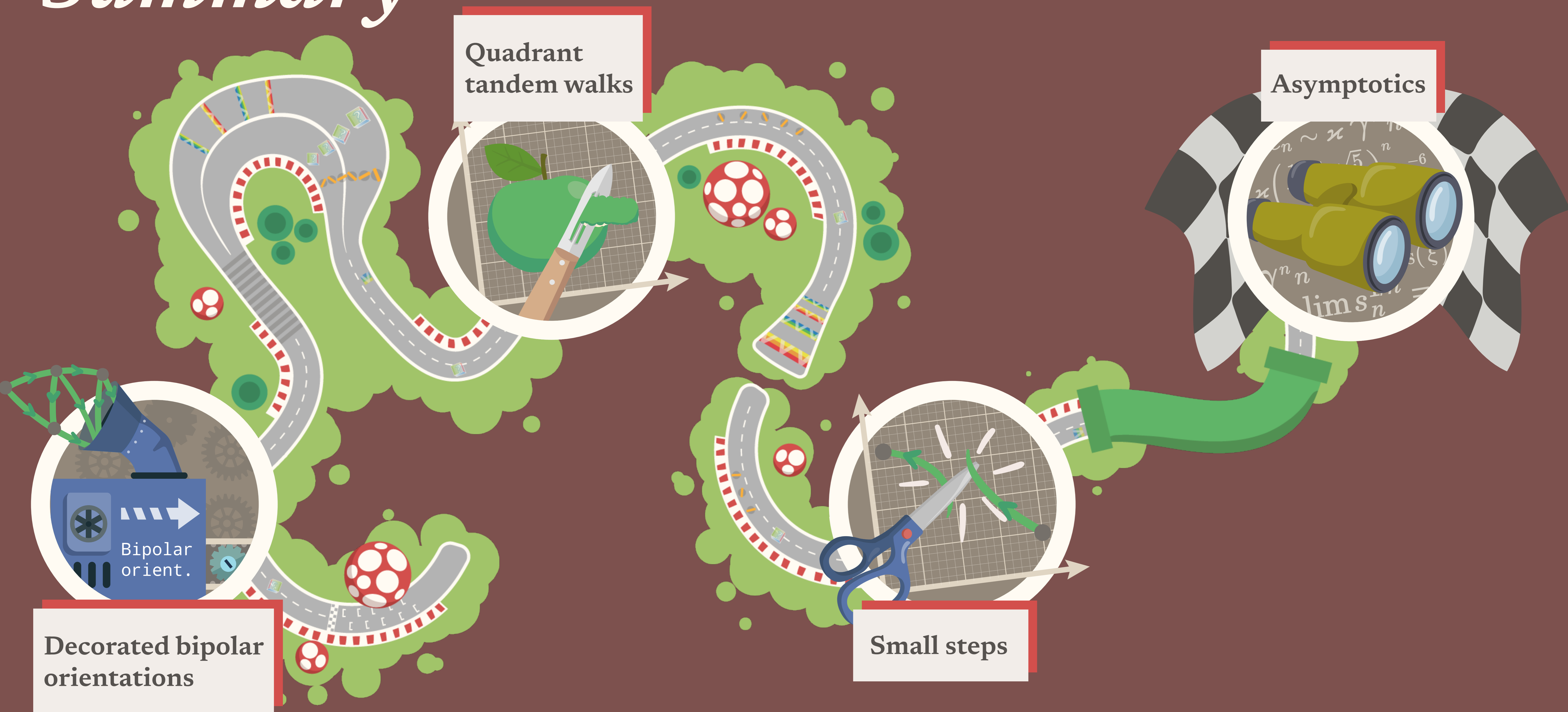
*bipolar posets
by edges*

*transversal
structures*

*polyhedral
orientations*

*Schnyder
labelings*

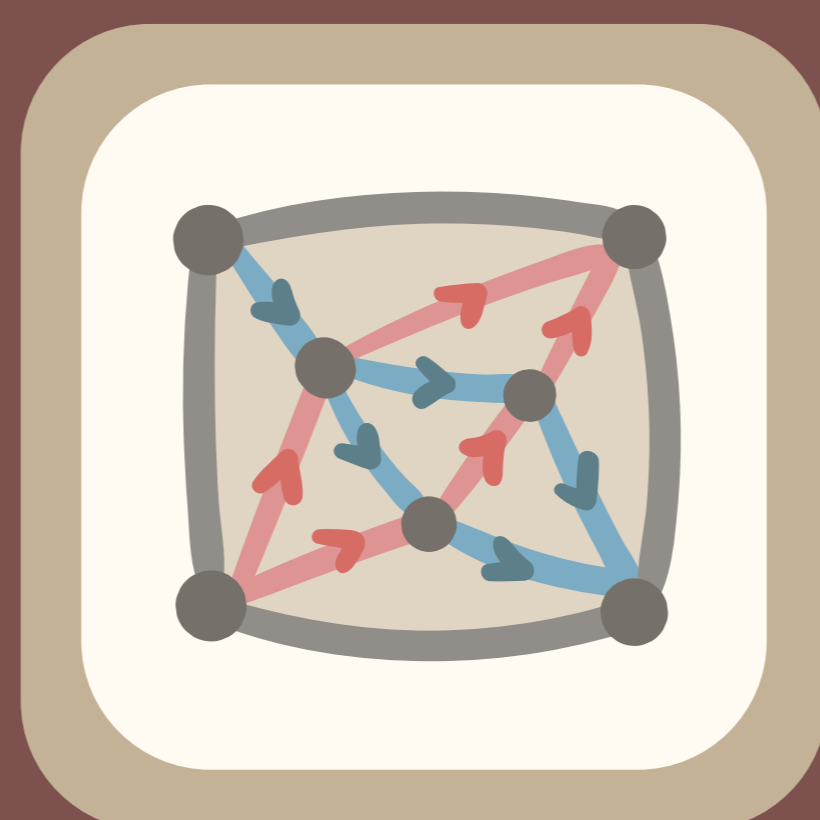
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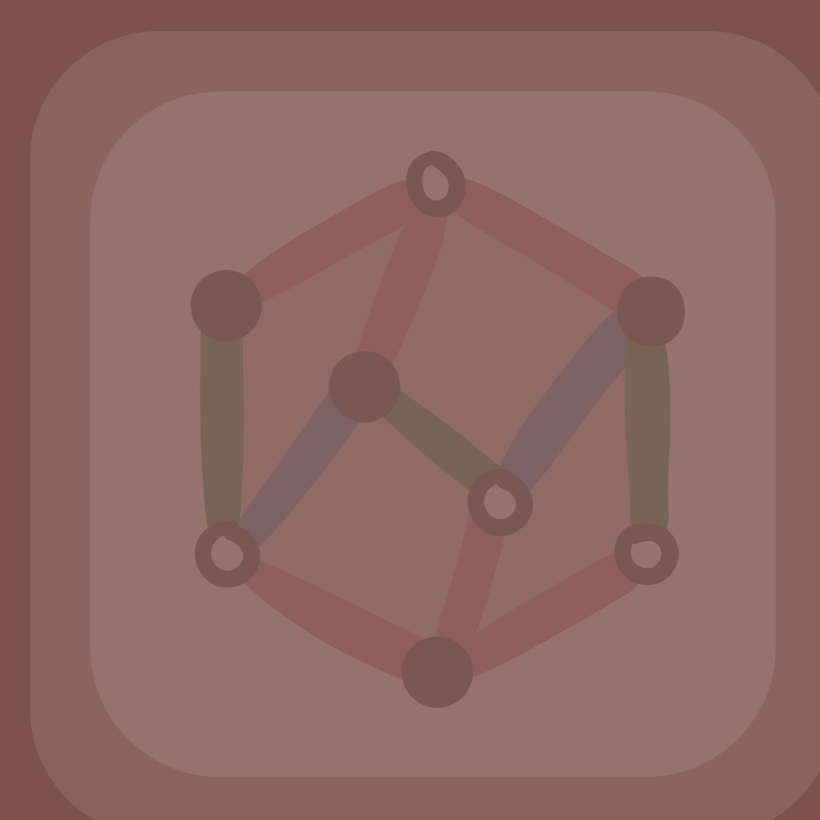
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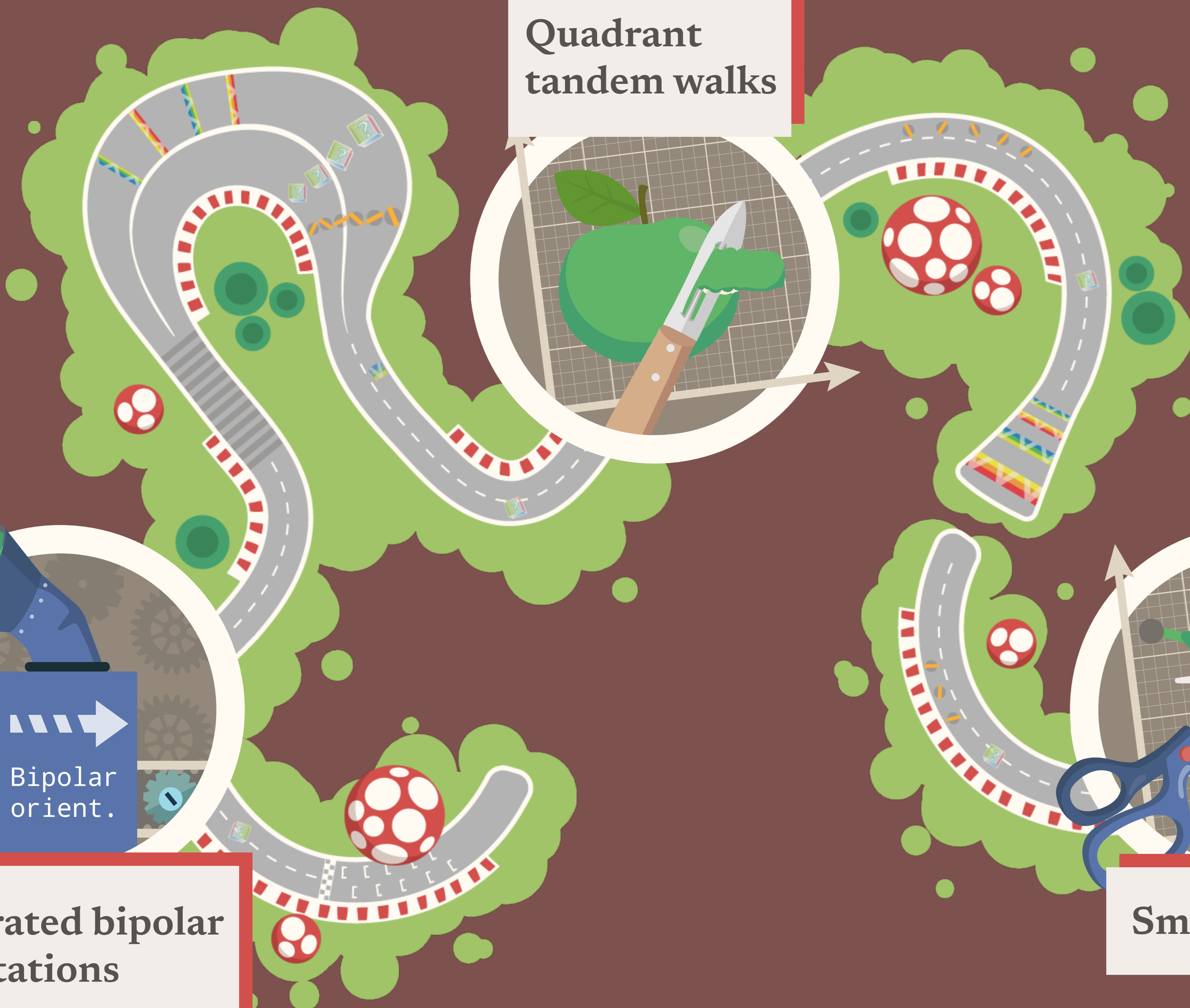


*Schnyder
labelings*

Summary

Quadrant tandem walks

Asymptotics



Decorated bipolar orientations



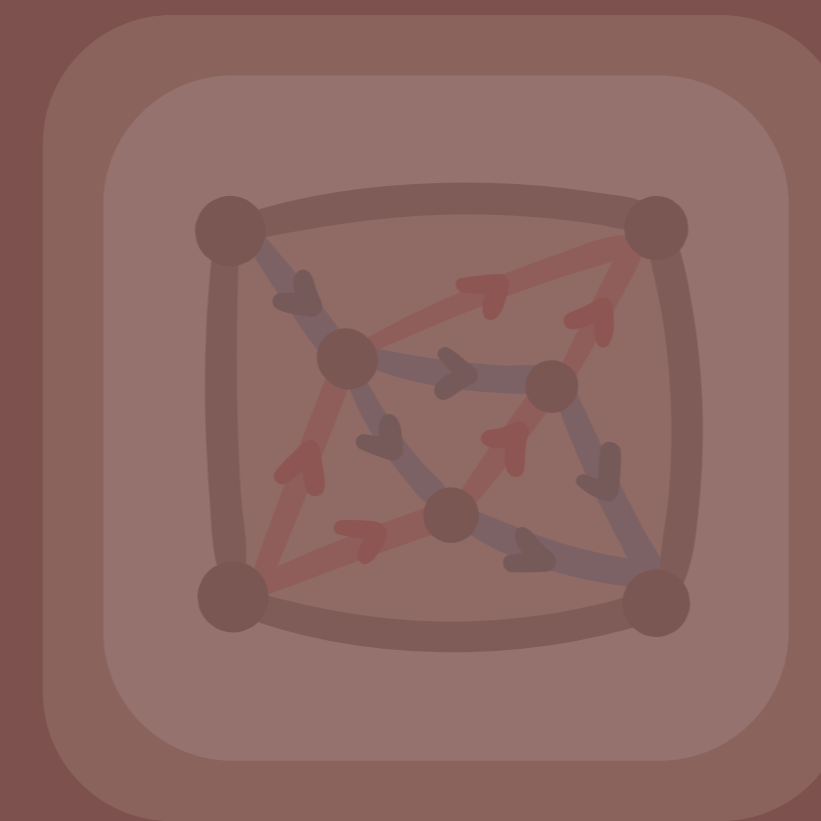
Small steps



bipolar posets by vertices



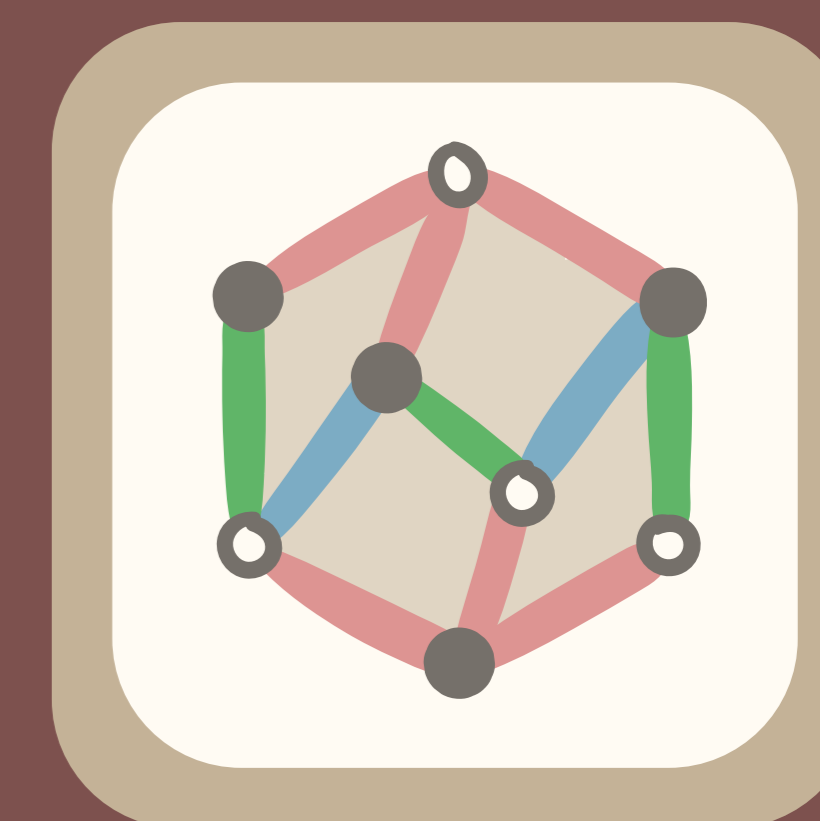
bipolar posets by edges



transversal structures



polyhedral orientations

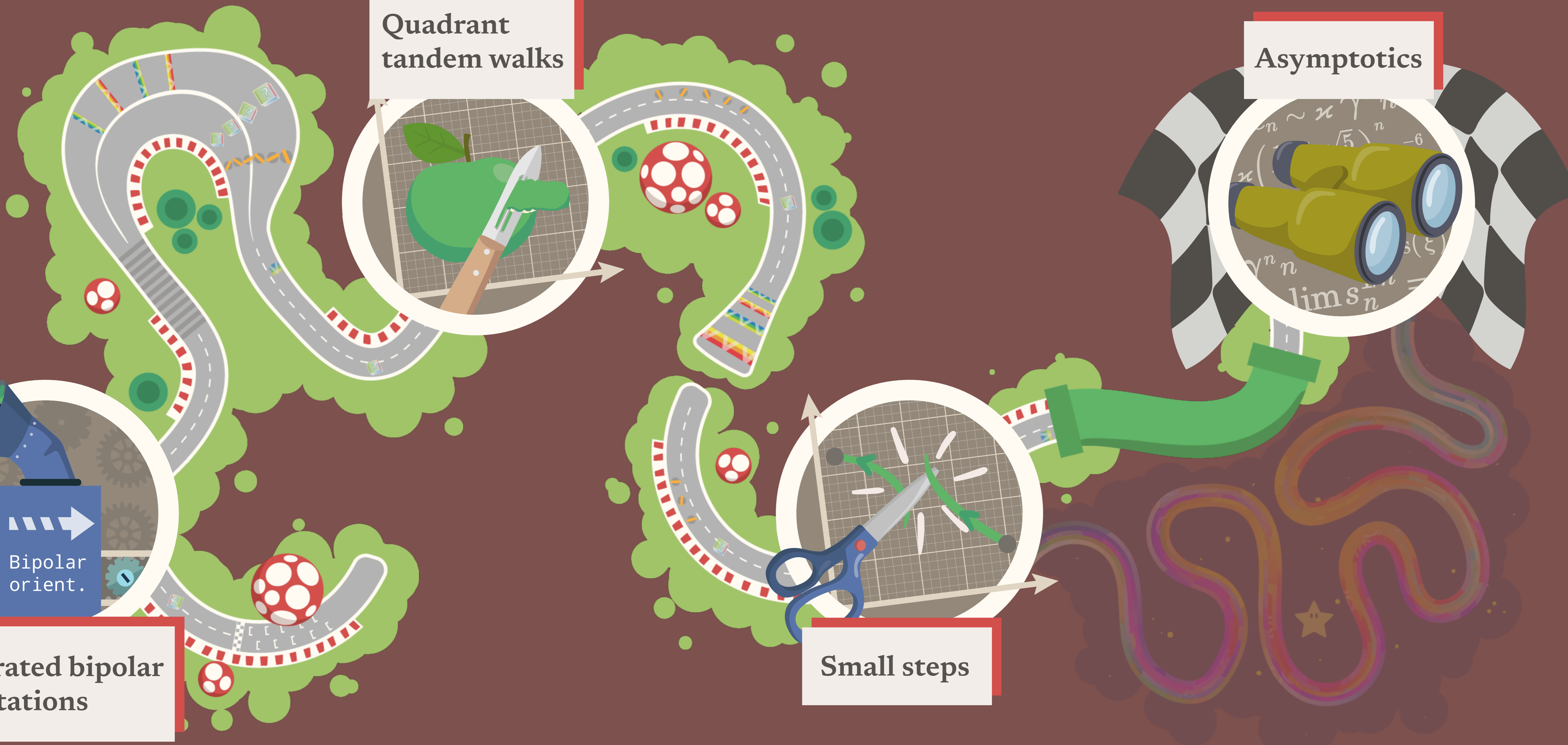


Schnyder labelings

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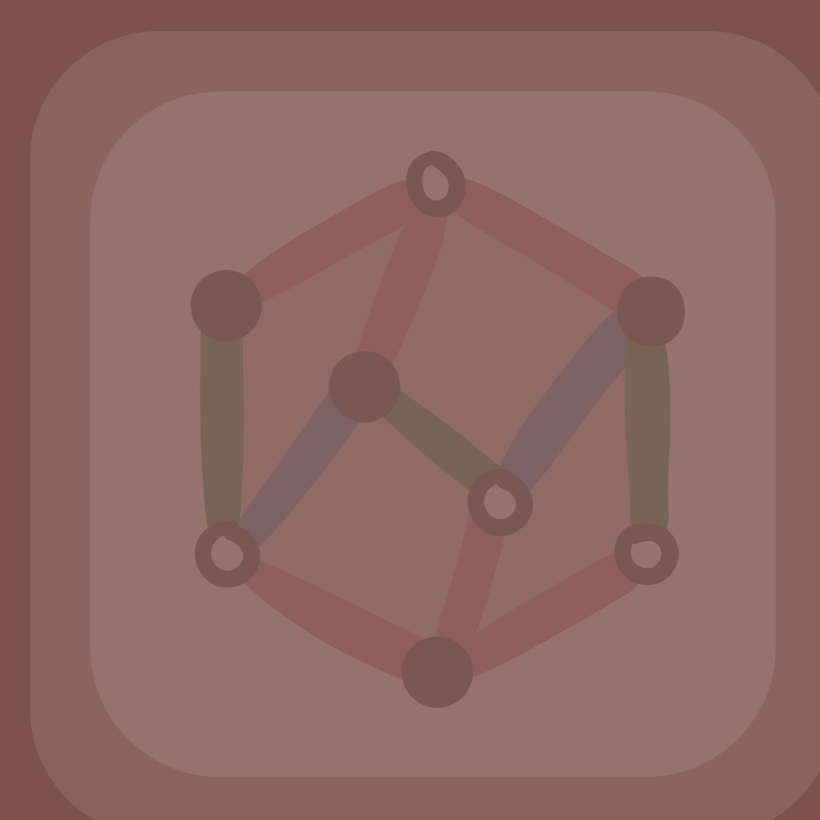
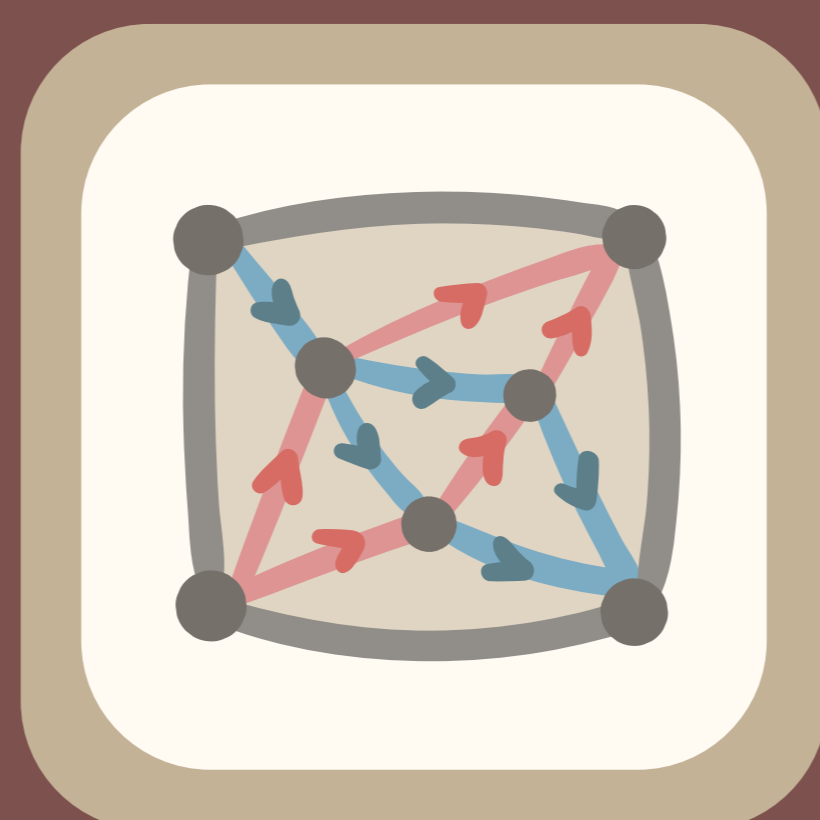
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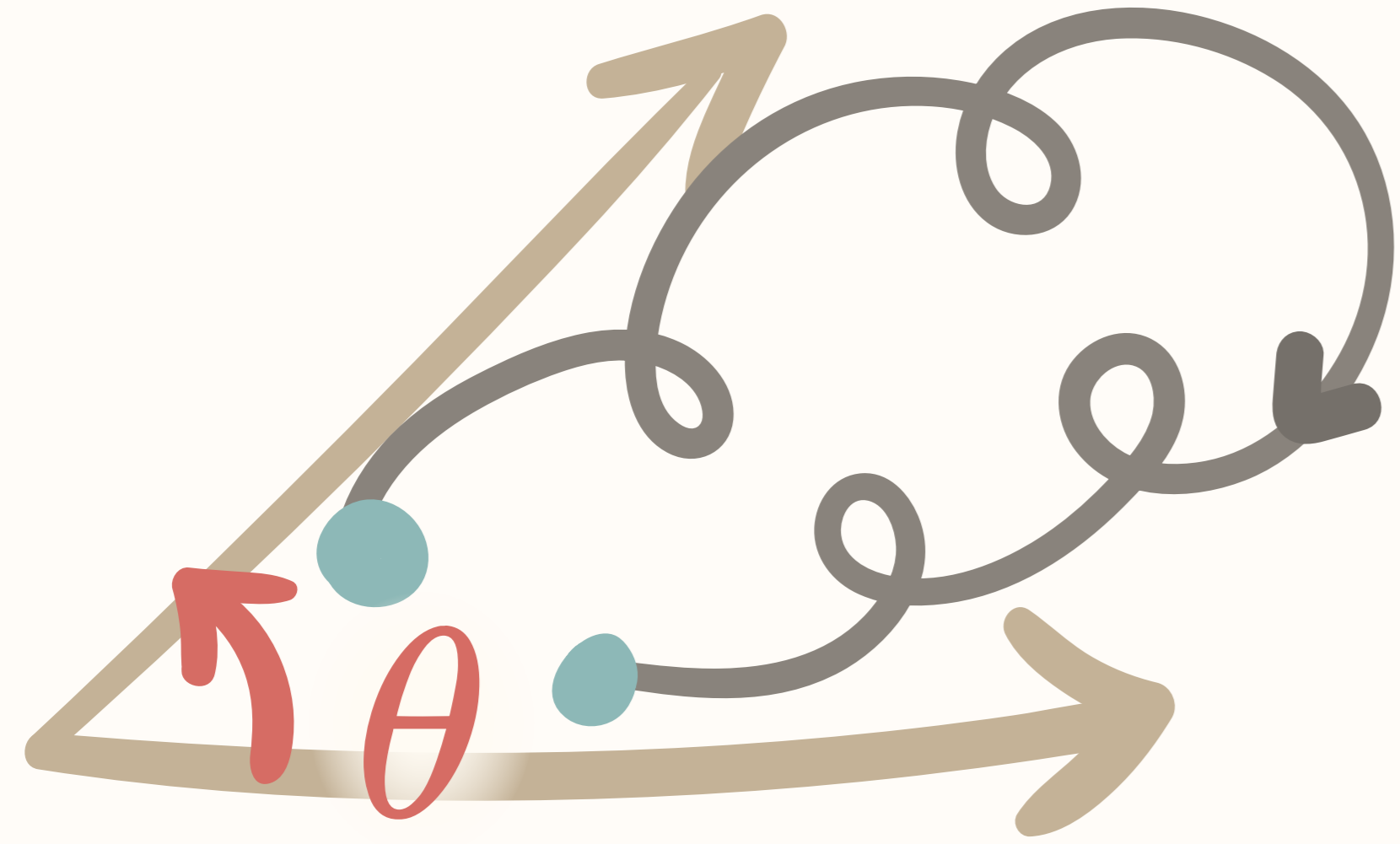
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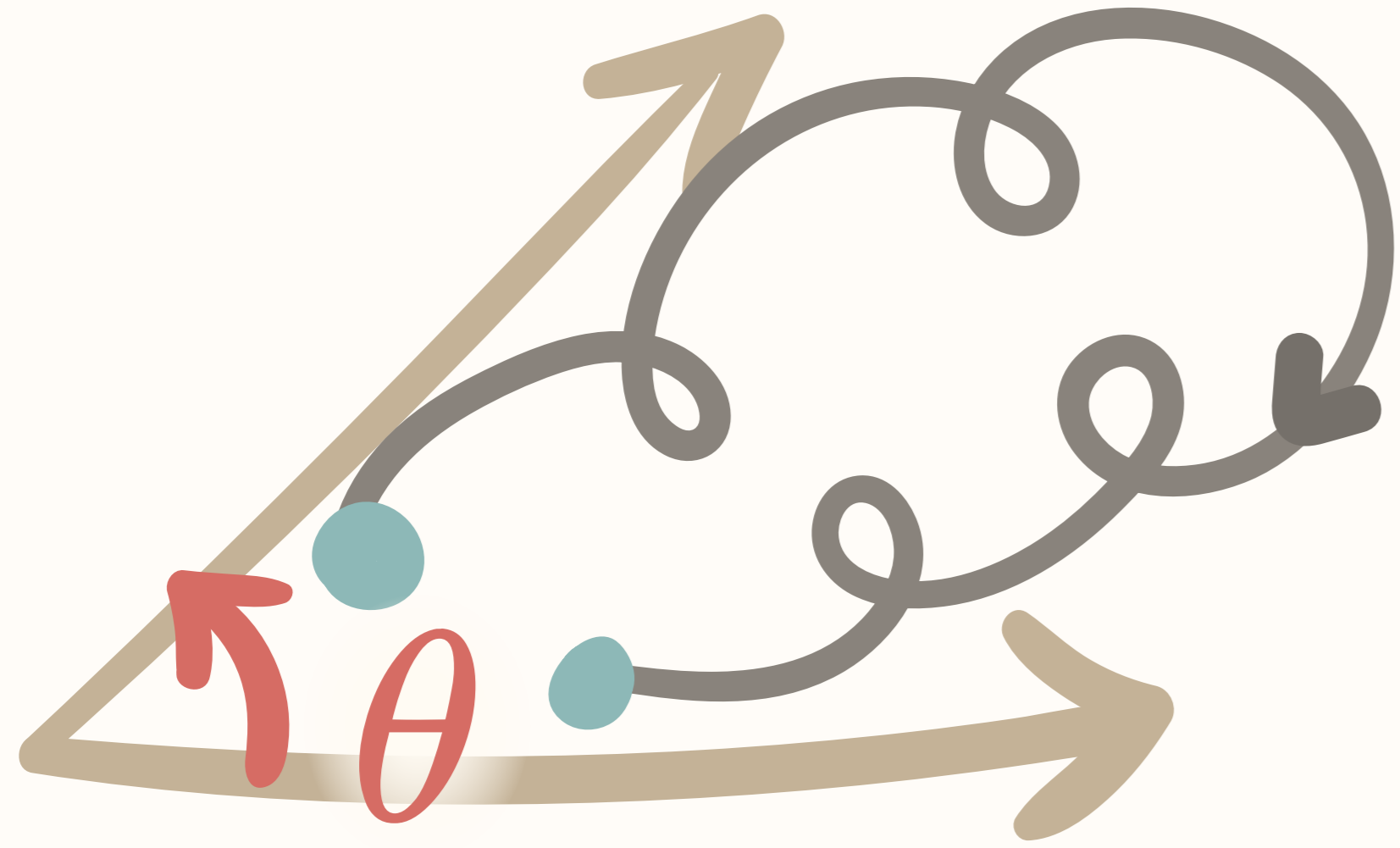
Asymptotic counting results



- ⇒ *Random walks in cone*, D. Denisov & V. Wachtel (2015)
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$$a_n \sim \kappa \cdot \gamma^n n^{-1-\pi/\theta}$$

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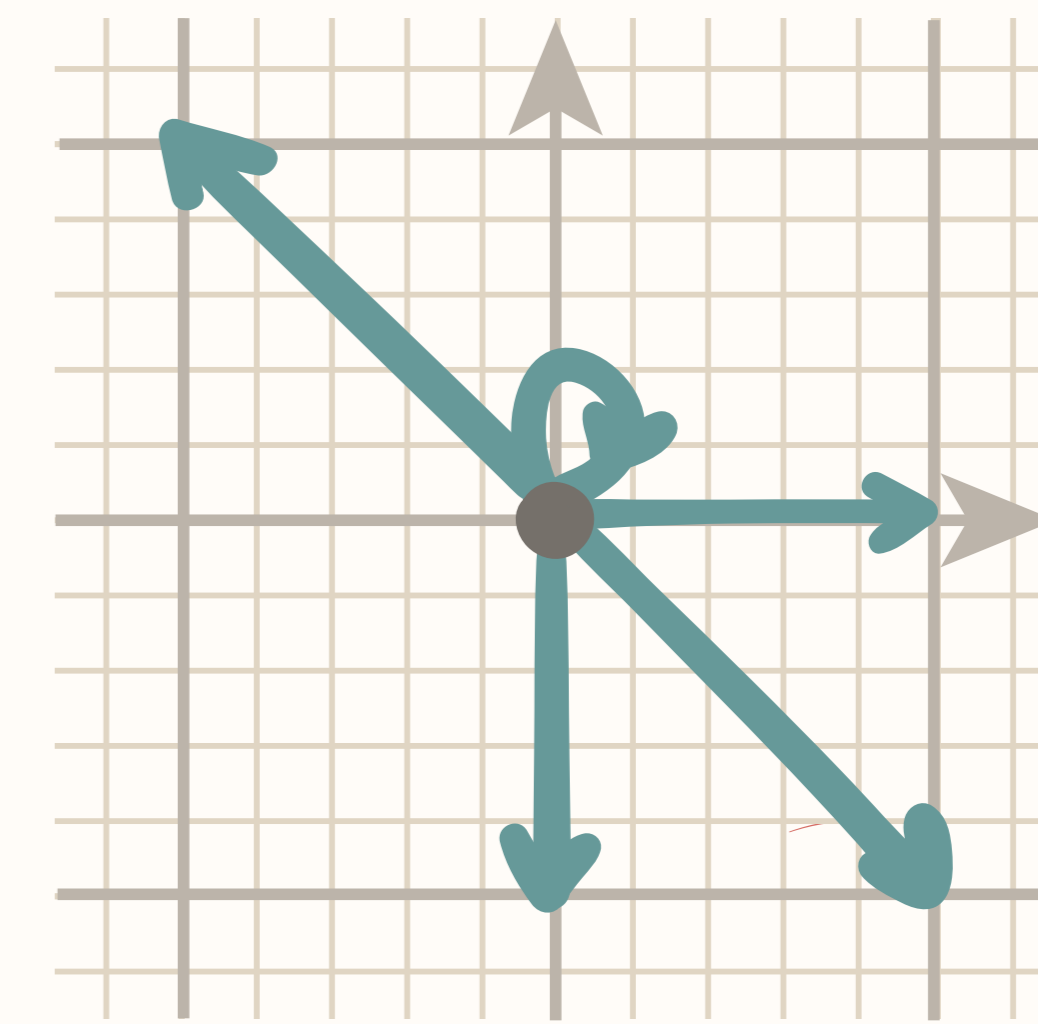
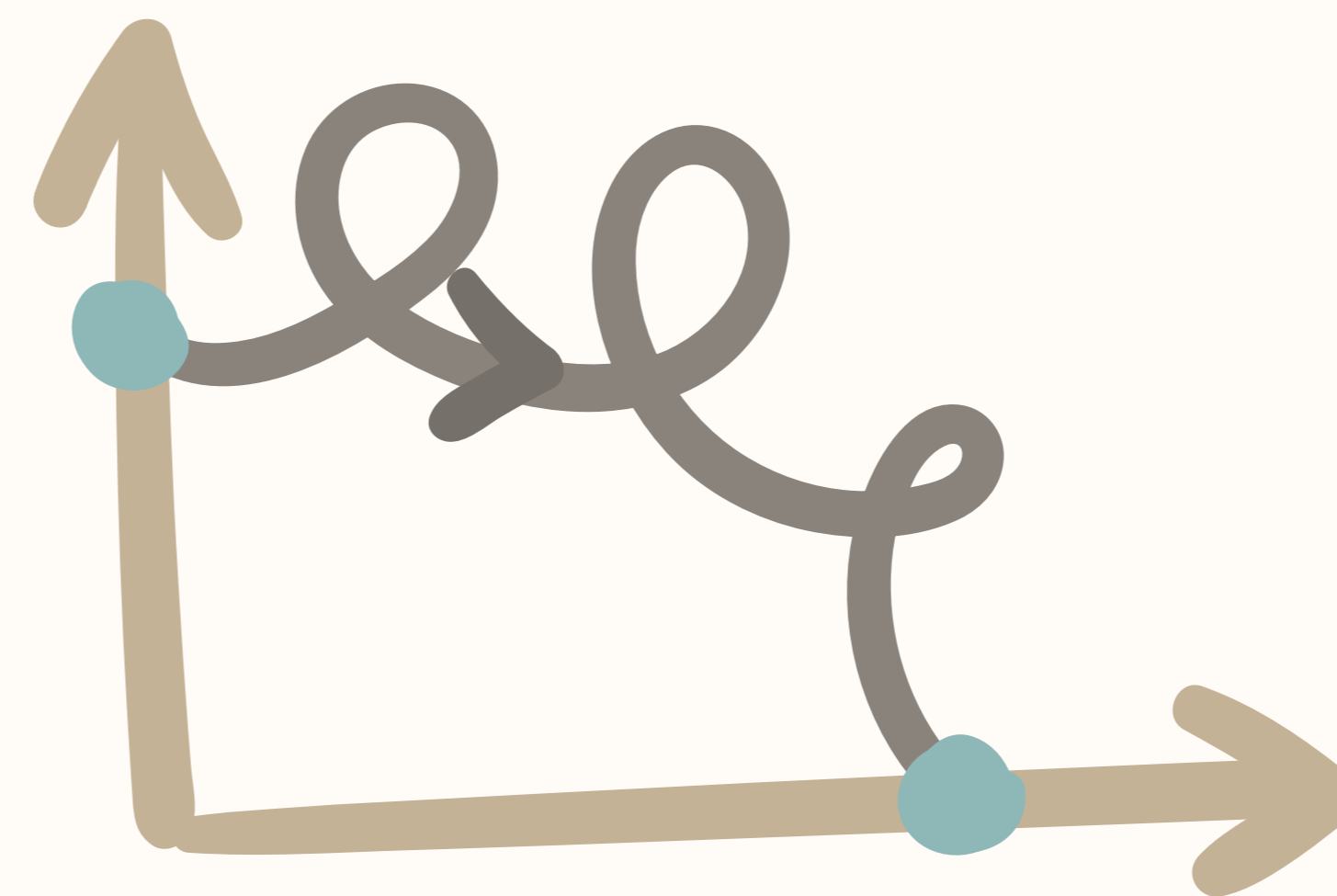
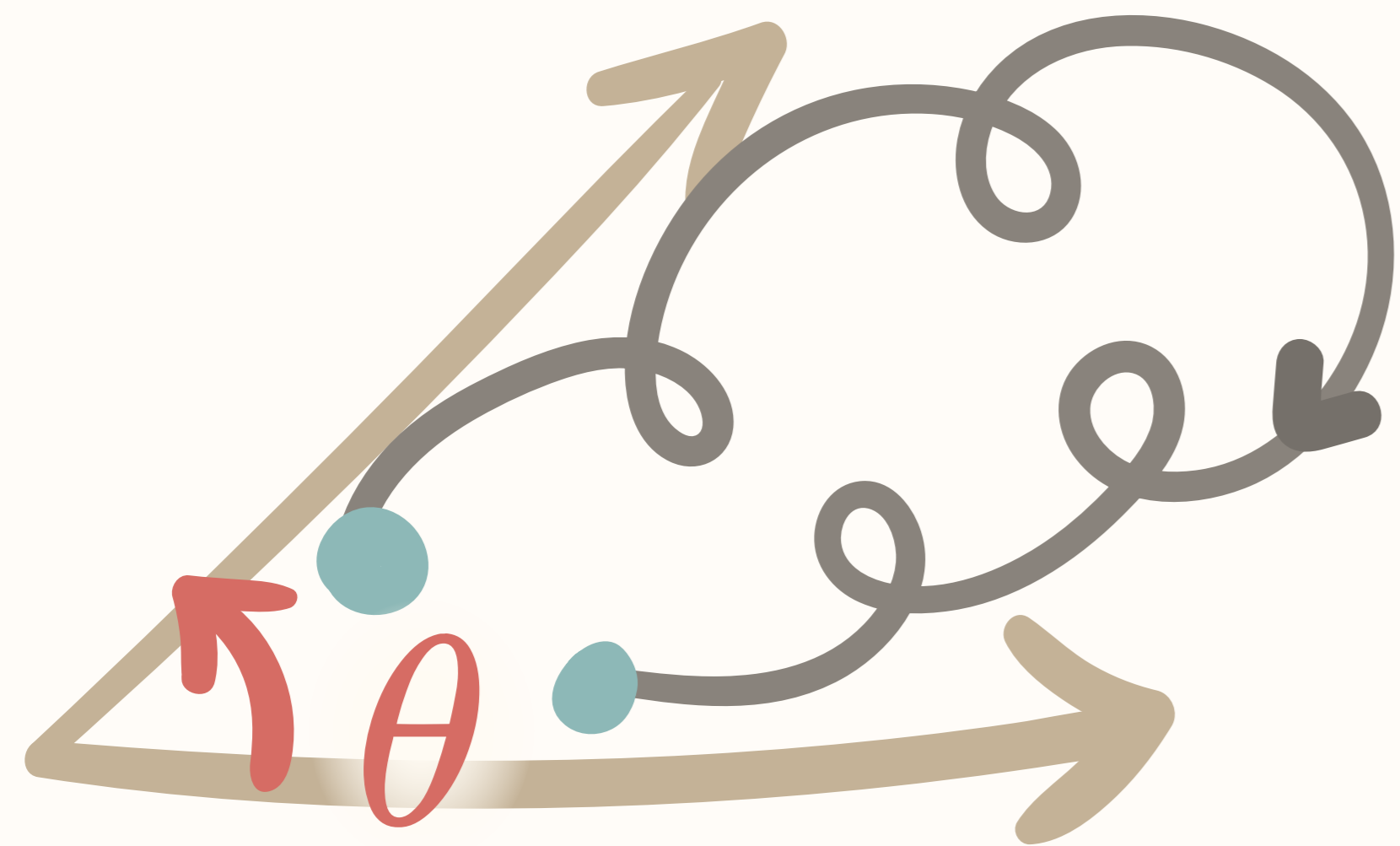
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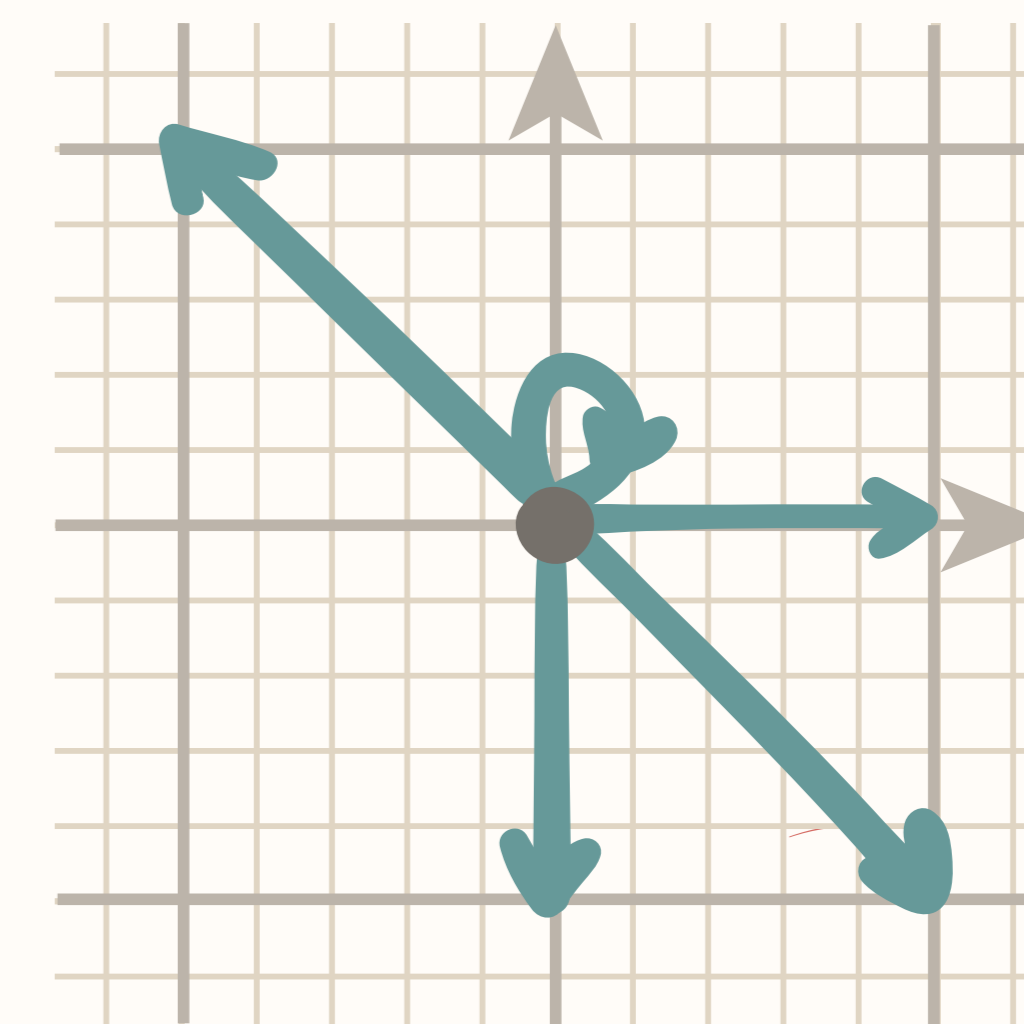
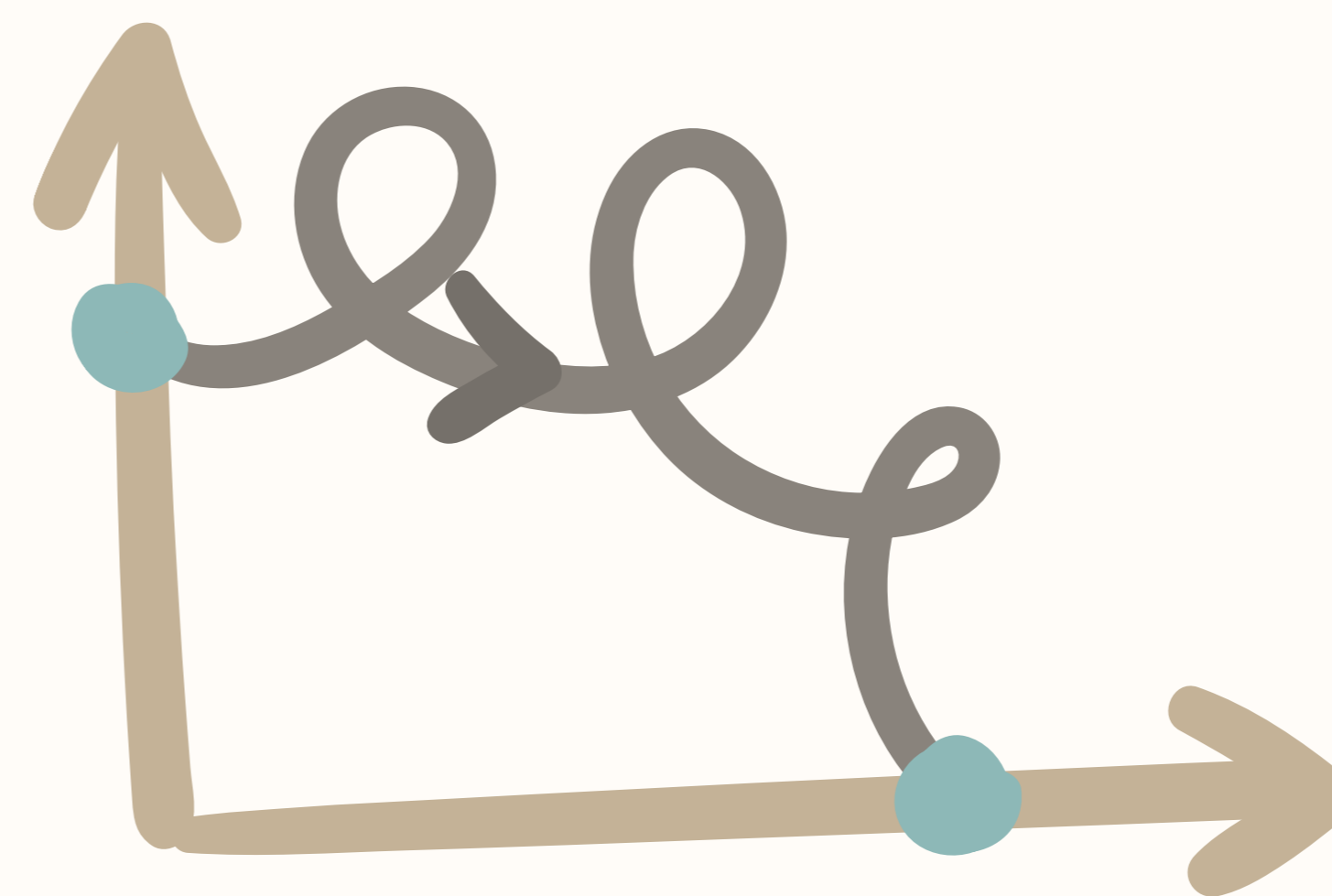
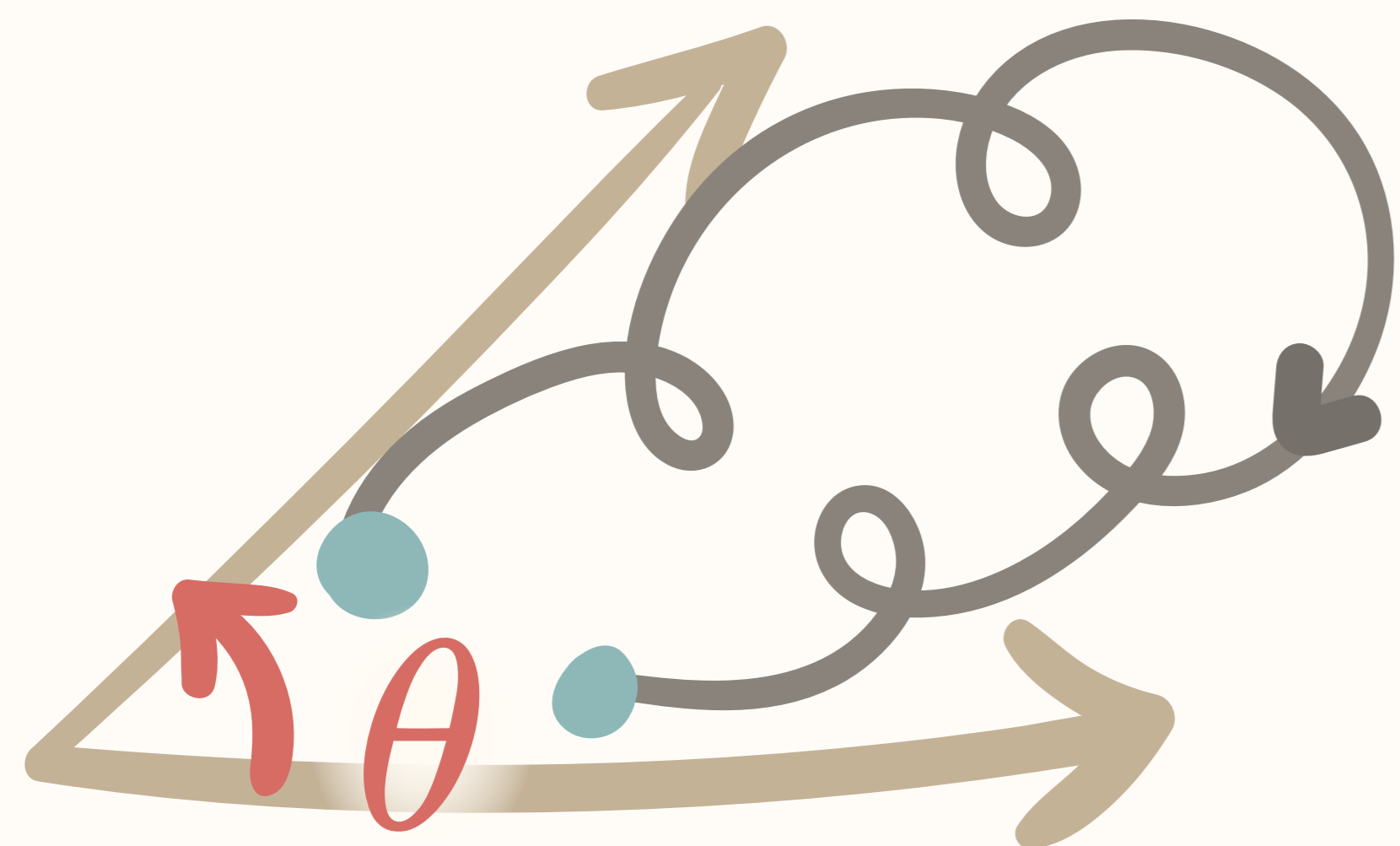
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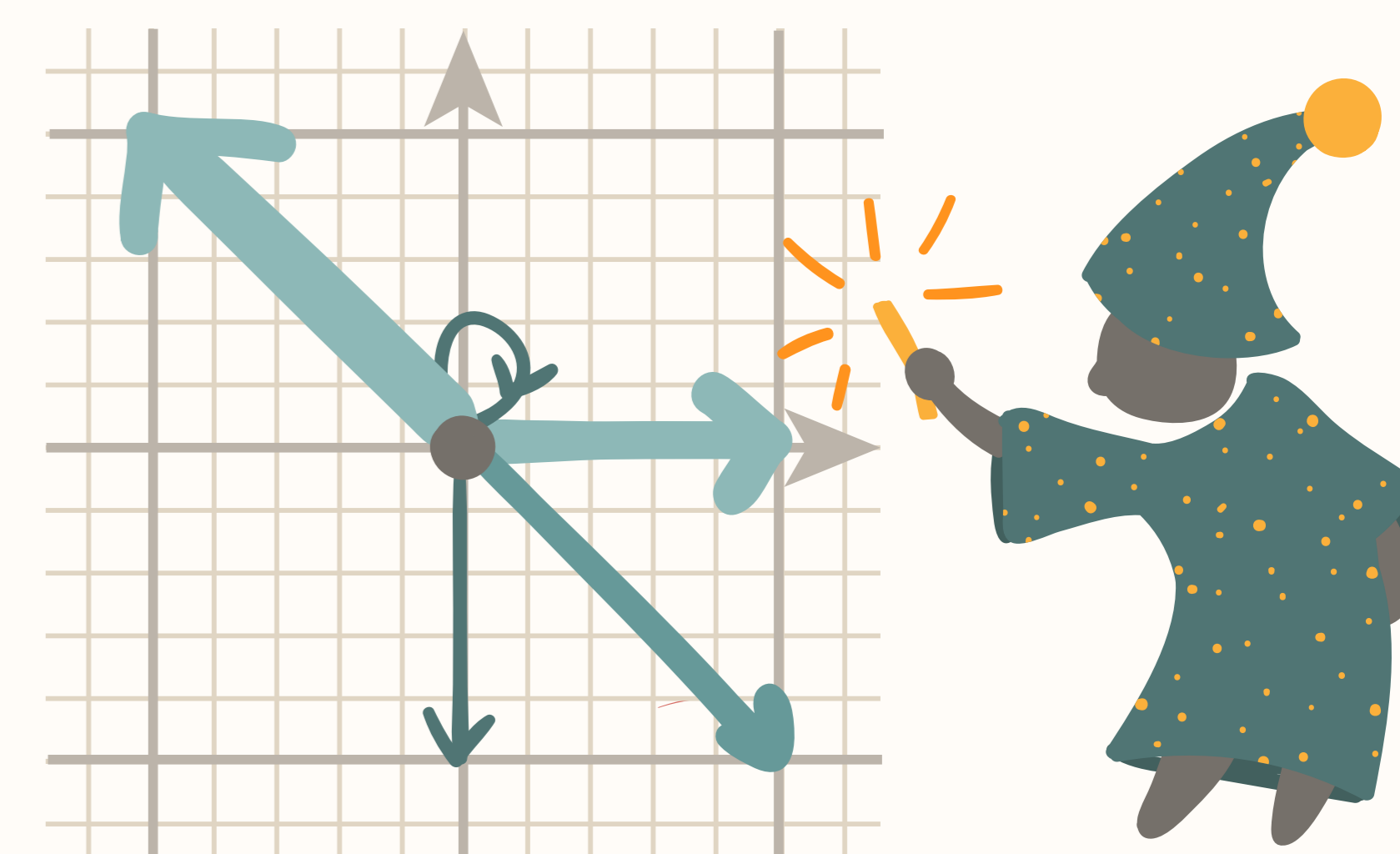
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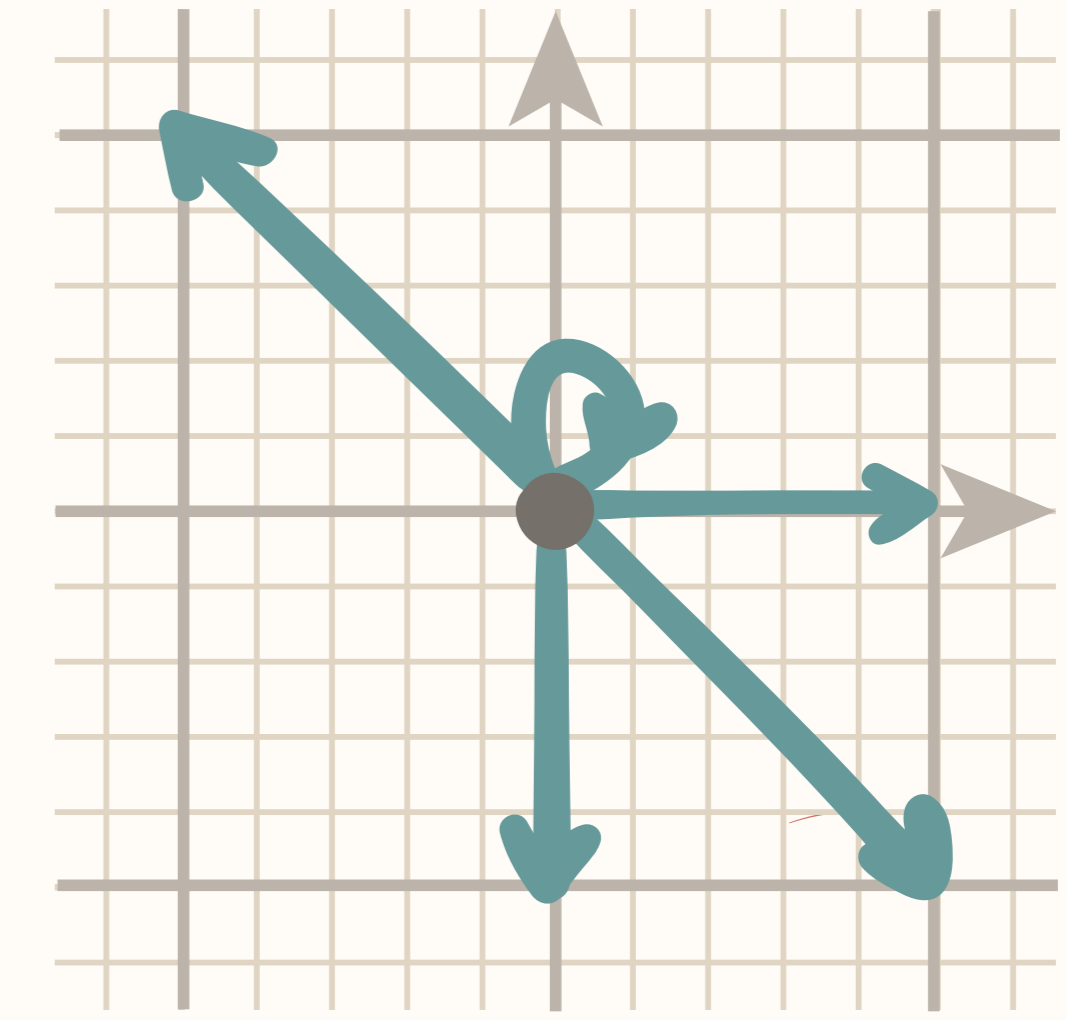
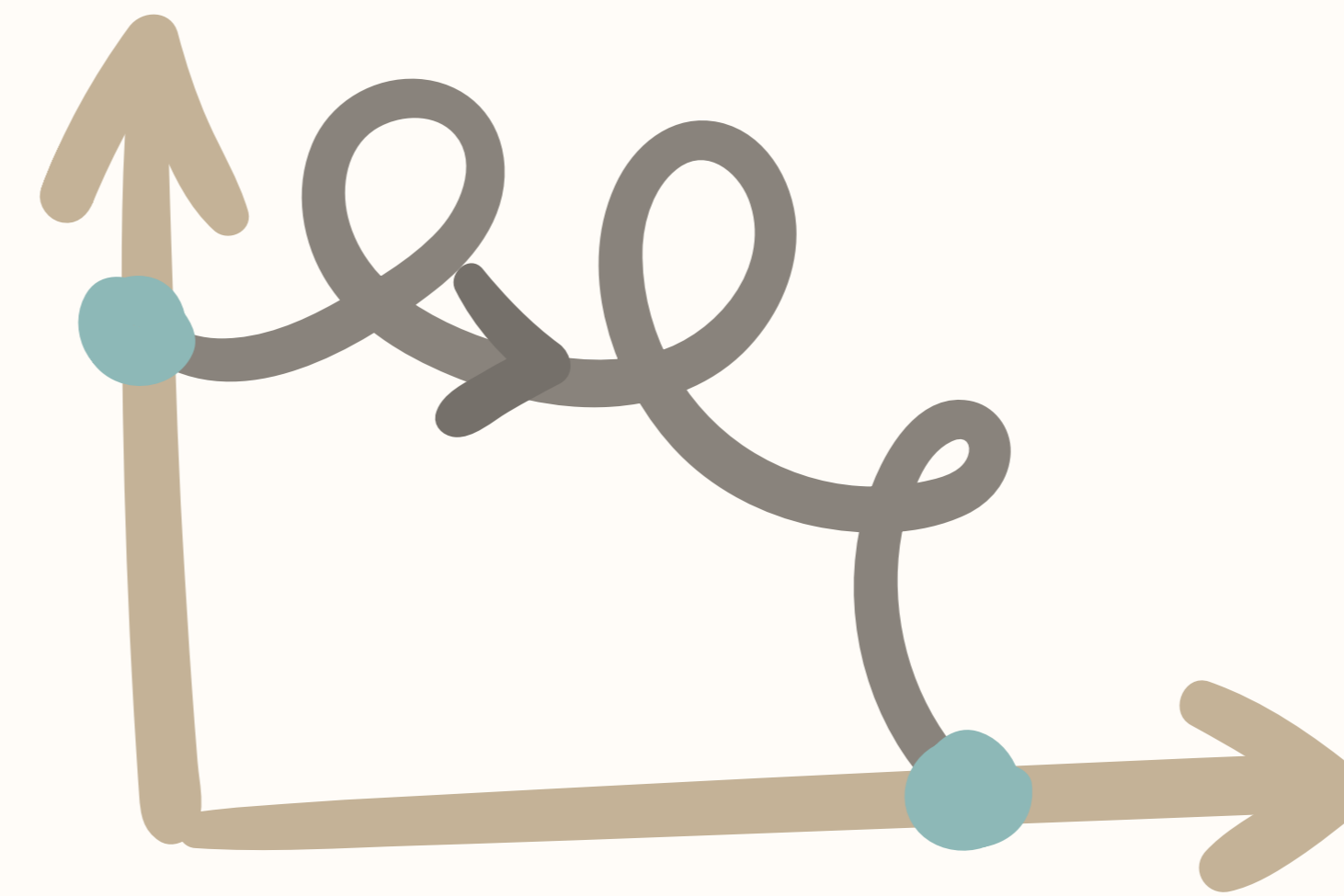
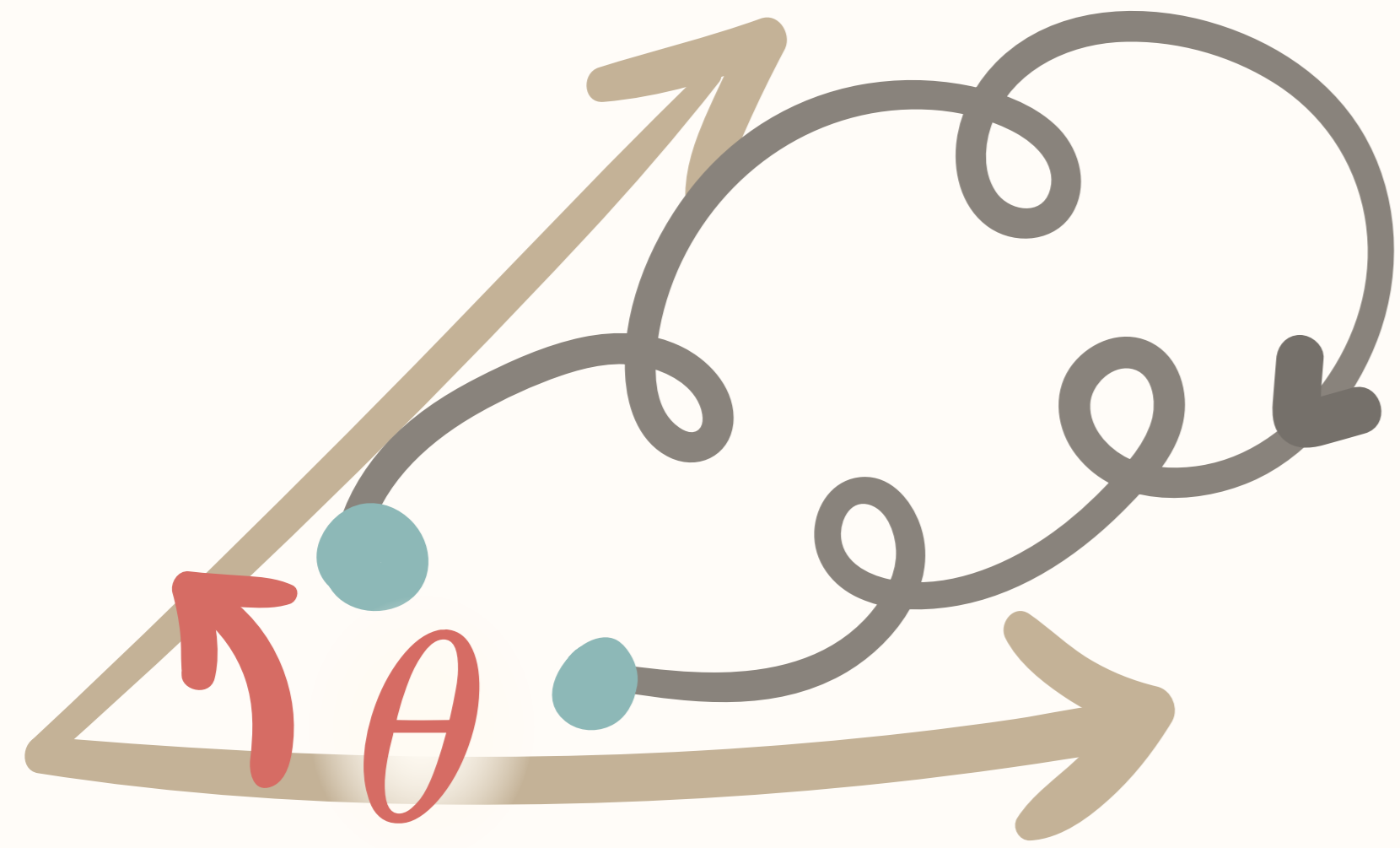
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Weighted steps

Asymptotic counting results

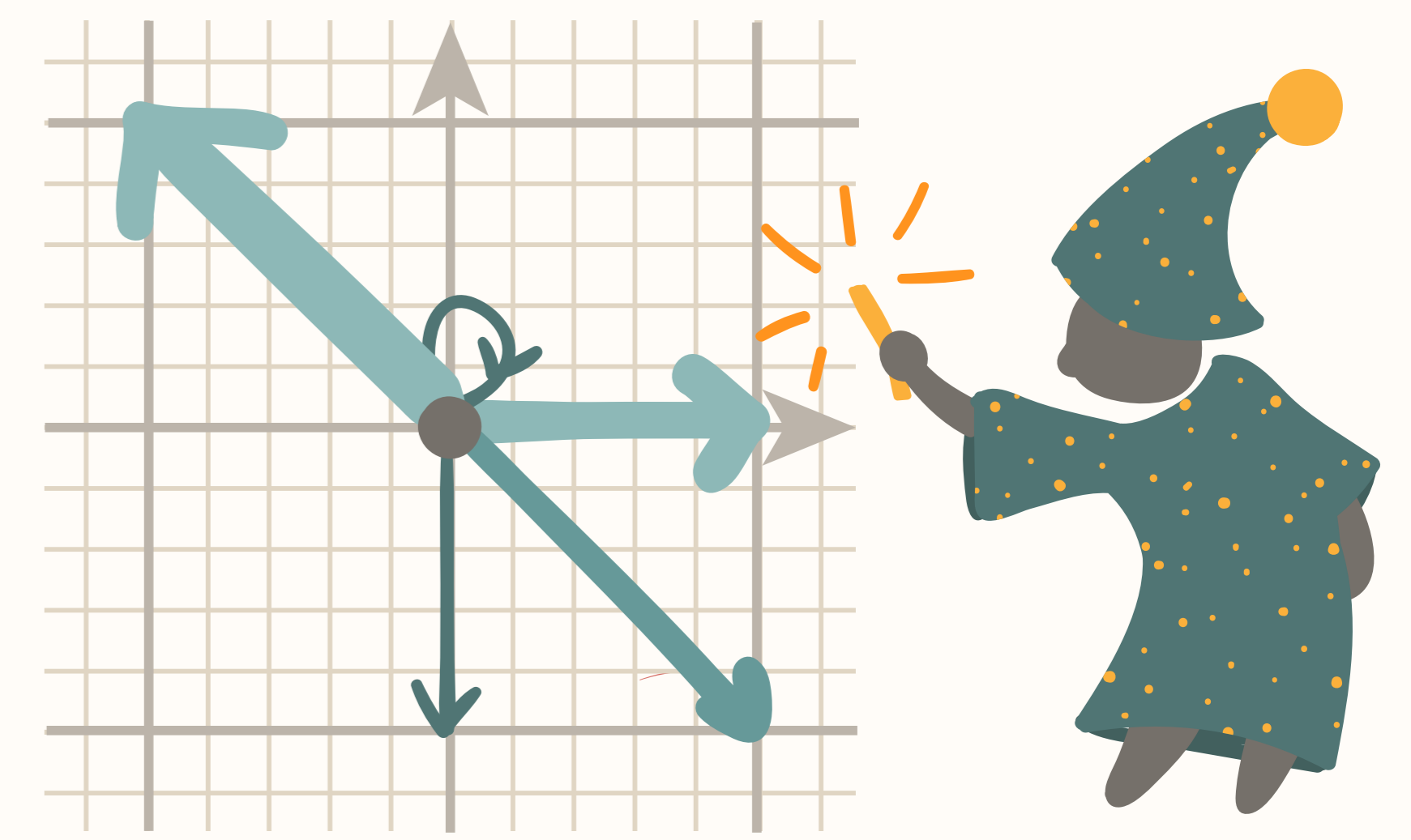


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Shear transformation



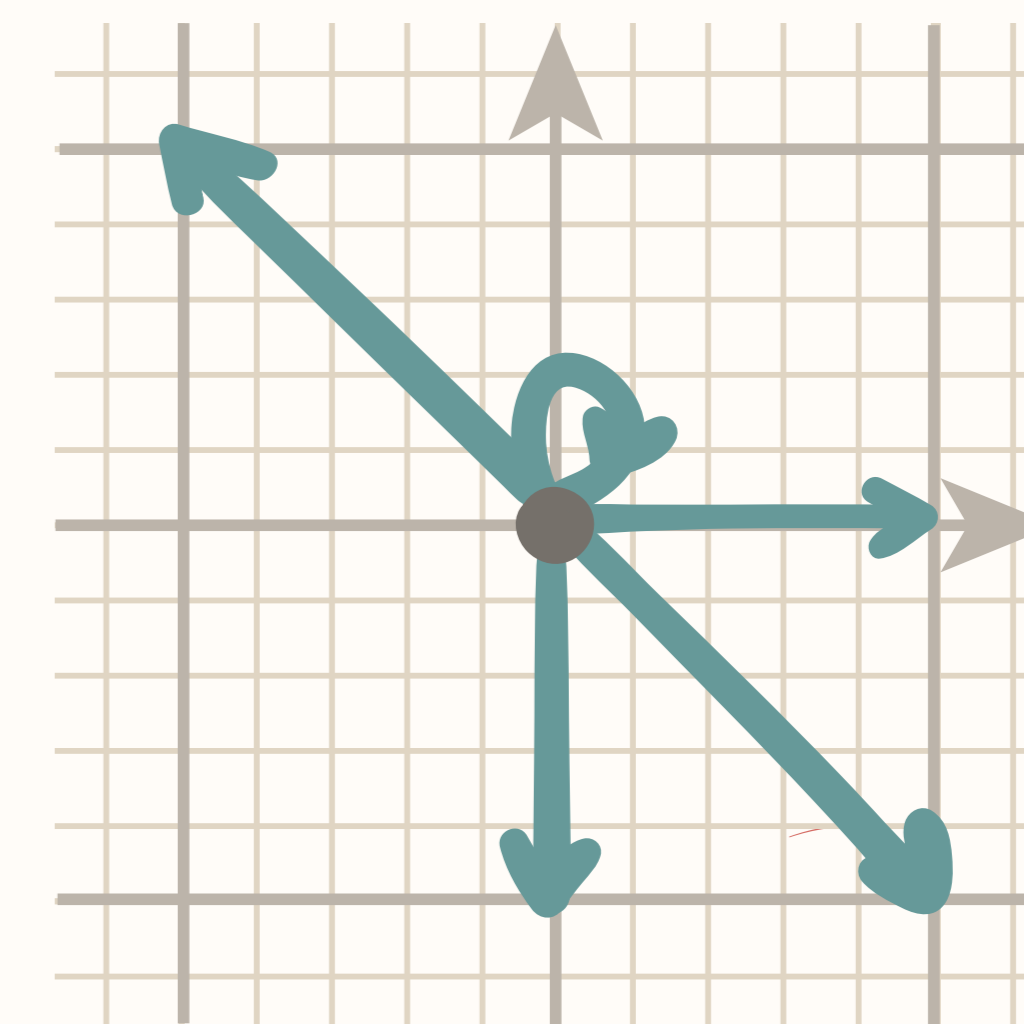
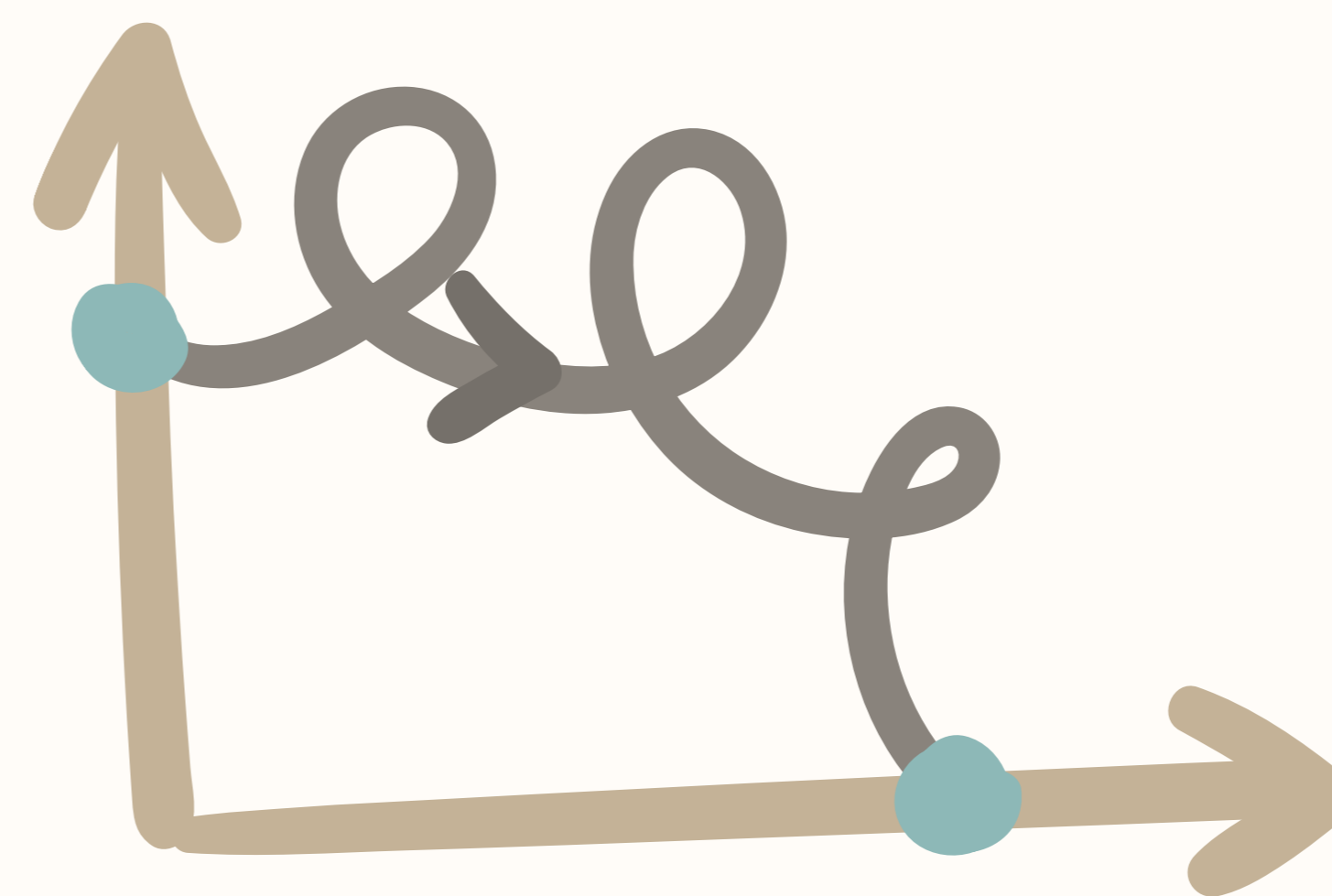
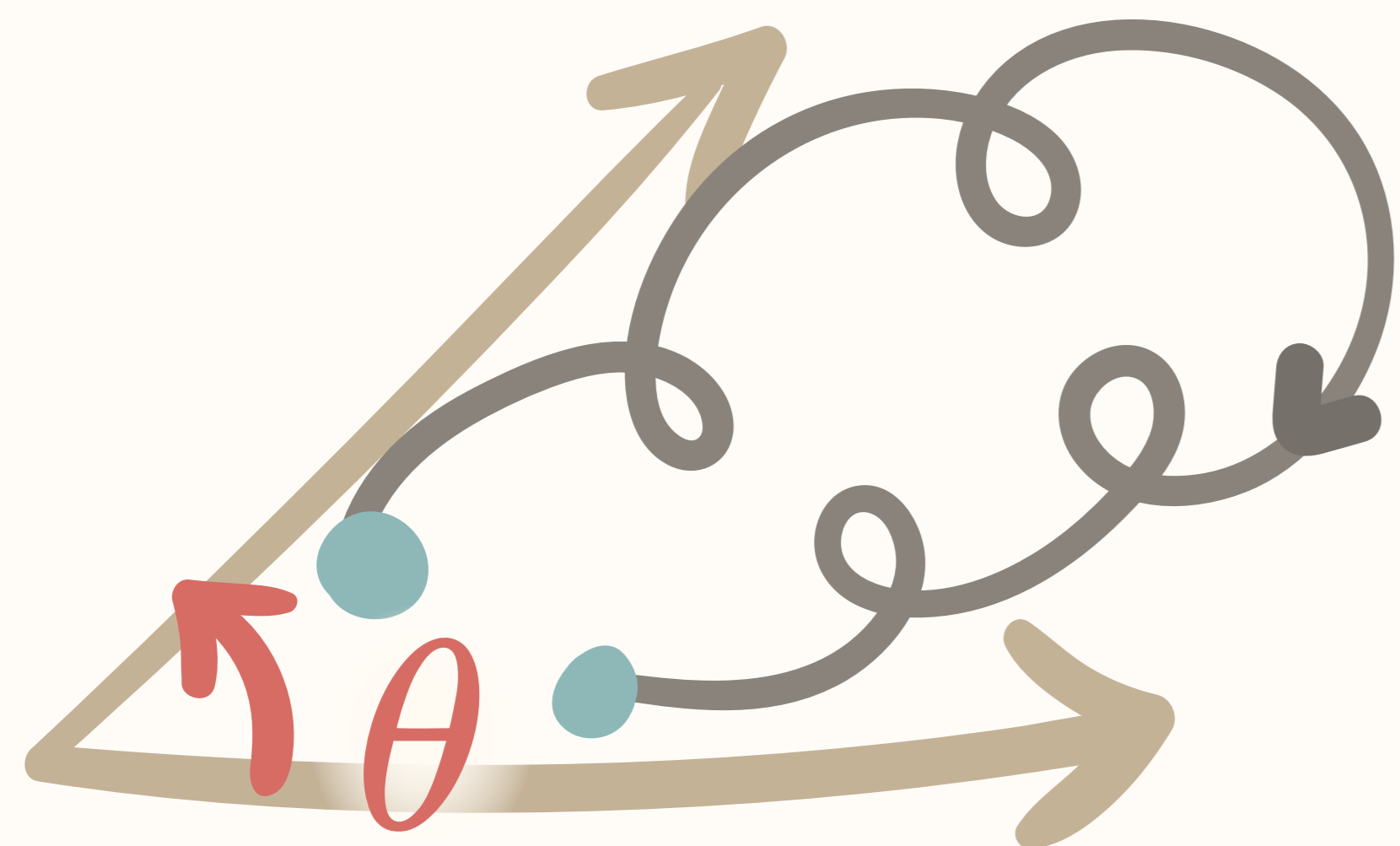
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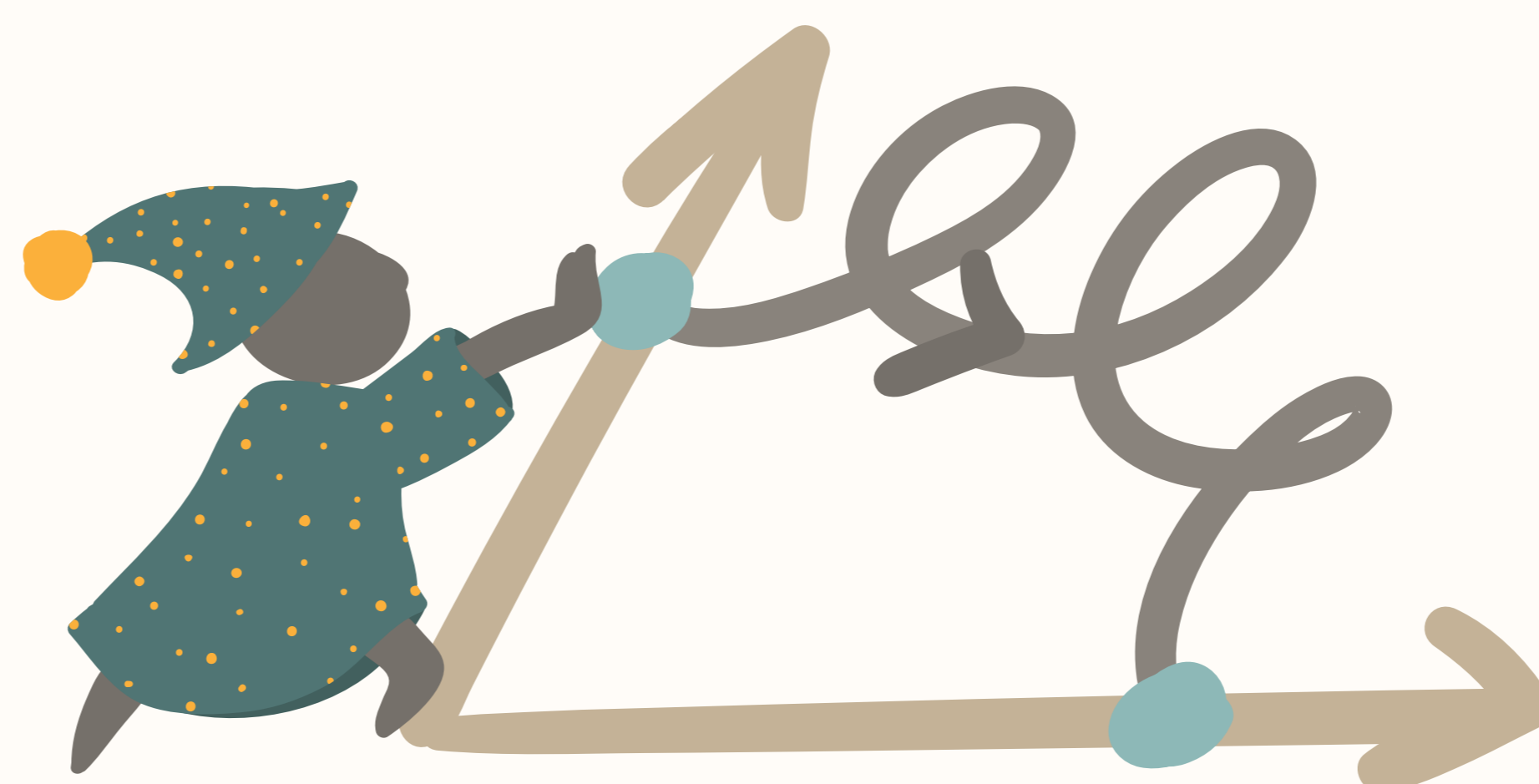
And the covariance matrix is identity.

Asymptotic counting results

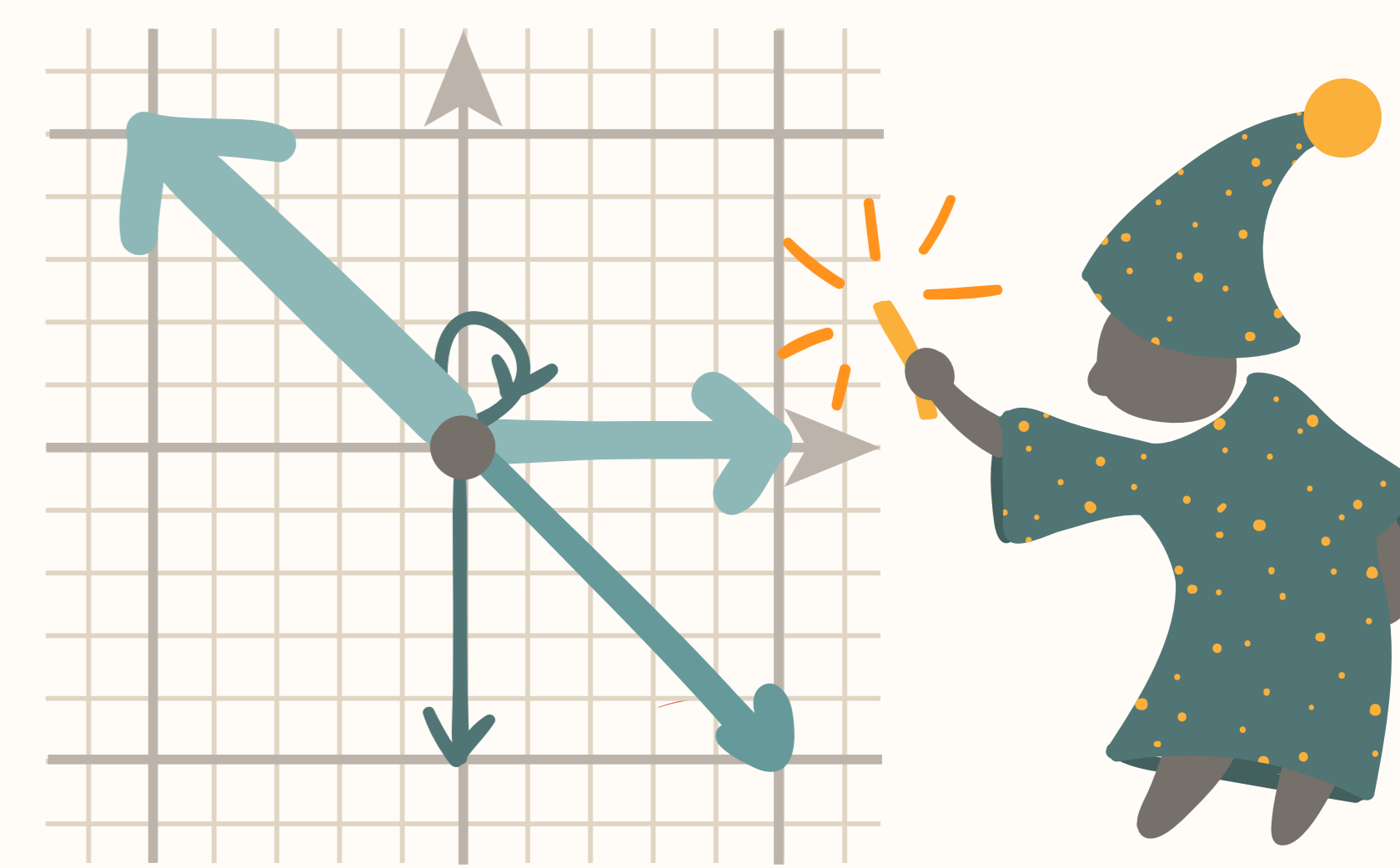


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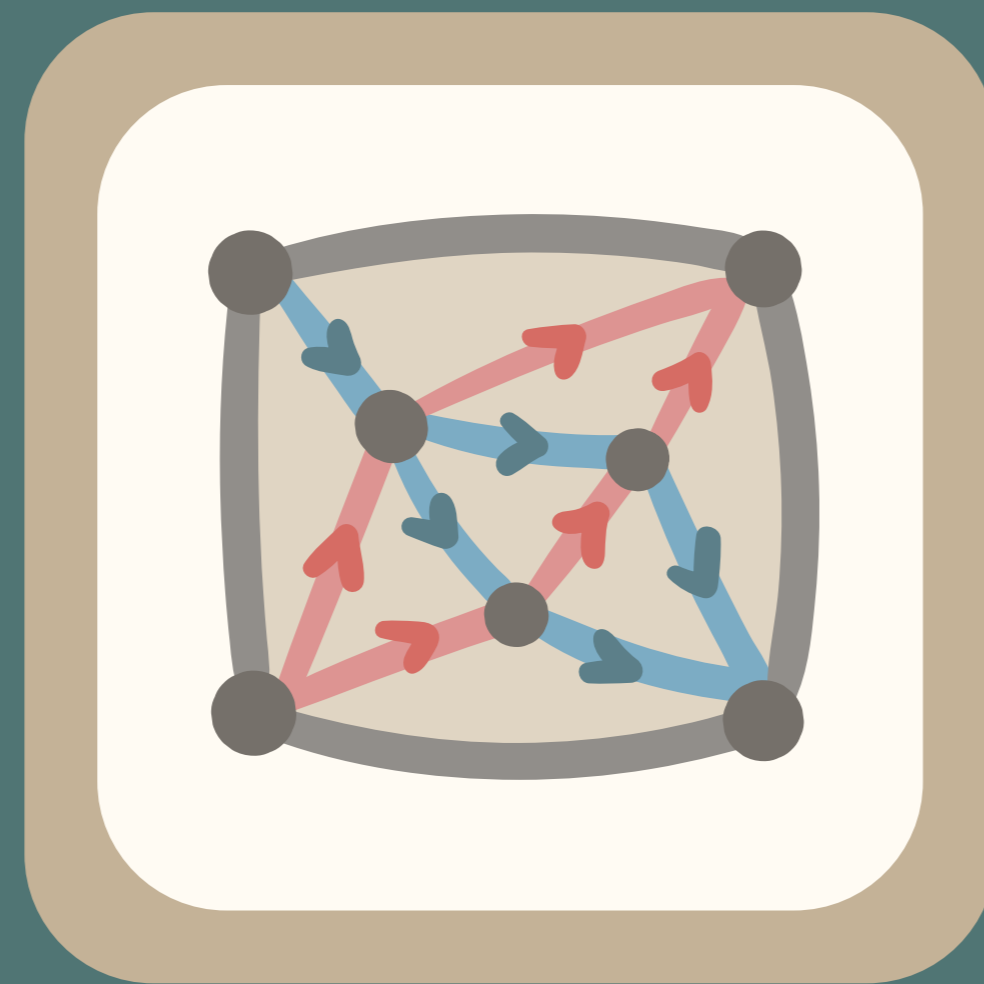
Asymptotics

Asymptotic counting results

Model

Asymptotics

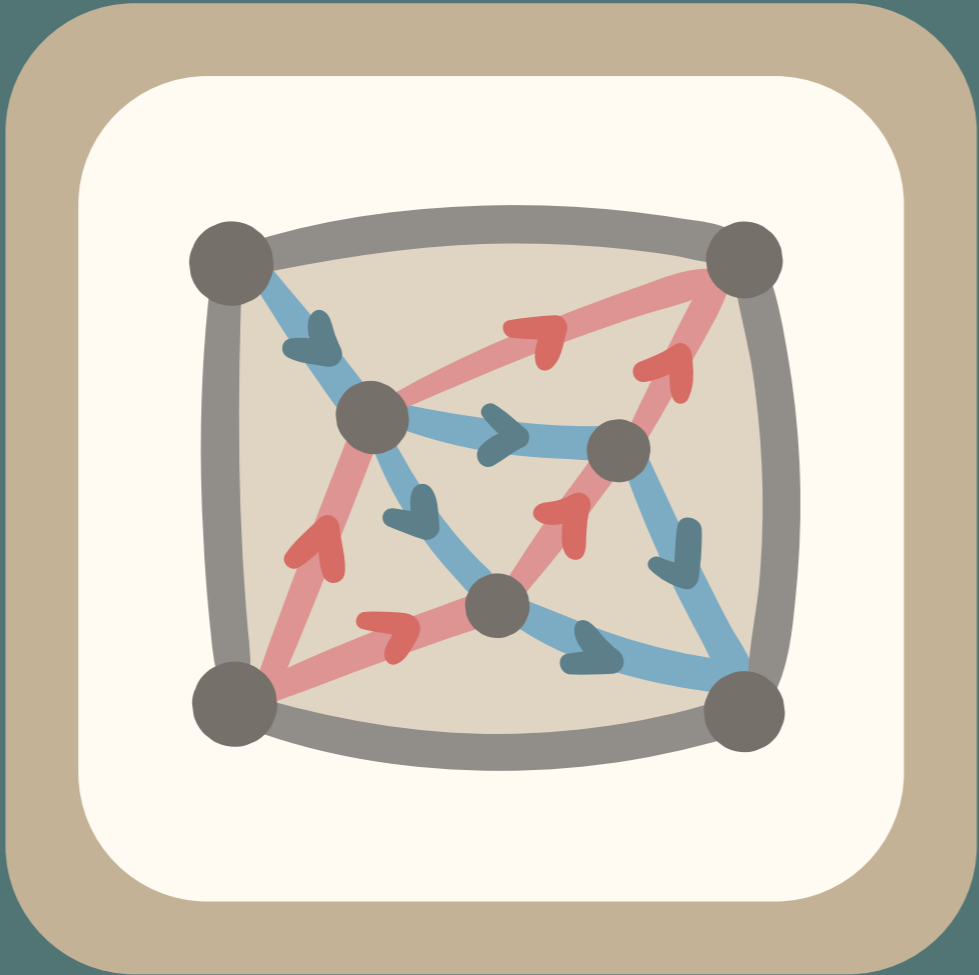


**Transversal
structures**
n vertices



$$t_n \sim \kappa \left(\frac{27}{2} \right)^n n^{-1 - \frac{\pi}{\arccos(7/8)}}$$

→ A $(4n-4)$ -Bit representation of a rectangular drawing or floorplan, Y. Inoue, T. Takahashi & R. Fujimaki (2009)

Asymptotic counting results

Model	Asymptotics
<p>Transversal structures <i>n</i> vertices</p> 	$t_n \sim \kappa \left(\frac{27}{2} \right)^n n^{-1 - \frac{\pi}{\arccos(7/8)}}$ <p>→ A $(4n-4)$-Bit representation of a rectangular drawing or floorplan, Y. Inoue, T. Takahashi & R. Fujimaki (2009)</p>
<p>Posets <i>n+2</i> edges</p> 	$e_n \sim \kappa \gamma^n n^{-\alpha} \quad \alpha = 1 + \frac{\pi}{\arccos(\xi)}$ <p>γ and ξ are algebraic $\gamma \approx 4.80 \dots$ $\alpha \approx 5.14 \dots$</p>
<p>Posets <i>n</i> vertices</p> 	$b_n \sim \kappa \left(\frac{11 + \sqrt{5}}{2} \right)^n n^{-6}$

PLANE PERMUTATIONS

$$p_n \sim \kappa \left(\frac{11 + \sqrt{5}}{2} \right)^n \cdot n^{-6}$$

⇒ Semi-Baxter and strong-Baxter :
two relatives of Baxter Sequences,
M. Bouvel, V. Guerrini, A. Rechnitzer
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s_n

Posets
 $n+2$ edges

Posets
 n vertices

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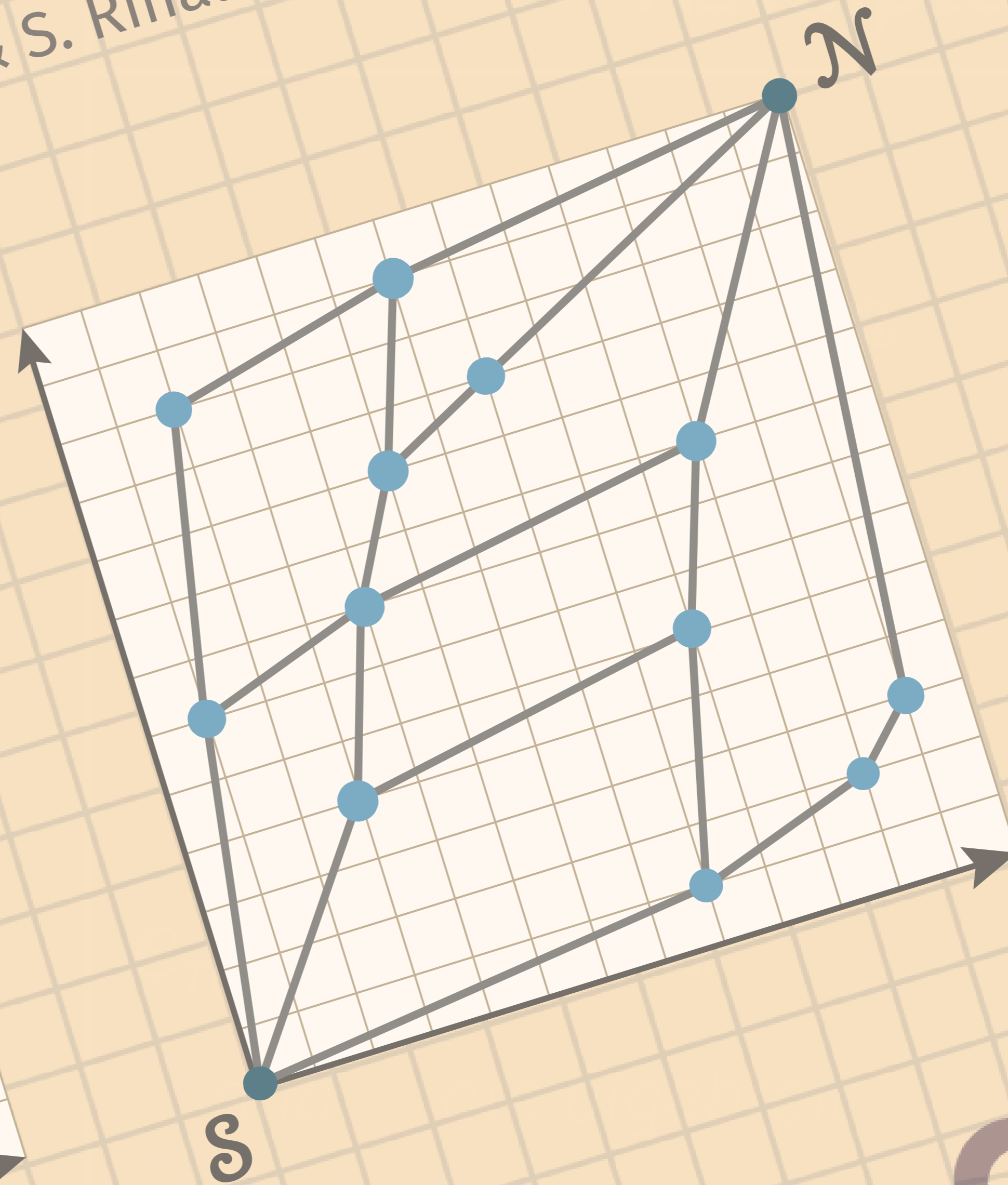
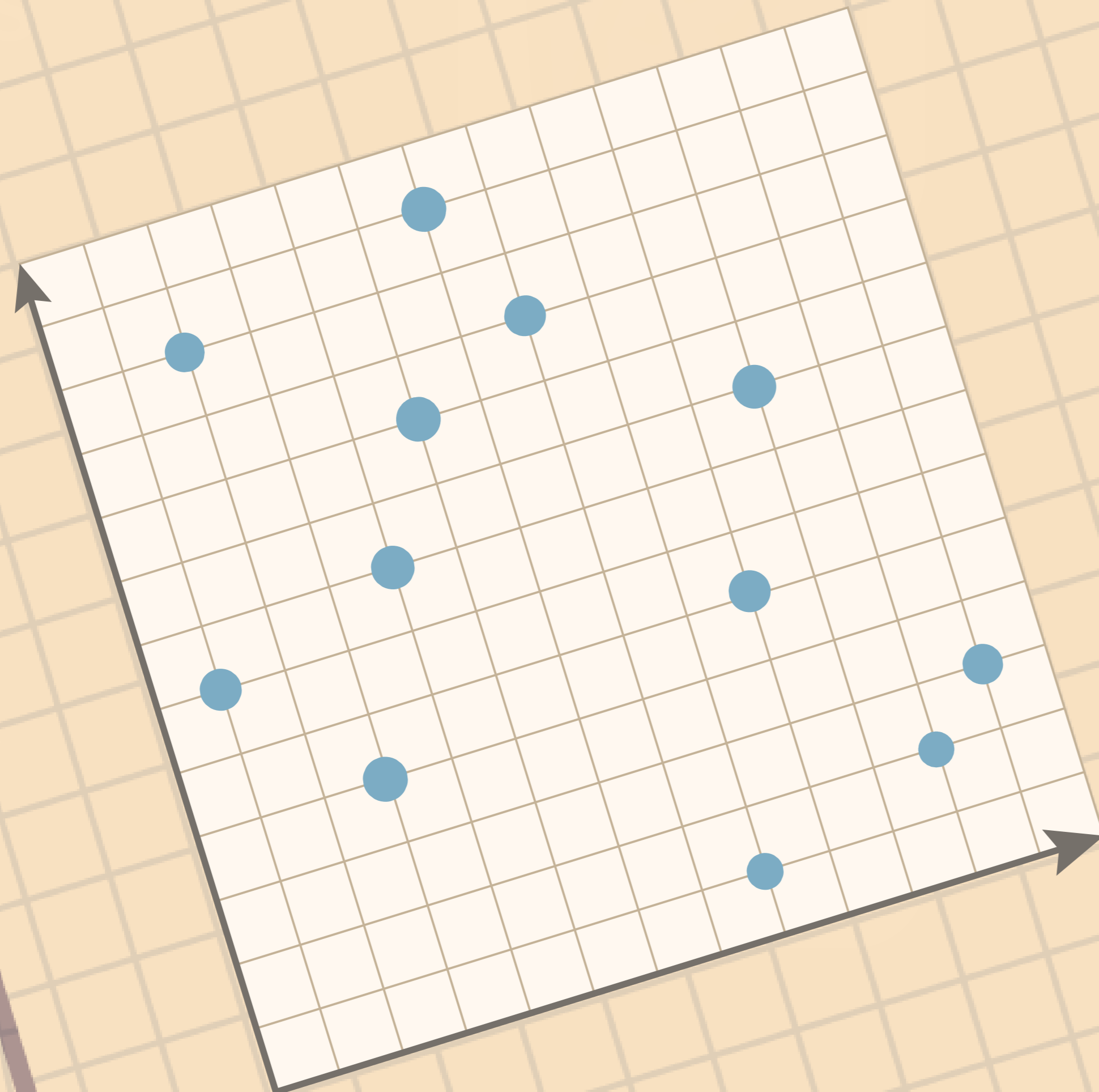
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Summary

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Asymptotics



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Small steps



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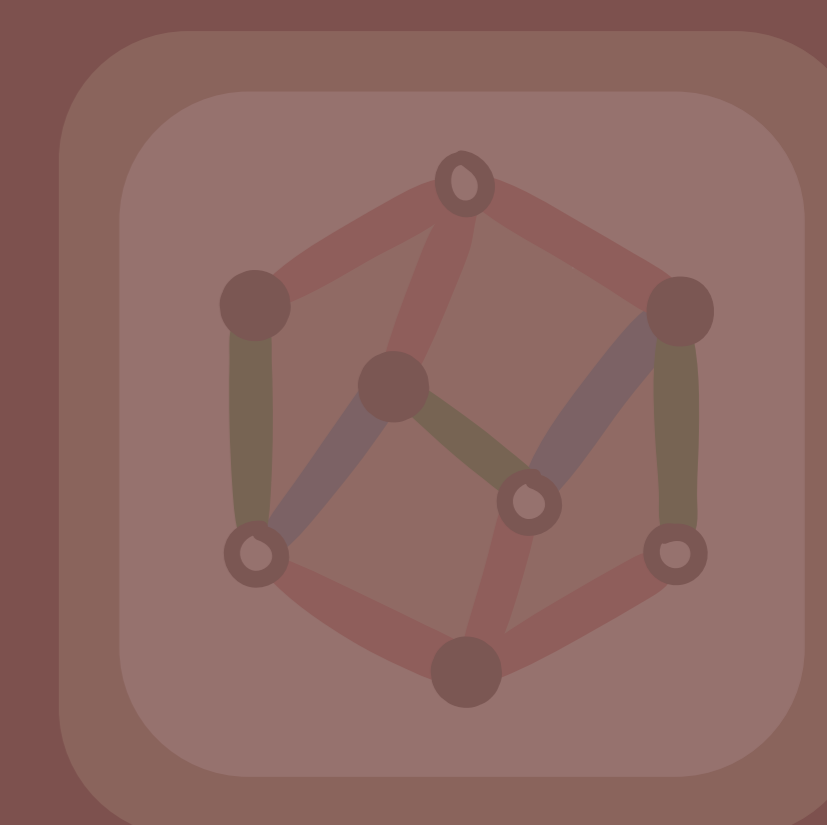
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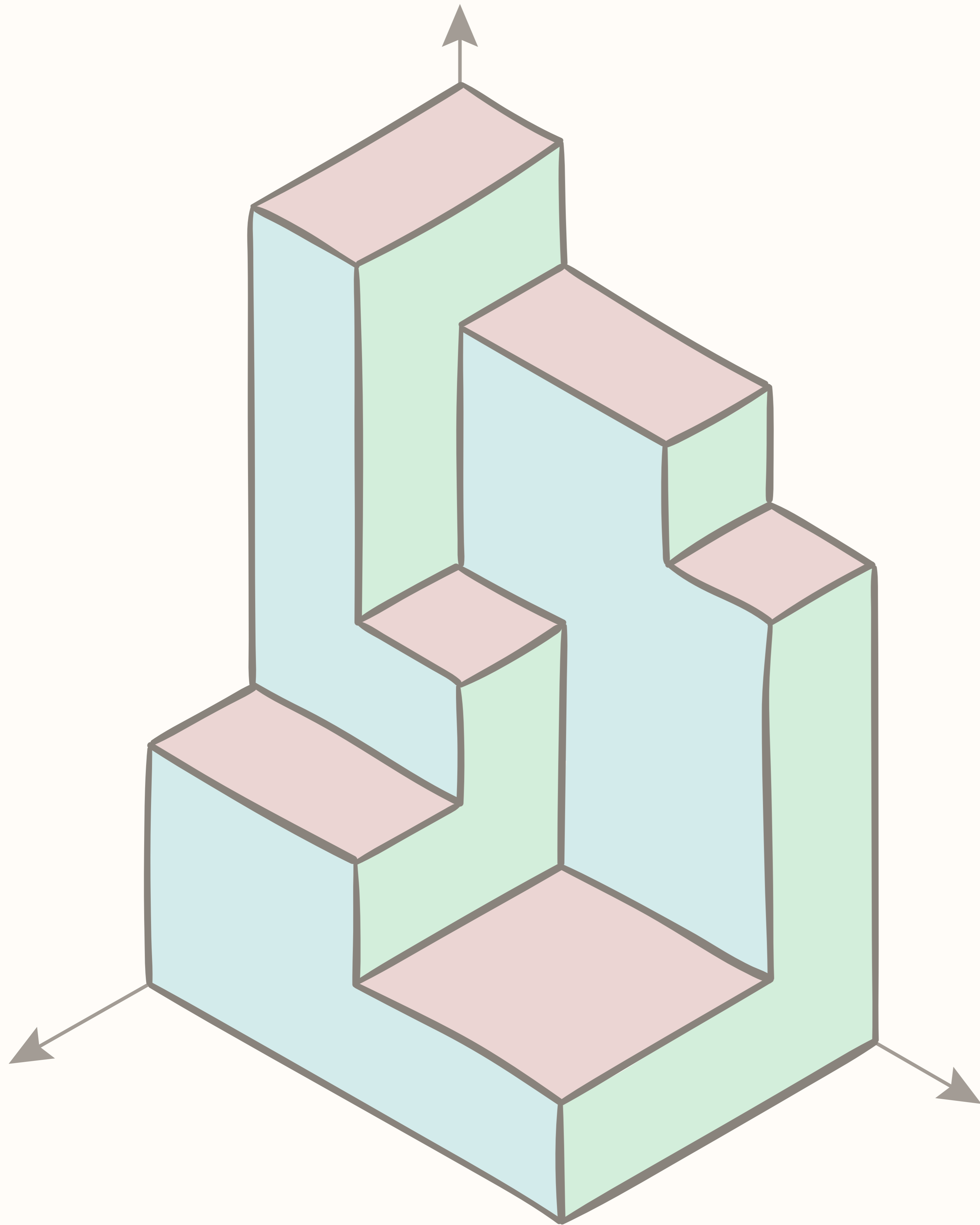


*polyhedral
orientations*

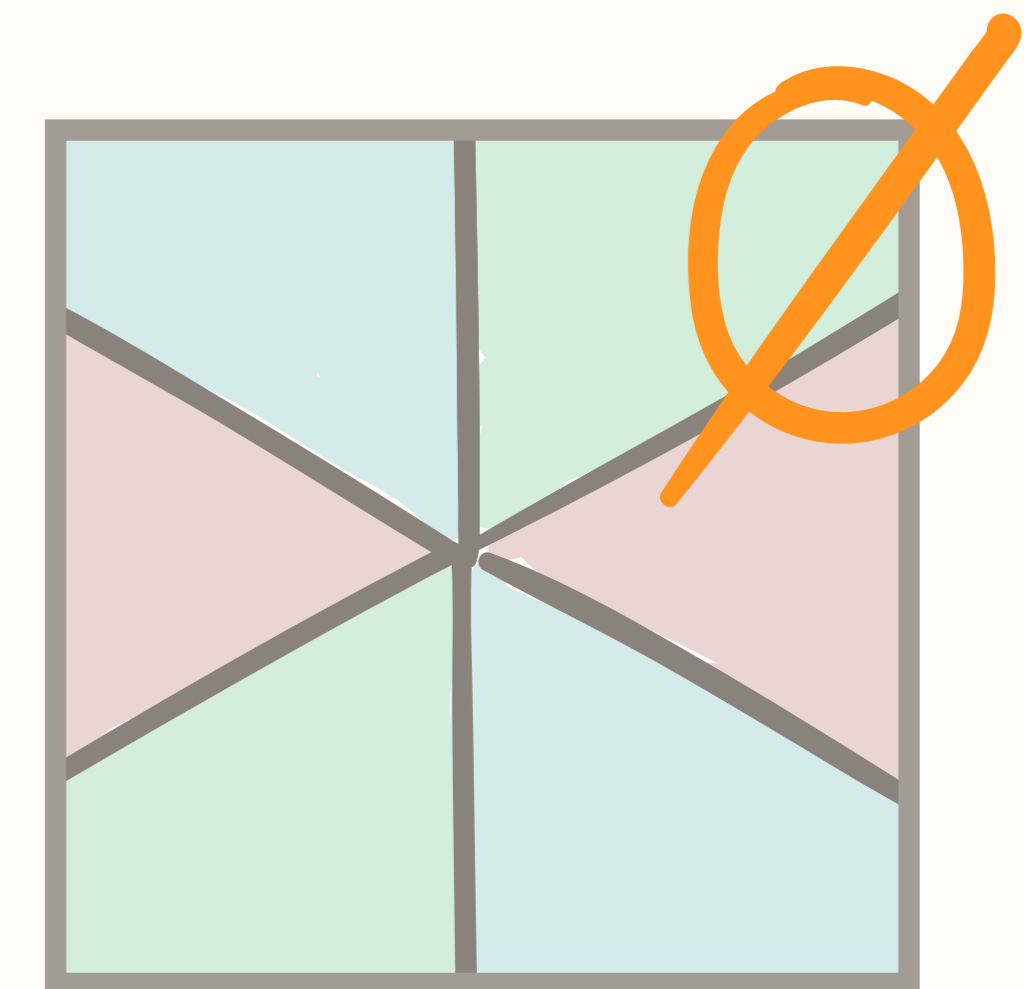
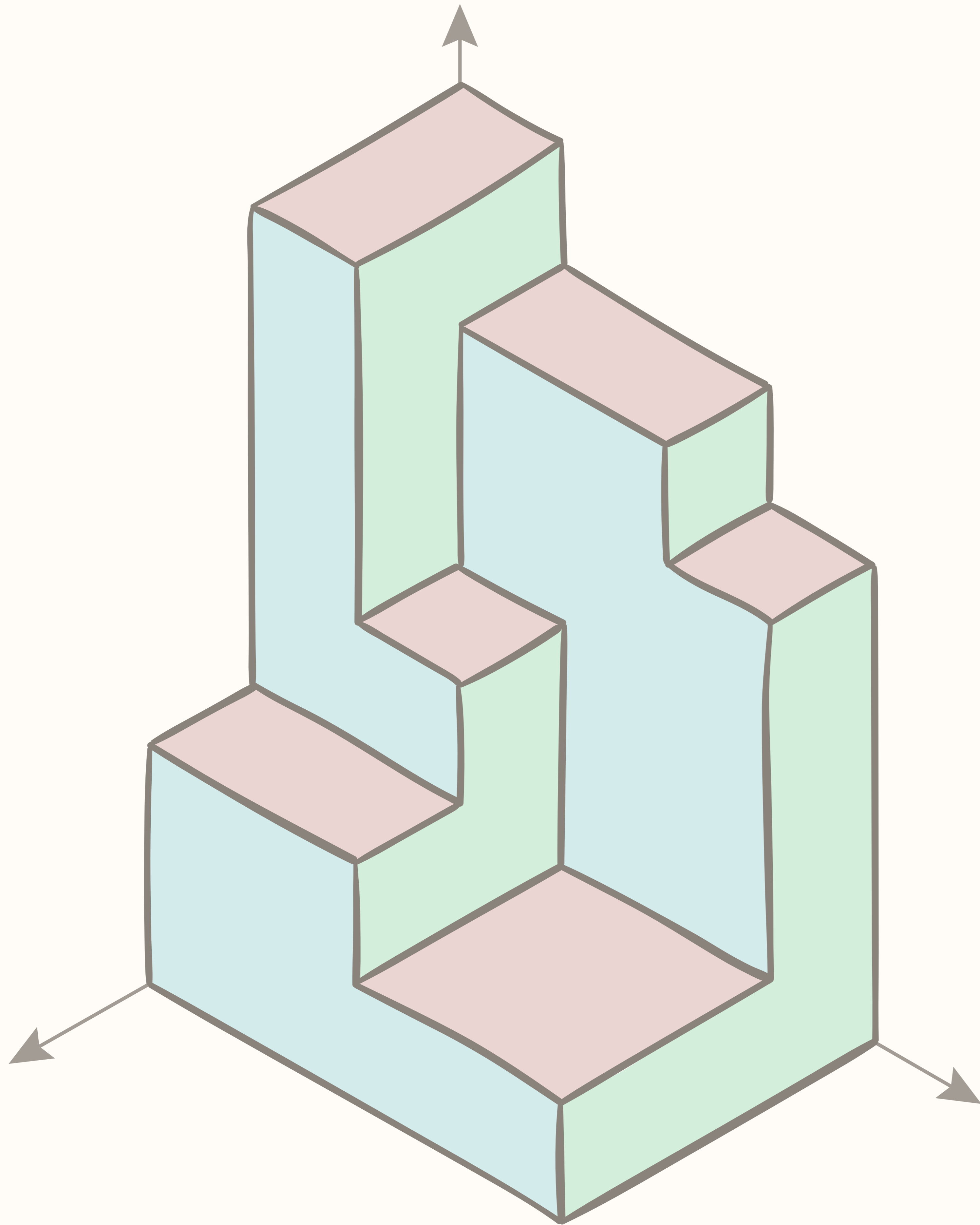


*Schnyder
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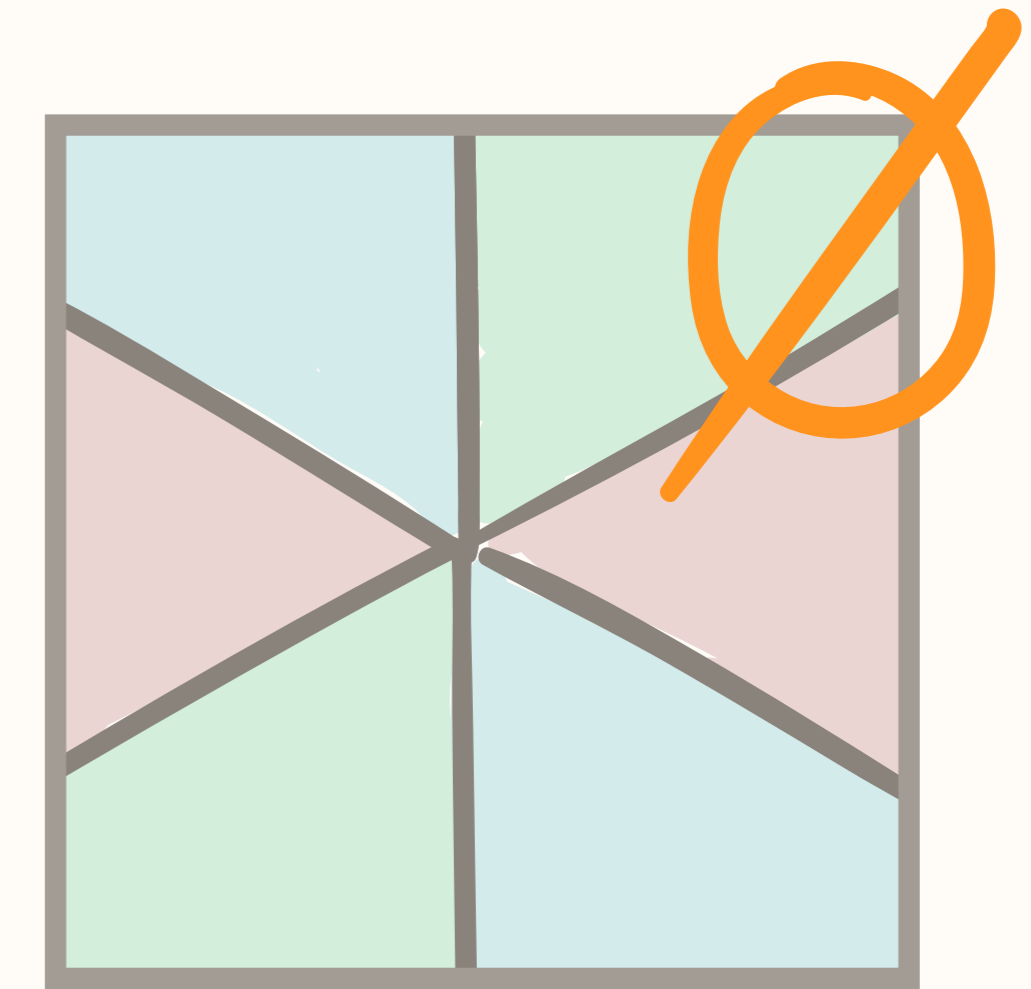
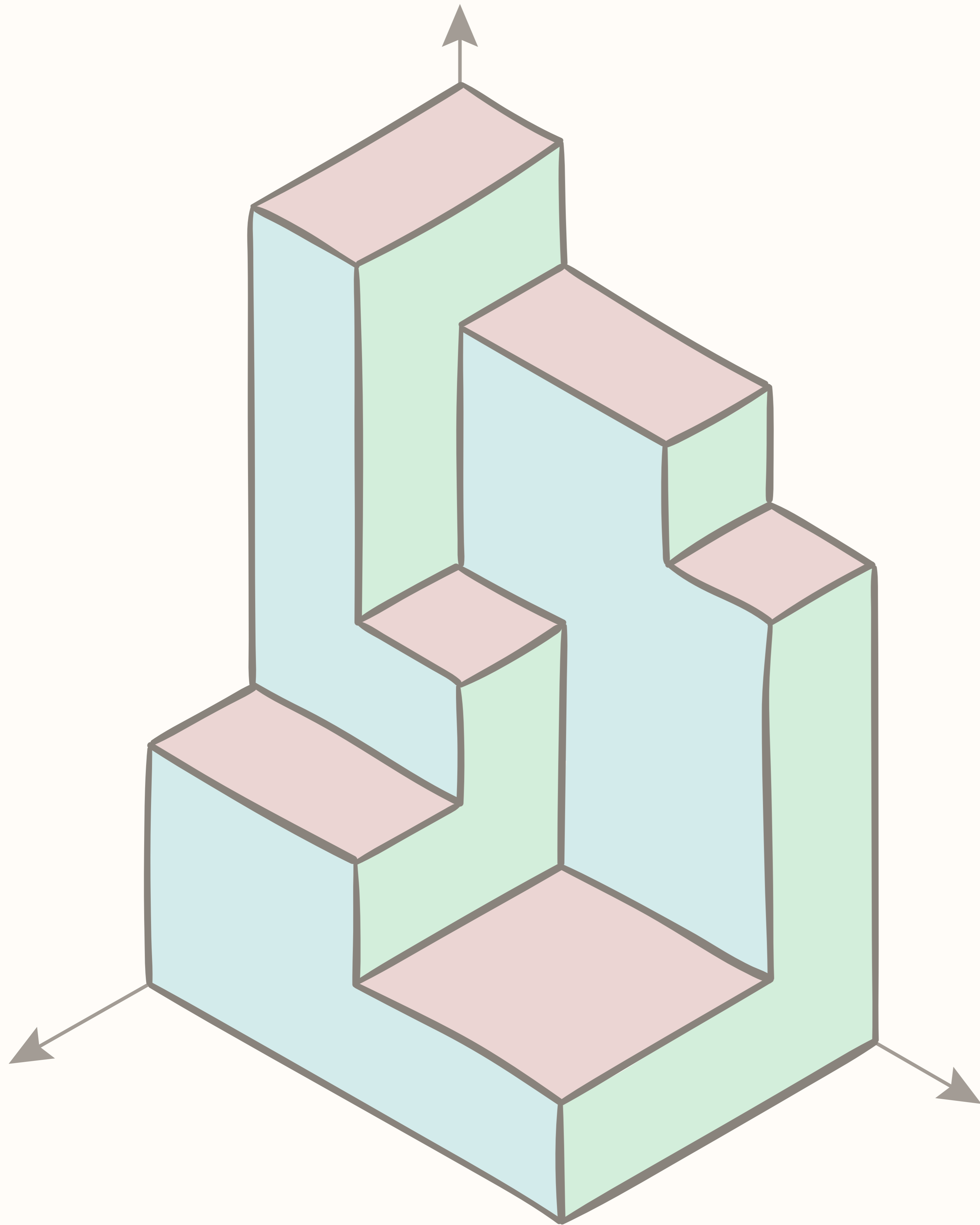
Corner polyhedra



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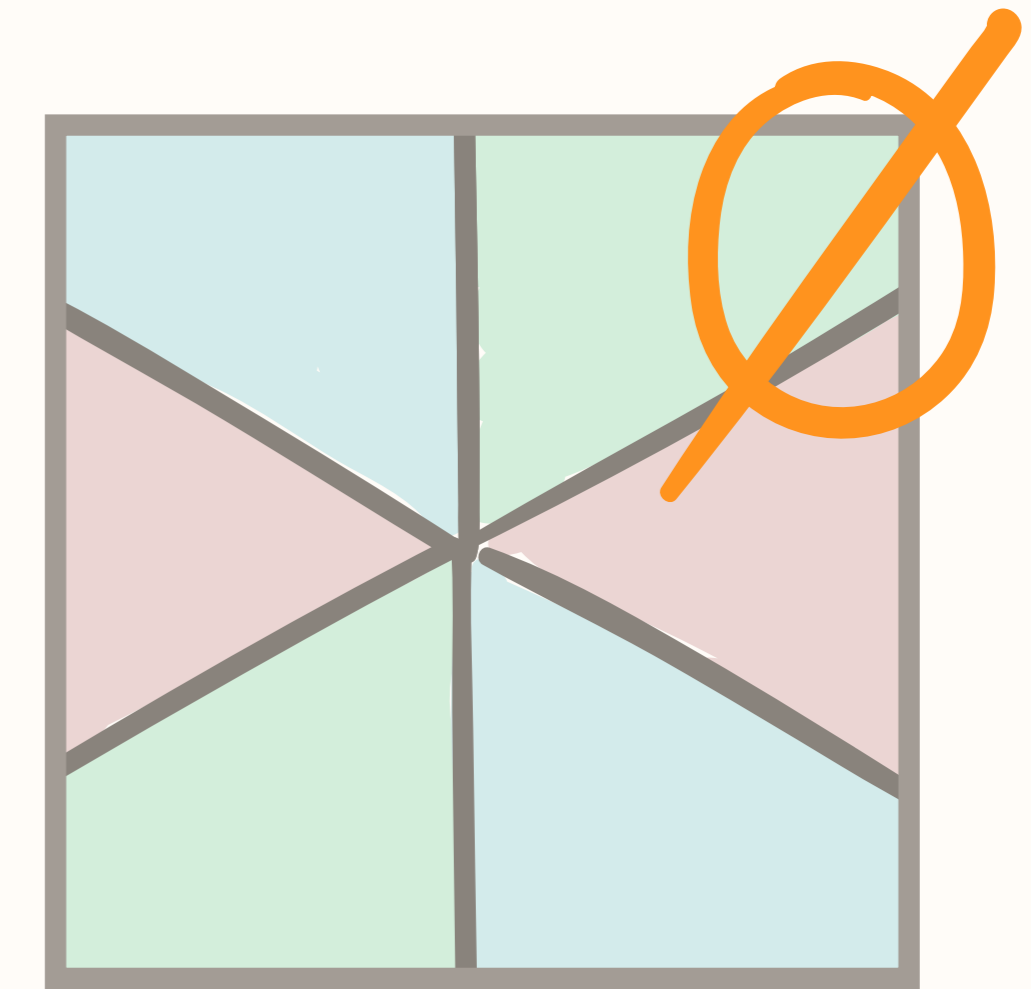
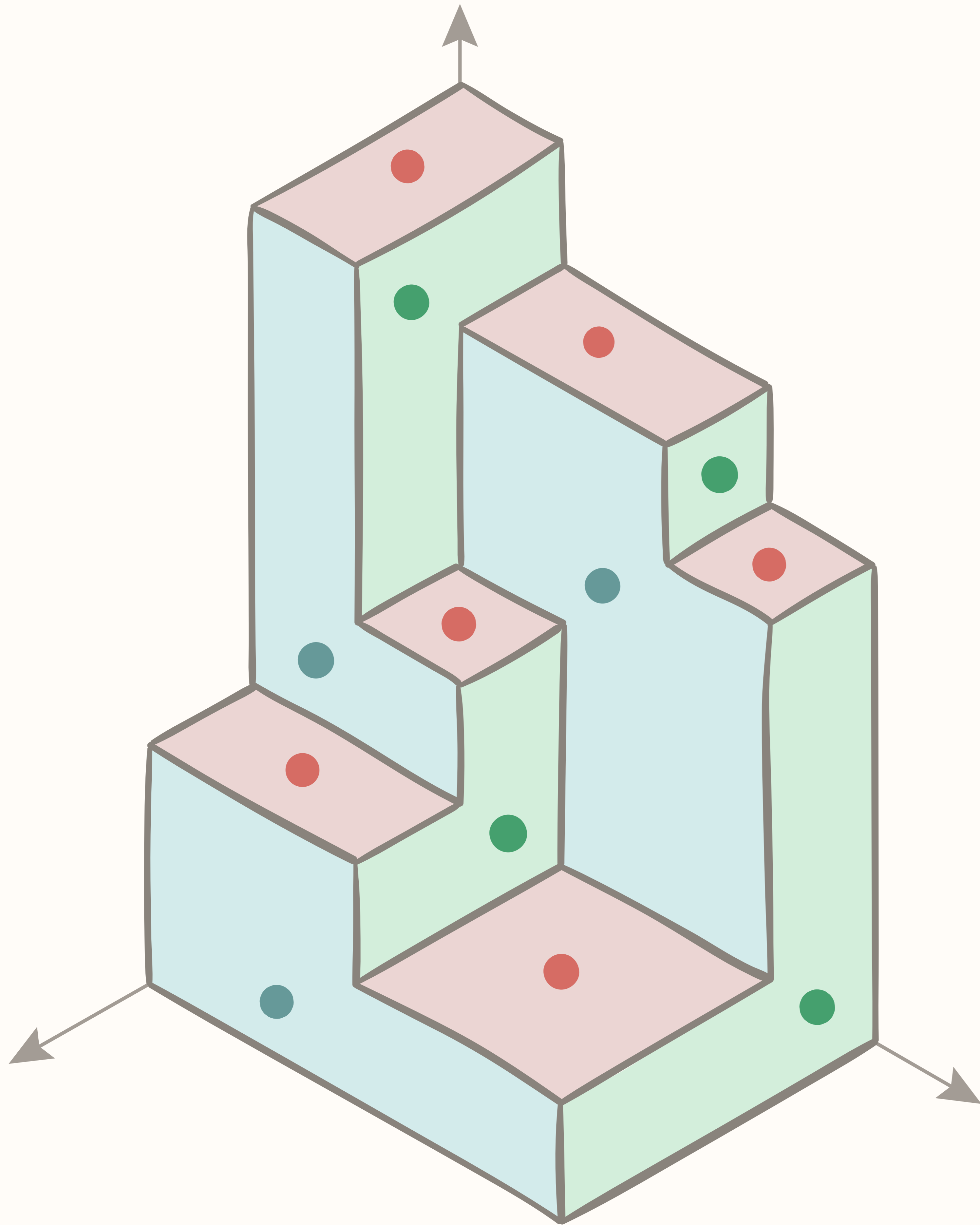
Corner polyhedra



corner polyhedra \leftrightarrow *polyhedral orientations*

\Rightarrow Steinitz theorems for orthogonal polyhedra,
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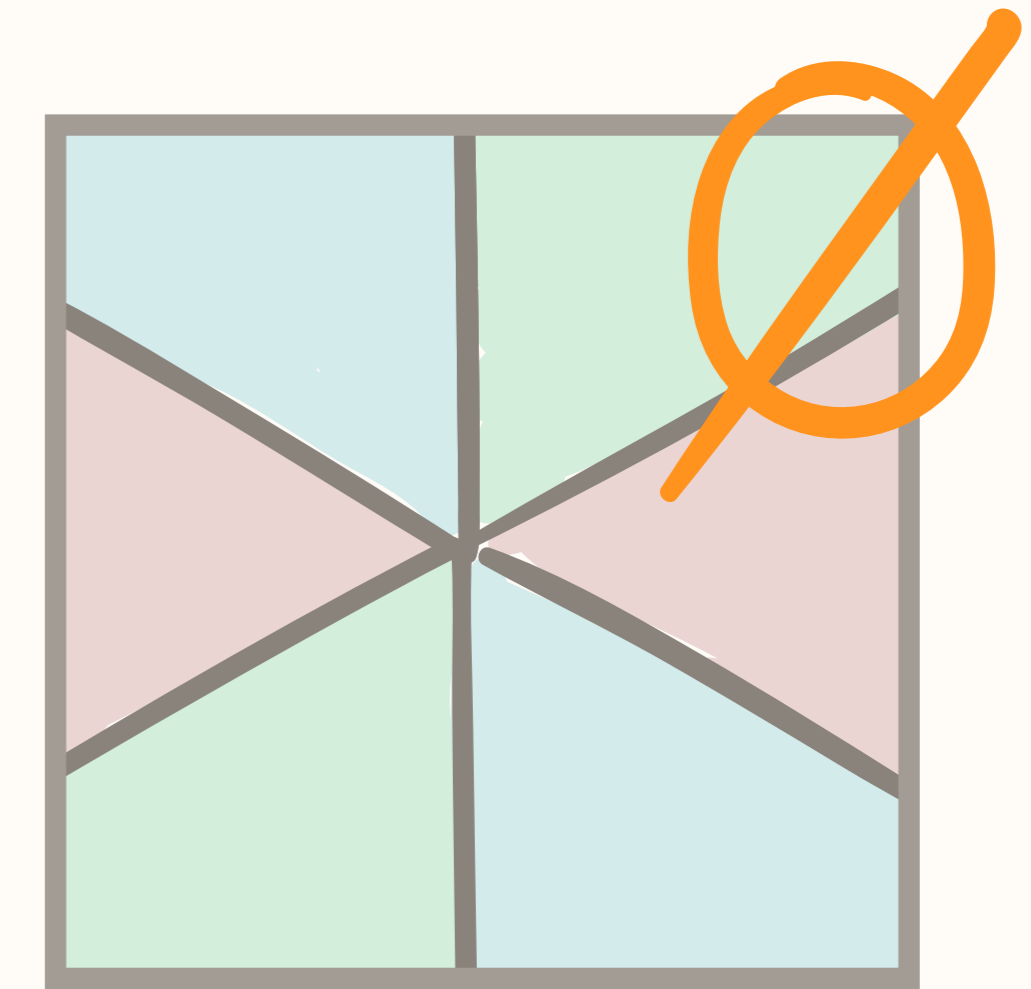
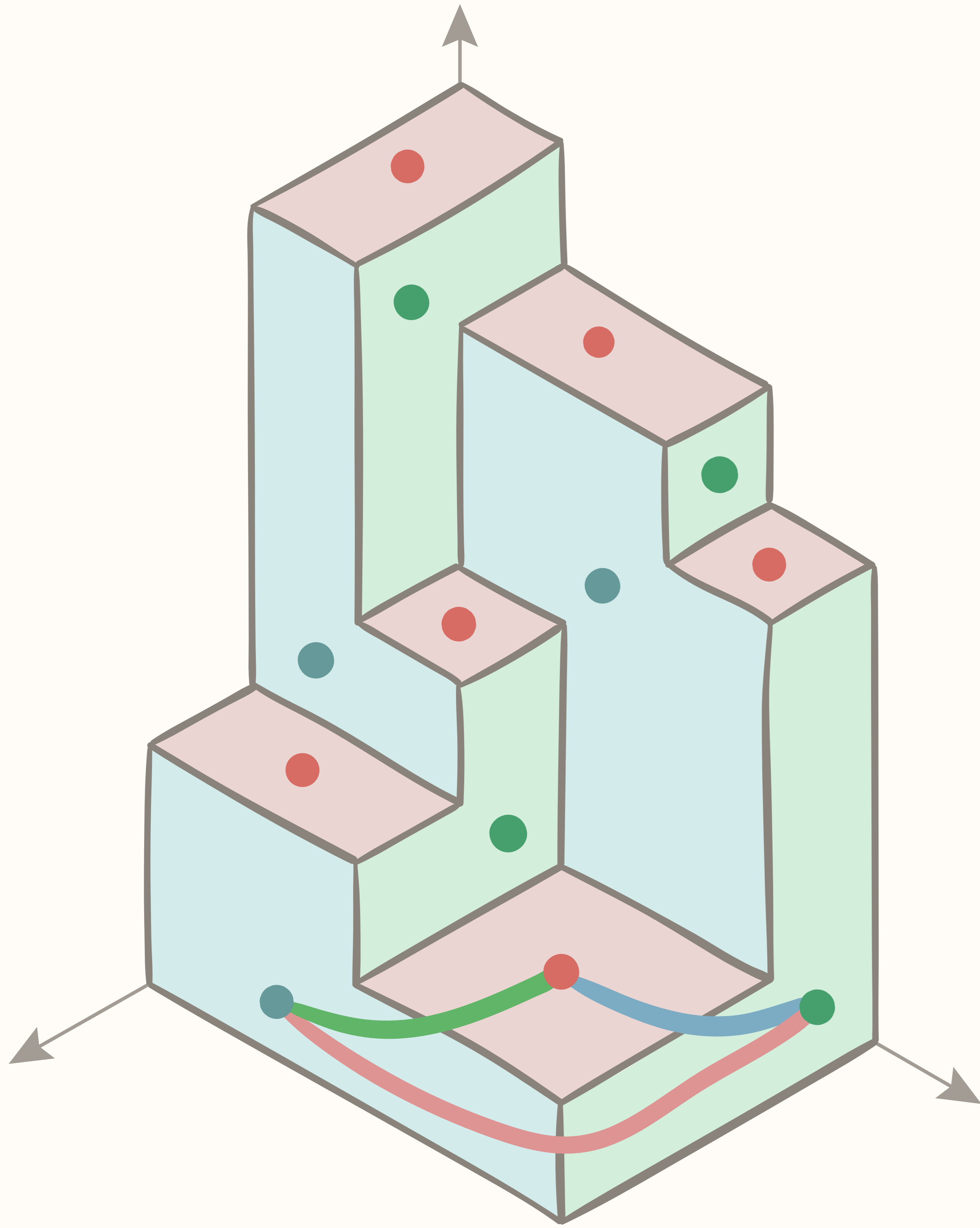
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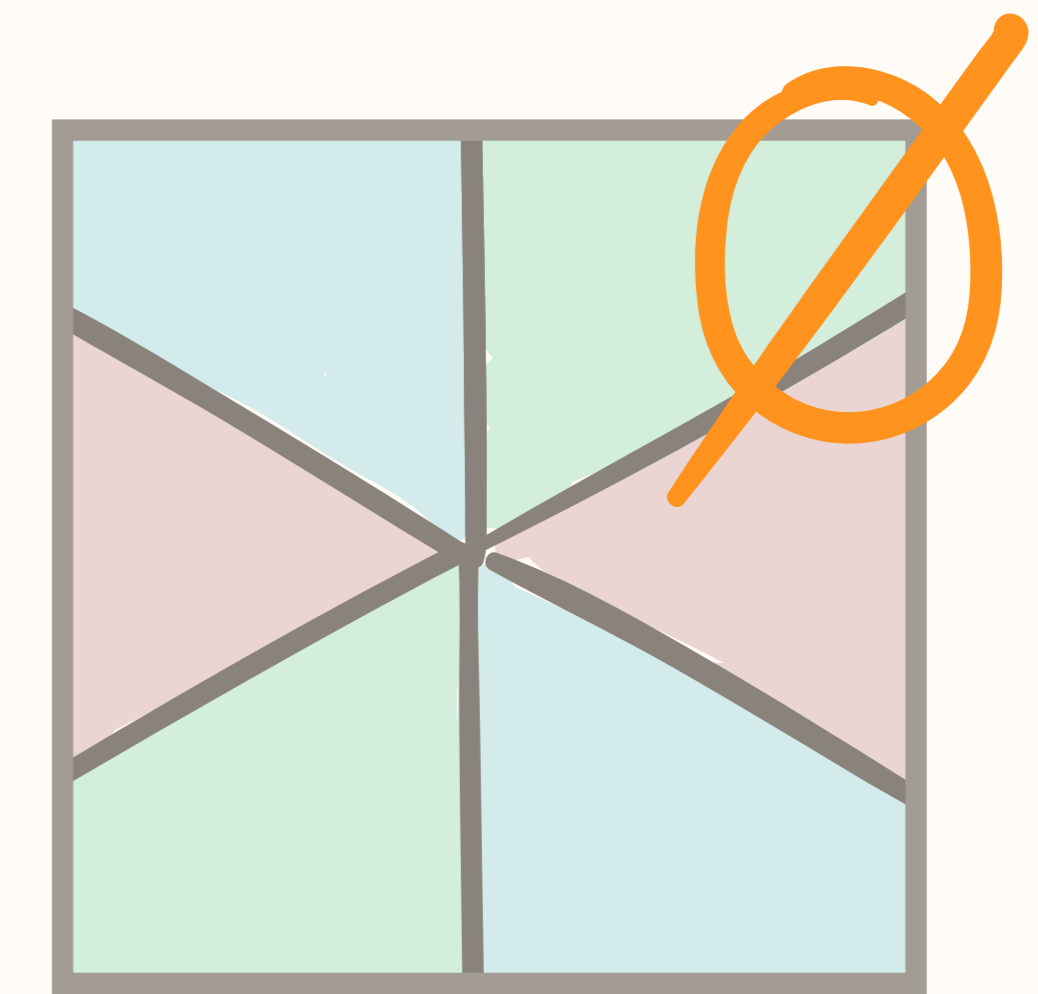
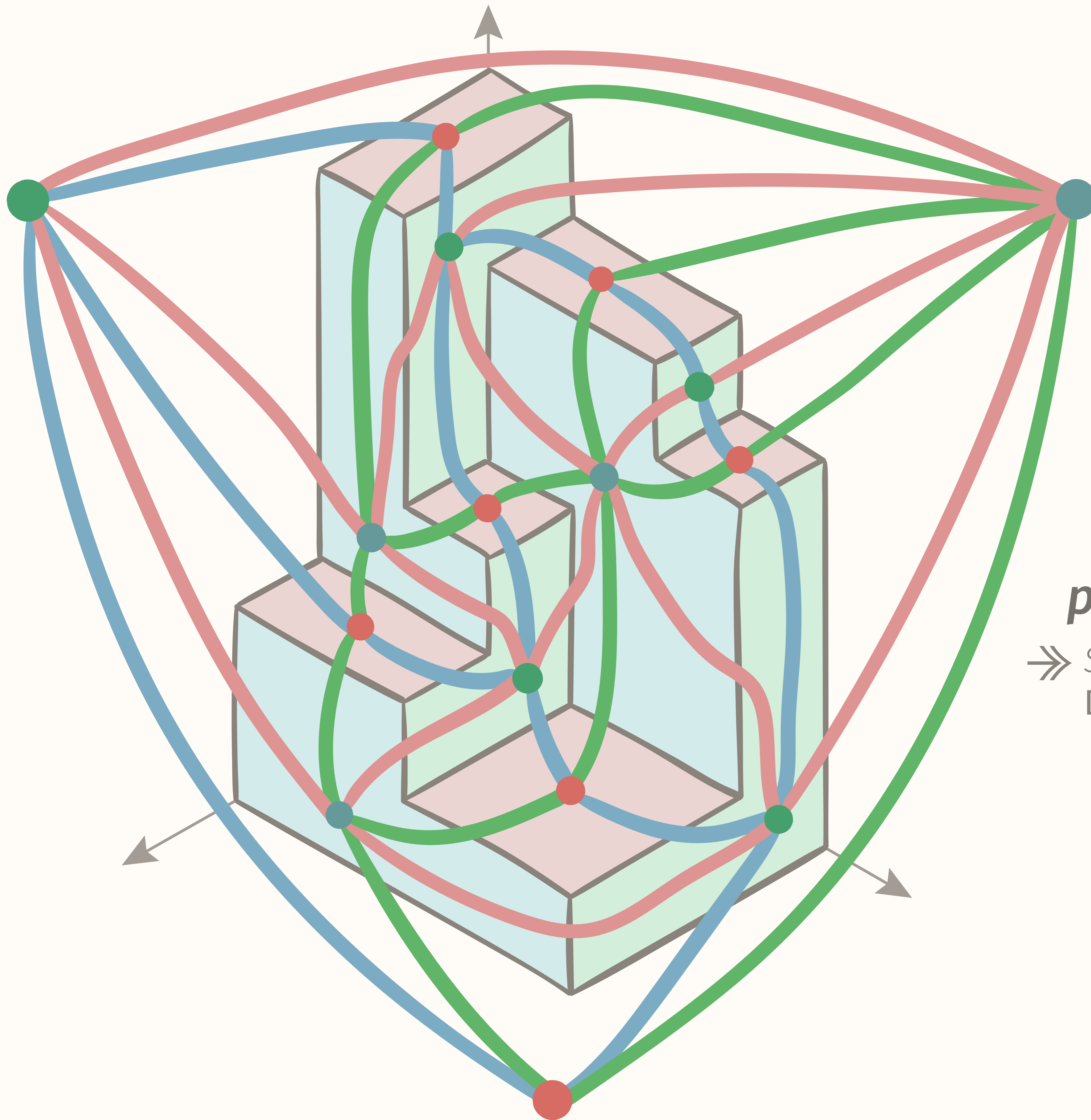
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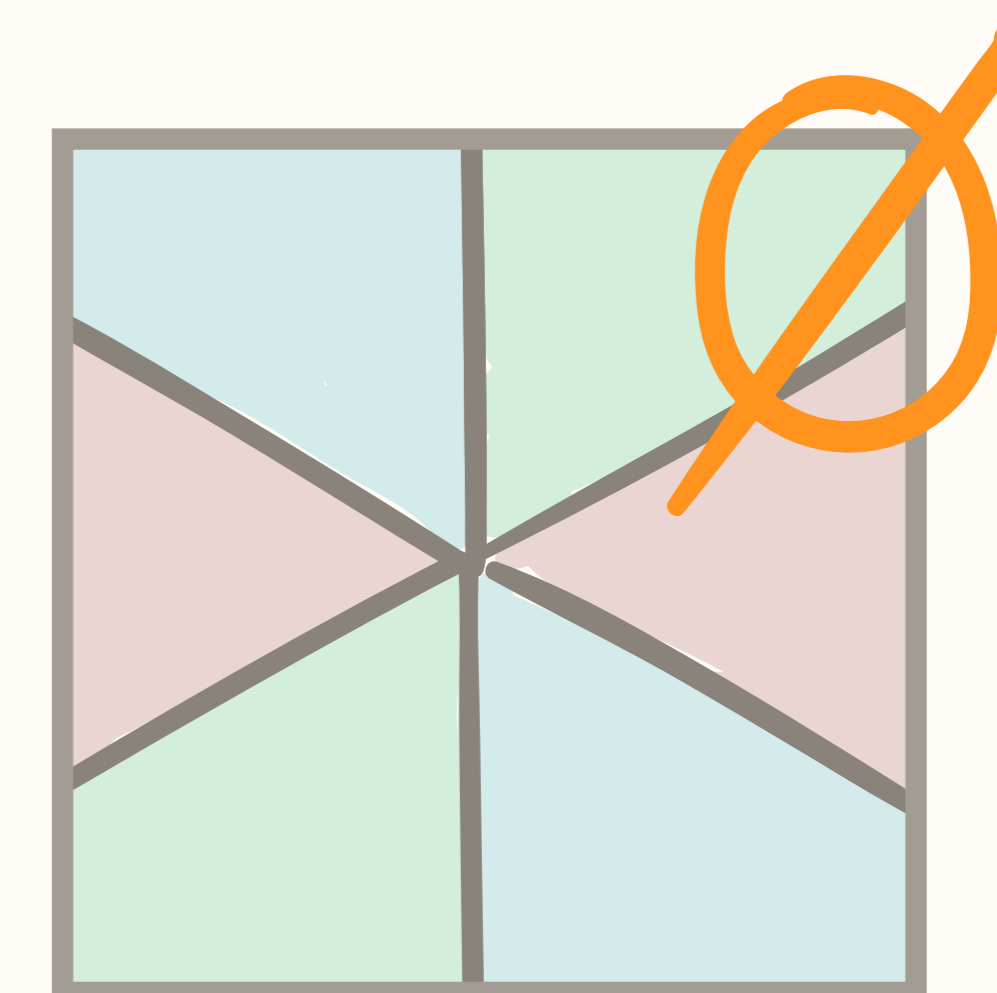
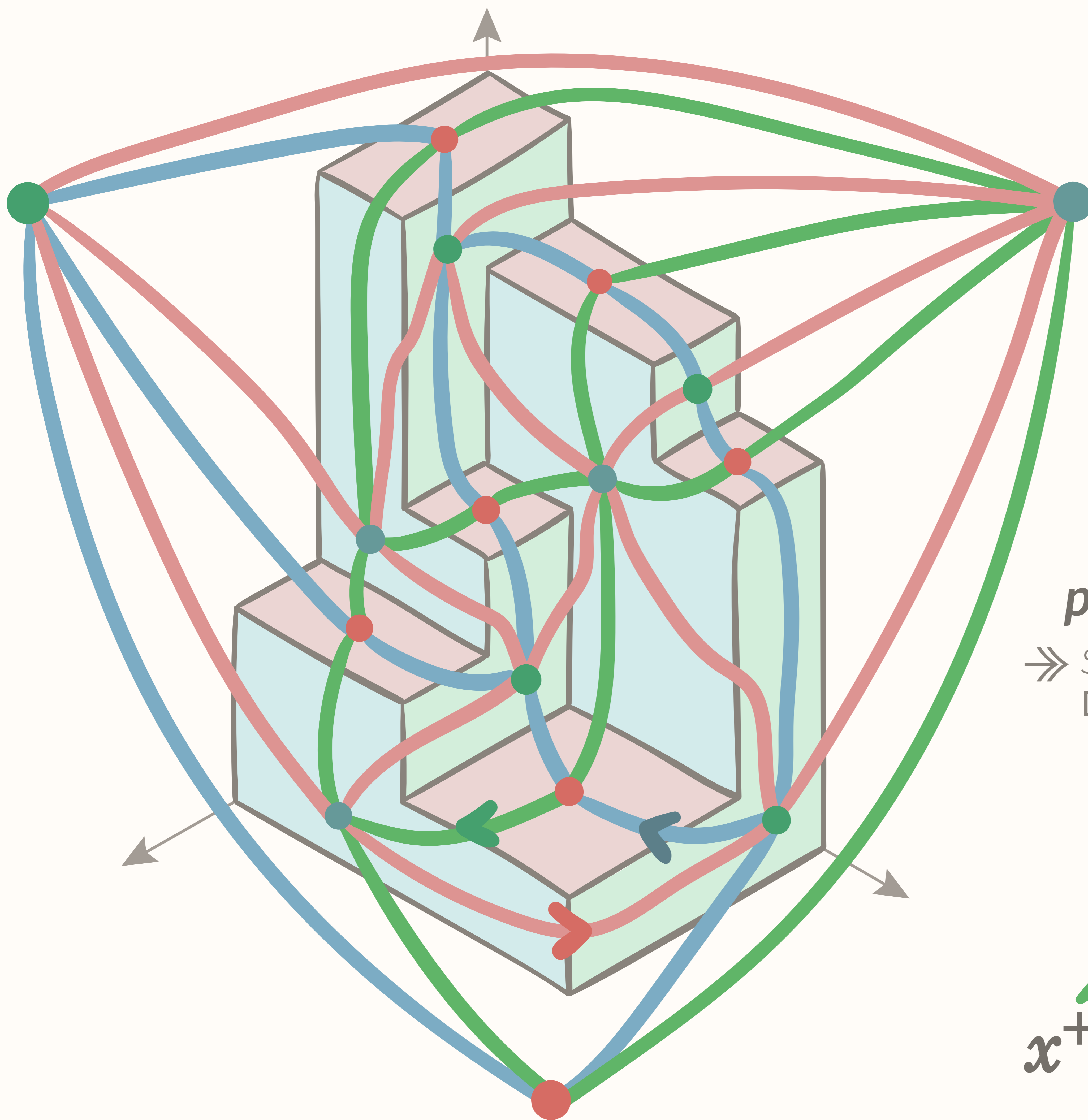
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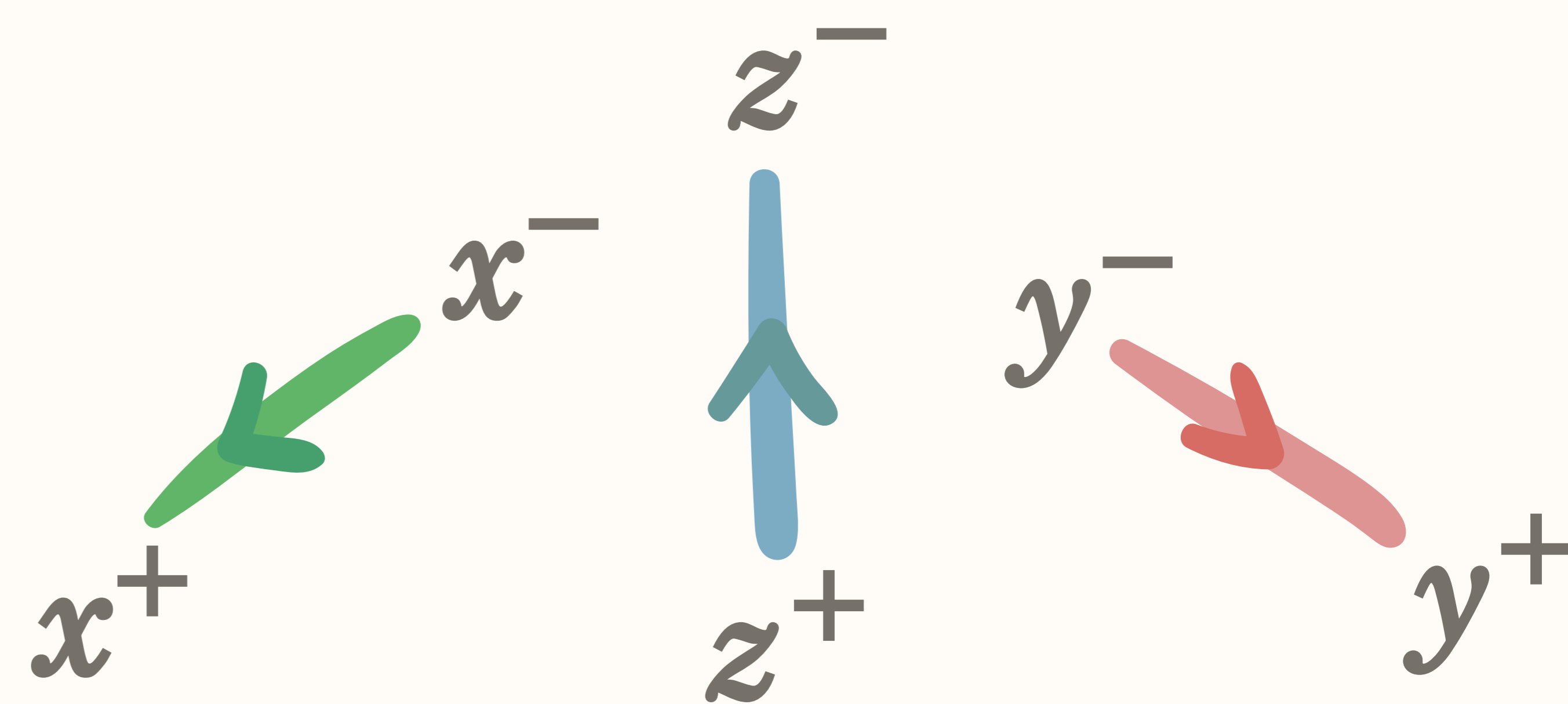
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Corner polyhedra

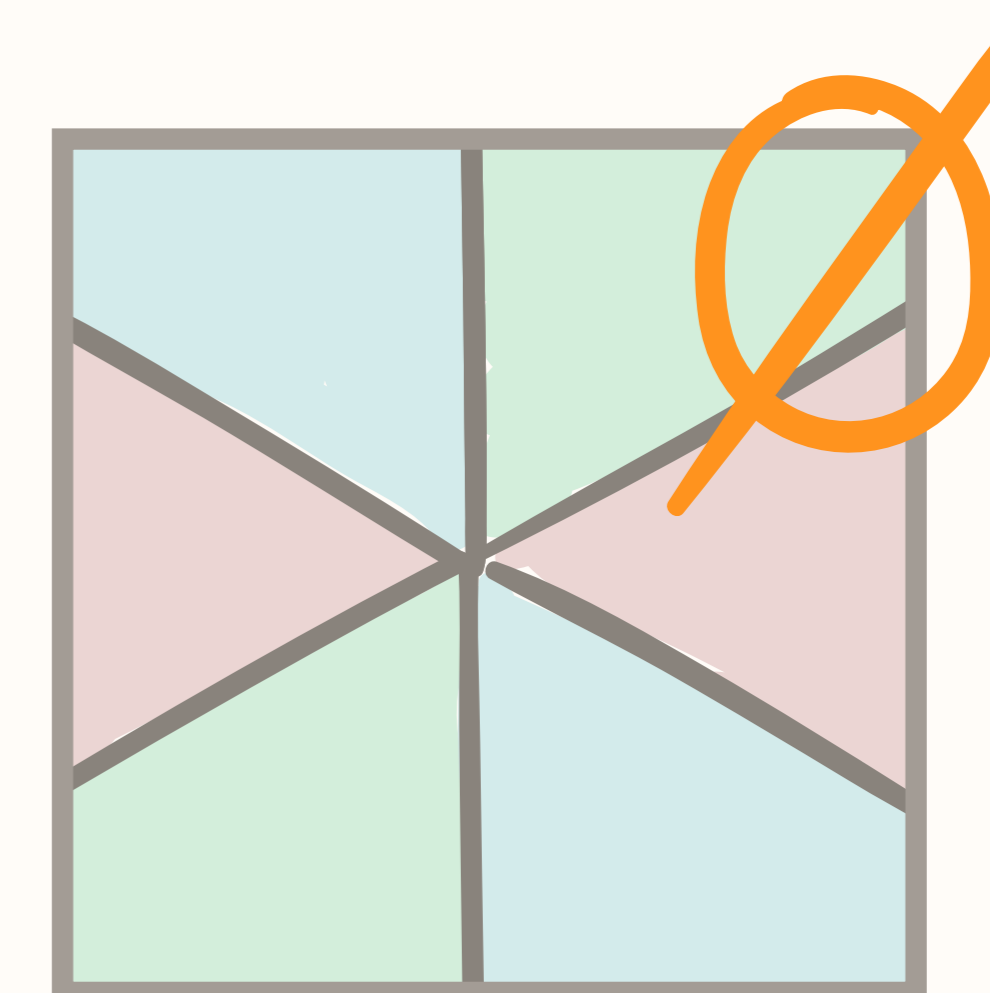
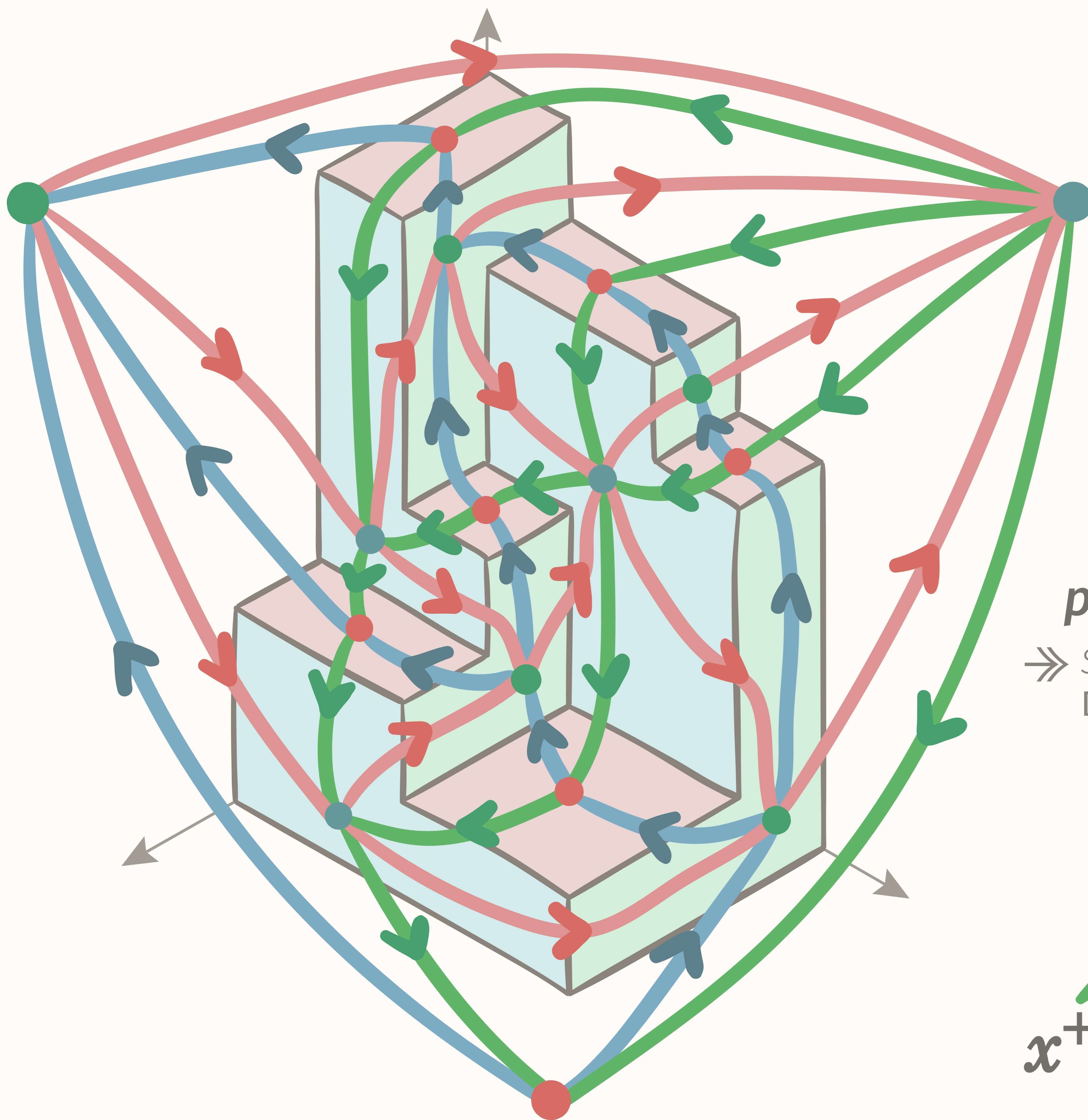


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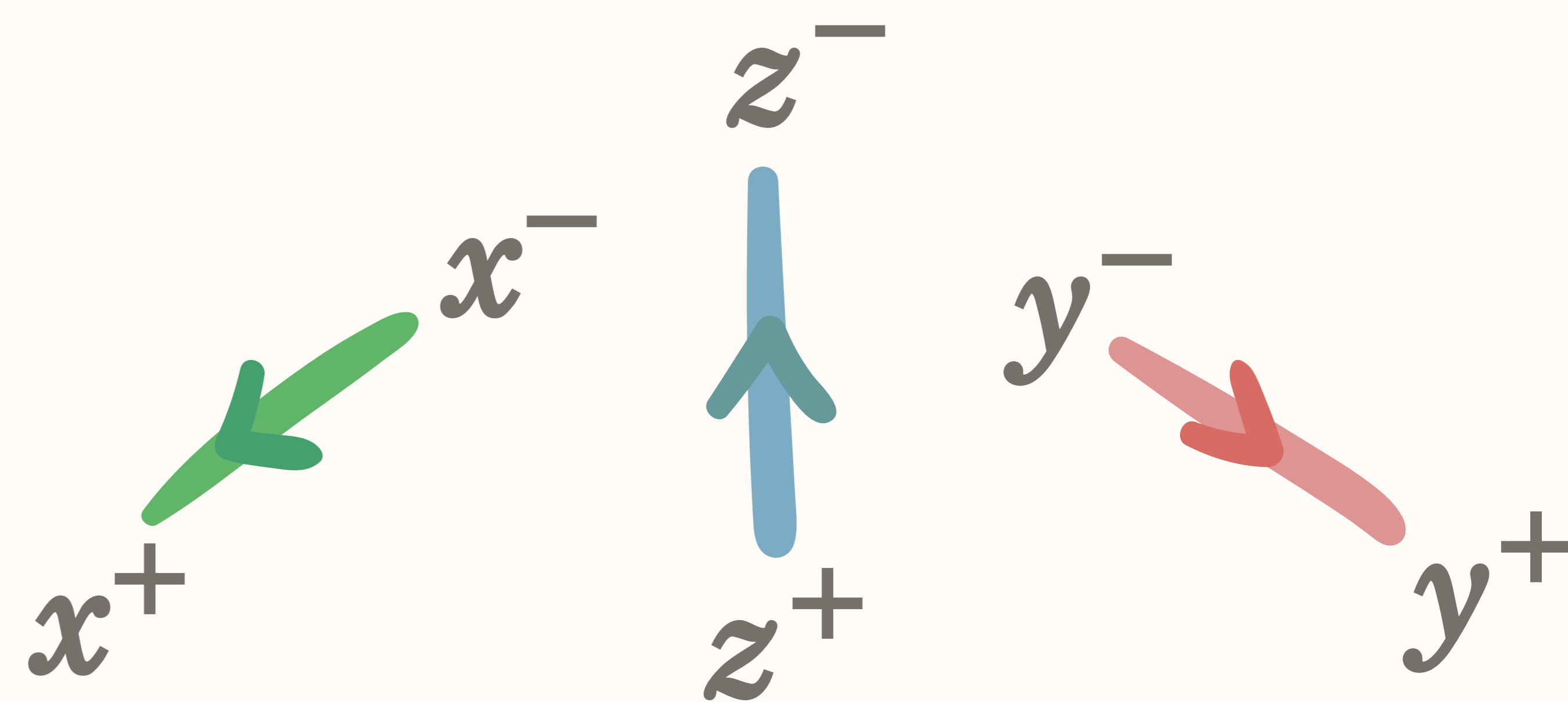


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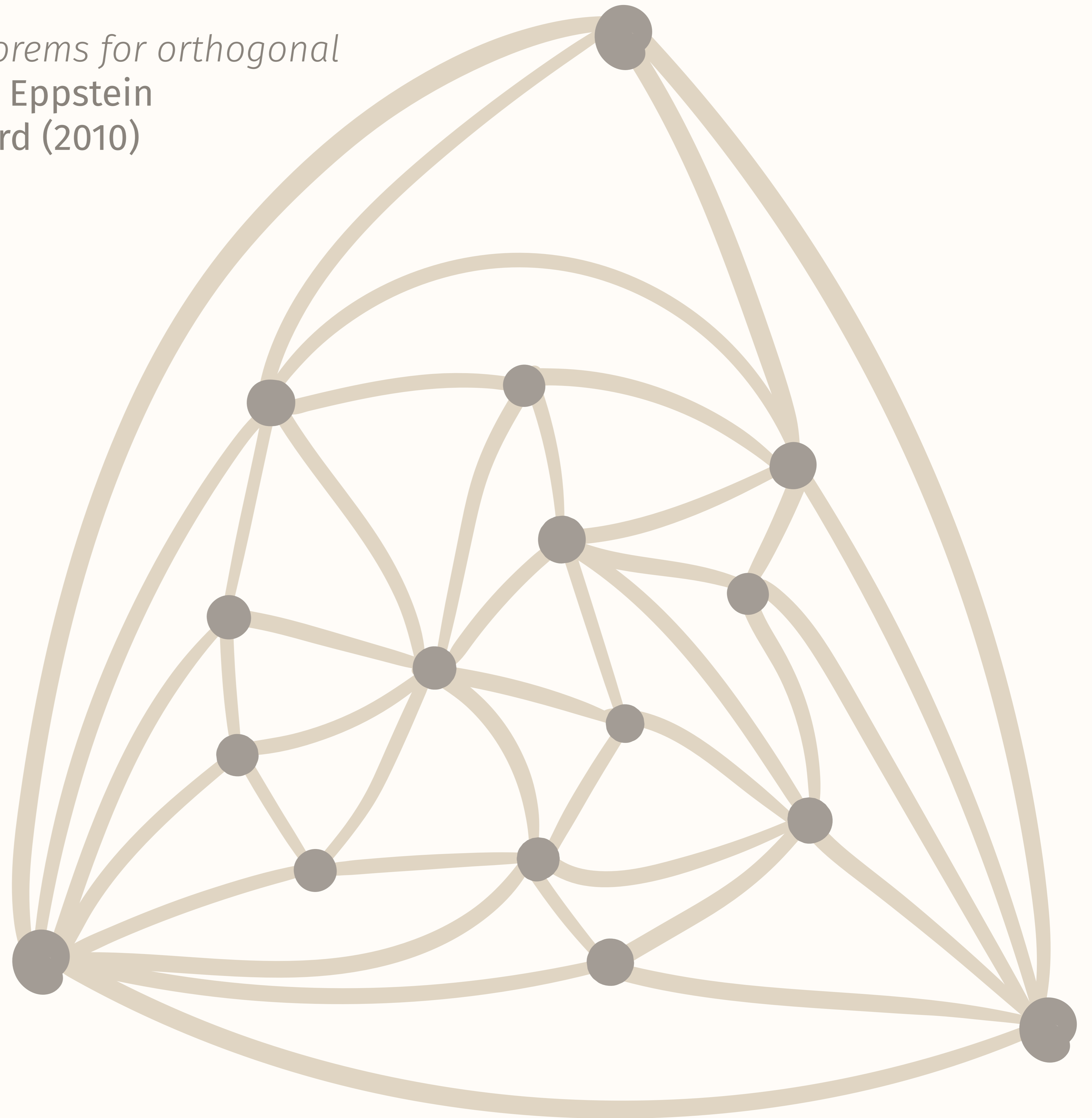
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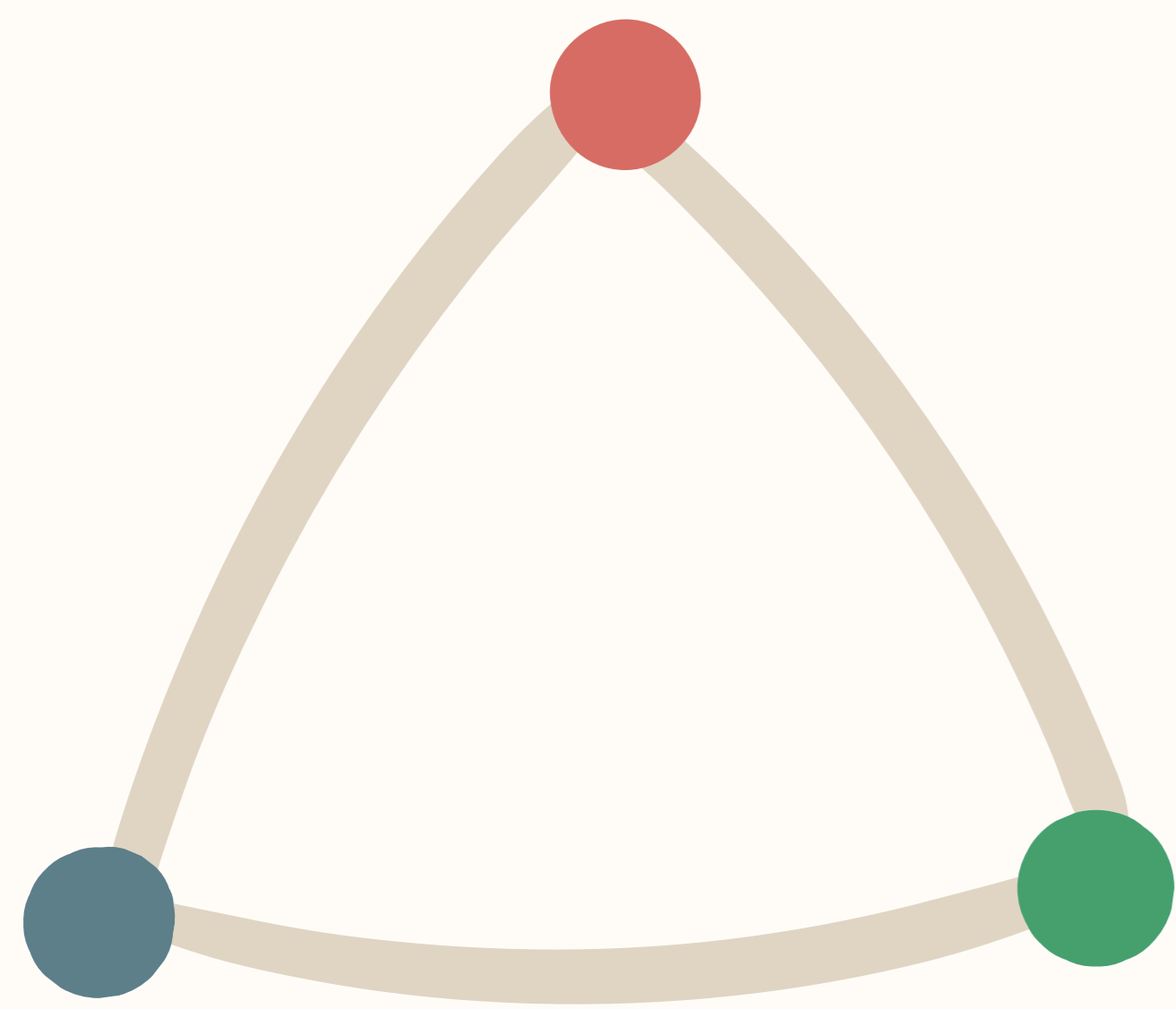


Polyhedral orientations

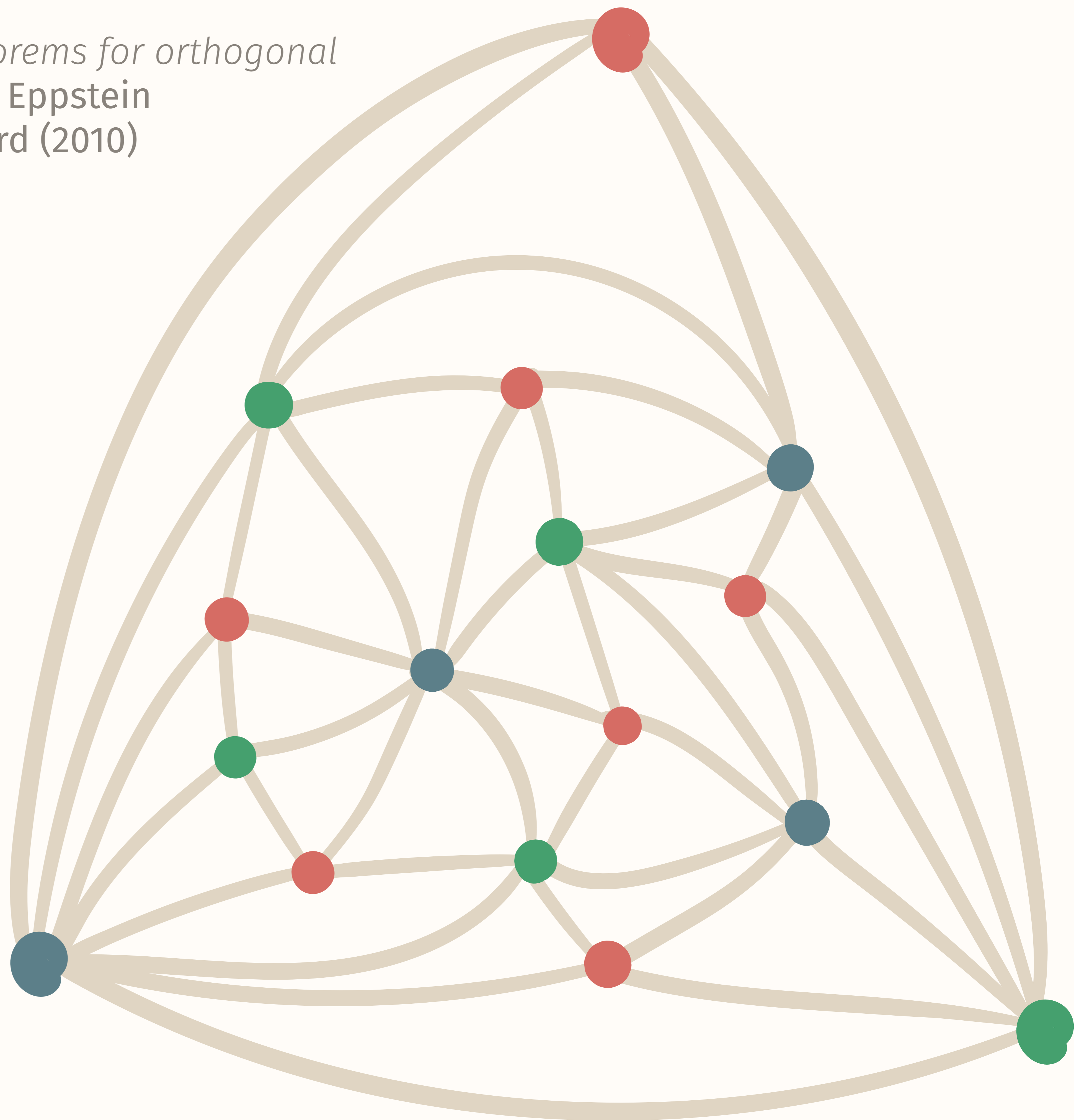
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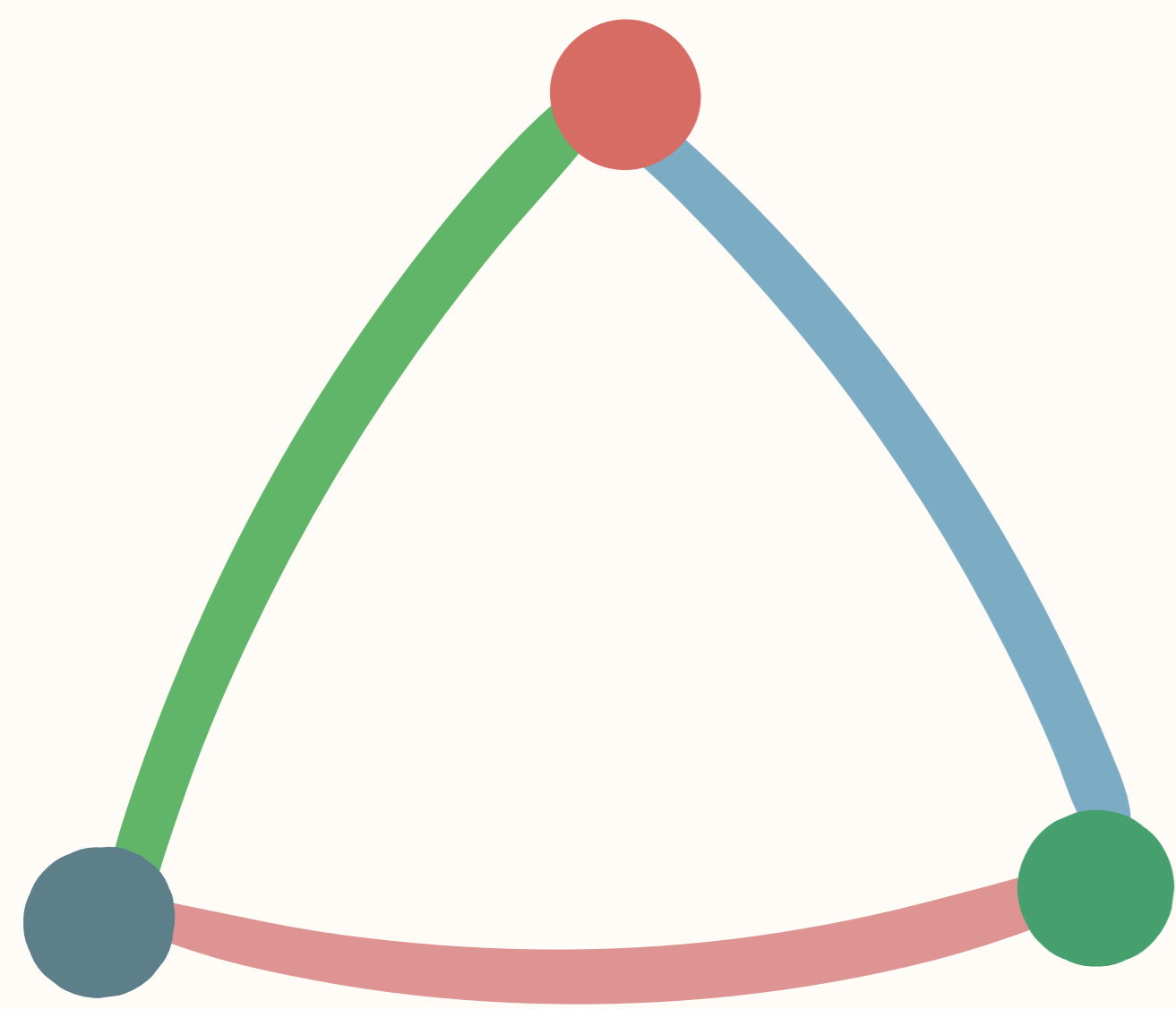
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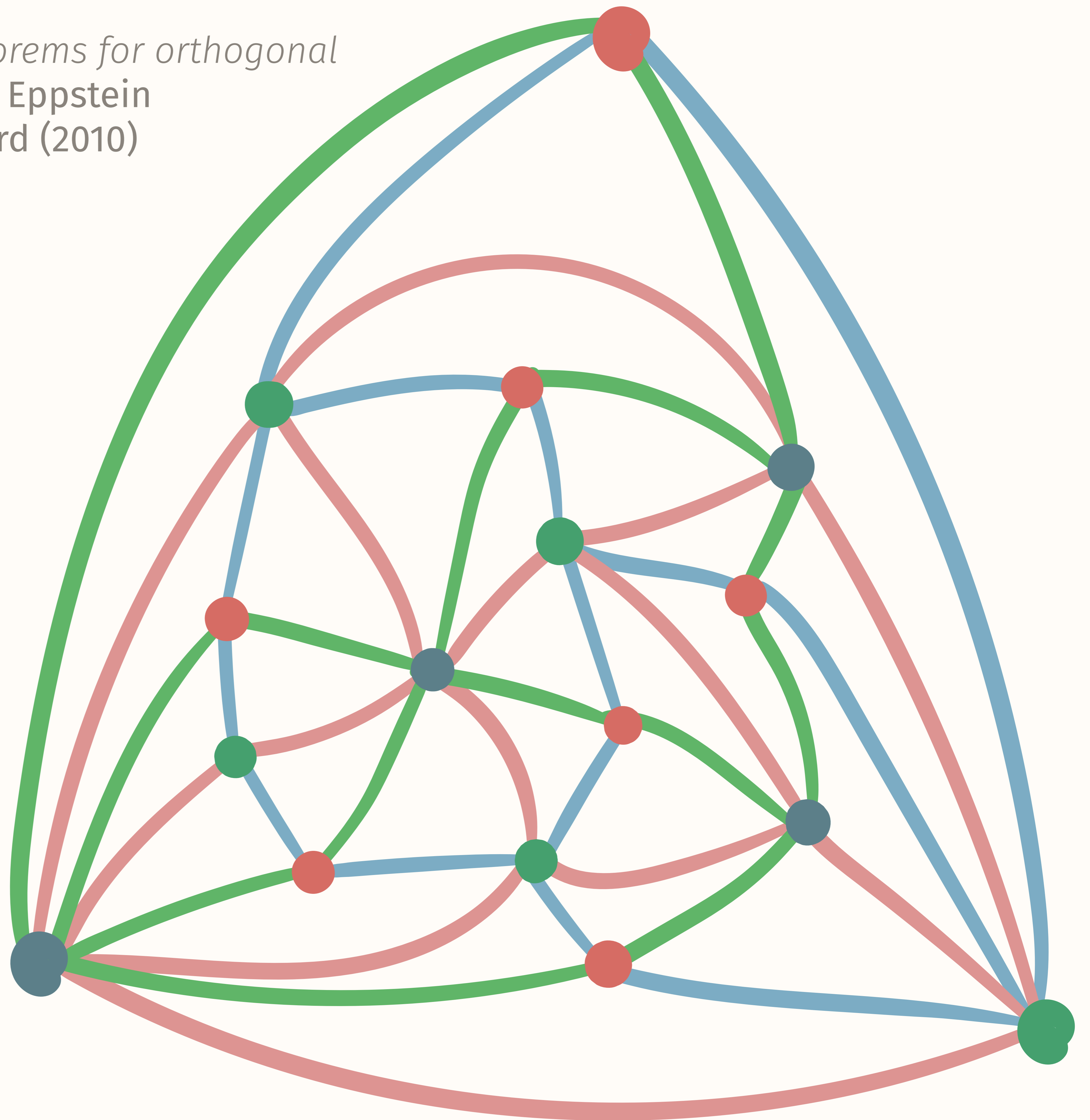
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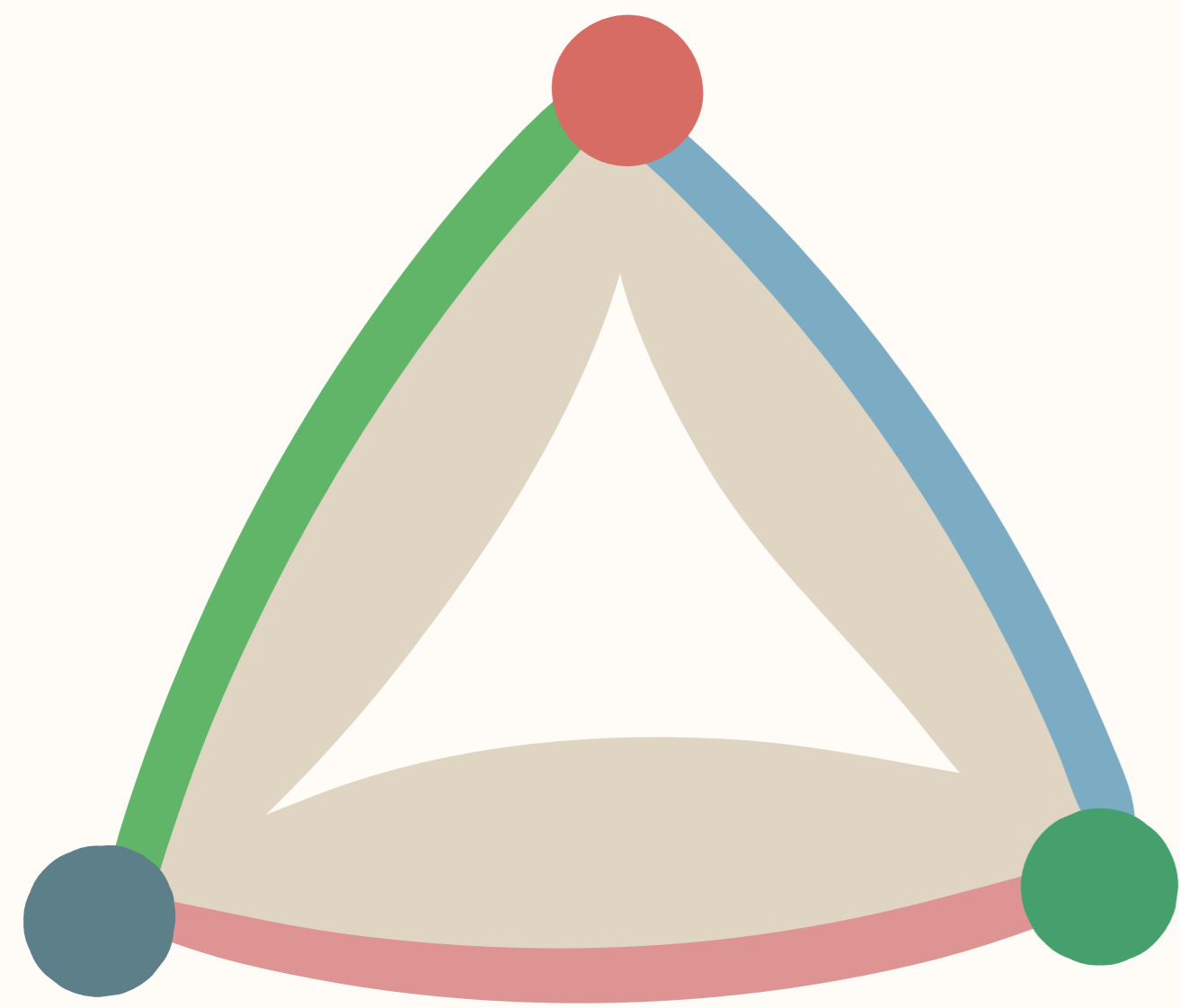
Polyhedral orientations



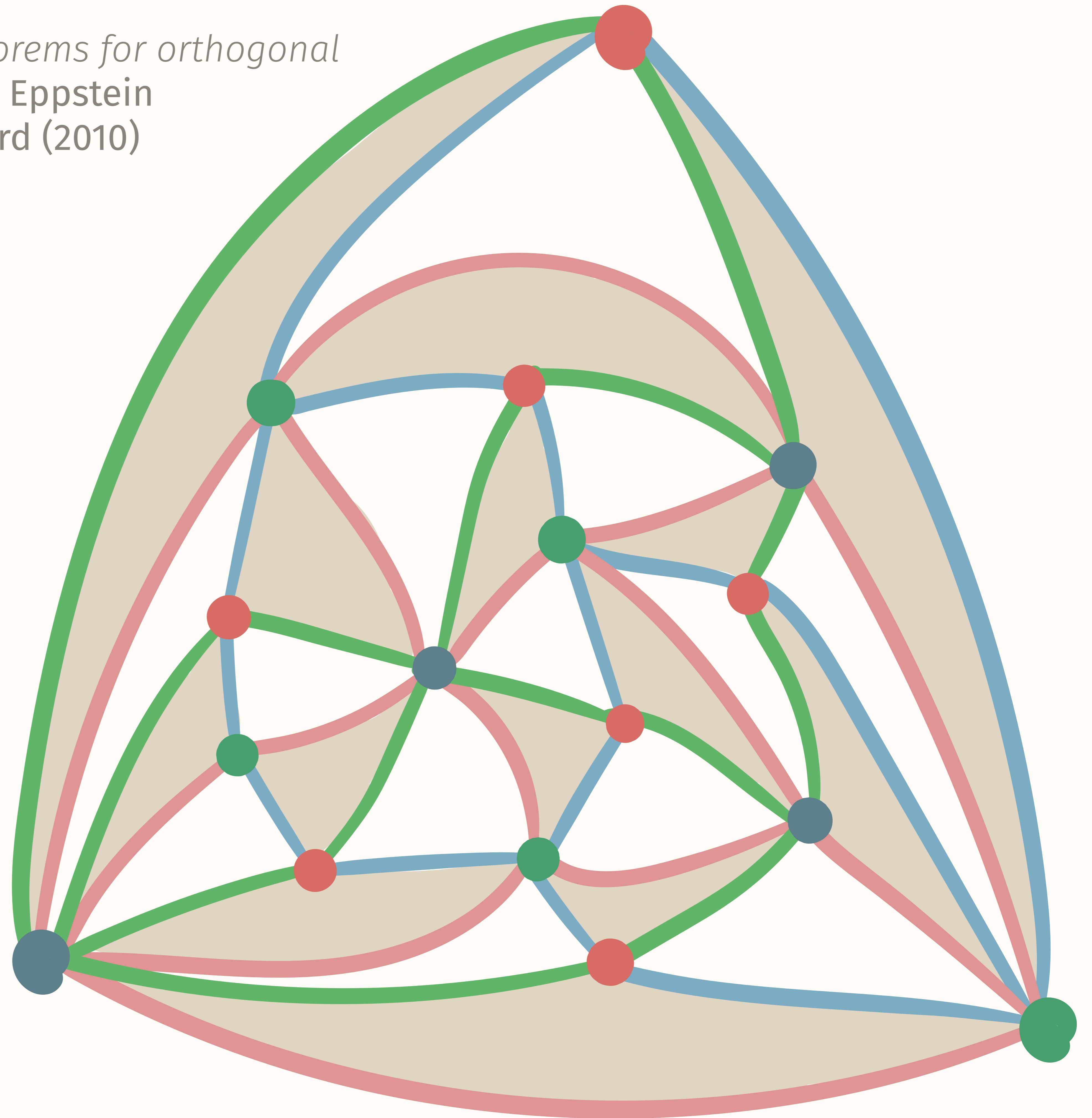
⇒ Steinitz theorems for orthogonal polyhedra, D. Eppstein & E. Mumford (2010)



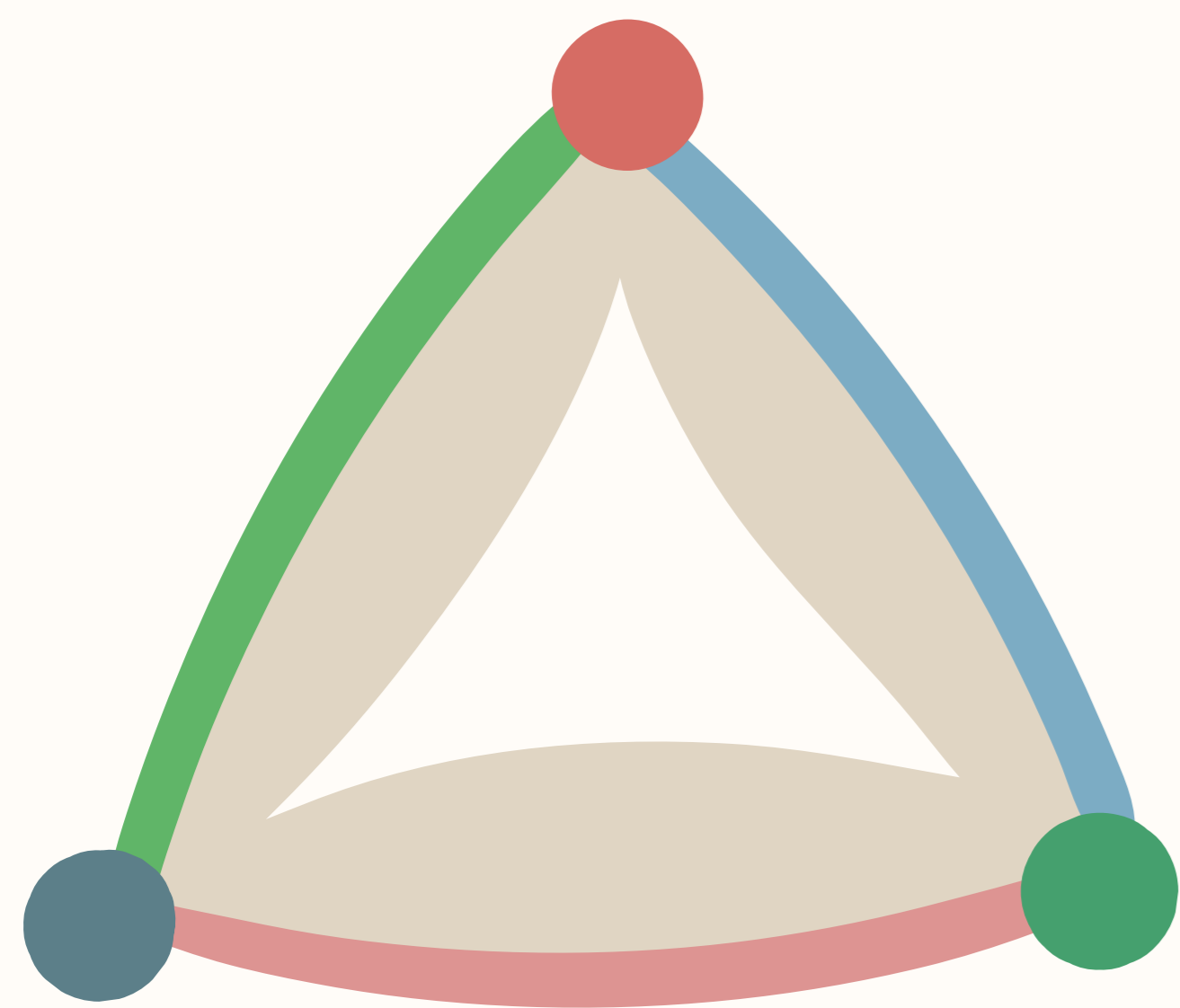
Polyhedral orientations



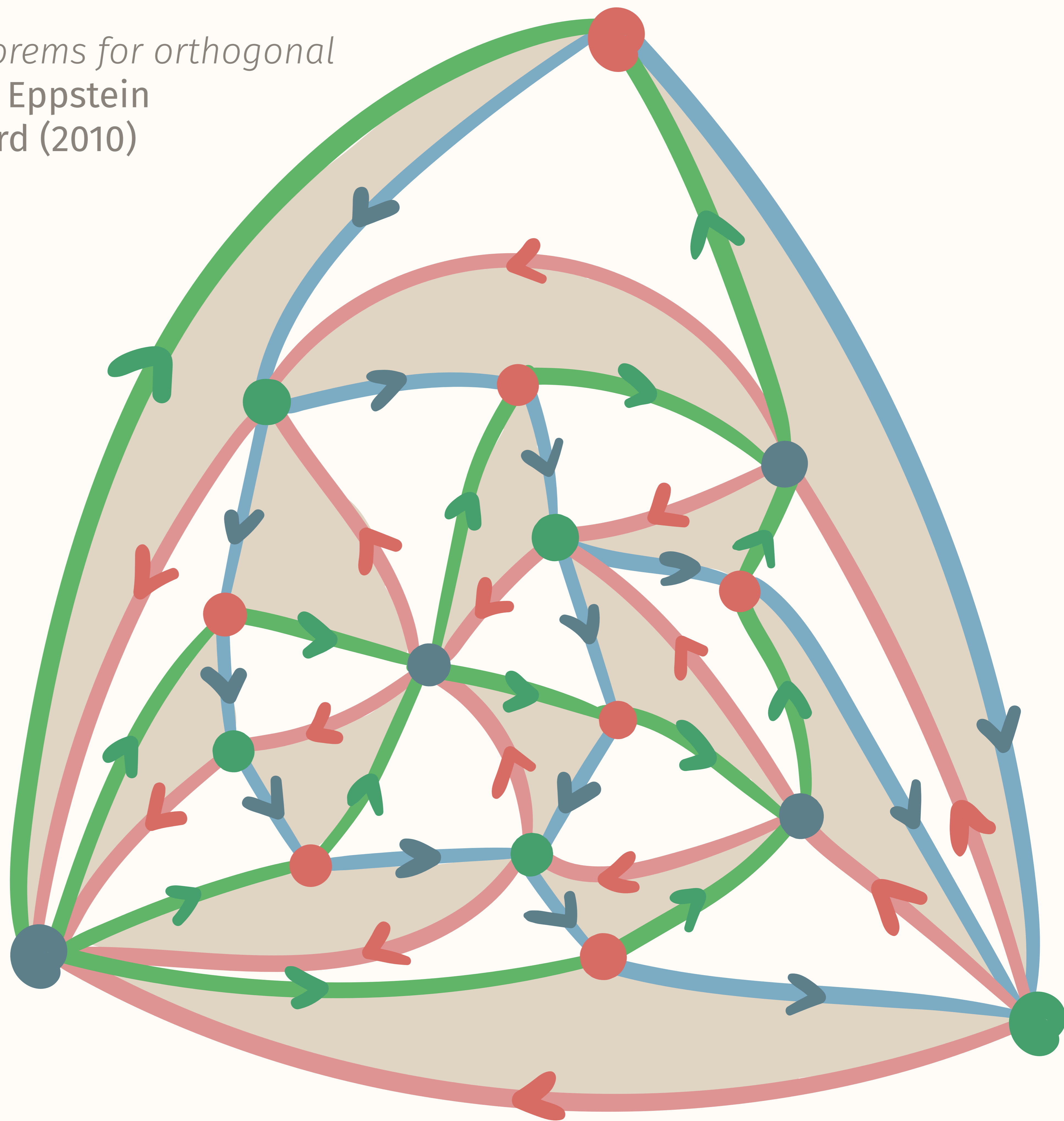
⇒ Steinitz theorems for orthogonal polyhedra, D. Eppstein & E. Mumford (2010)



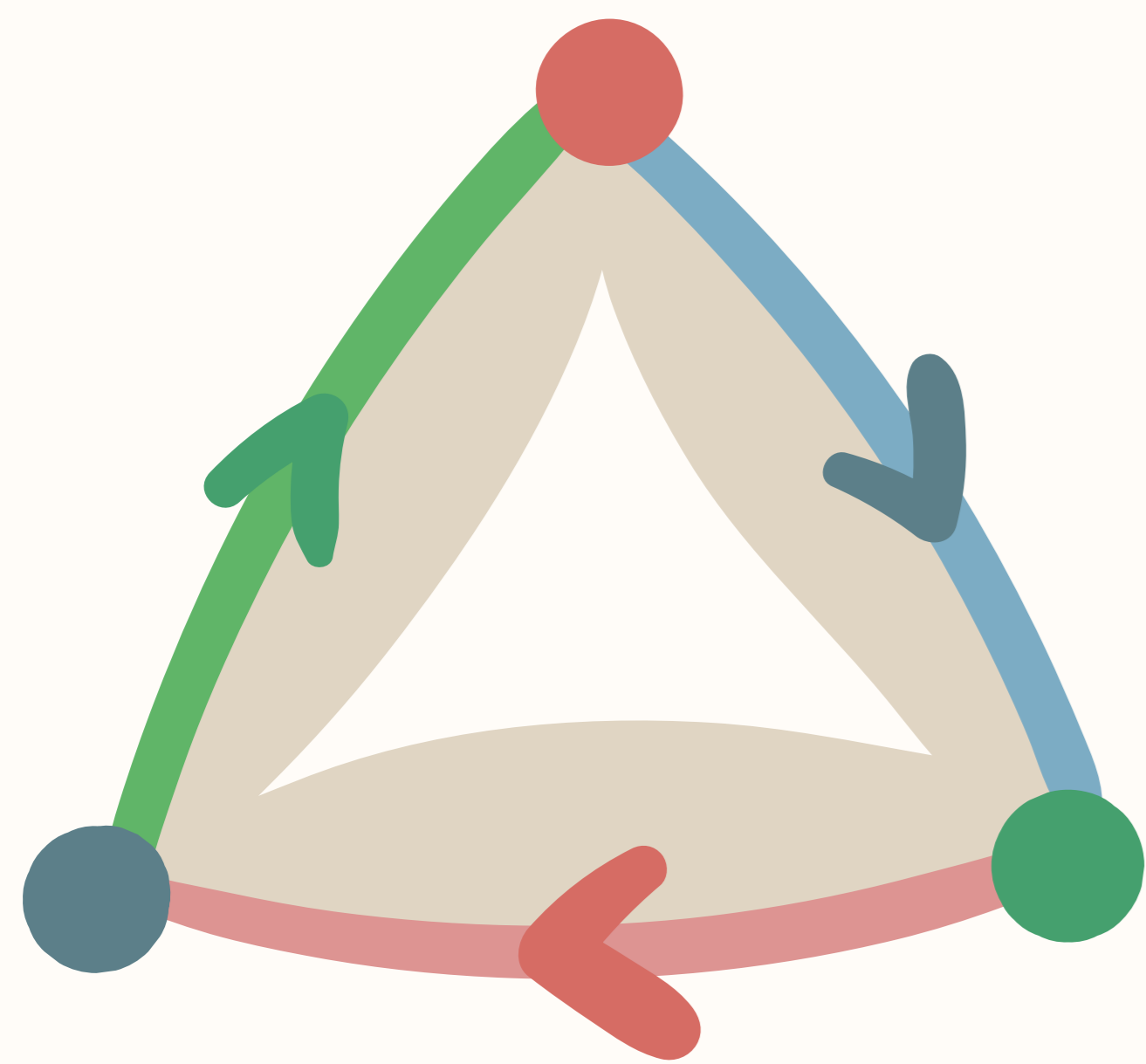
Polyhedral orientations



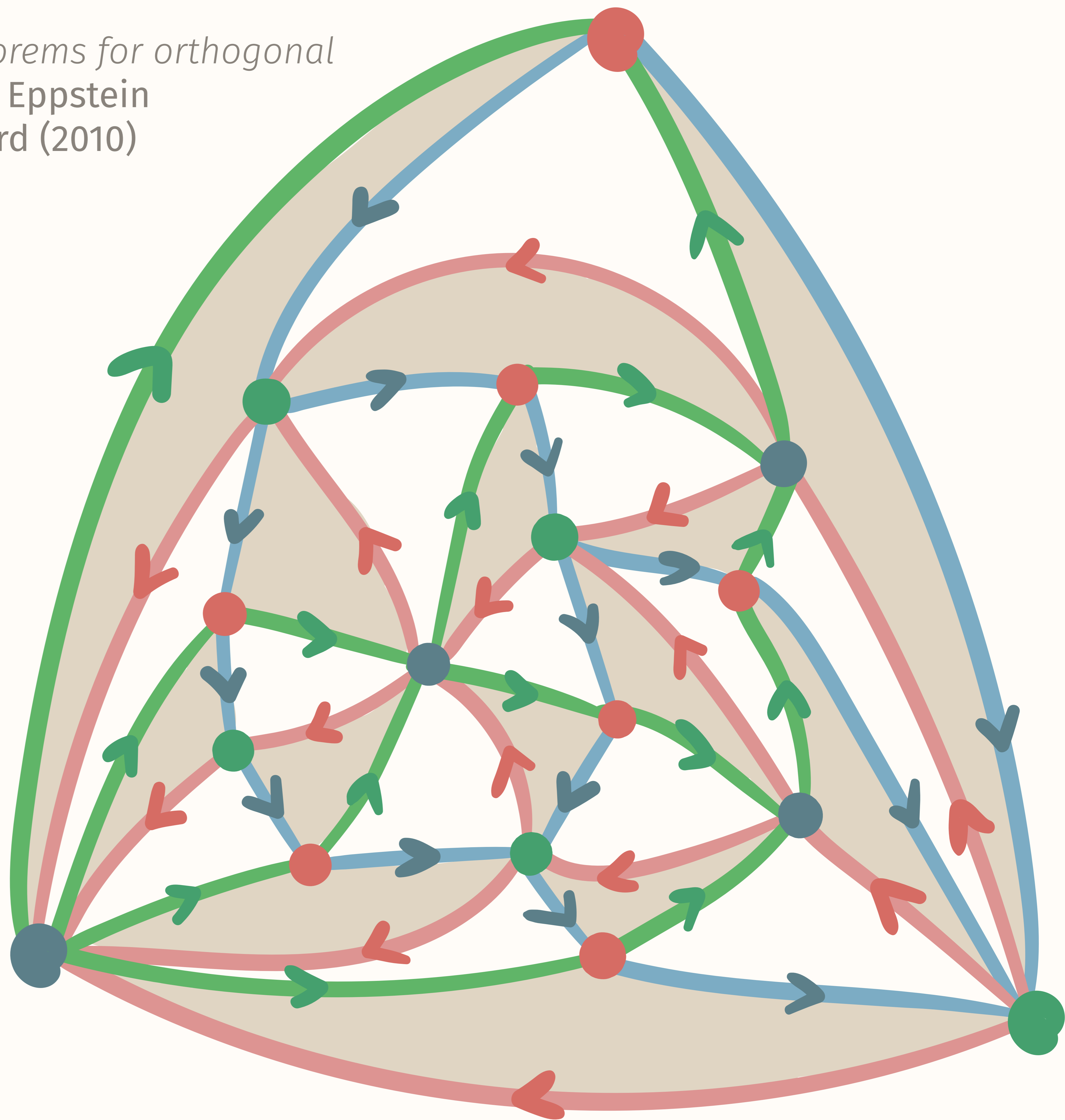
⇒ Steinitz theorems for orthogonal polyhedra, D. Eppstein & E. Mumford (2010)



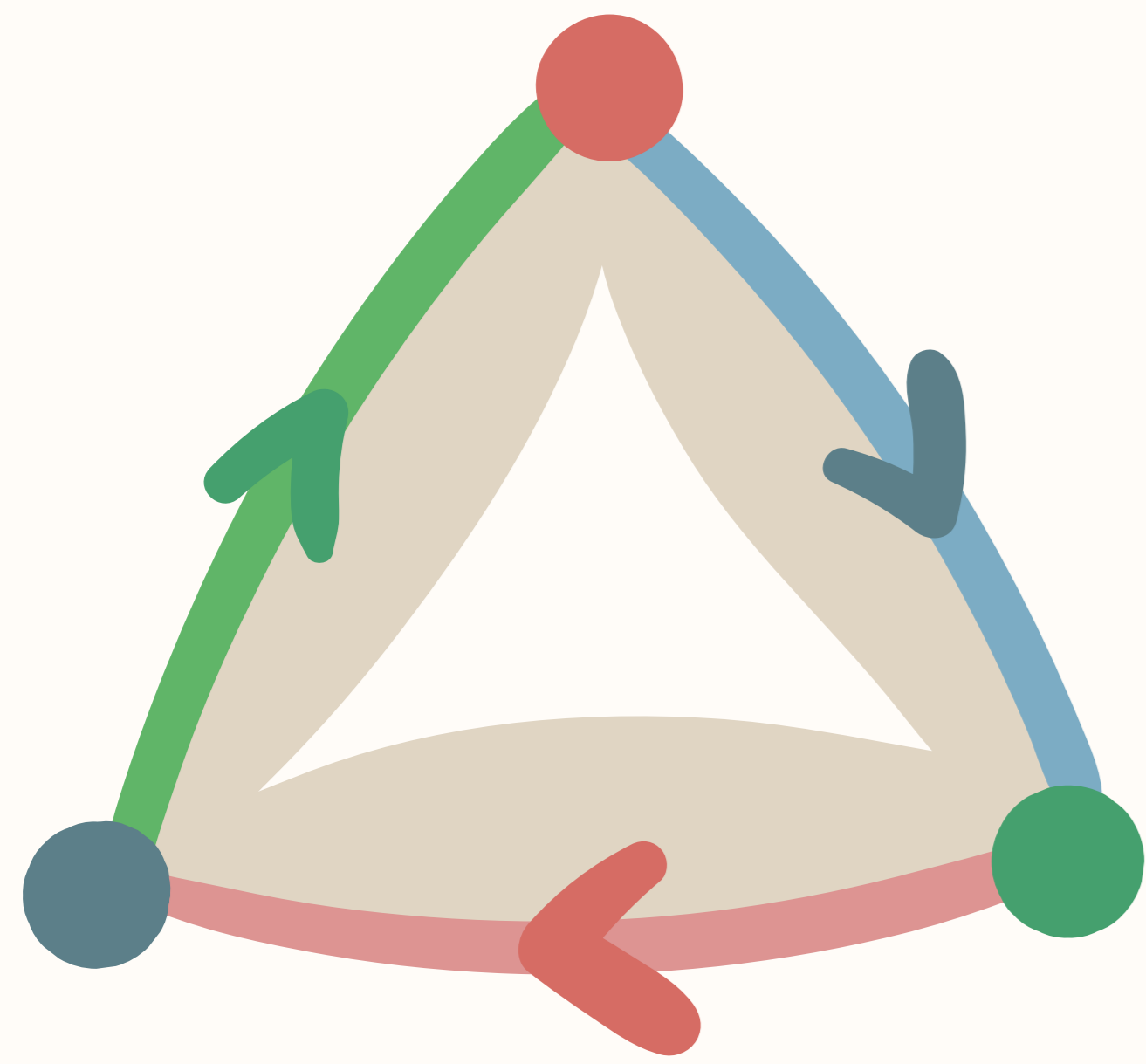
Polyhedral orientations



⇒ Steinitz theorems for orthogonal polyhedra, D. Eppstein & E. Mumford (2010)

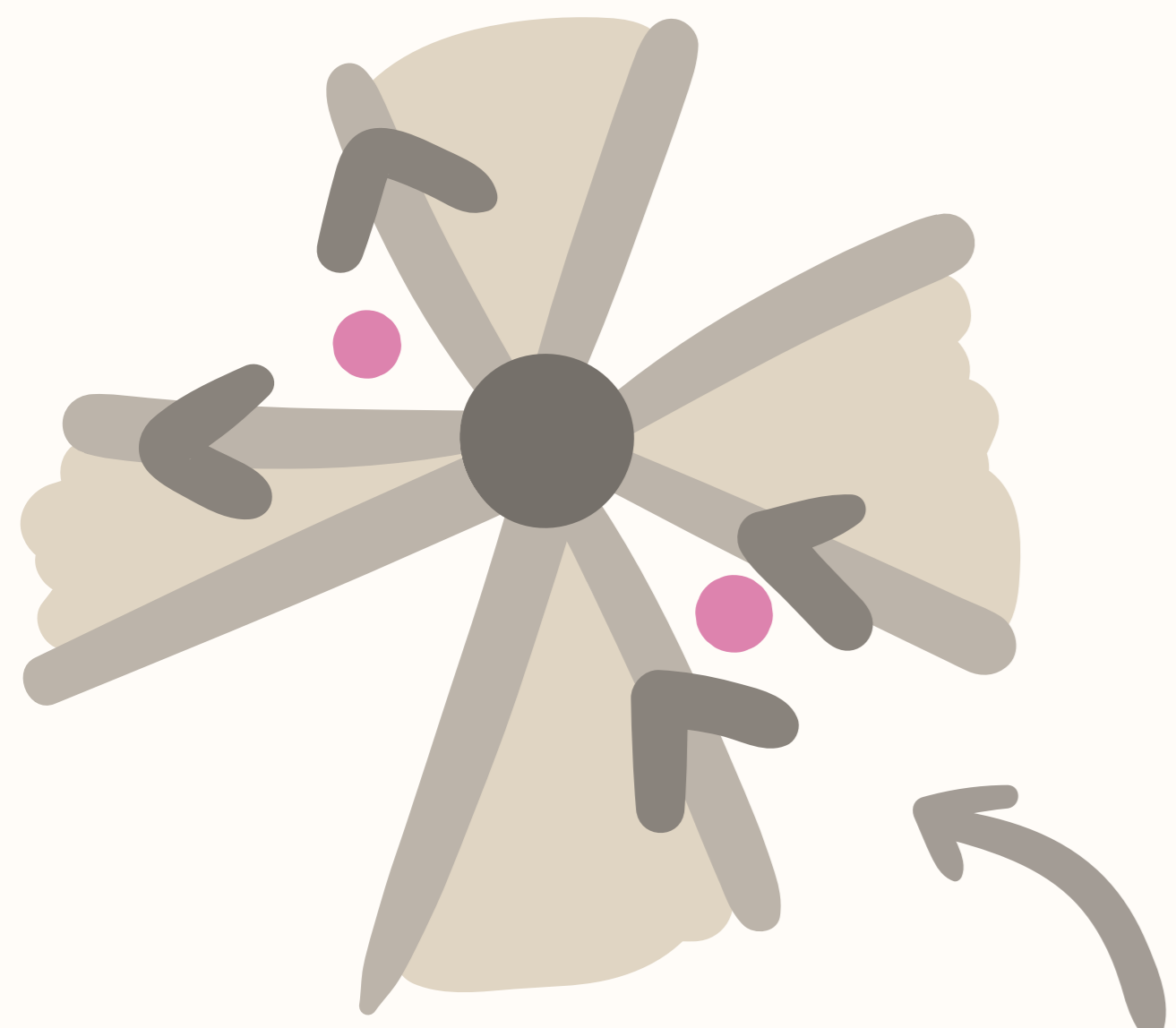


Polyhedral orientations

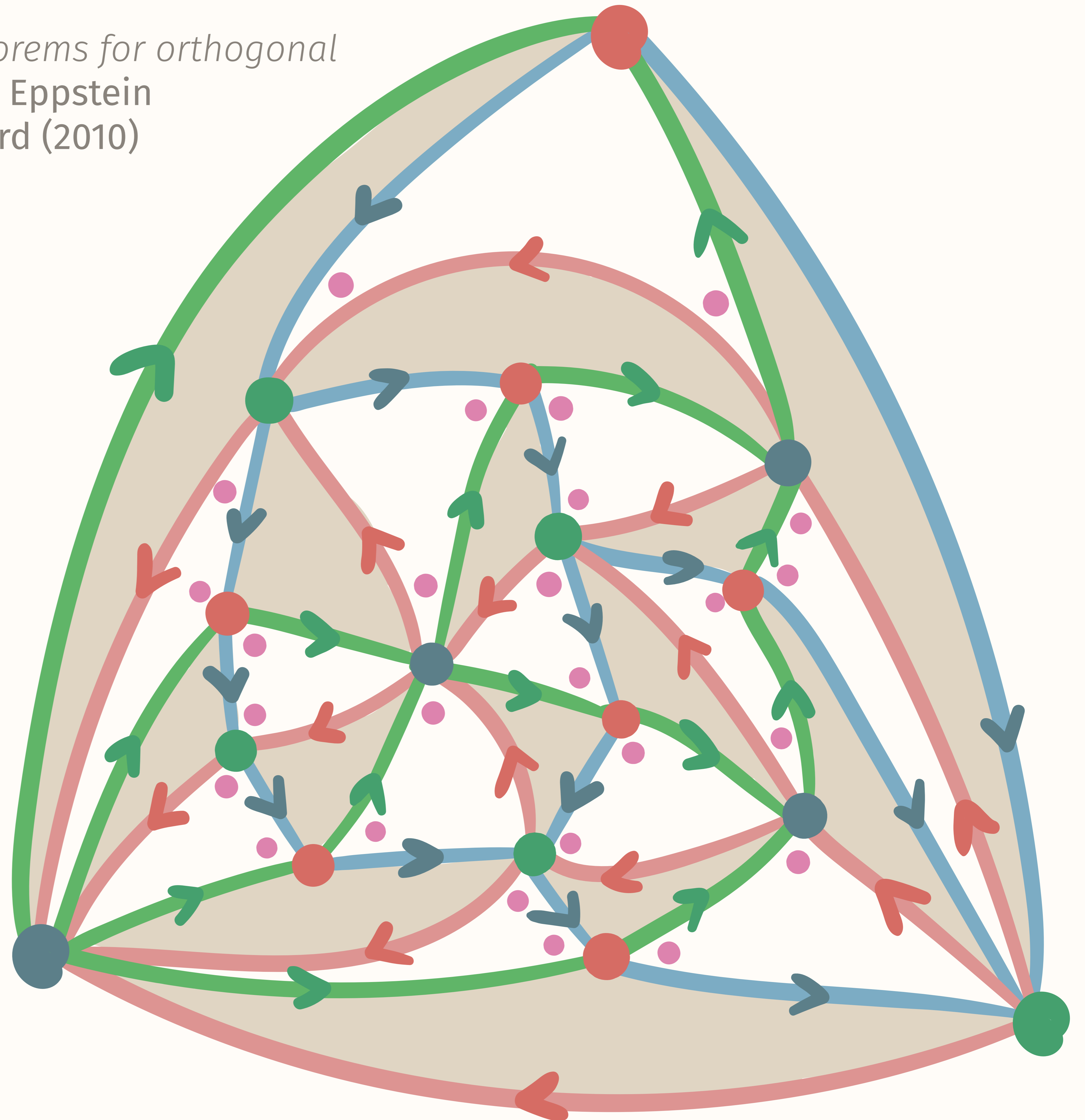


⇒ Steinitz theorems for orthogonal polyhedra, D. Eppstein & E. Mumford (2010)

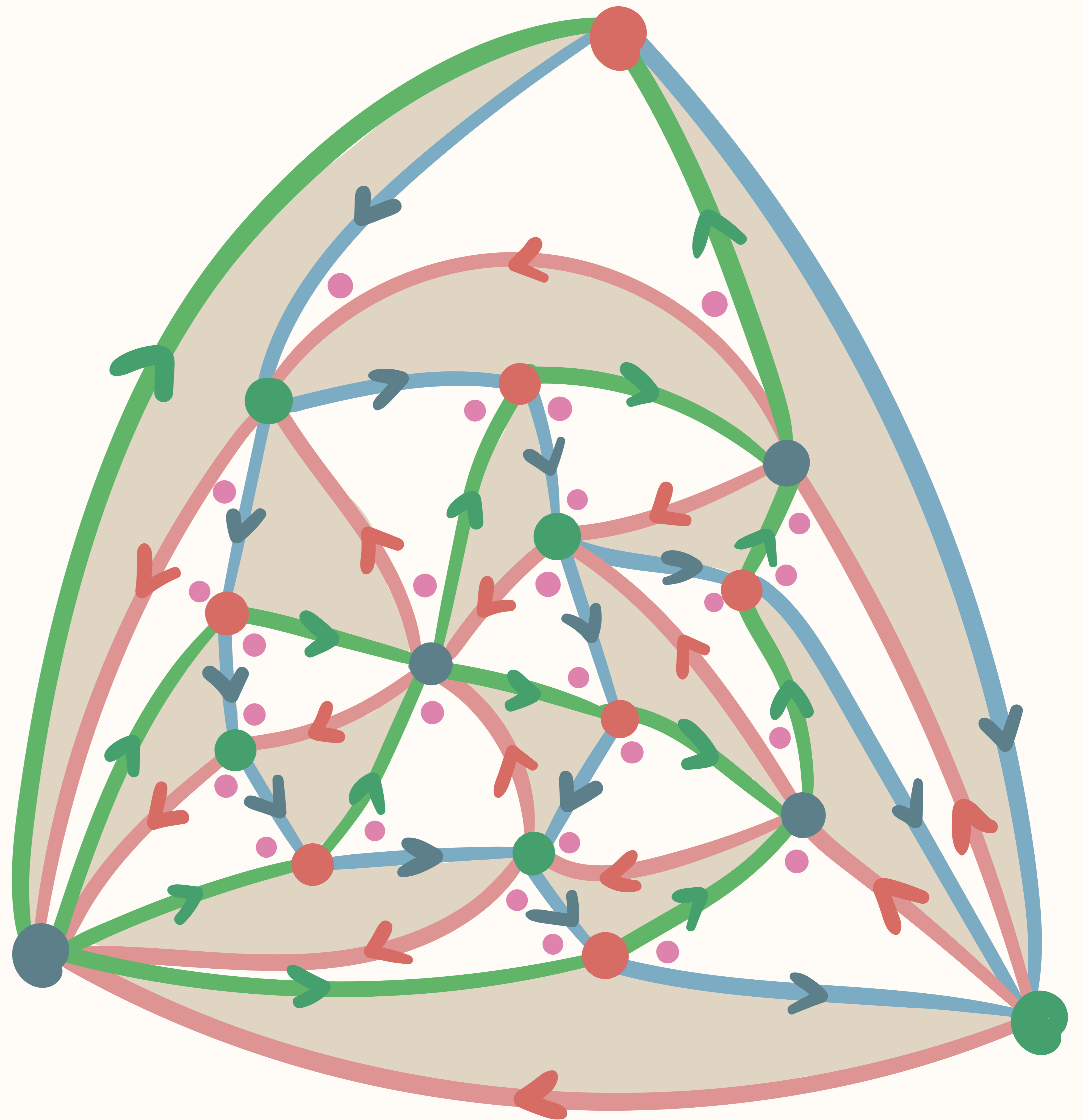
inner:



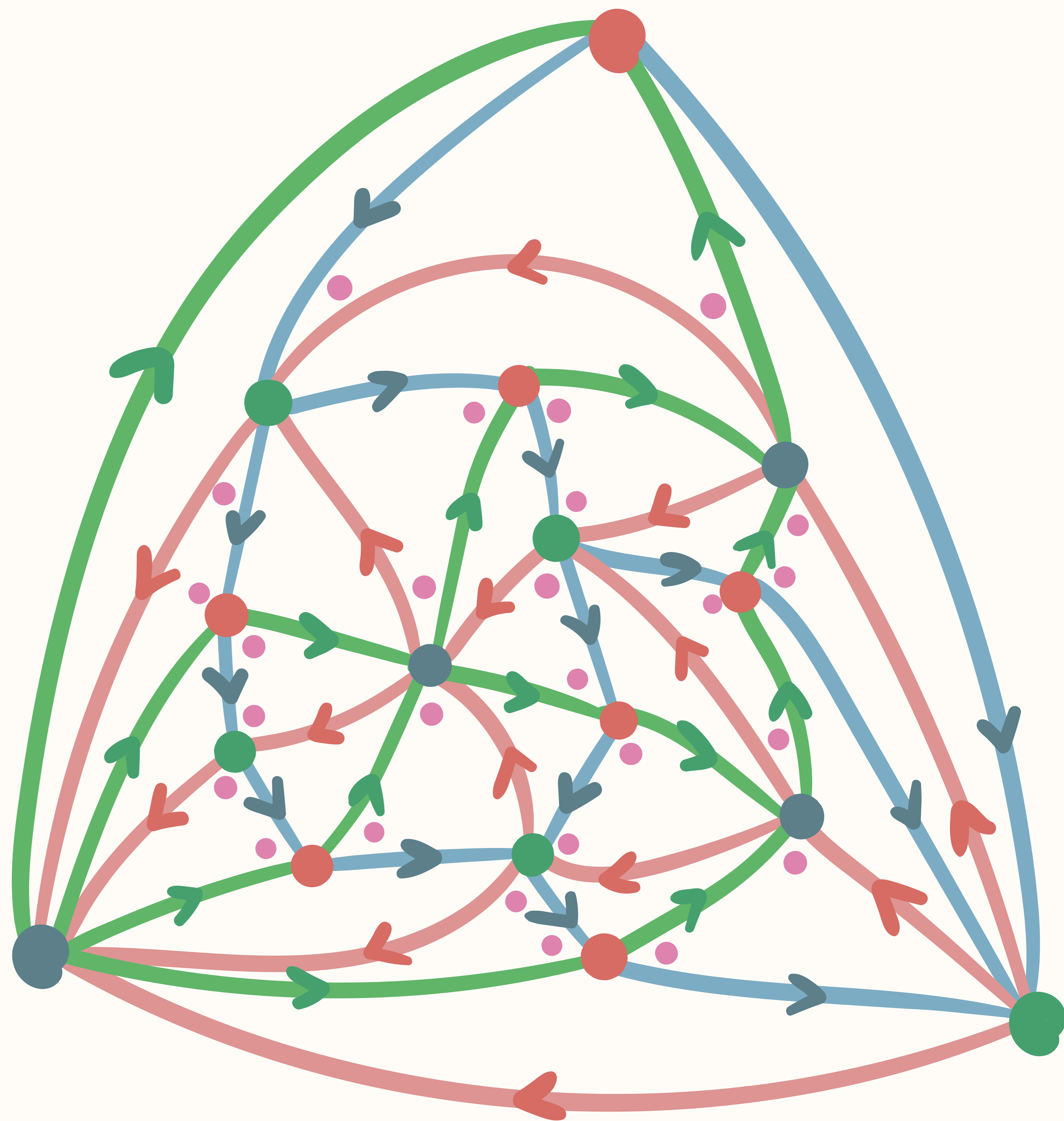
extremal x 2



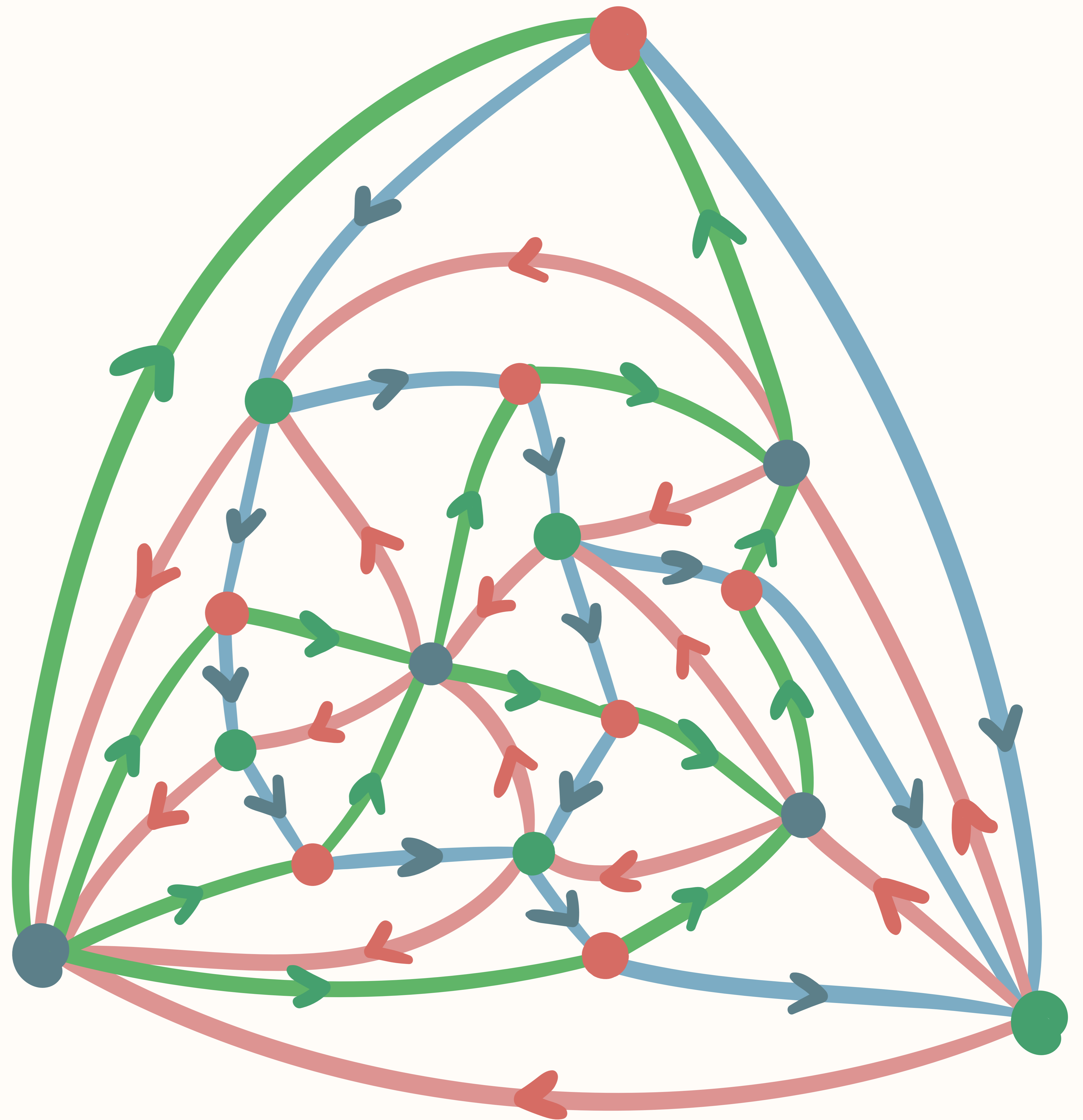
Polyhedral orientations



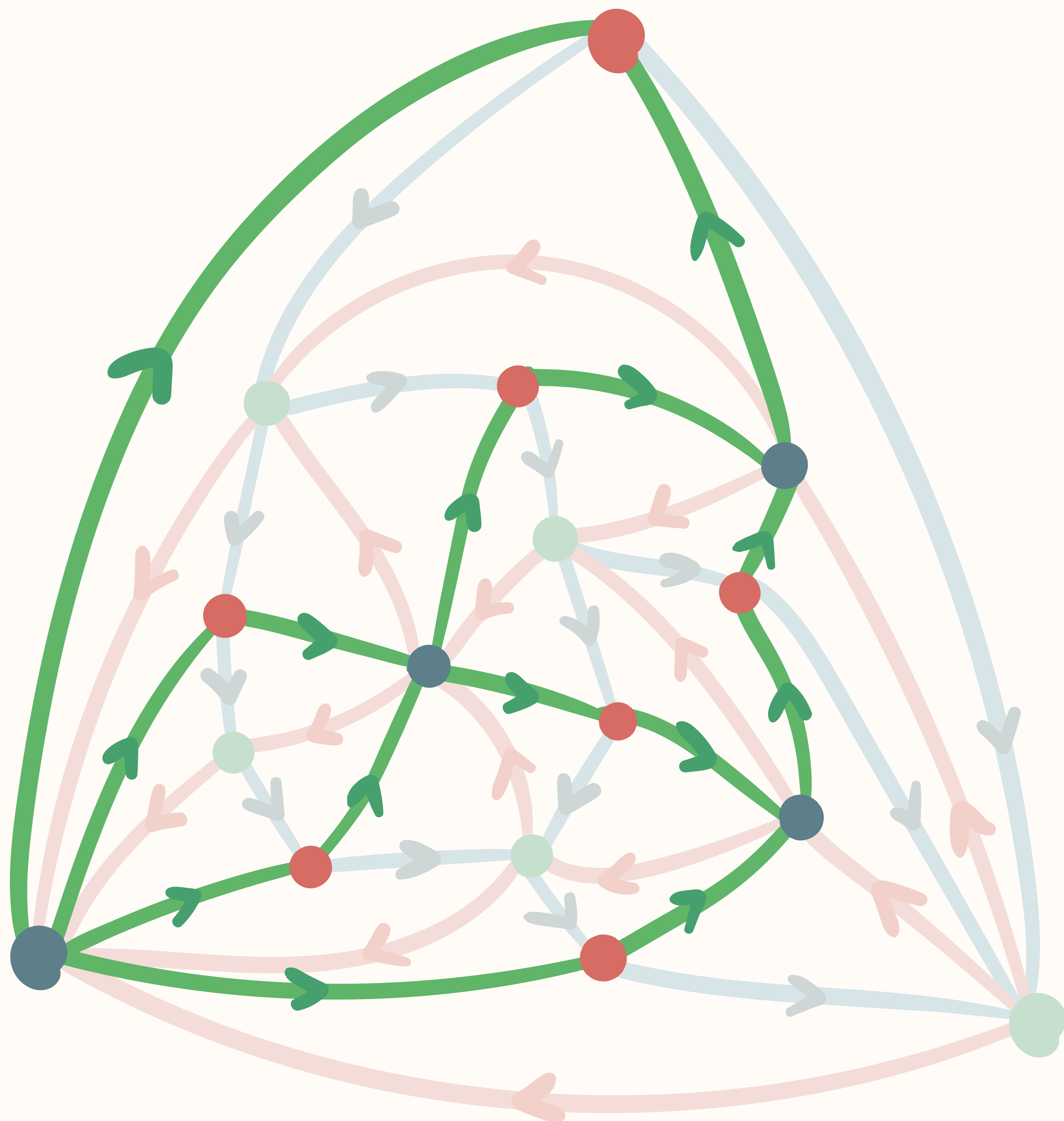
Polyhedral orientations



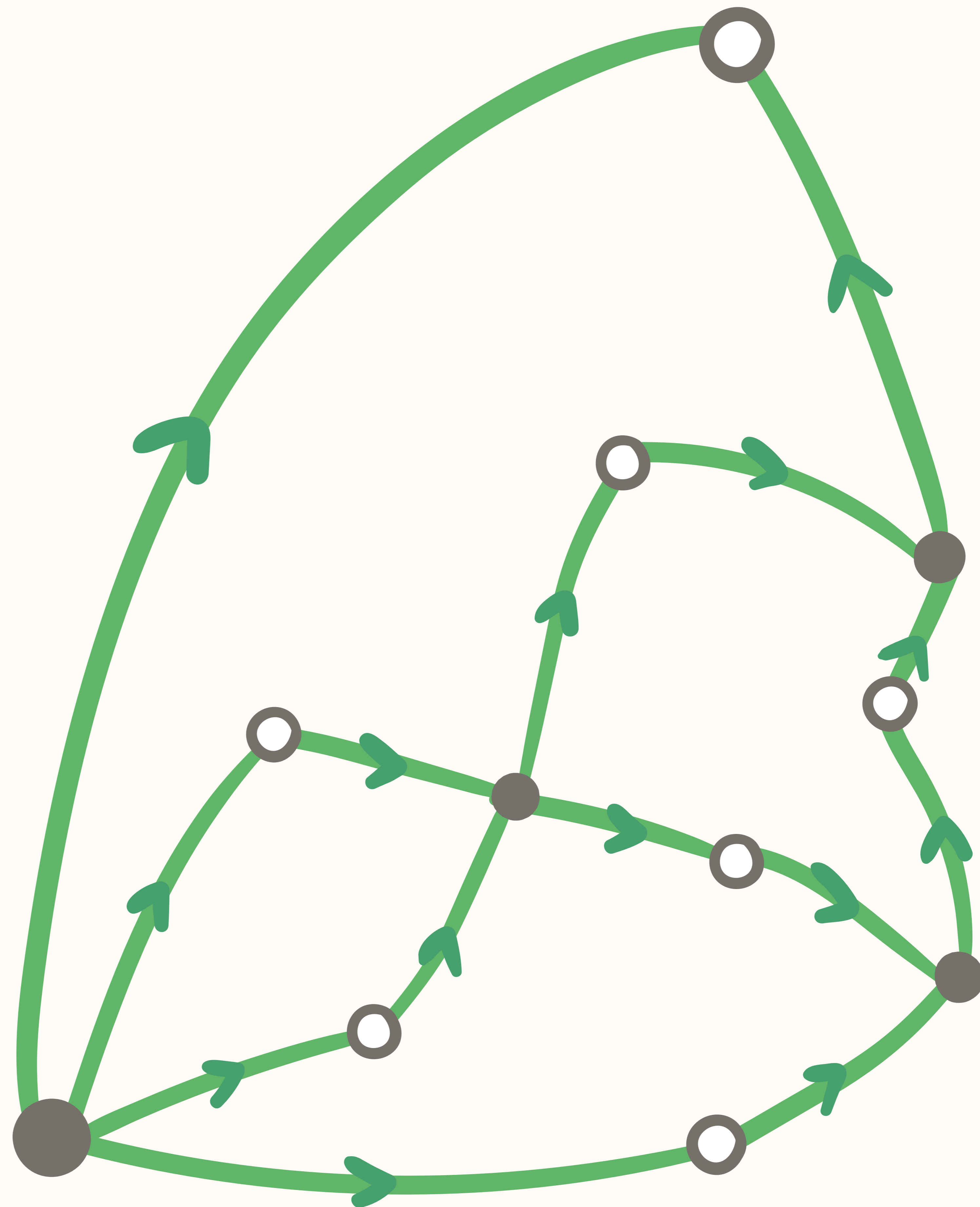
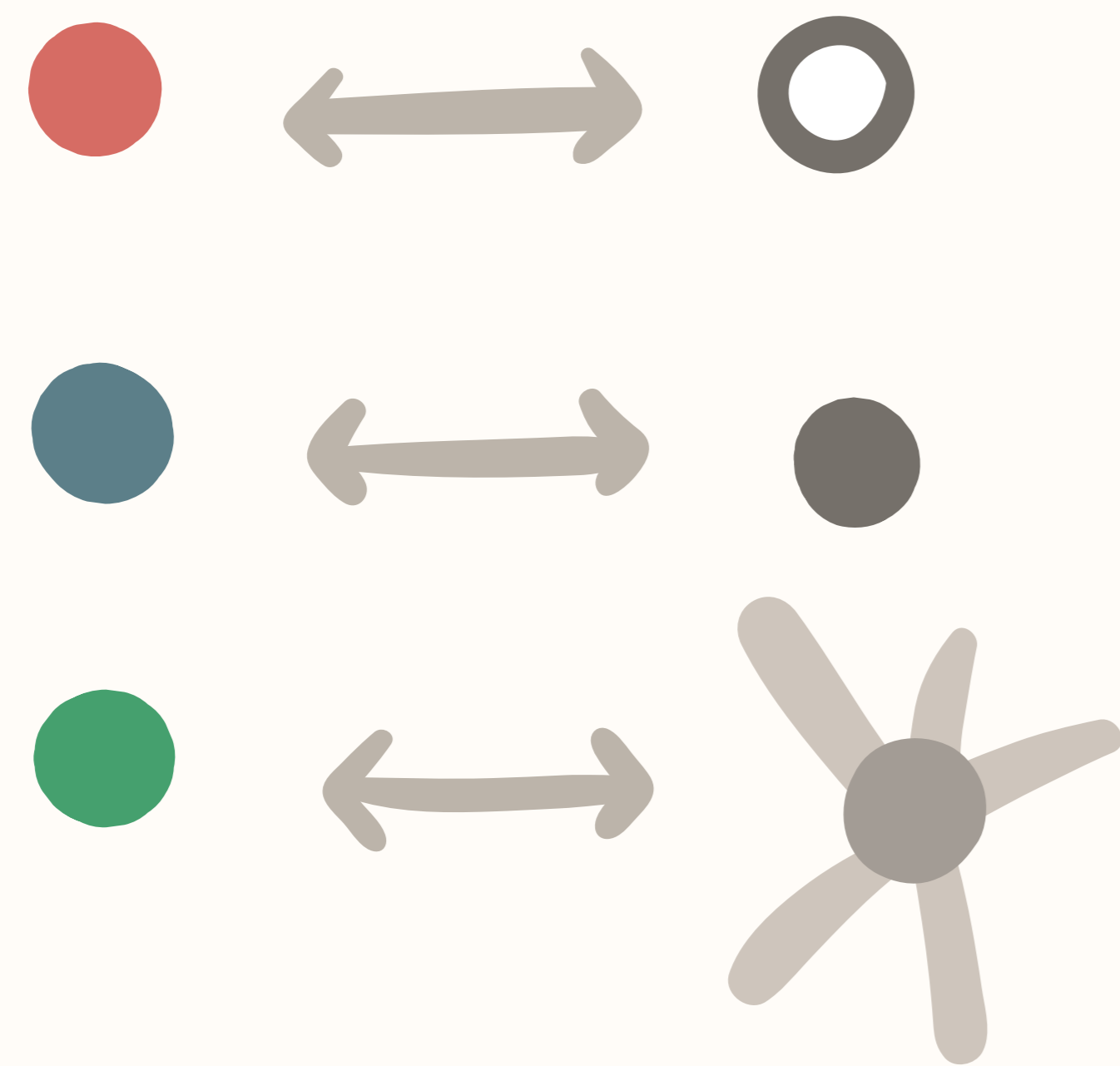
Polyhedral orientations



Polyhedral orientations

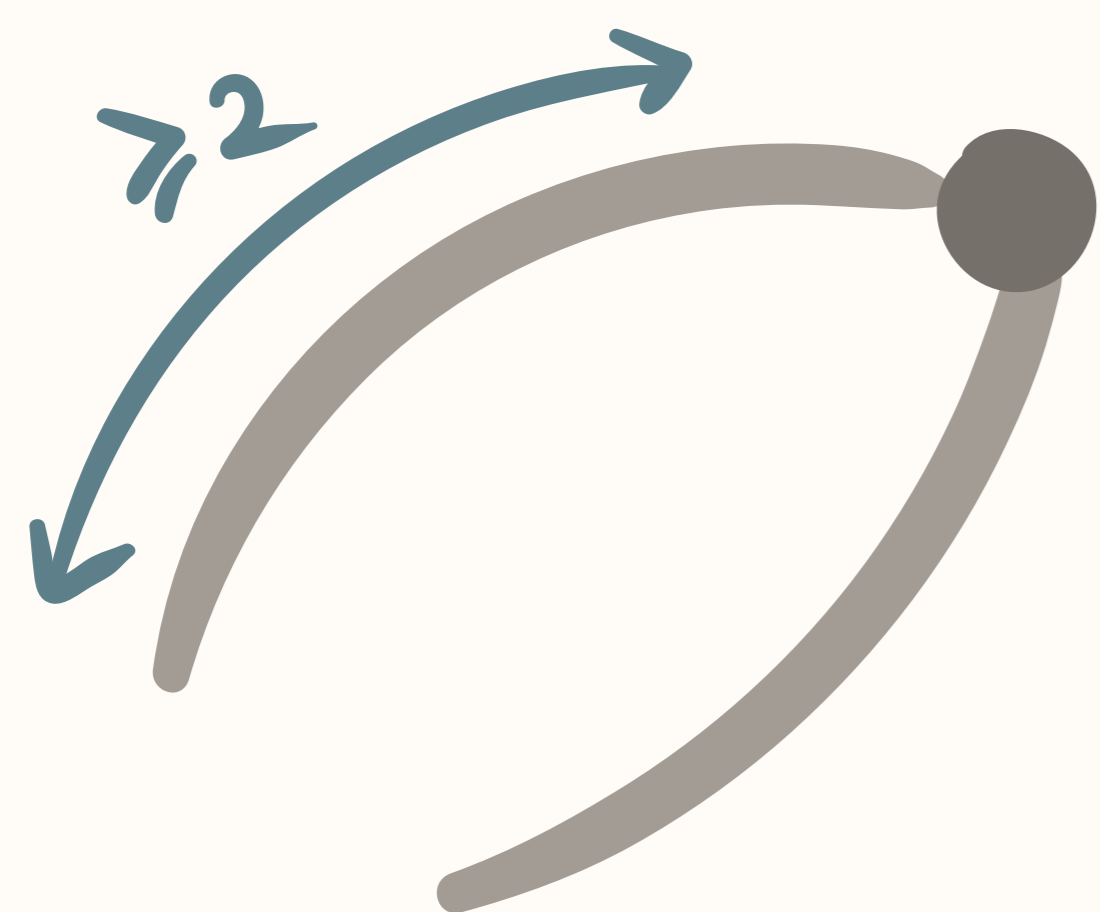
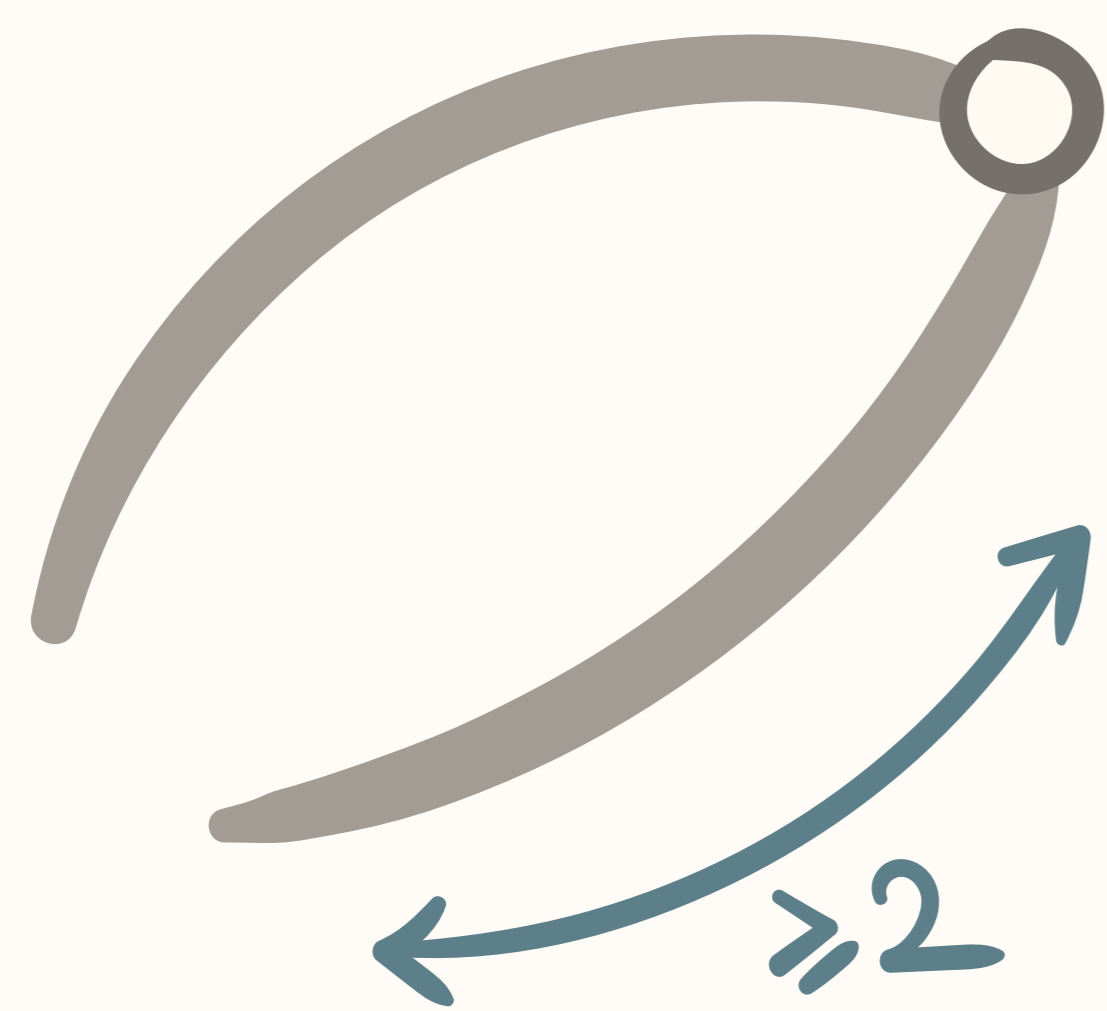
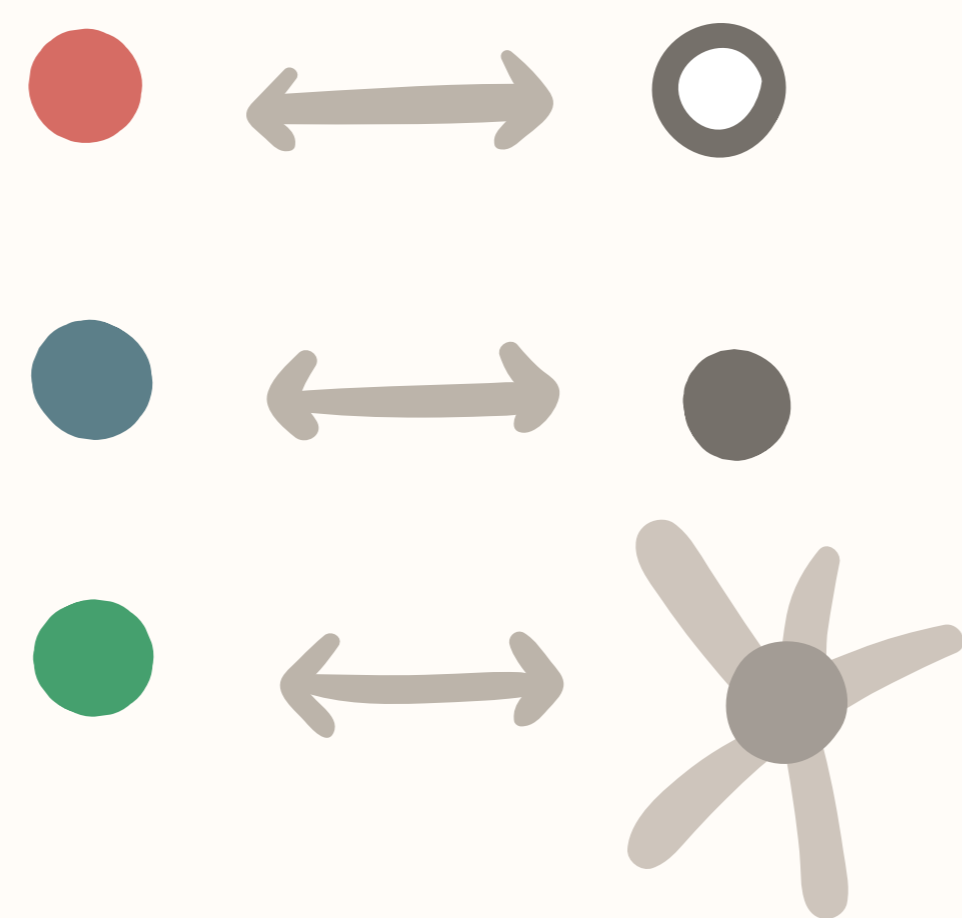
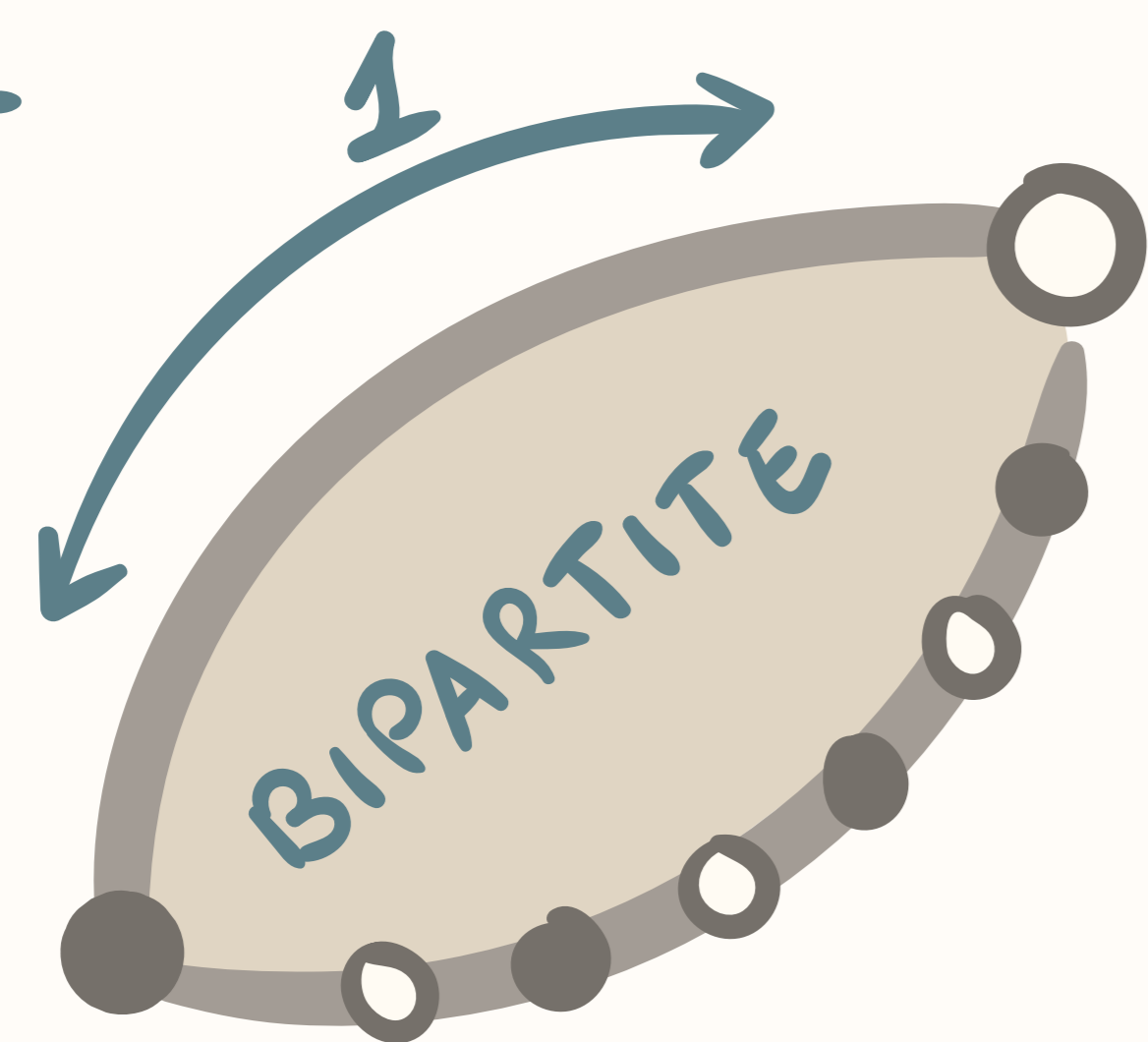


Polyhedral orientations

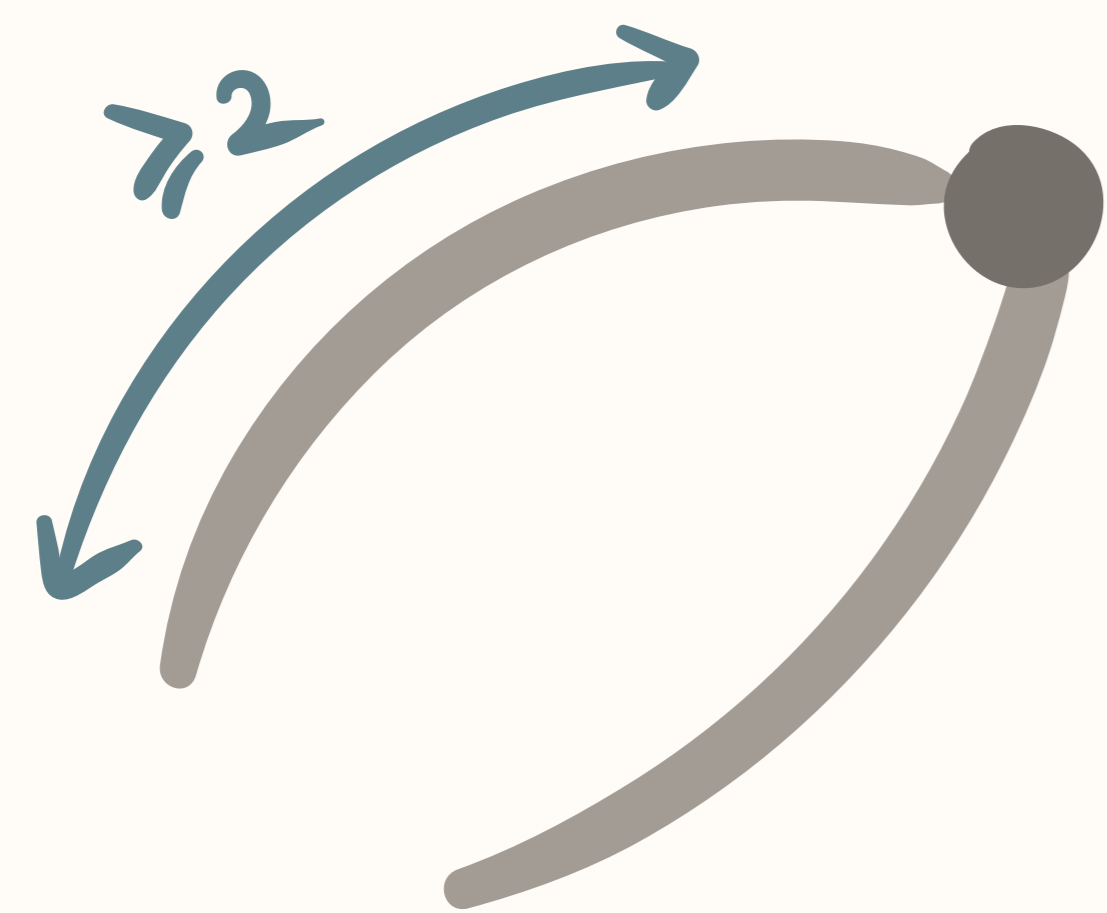
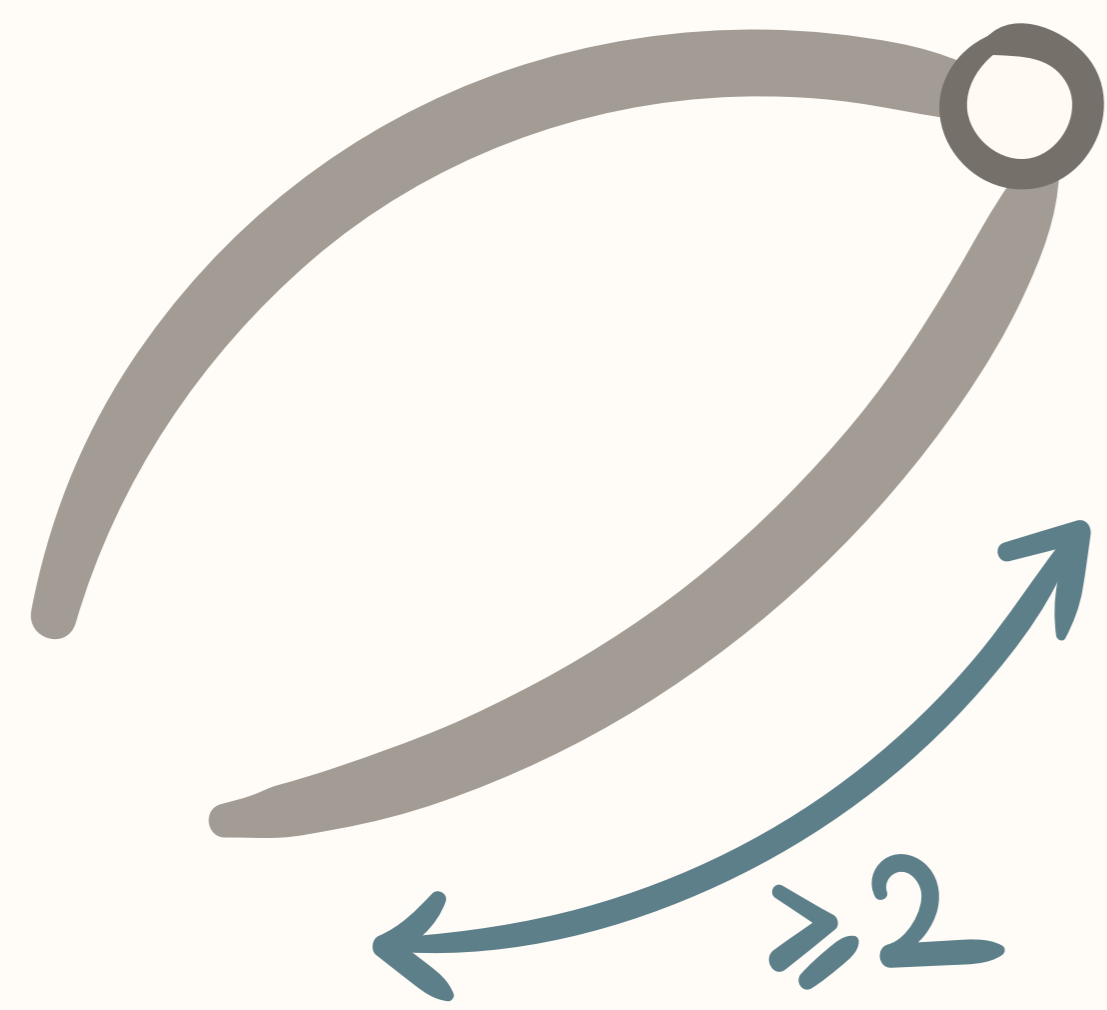
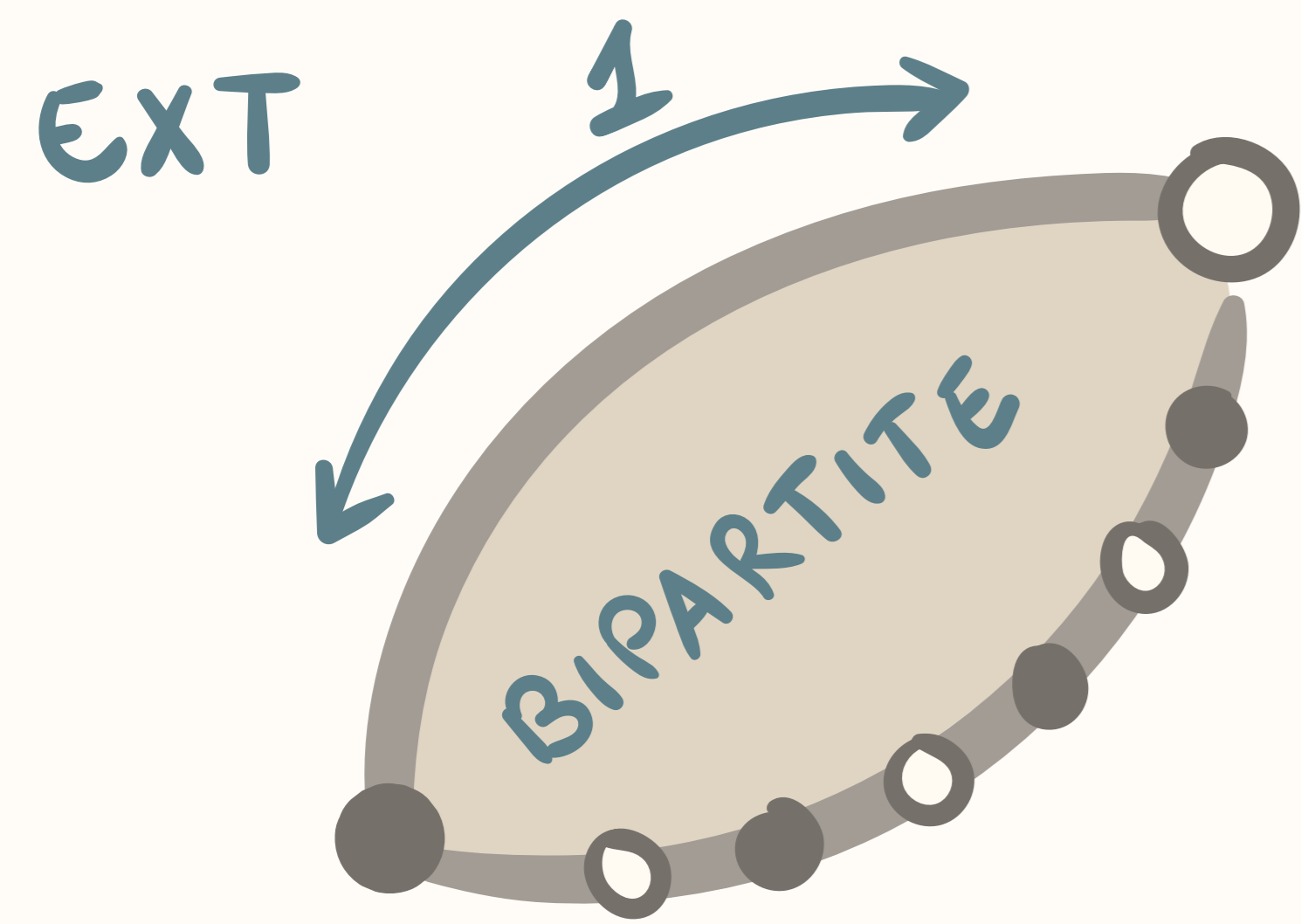


Polyhedral orientations

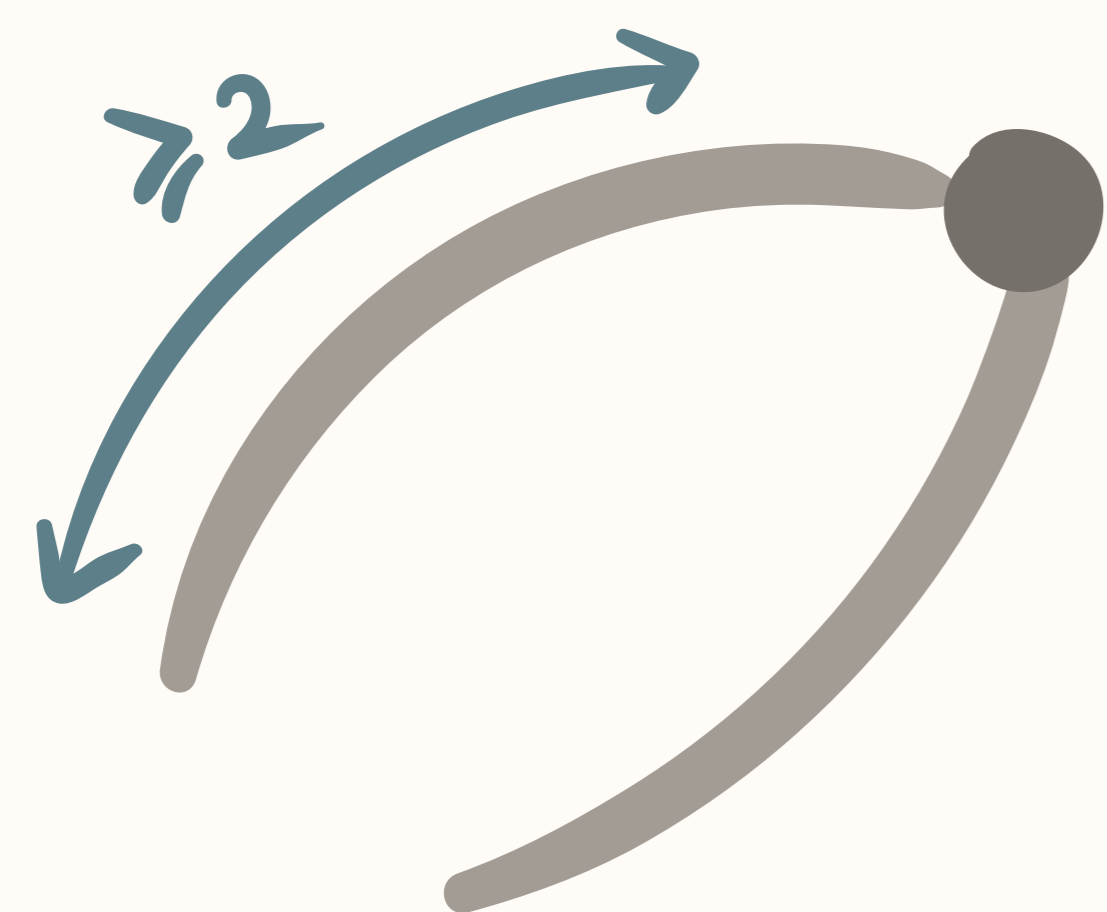
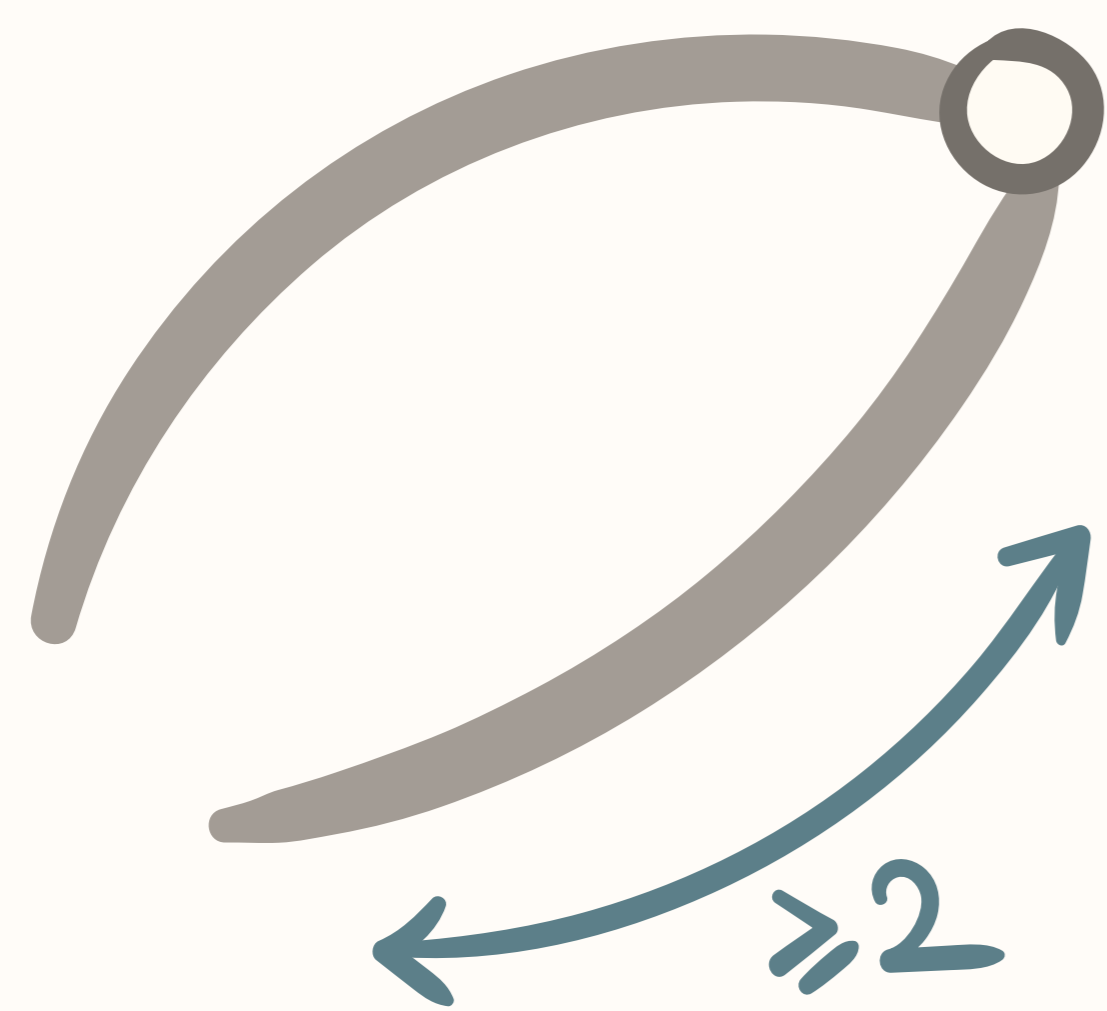
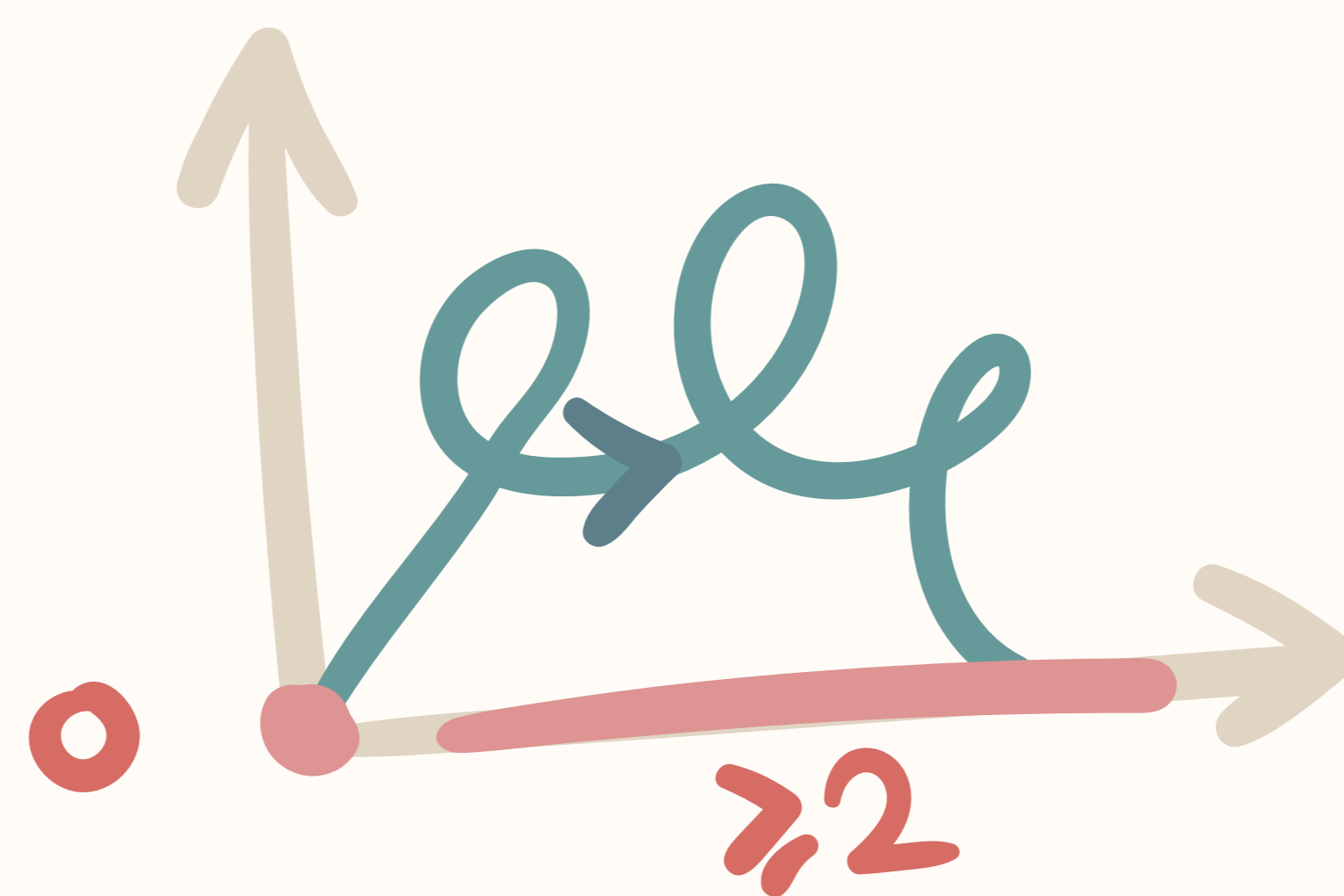
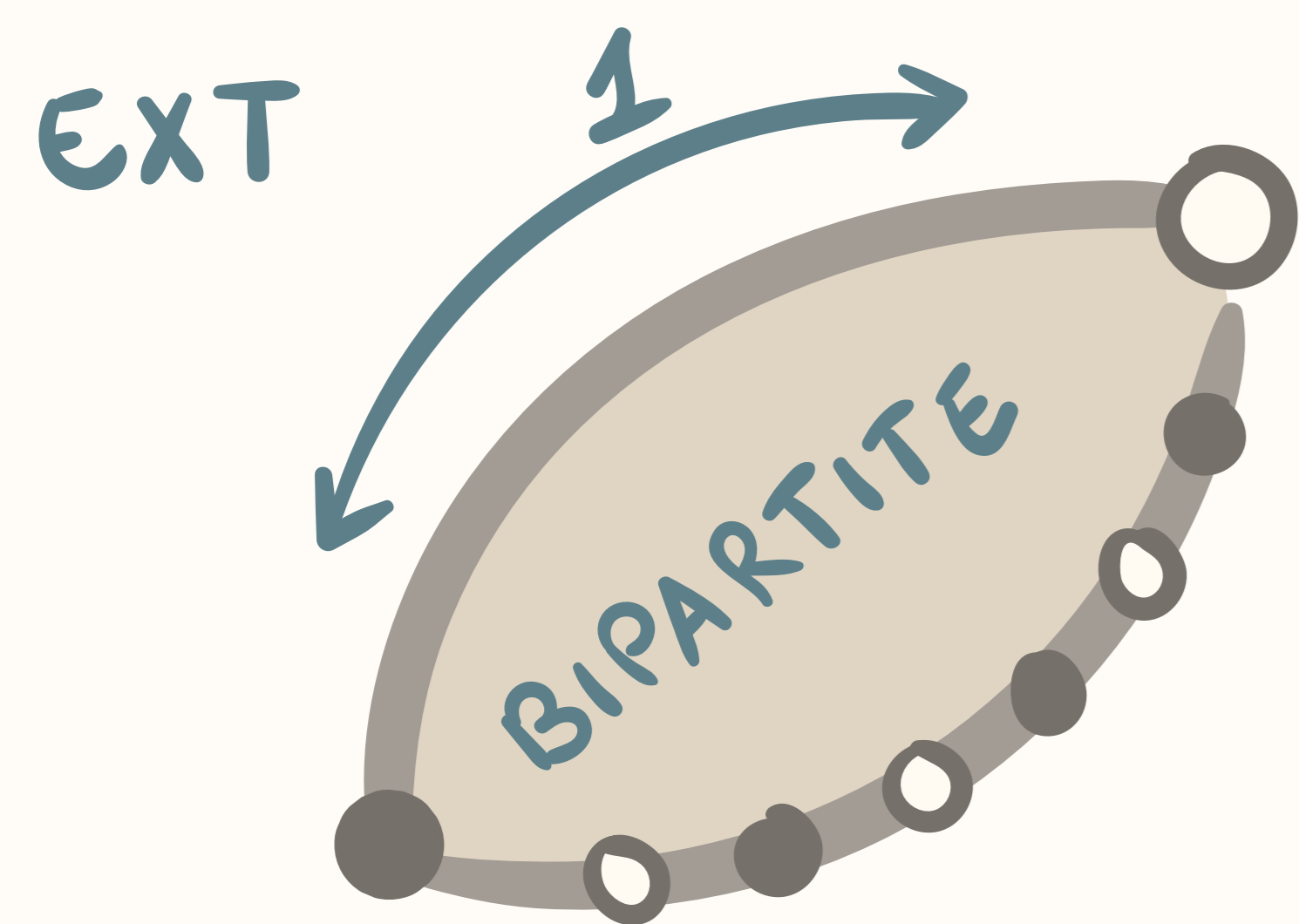
EXT



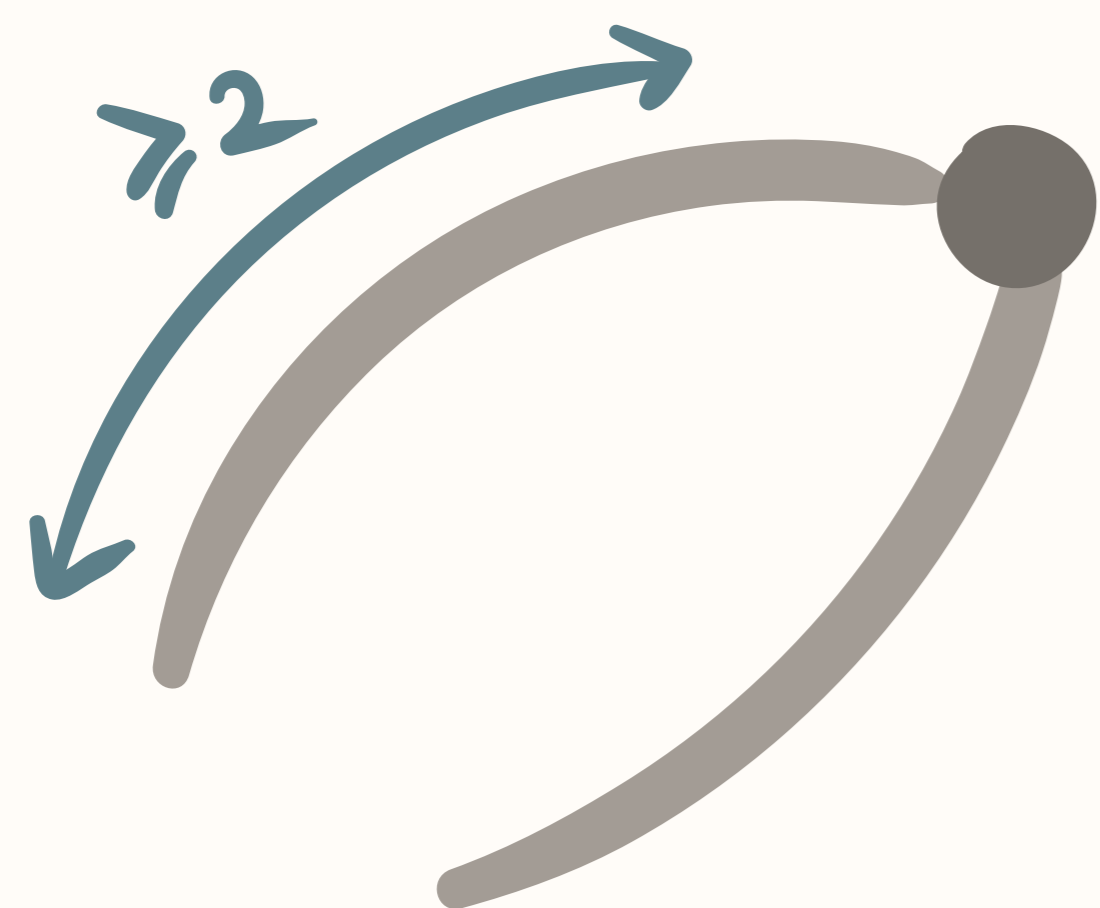
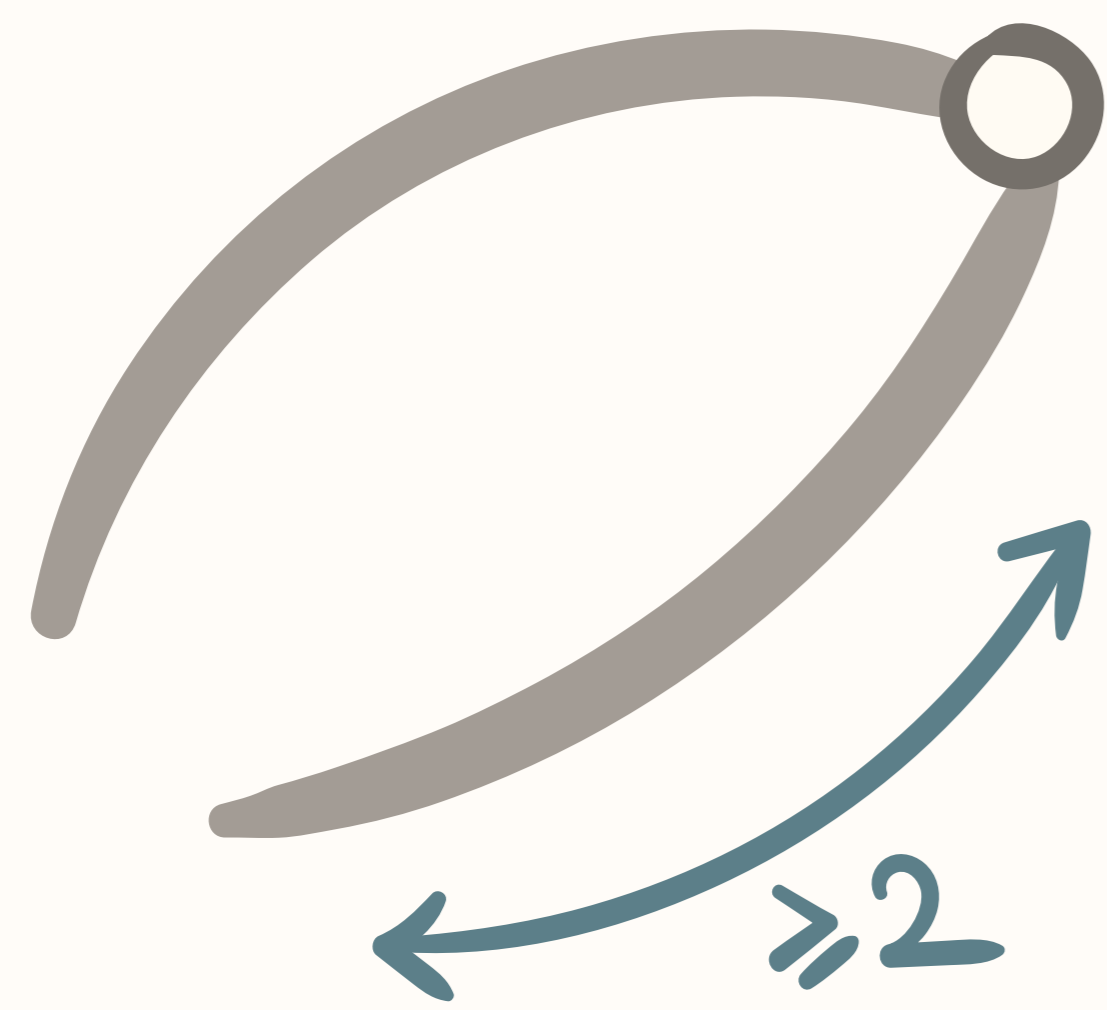
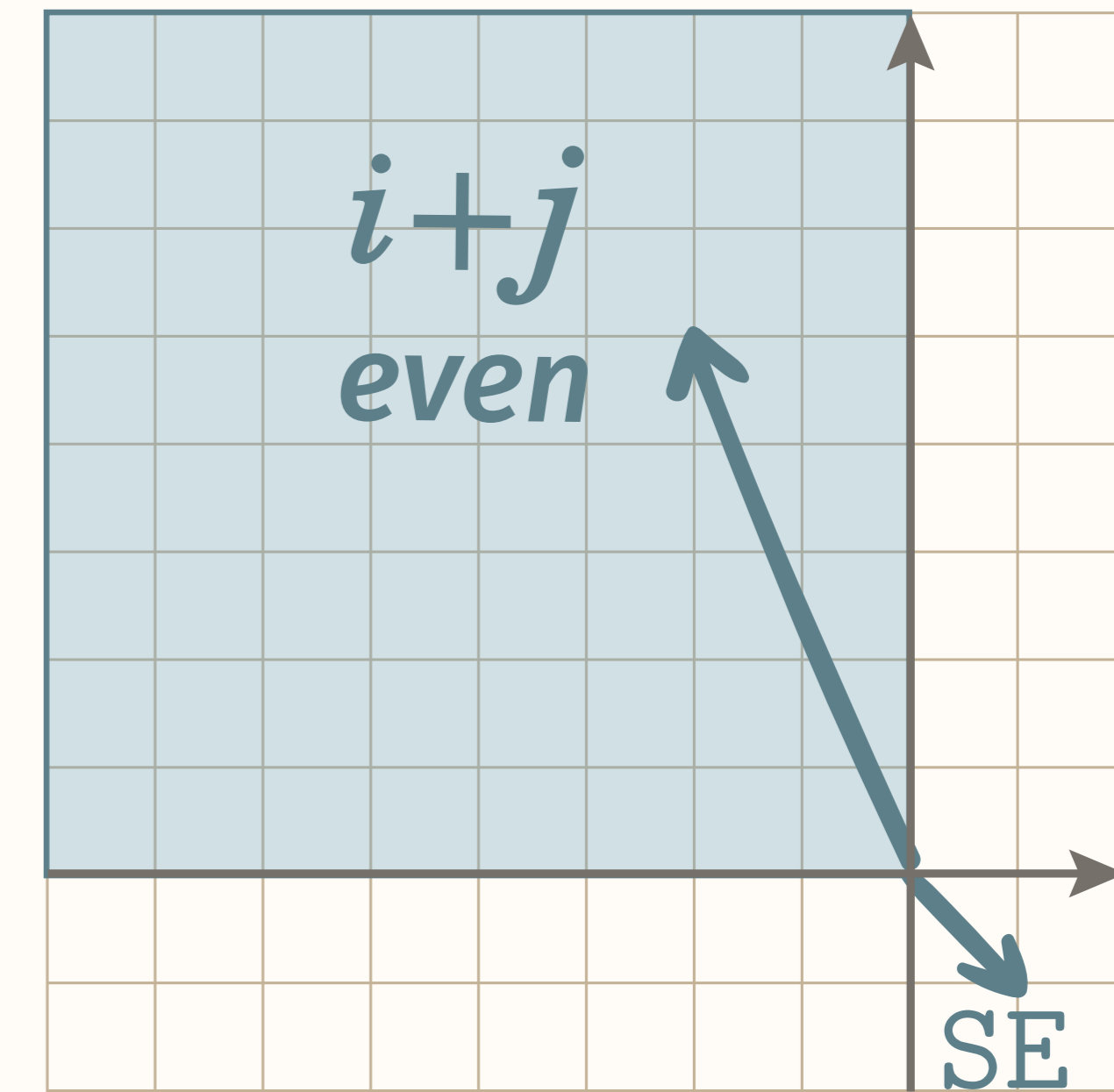
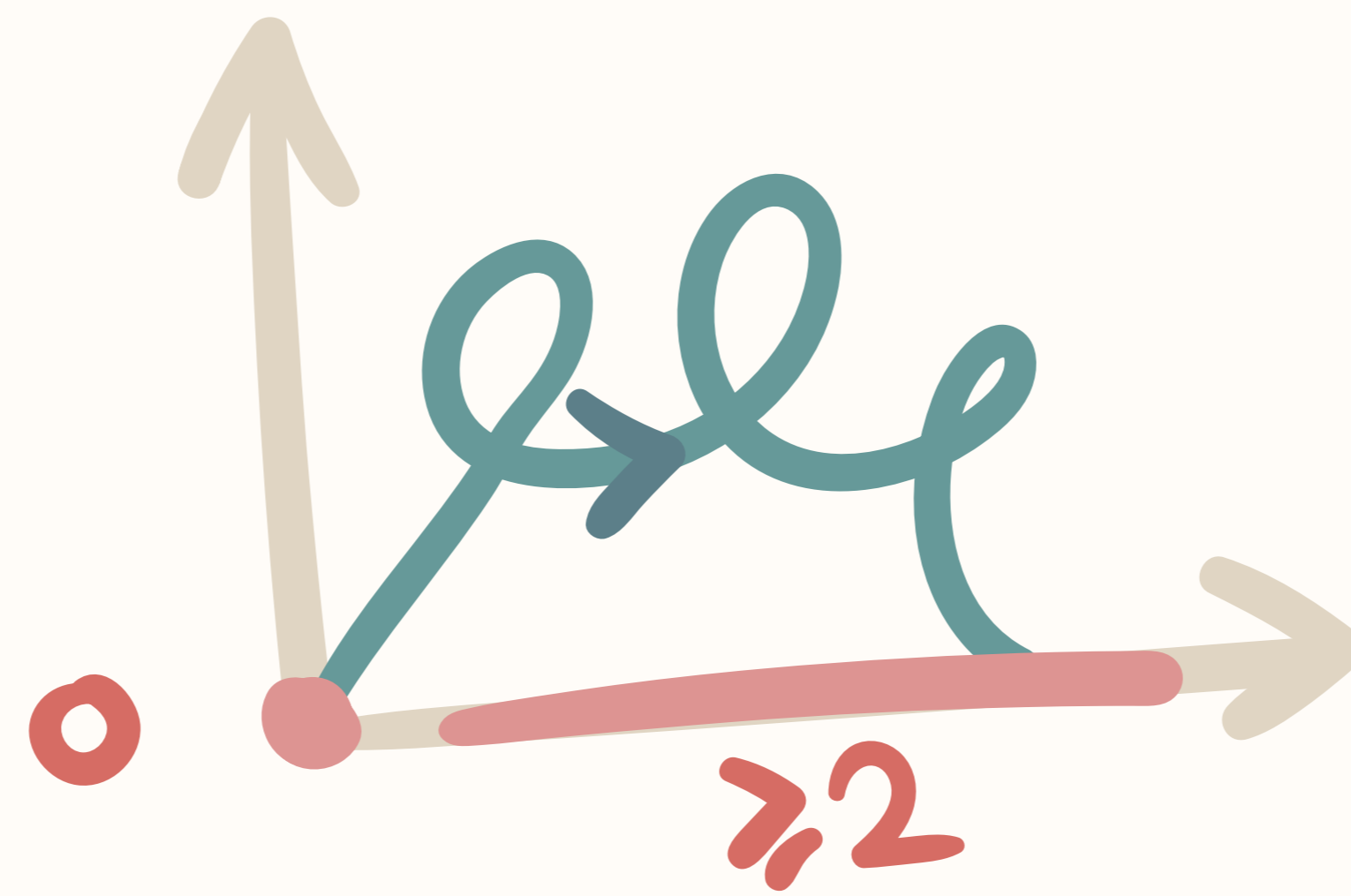
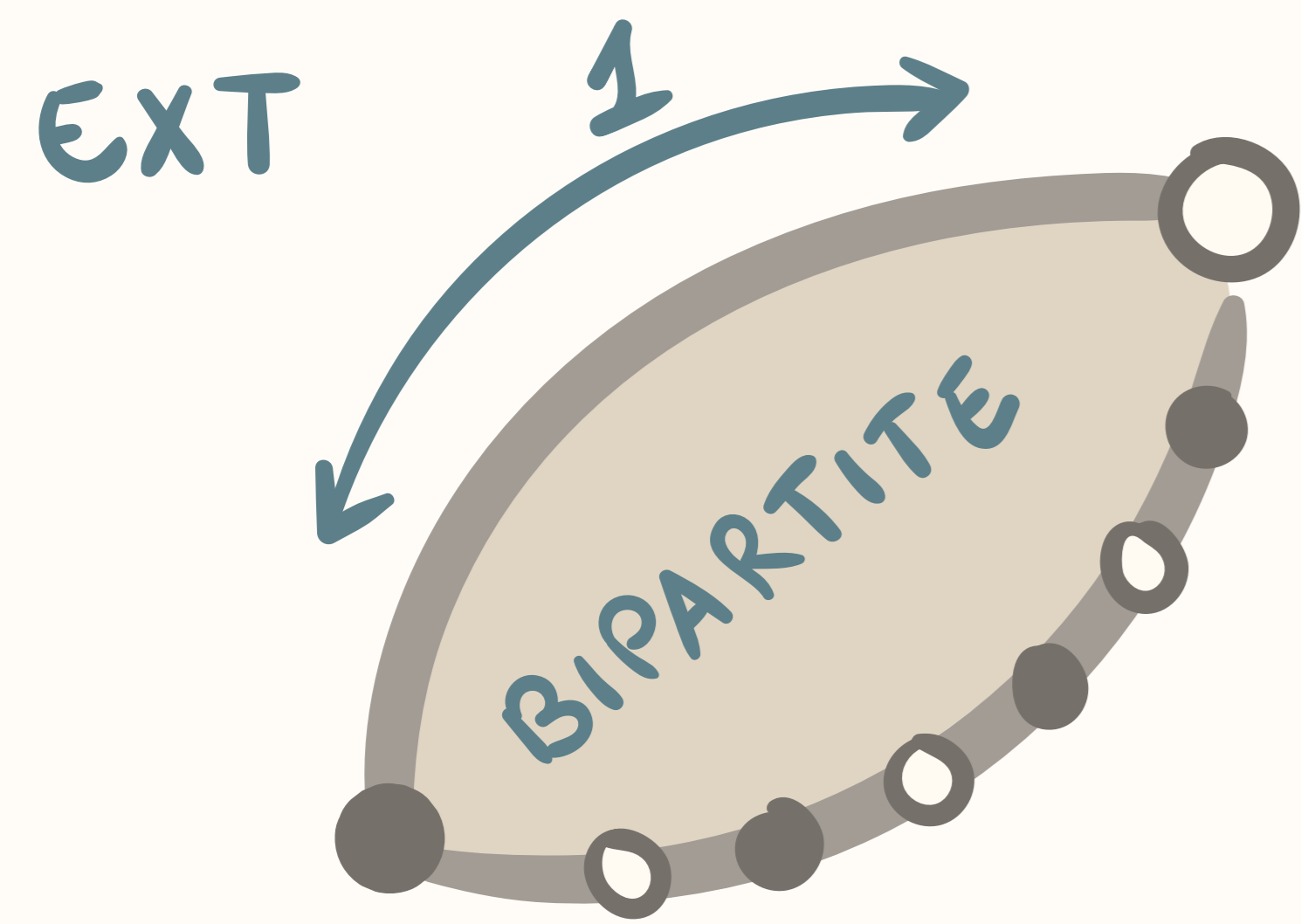
Polyhedral orientations via KMSW



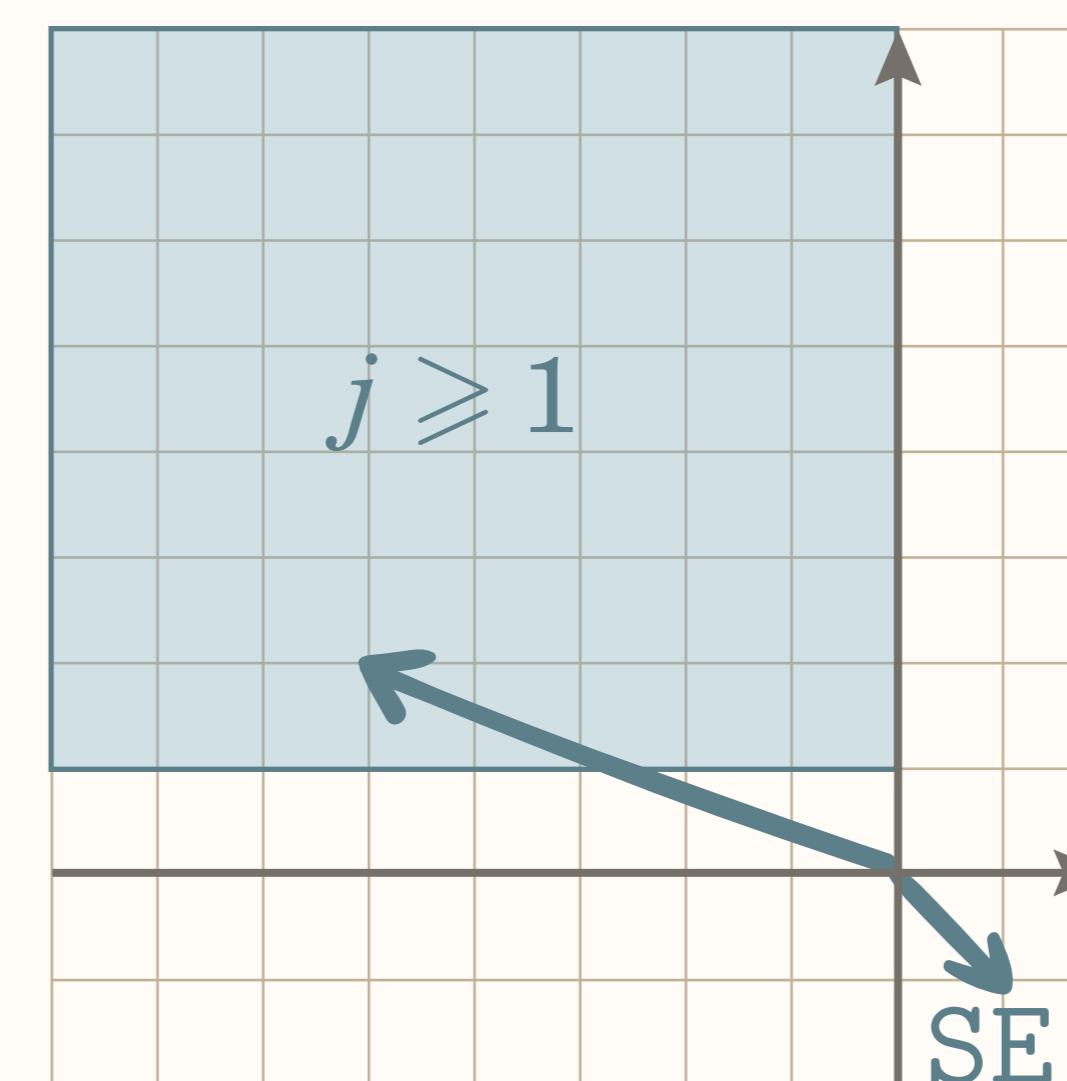
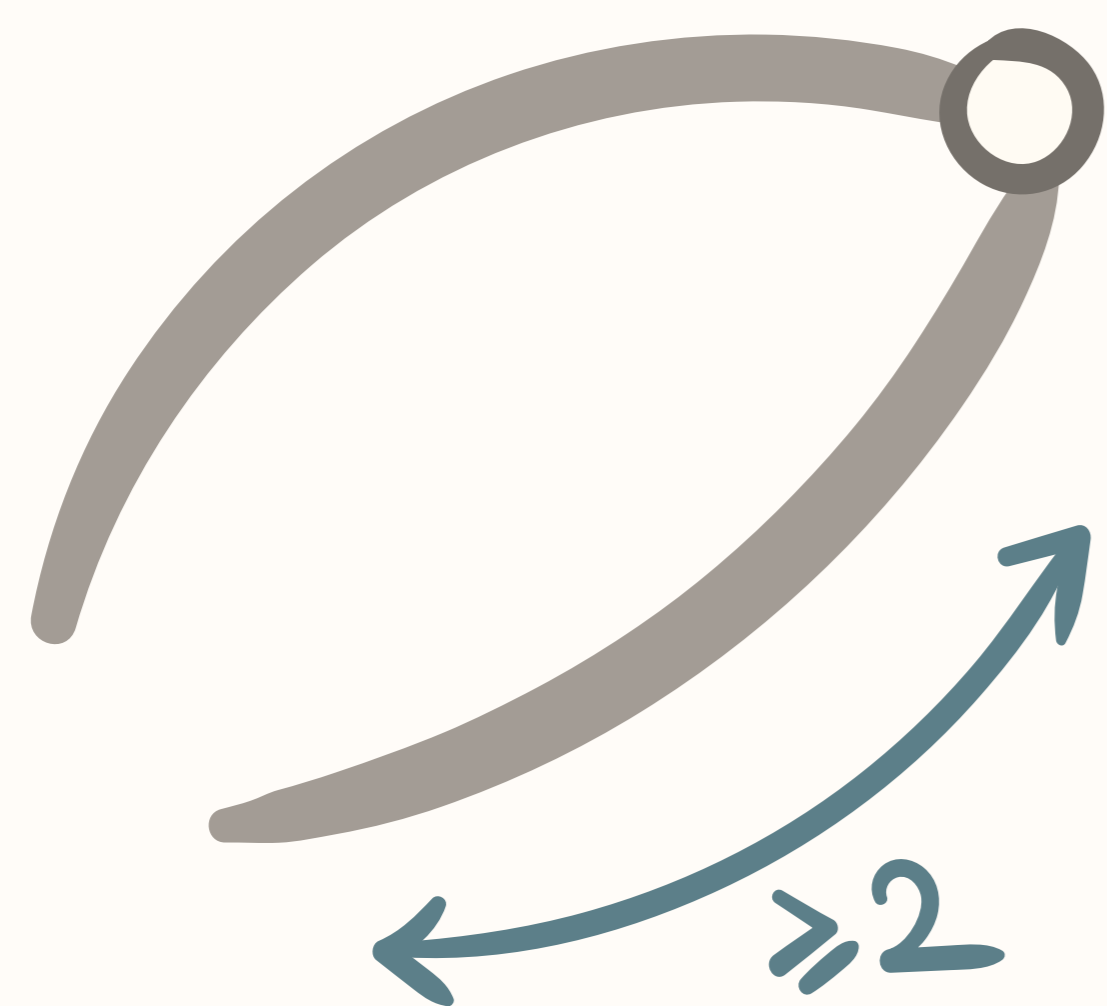
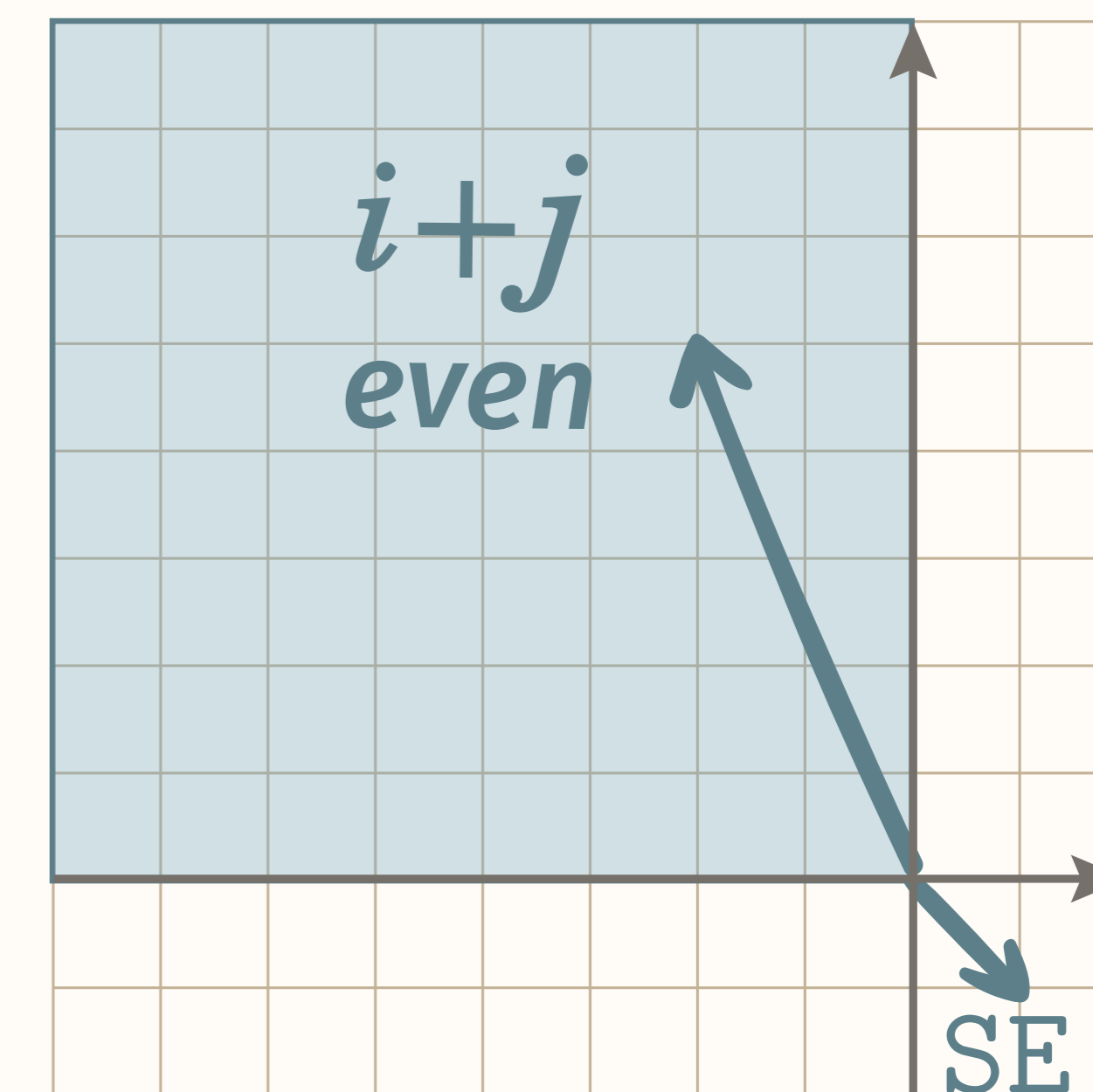
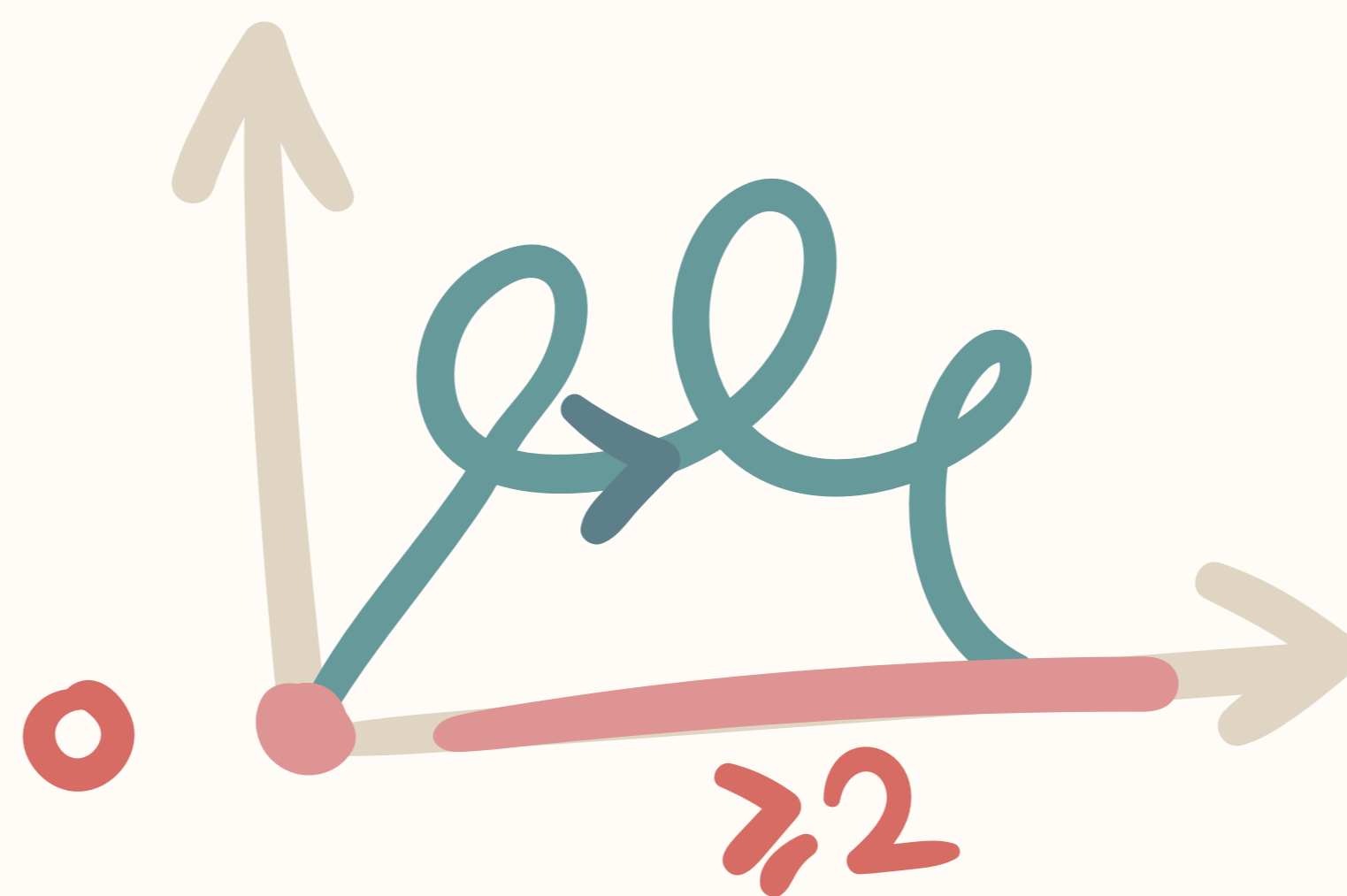
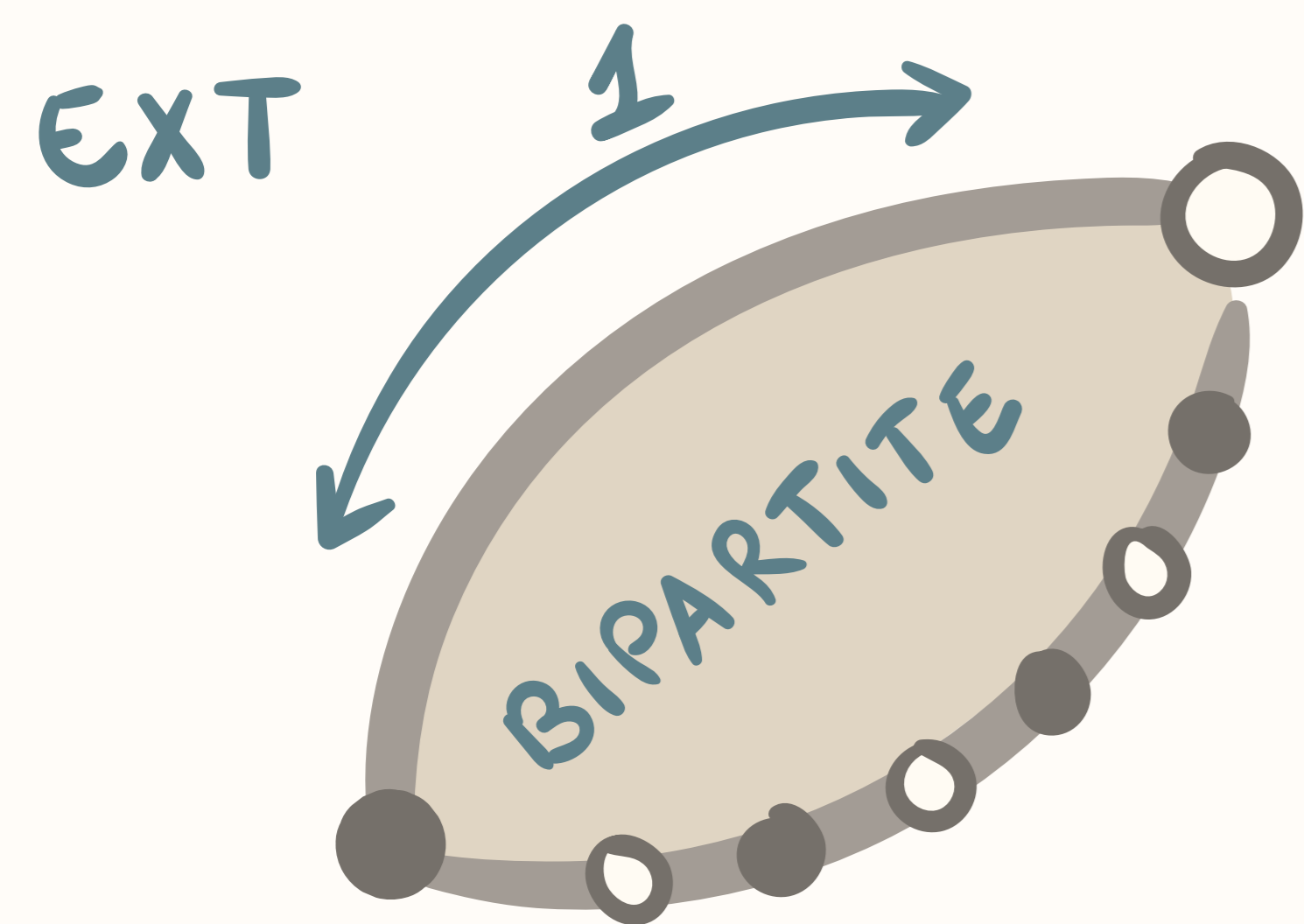
Polyhedral orientations via KMSW



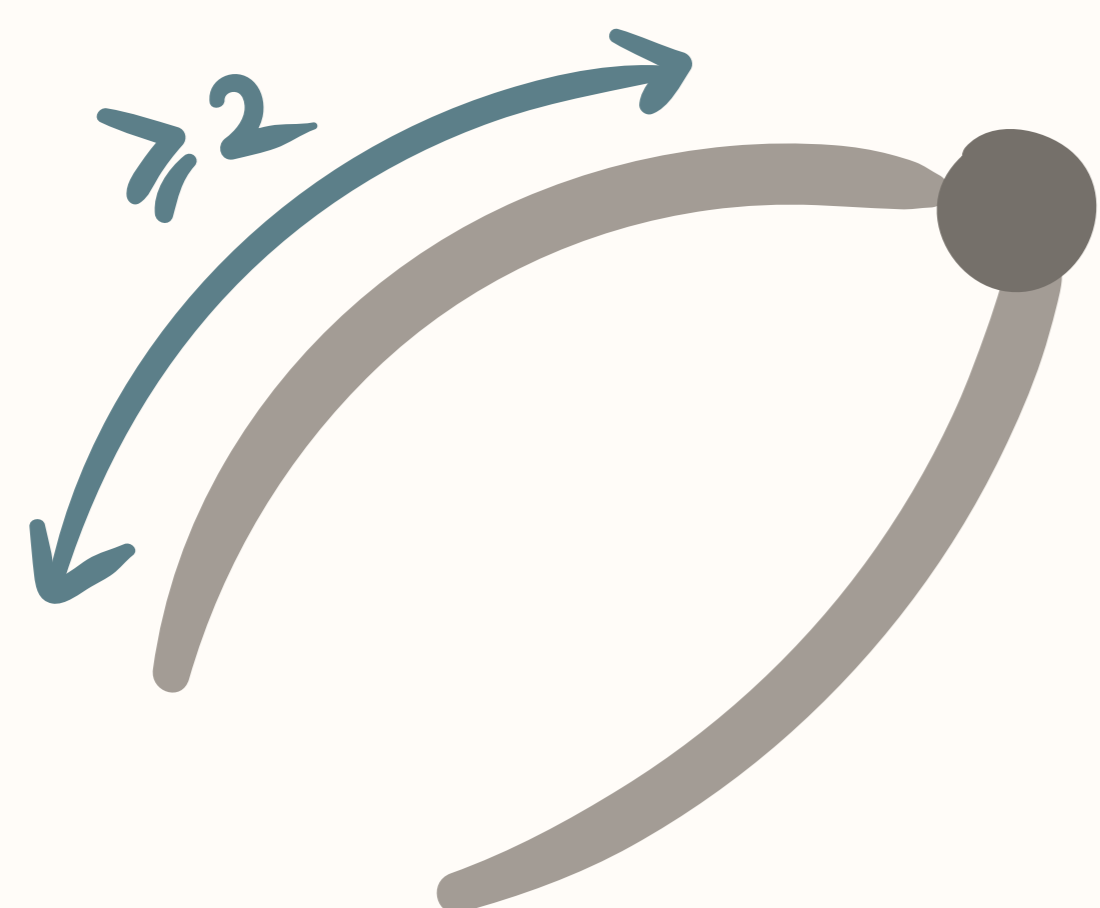
Polyhedral orientations via KMSW



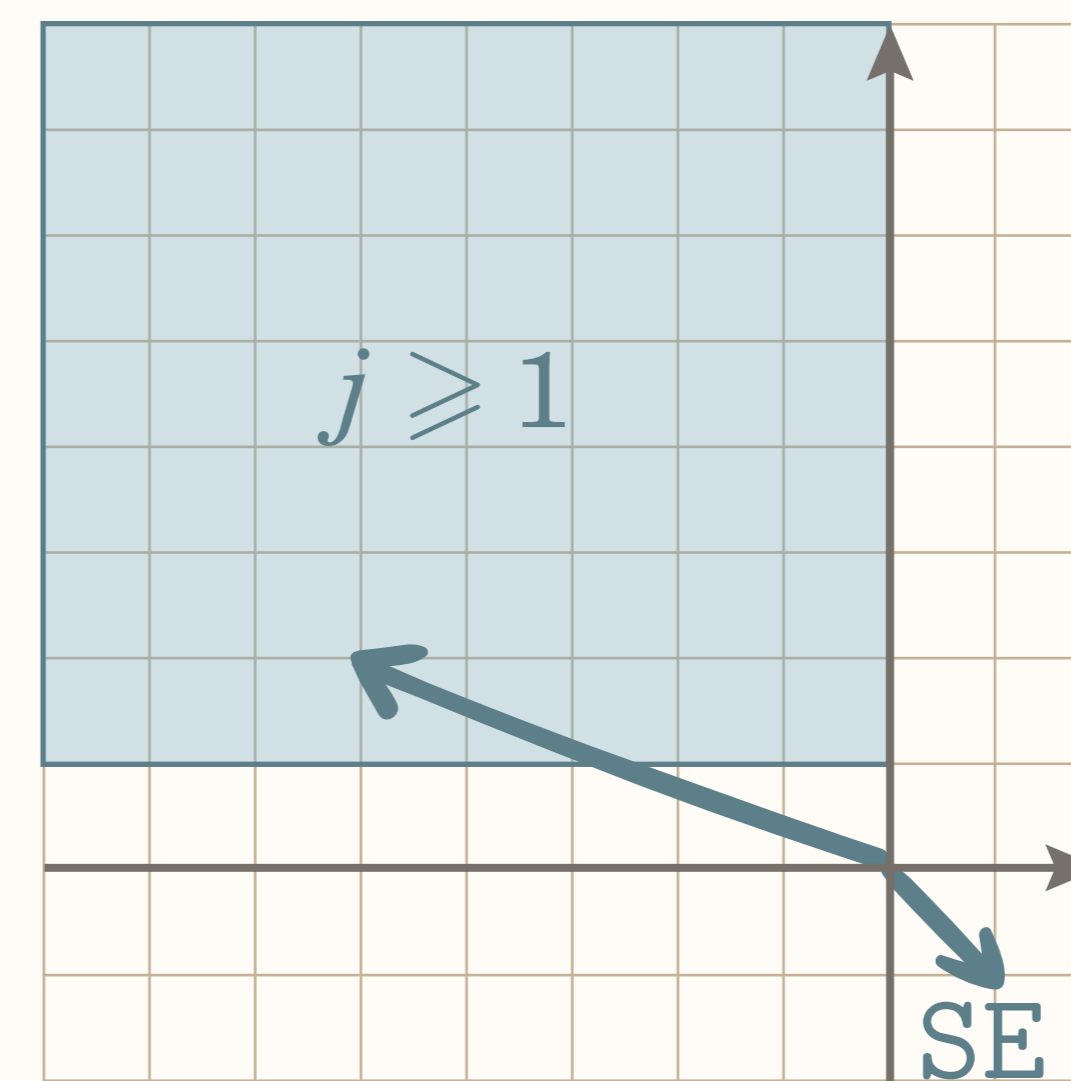
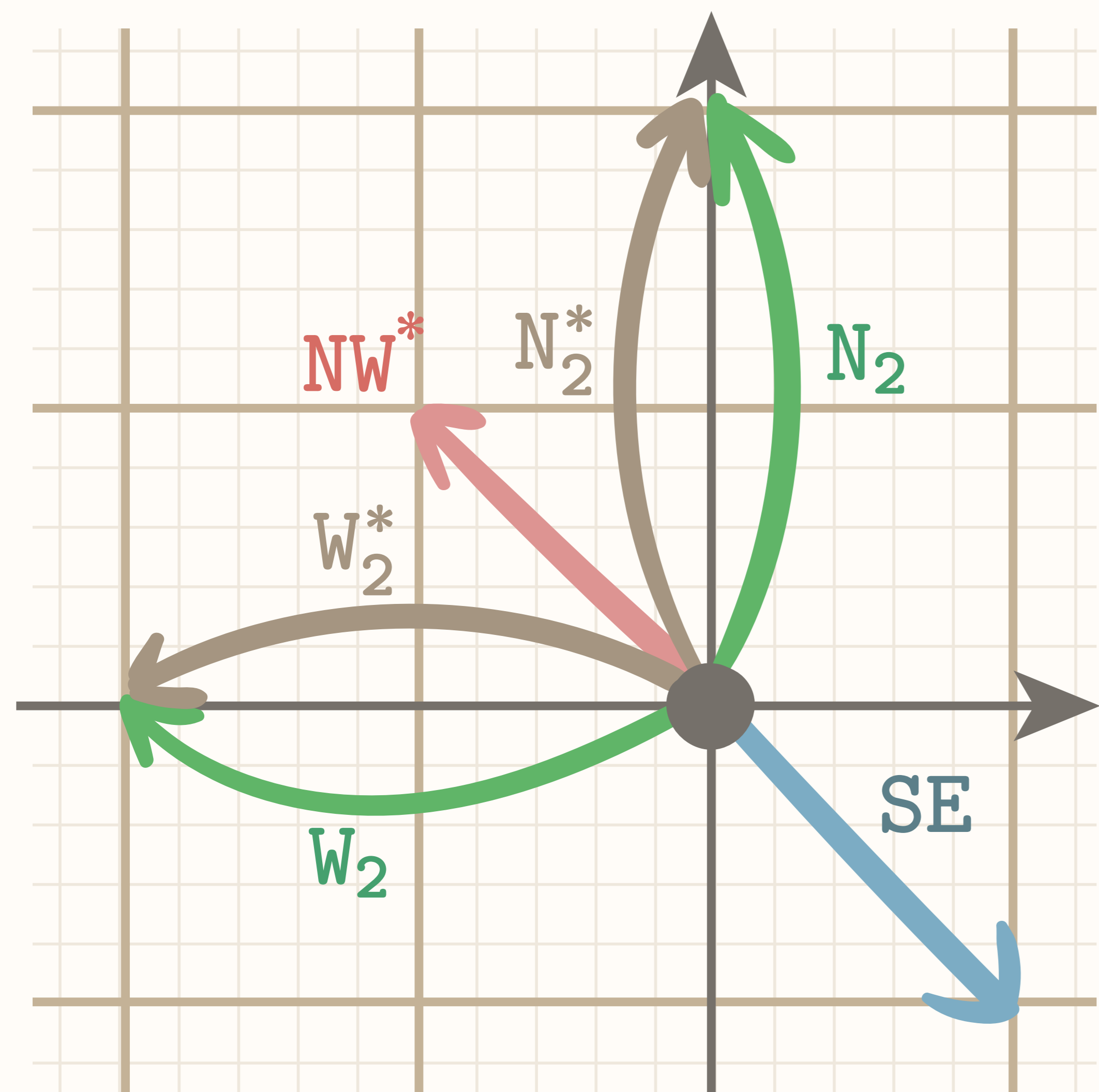
Polyhedral orientations via KMSW



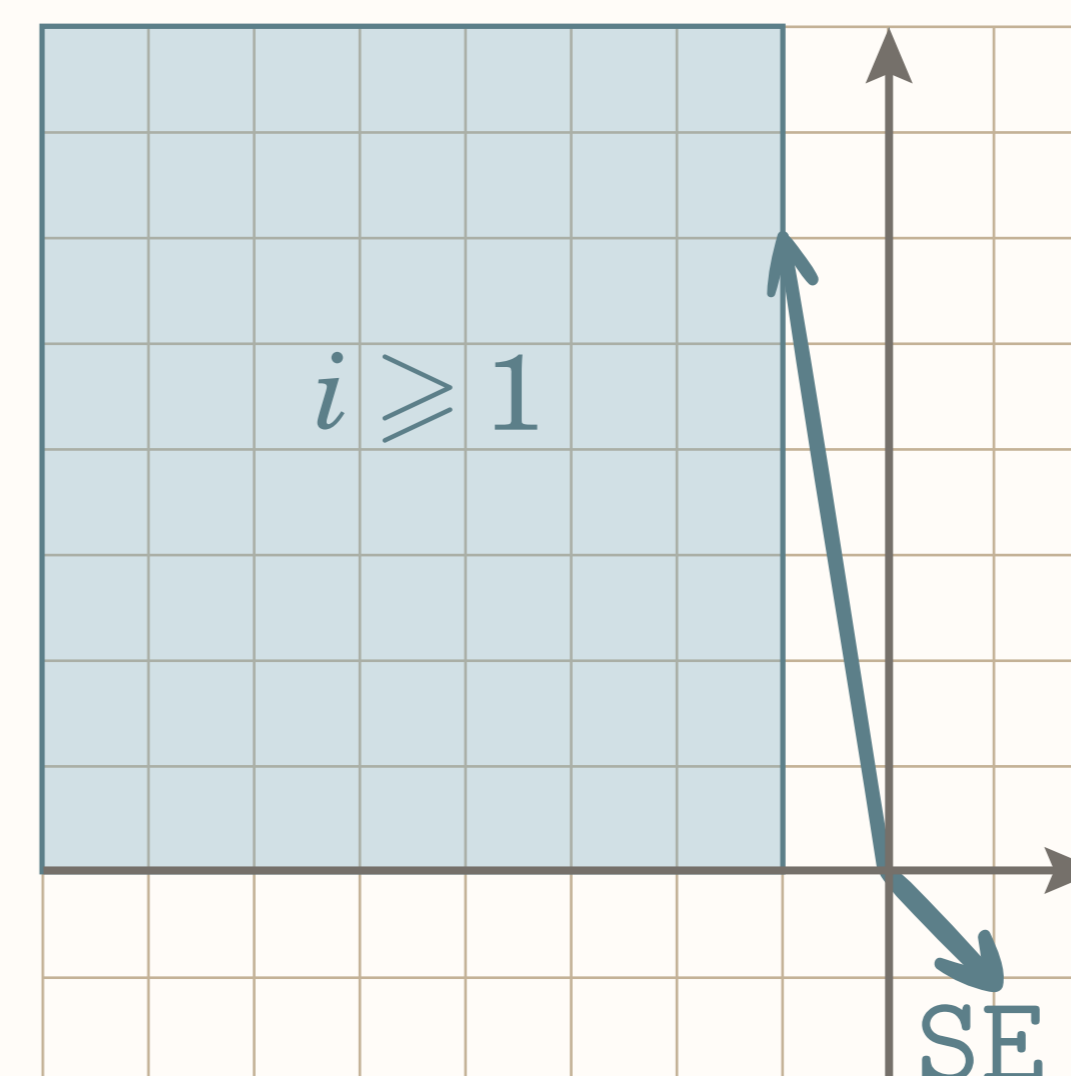
when x, y
are even



Polyhedral orientations via KMSW

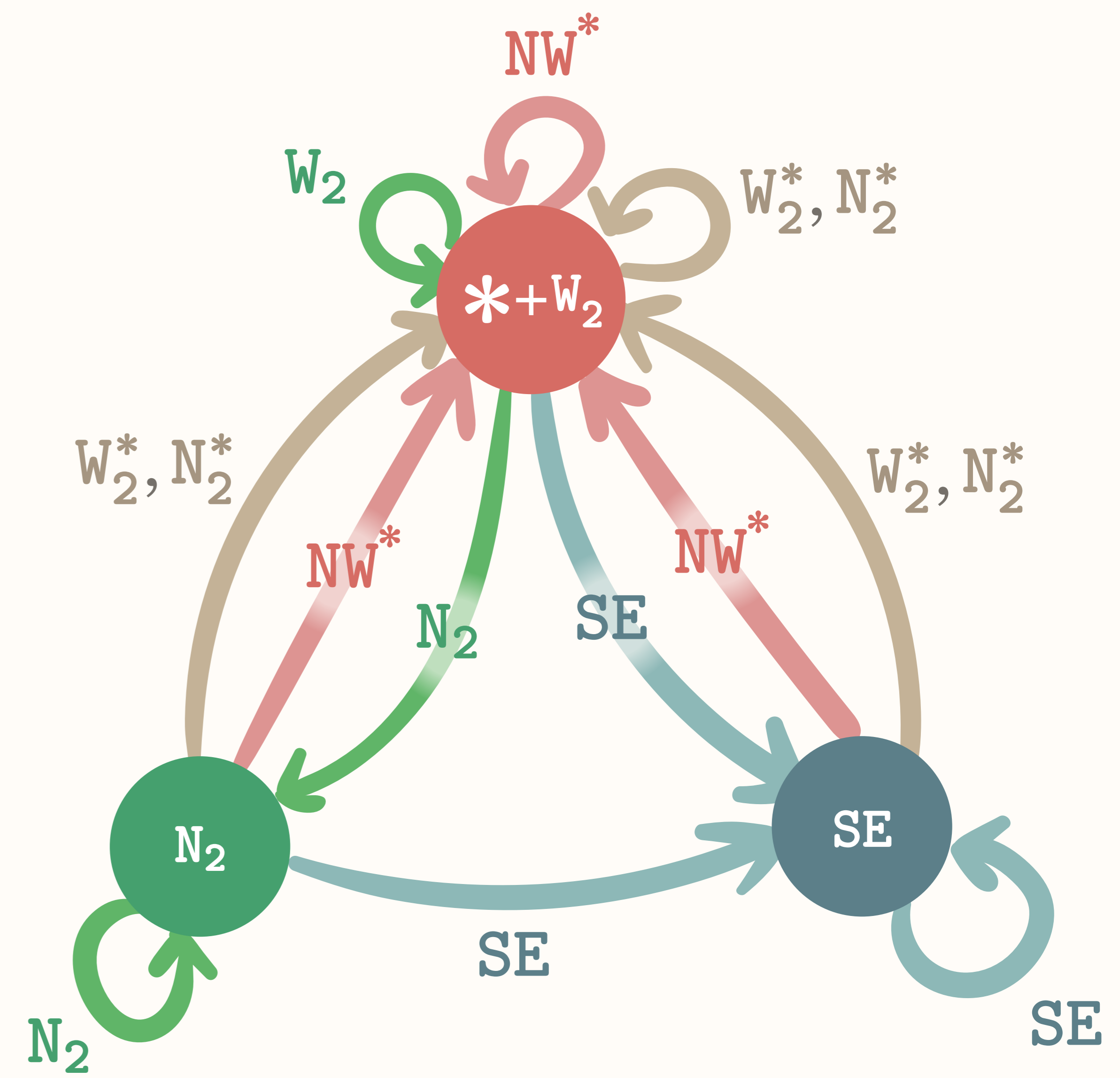
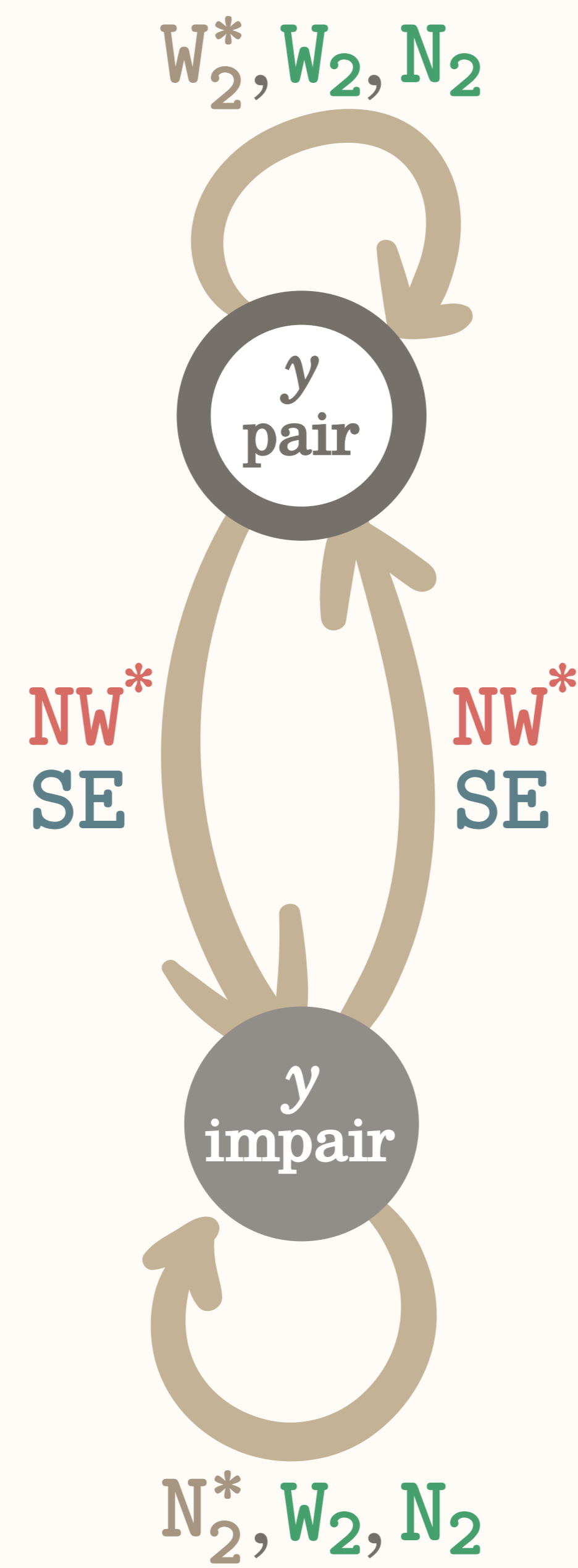
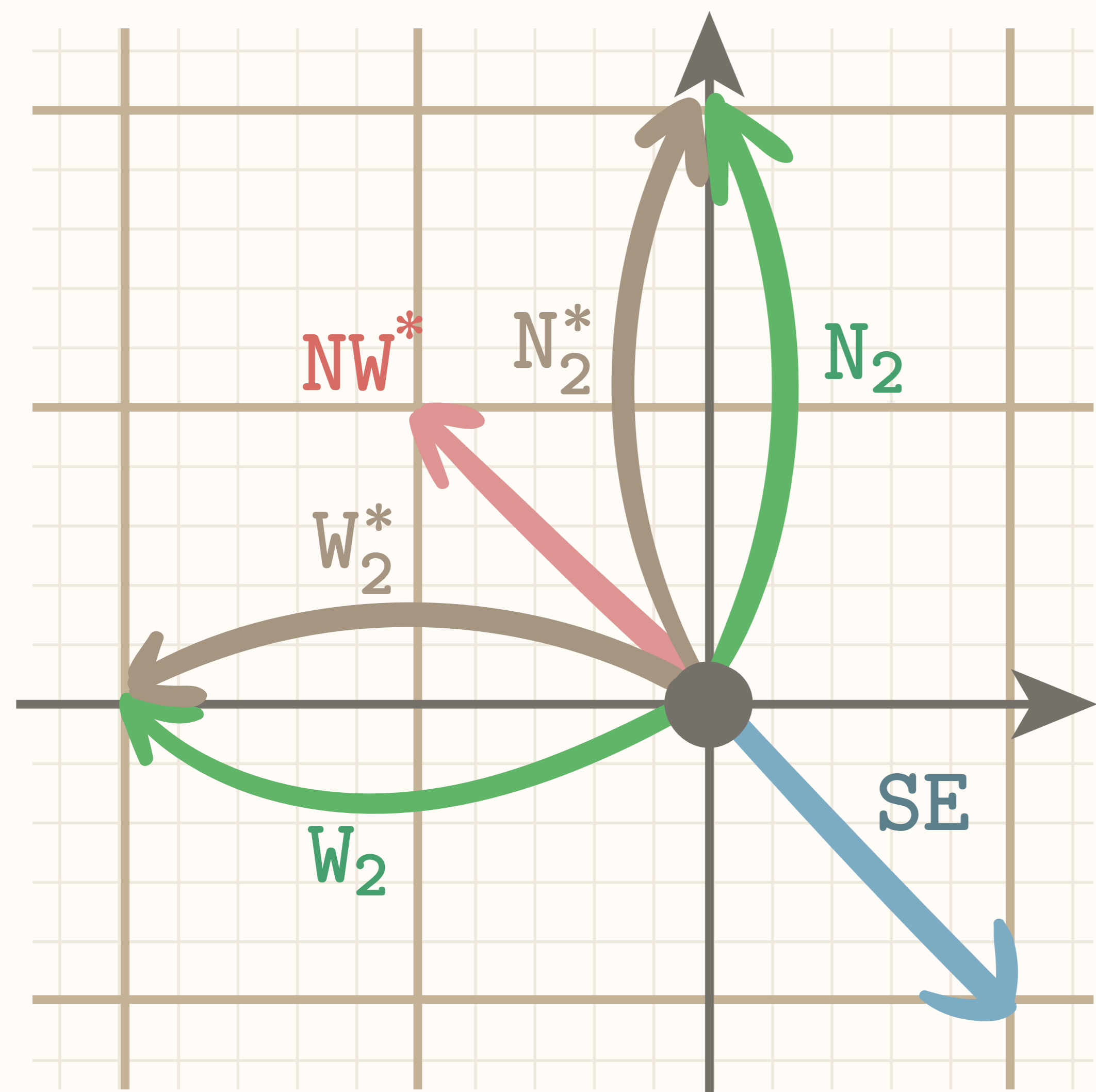


*when x, y
are even*

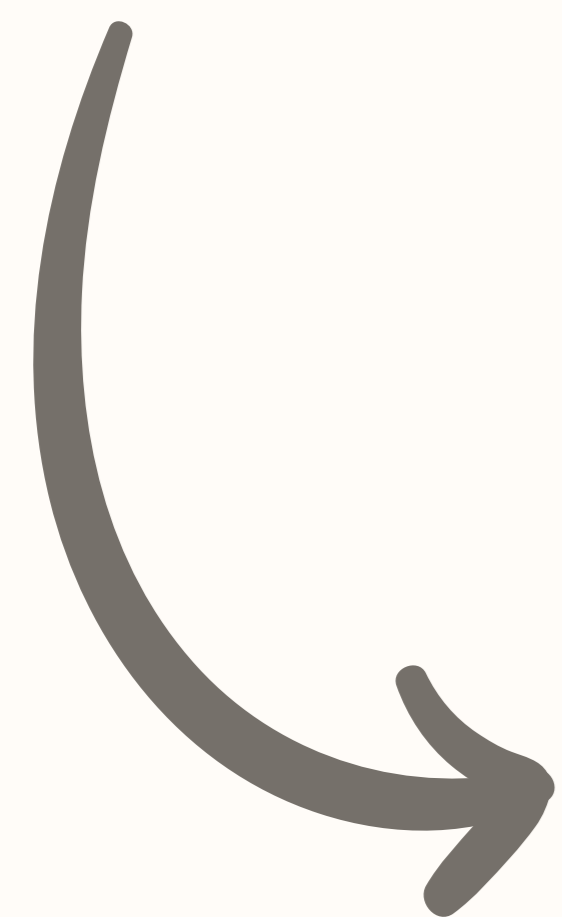
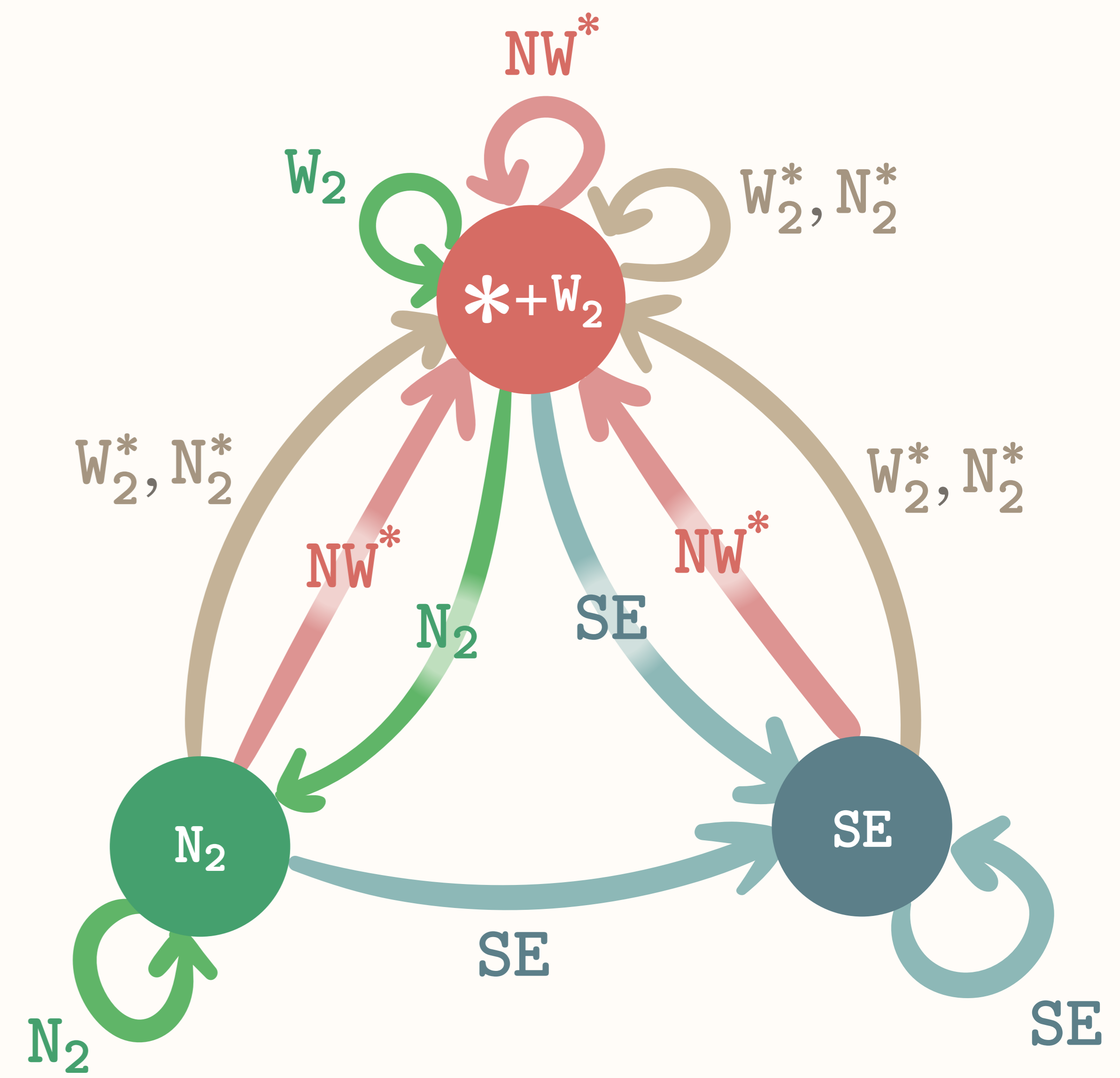
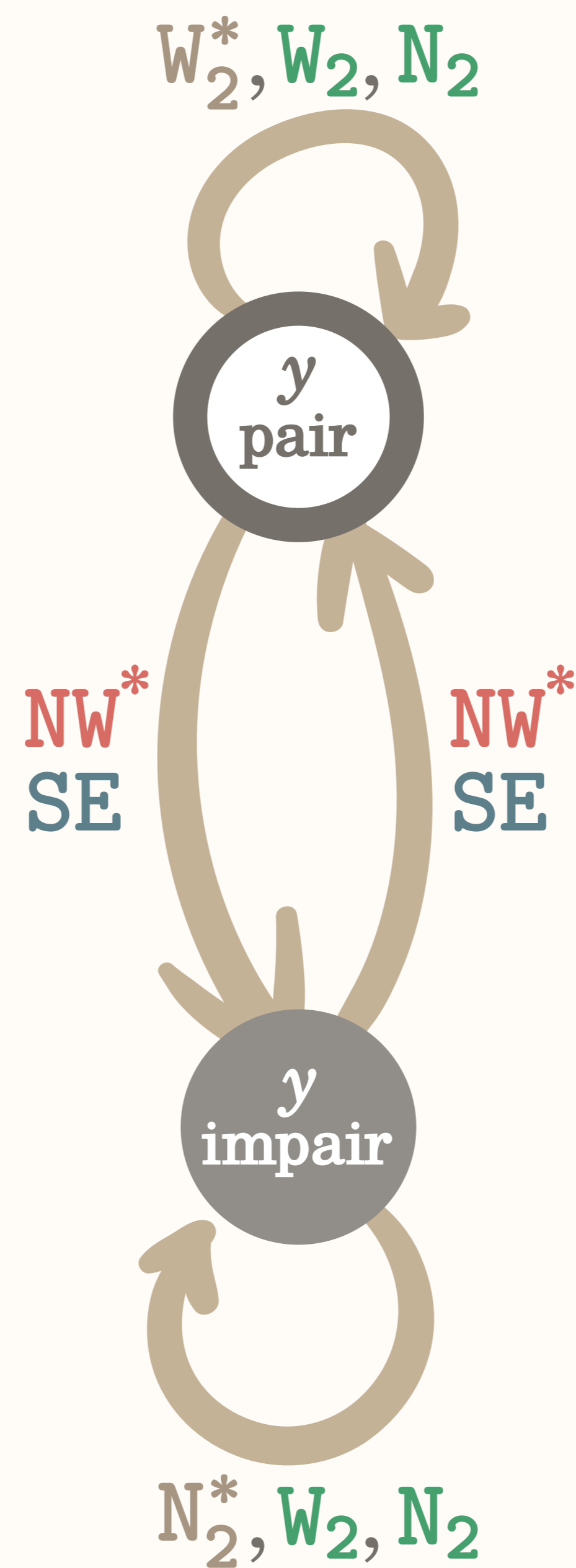
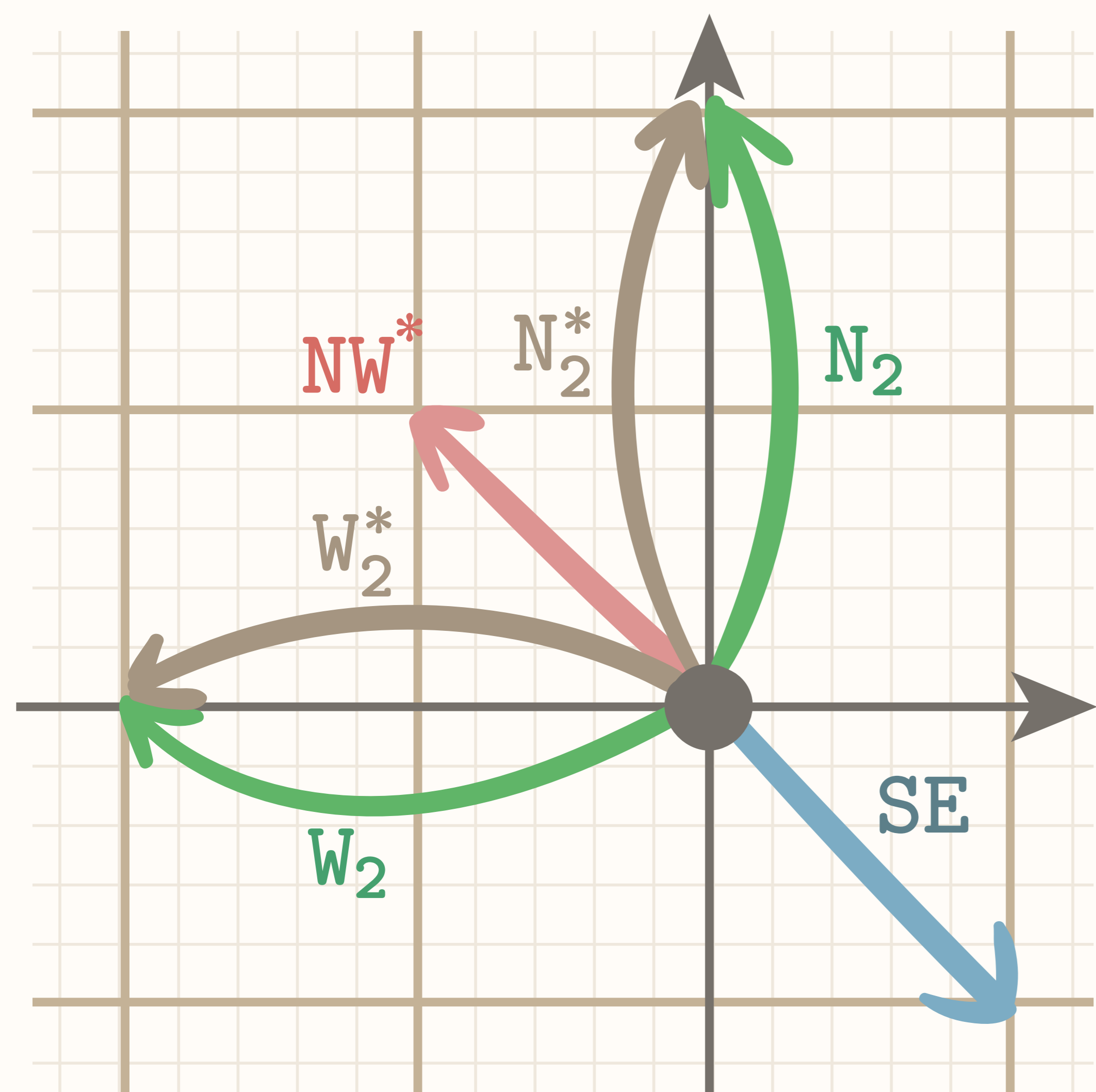


*when x, y
are odd*

Polyhedral orientations via KMSW

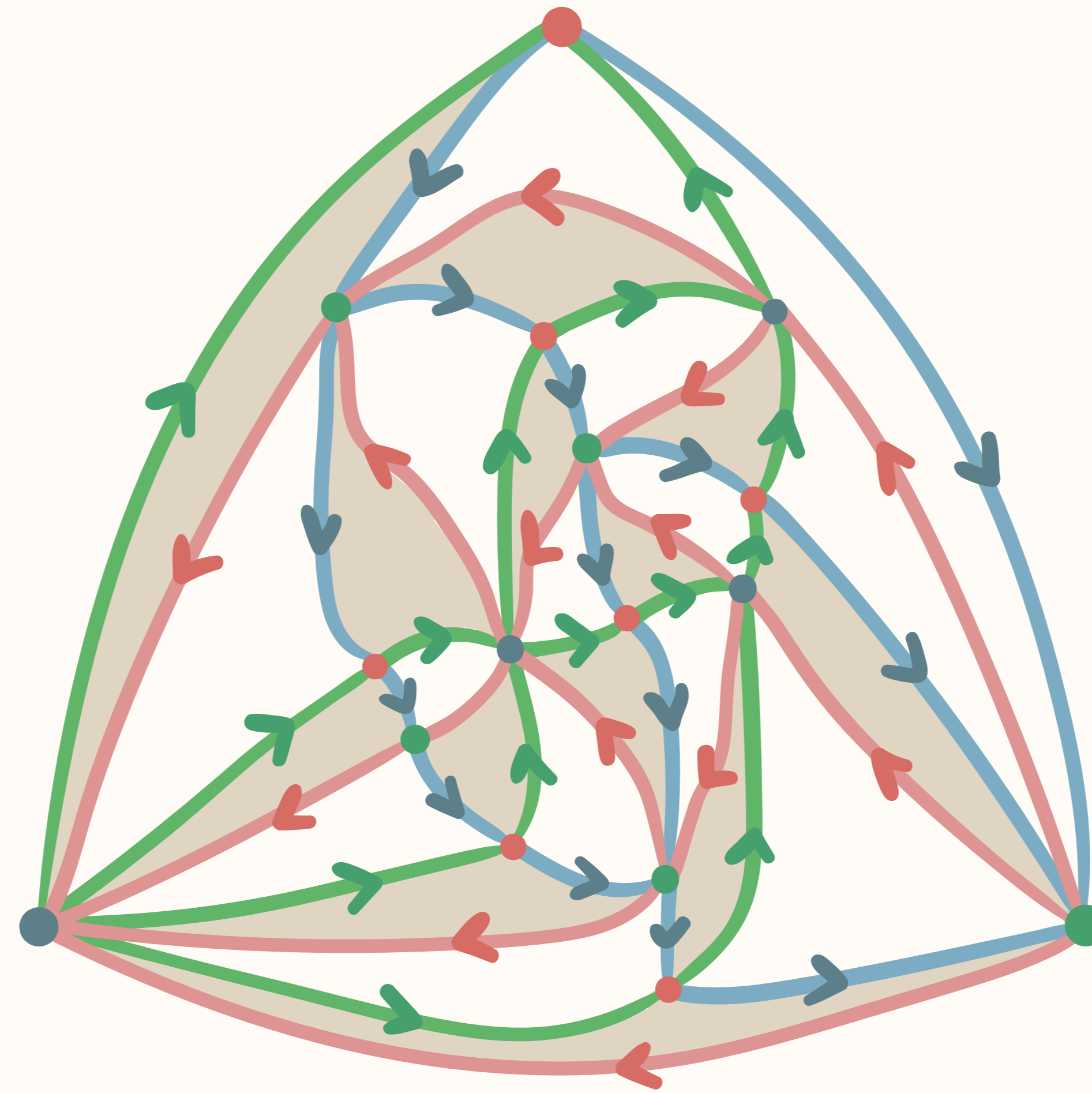
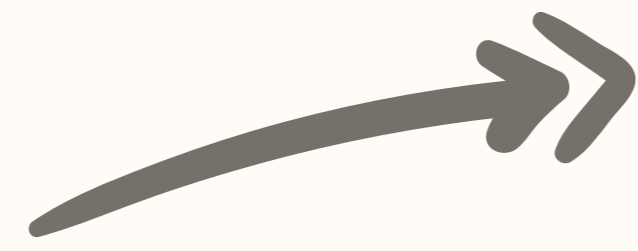
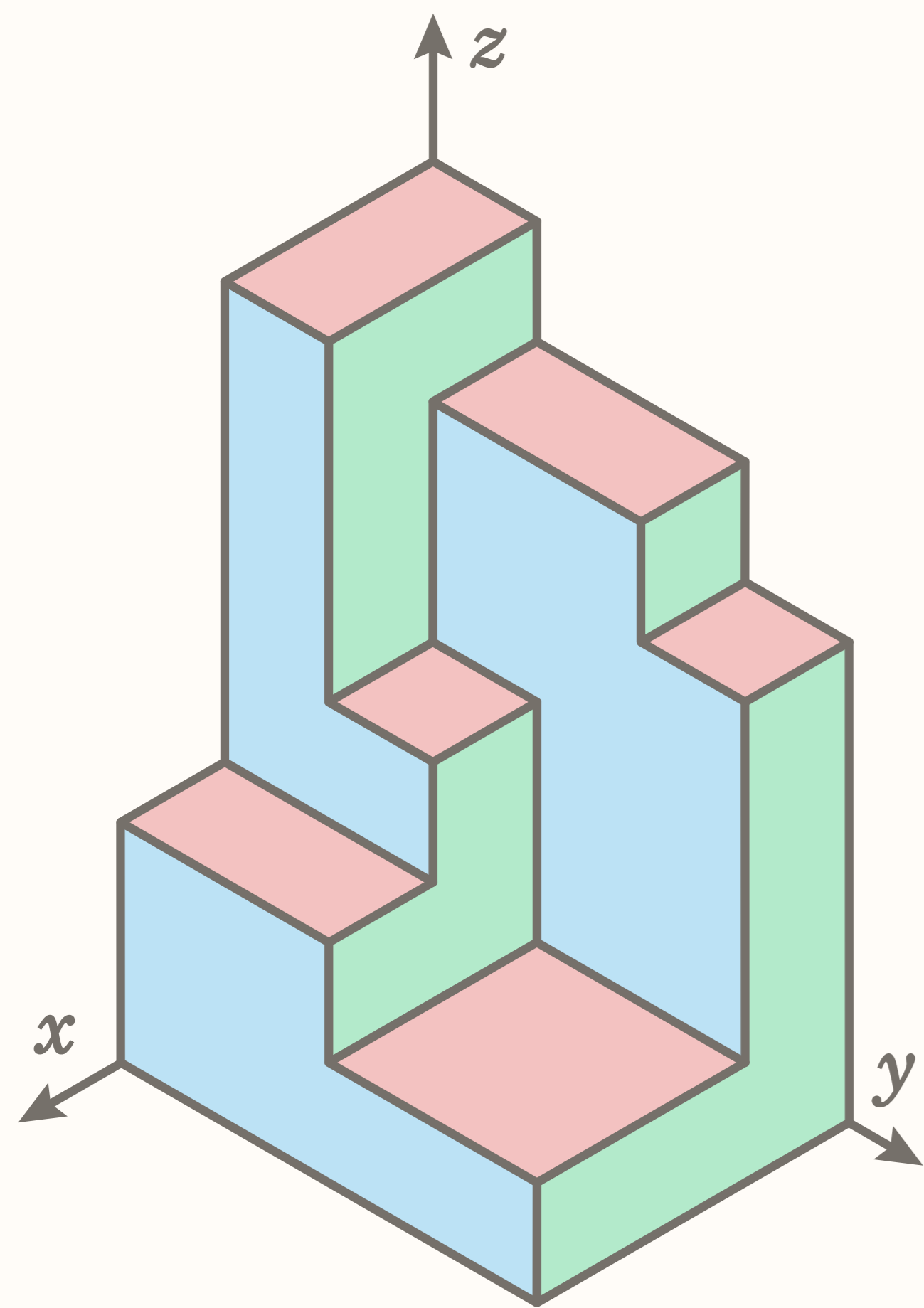


Polyhedral orientations via KMSW

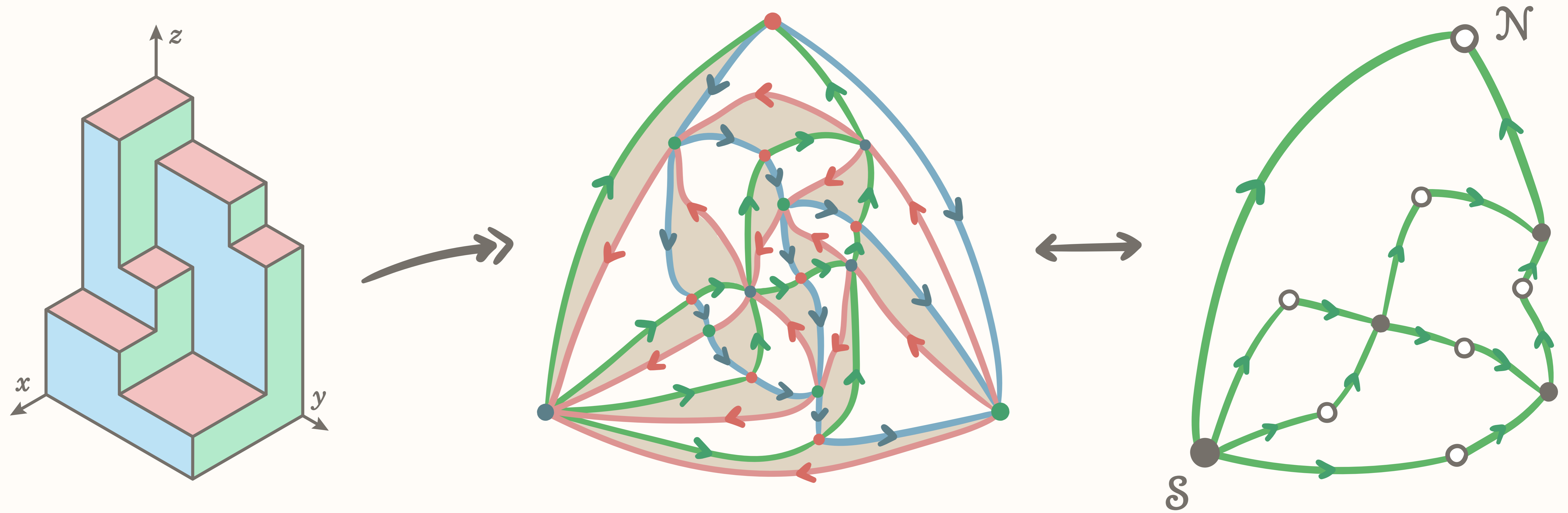


$$t^3 + 3t^5 + 4t^6 + 15t^7 + 39t^8 + 122t^9 + 375t^{10} + 1212t^{11} + 3980t^{12} + O(t^{13})$$

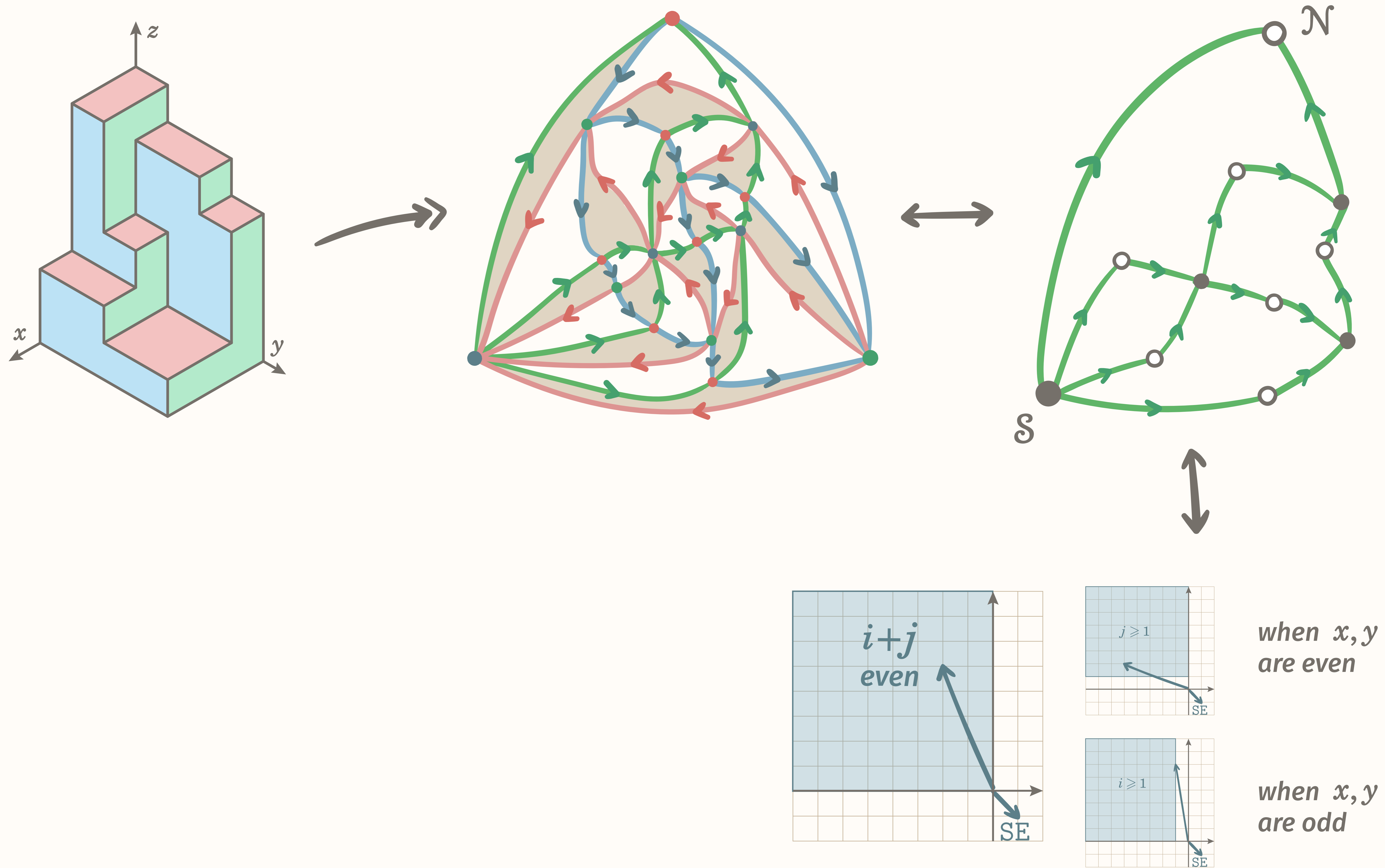
Recap polyhedral orientations



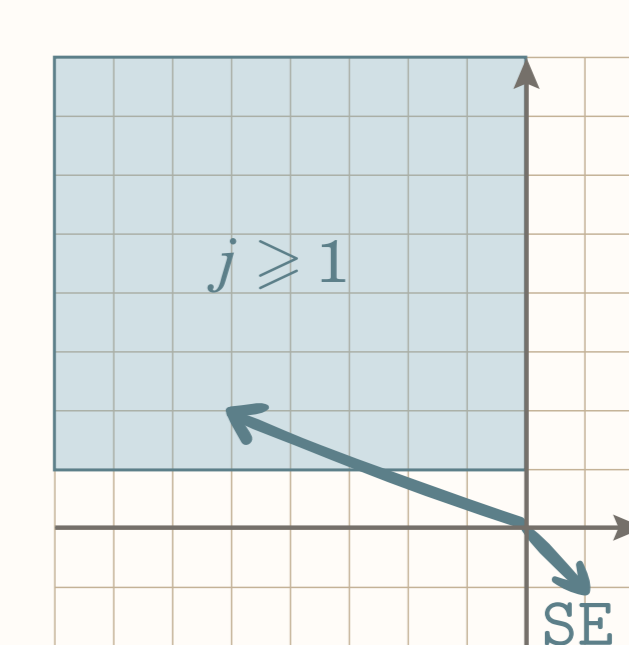
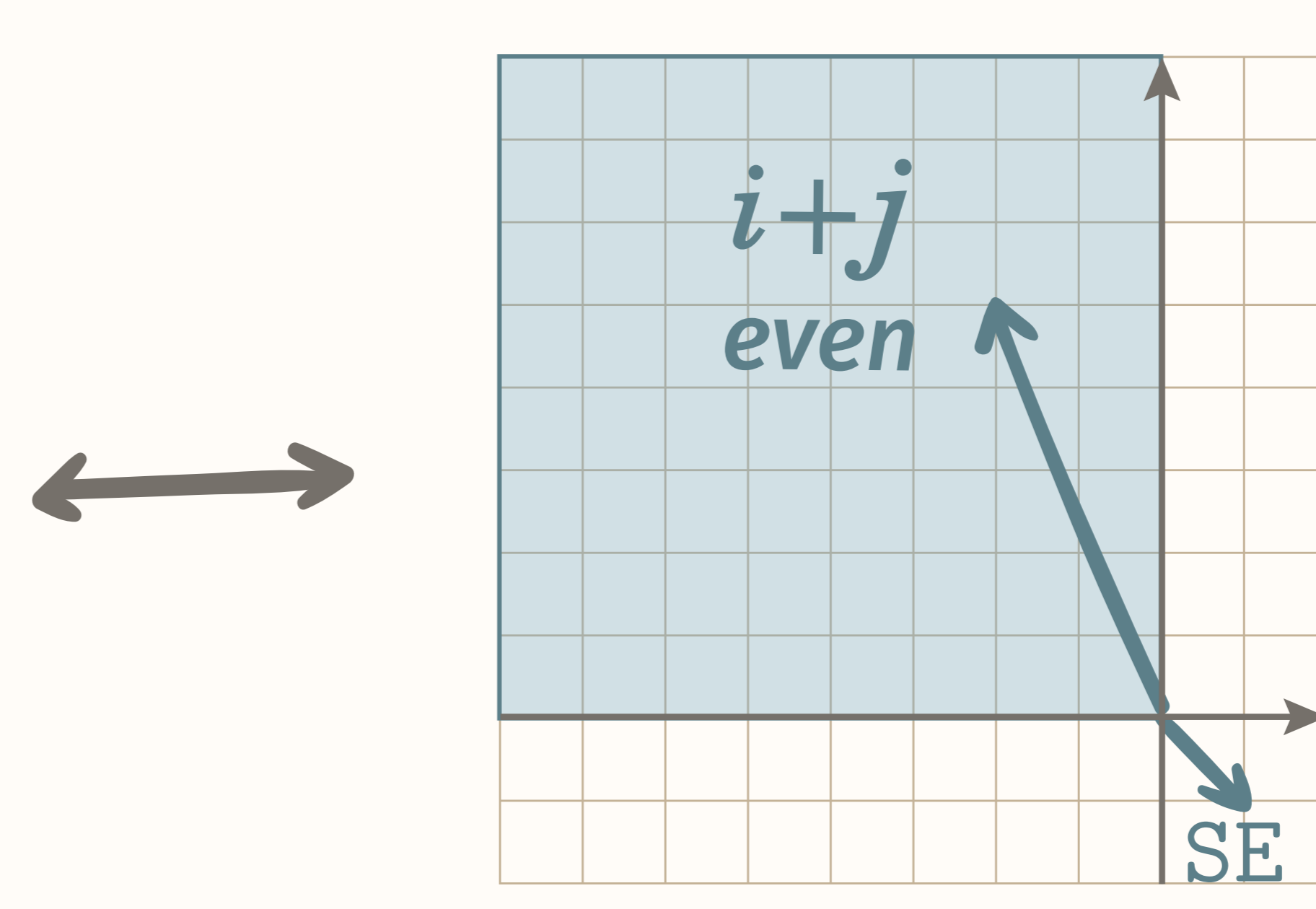
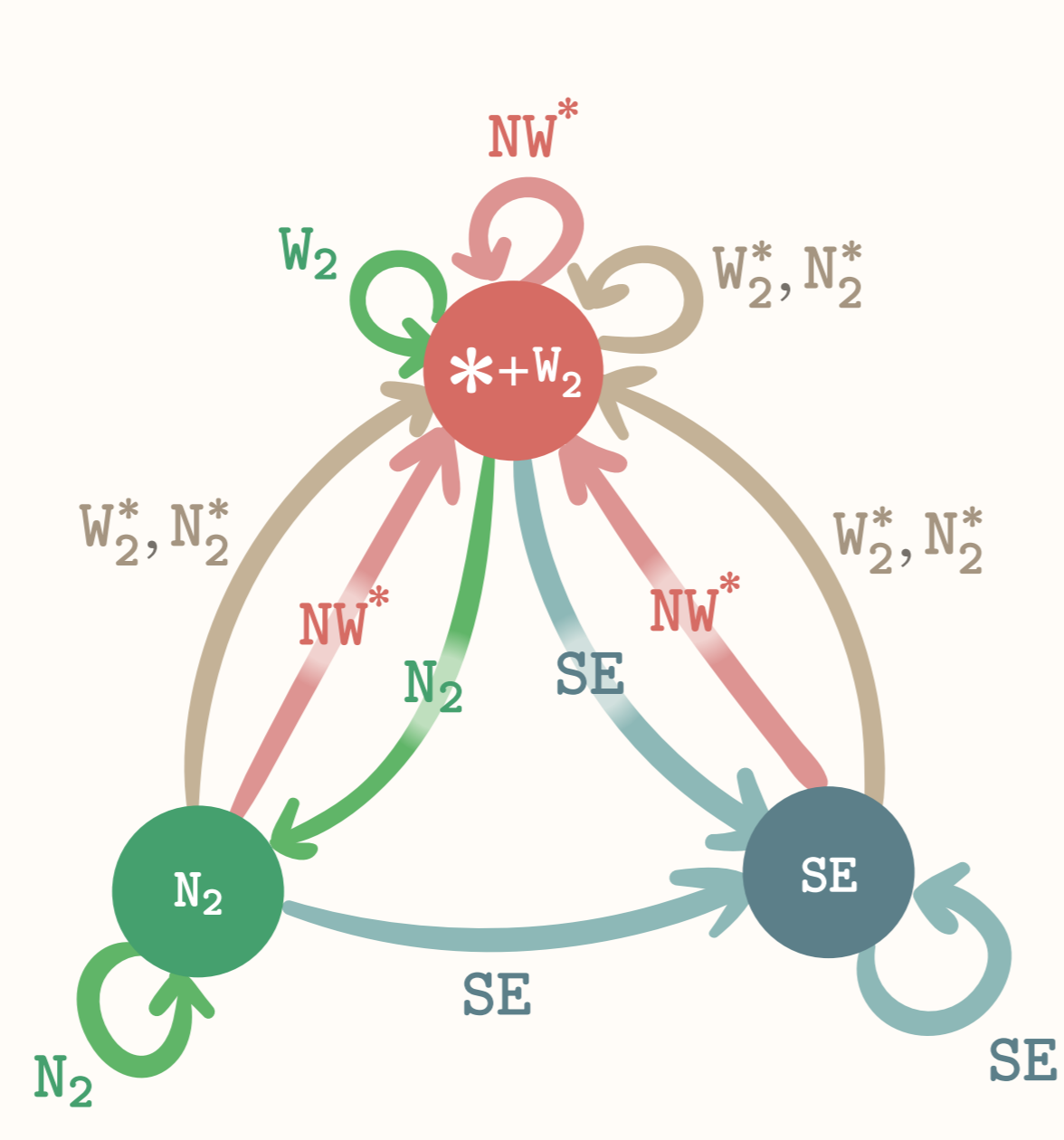
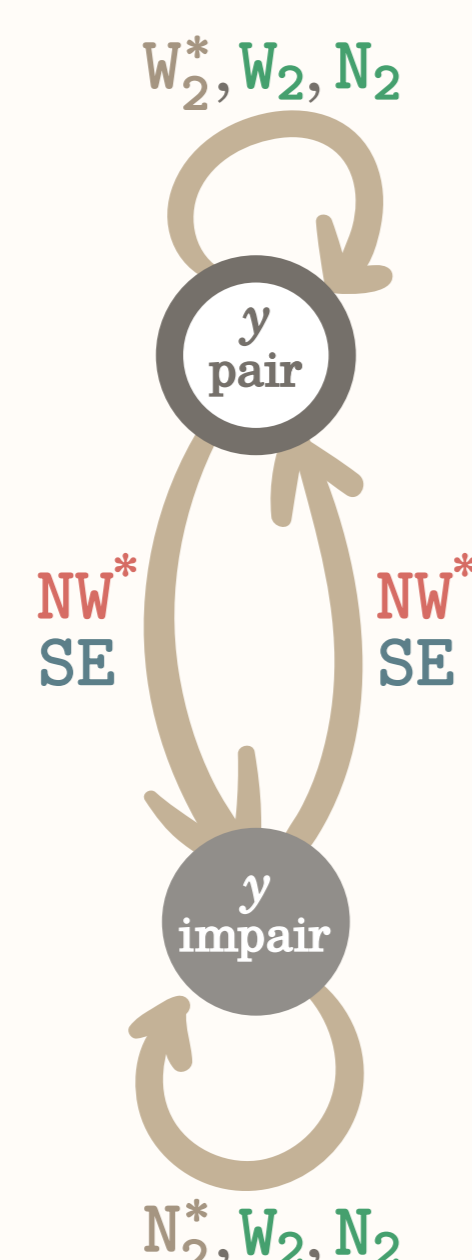
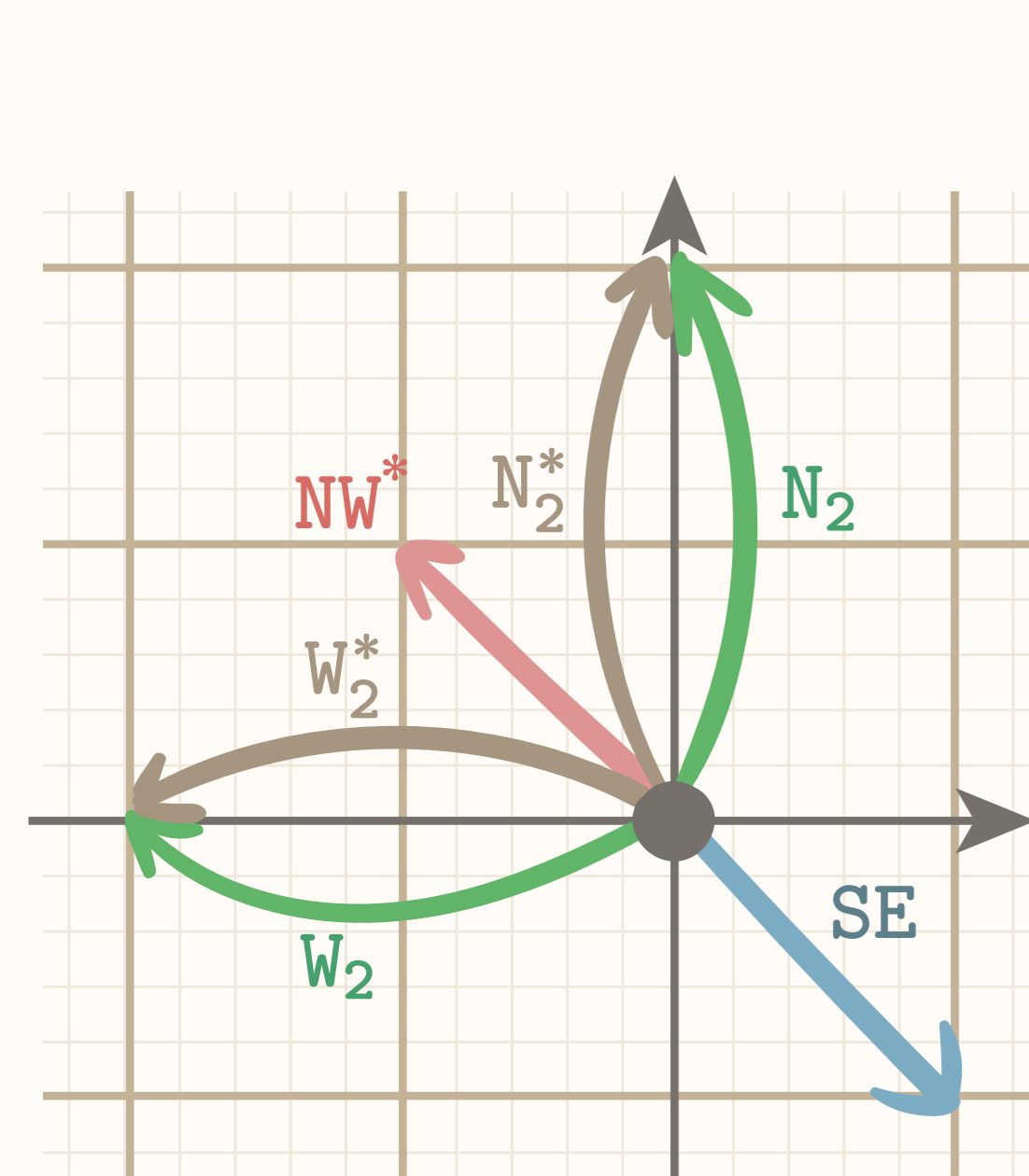
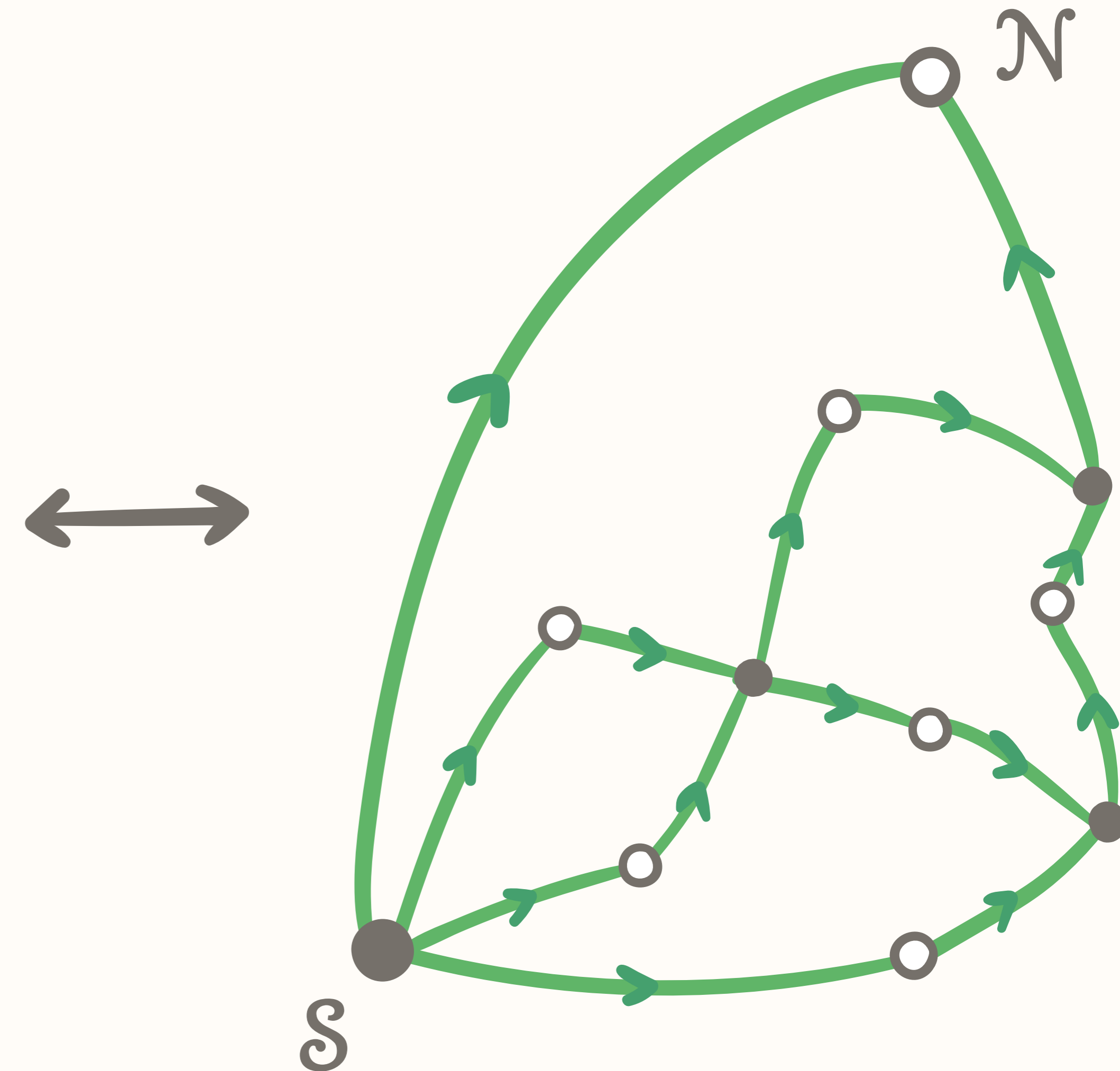
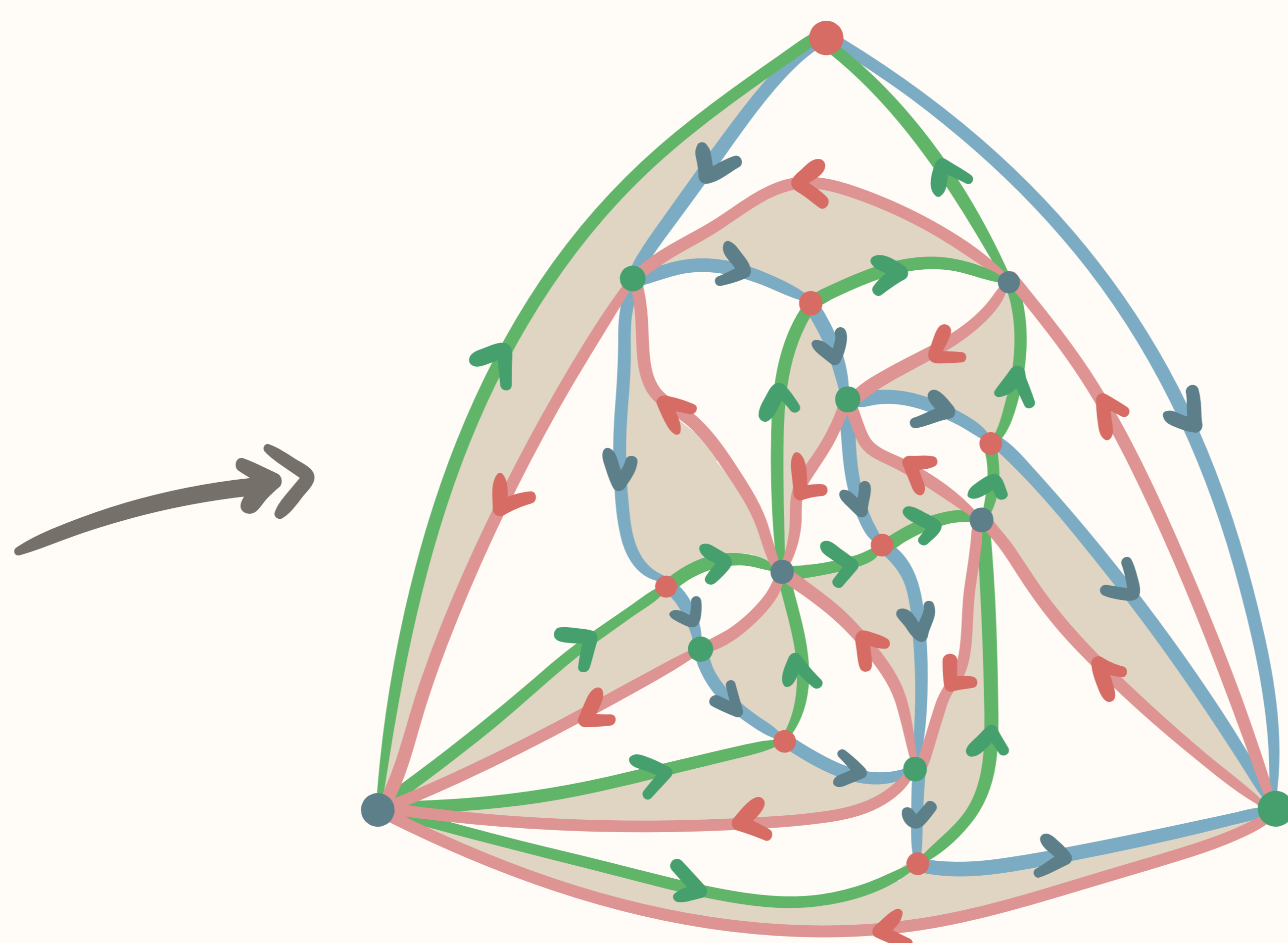
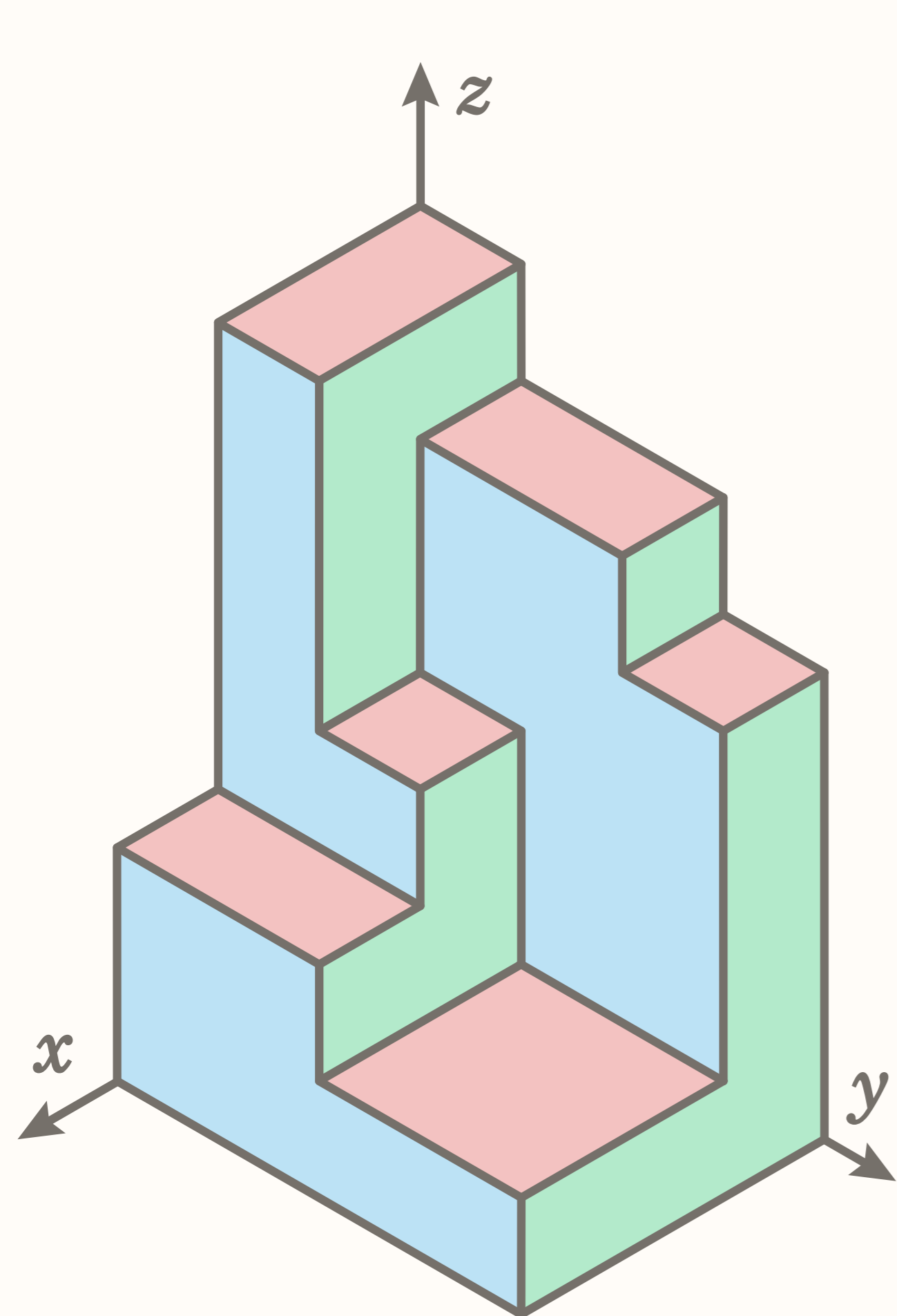
Recap polyhedral orientations



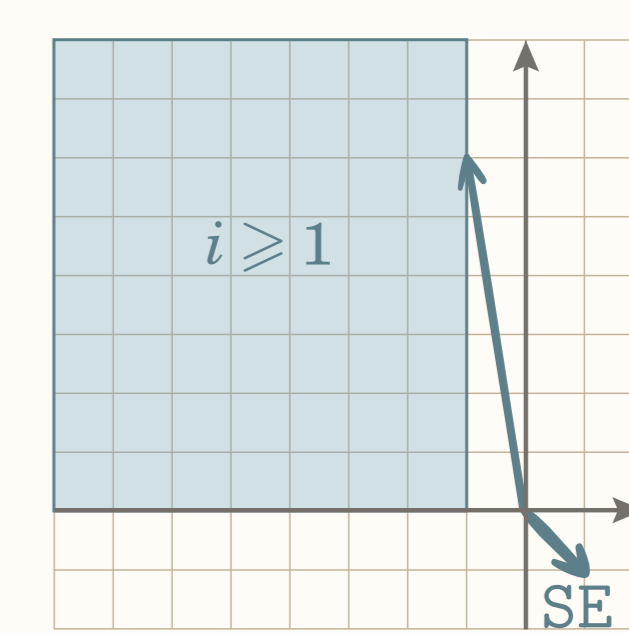
Recap polyhedral orientations



Recap polyhedral orientations



when x, y are even



when x, y are odd

Summary

Quadrant tandem walks

Asymptotics



Decorated bipolar orientations

Small steps



bipolar posets by vertices



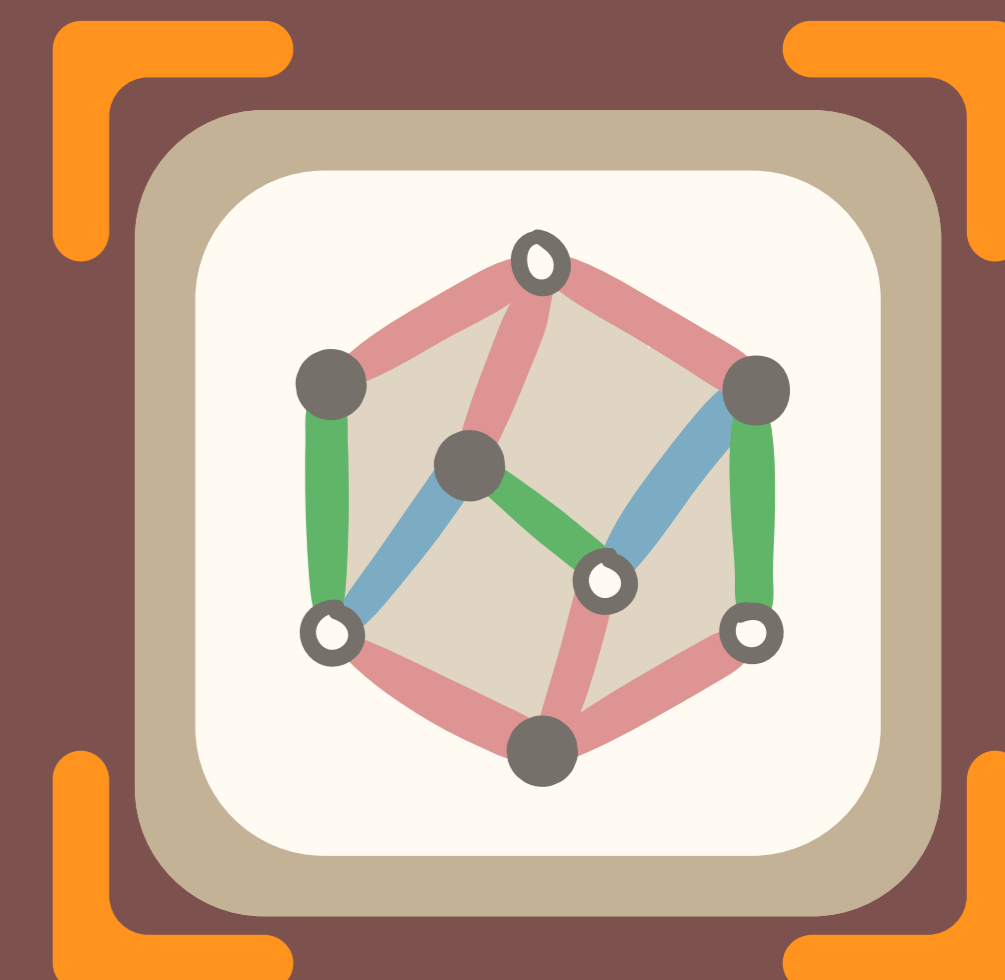
bipolar posets by edges



transversal structures

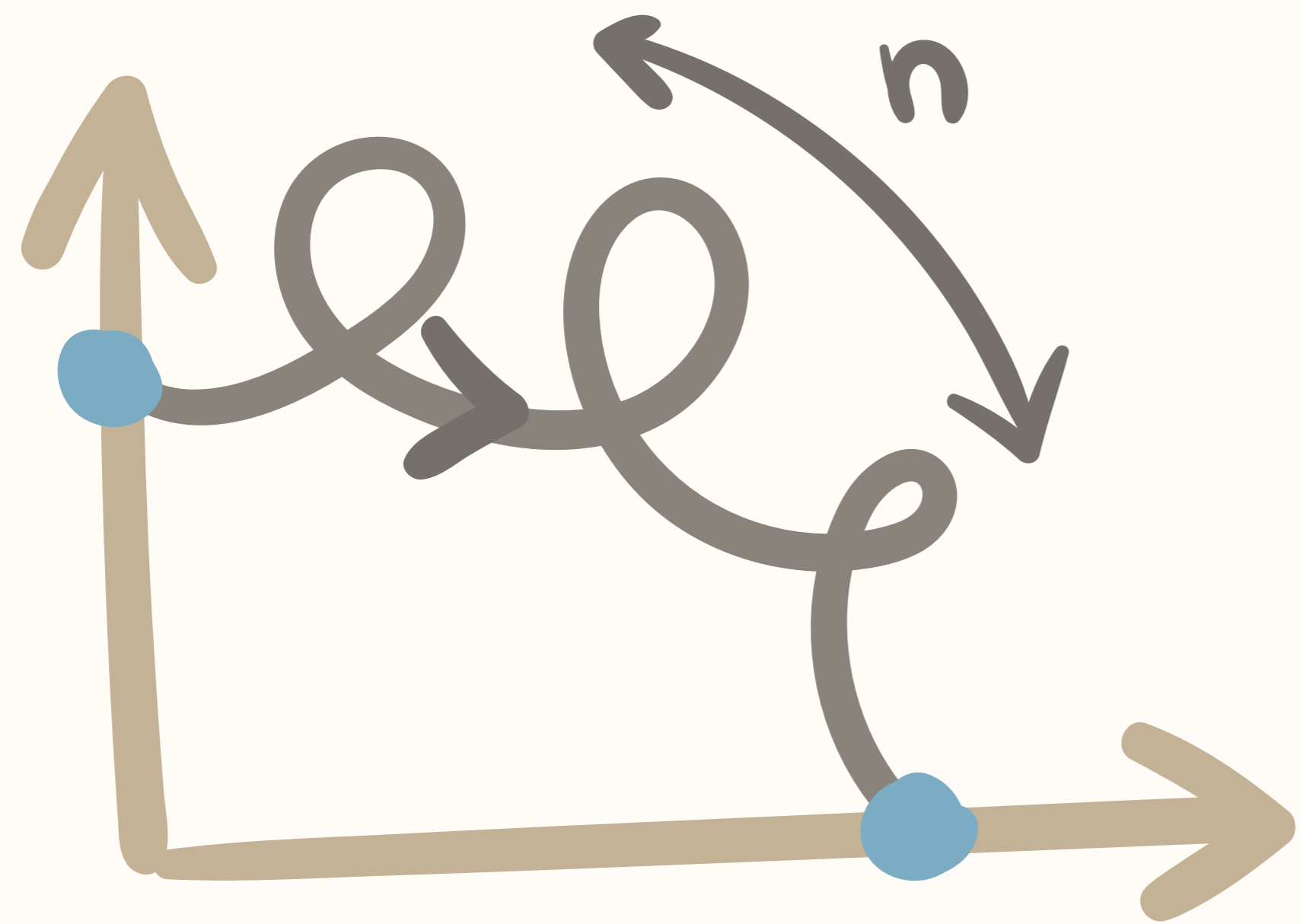


polyhedral orientations

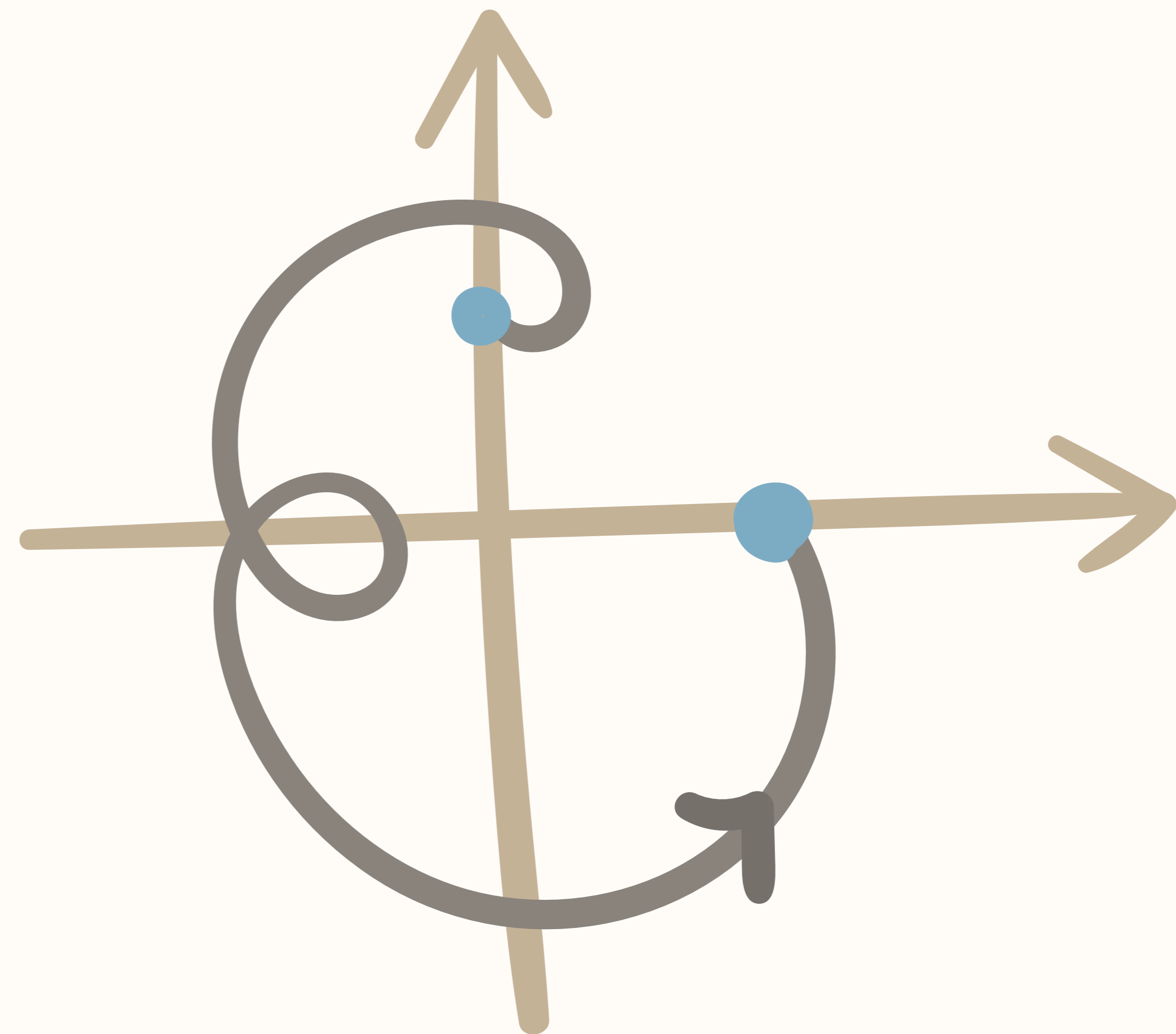
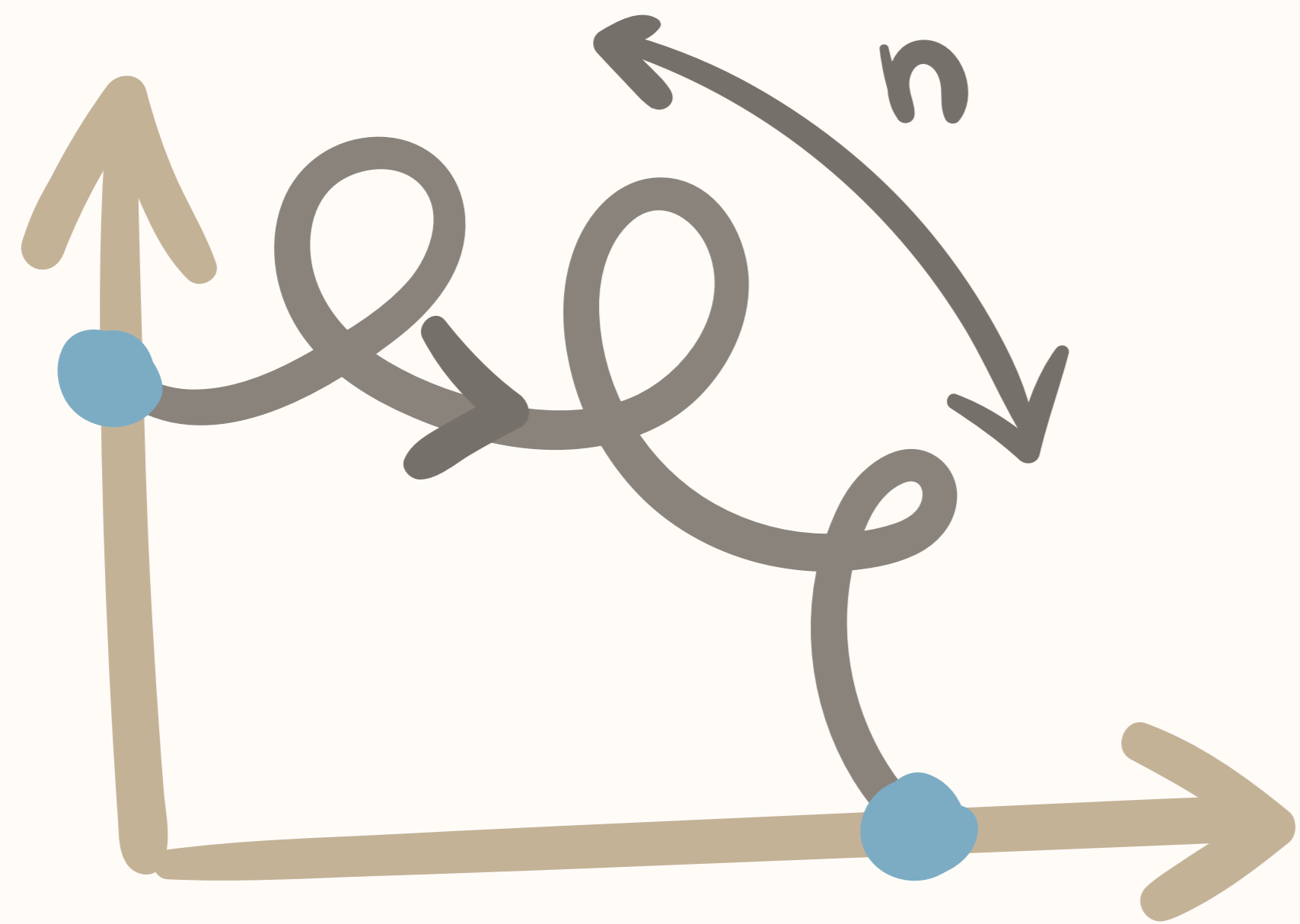


Schnyder labelings

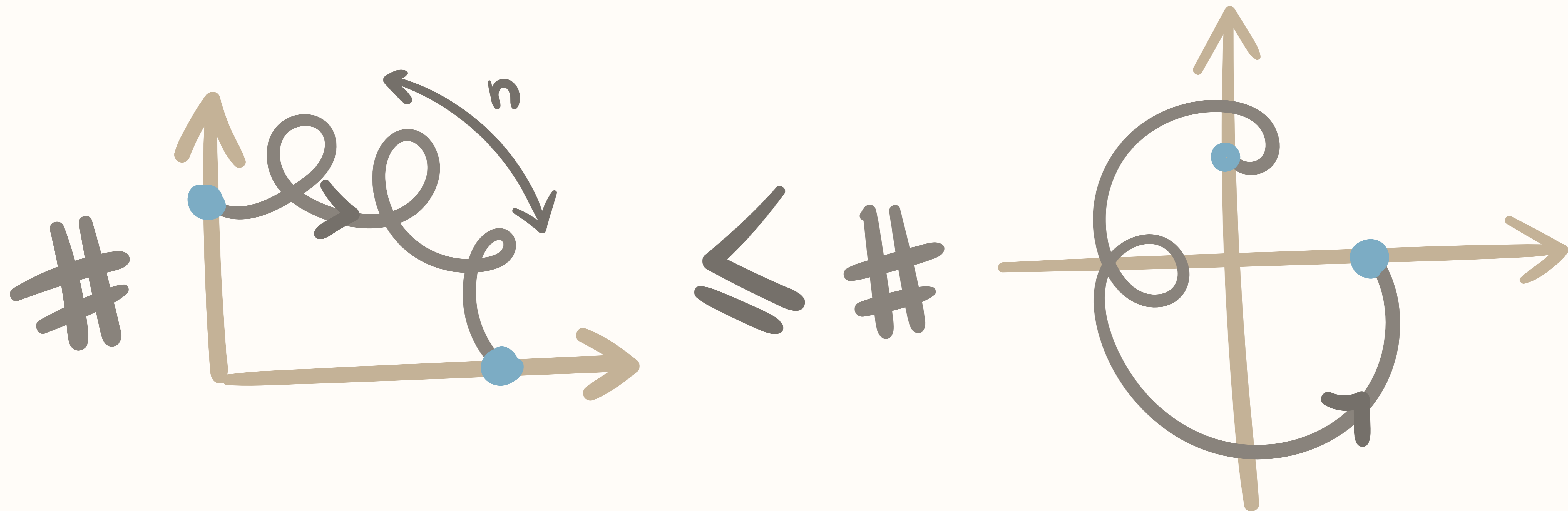
Asymptotic counting results



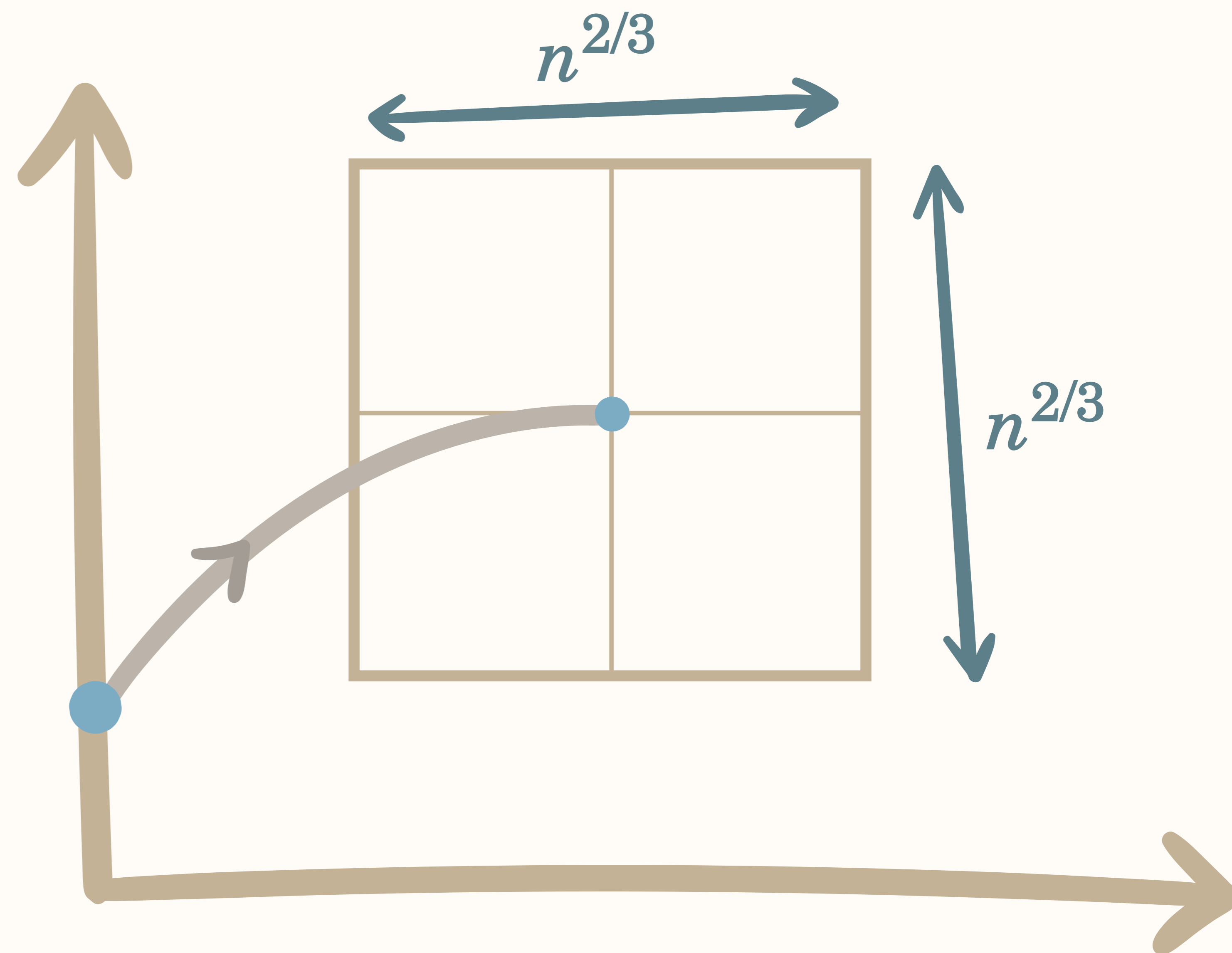
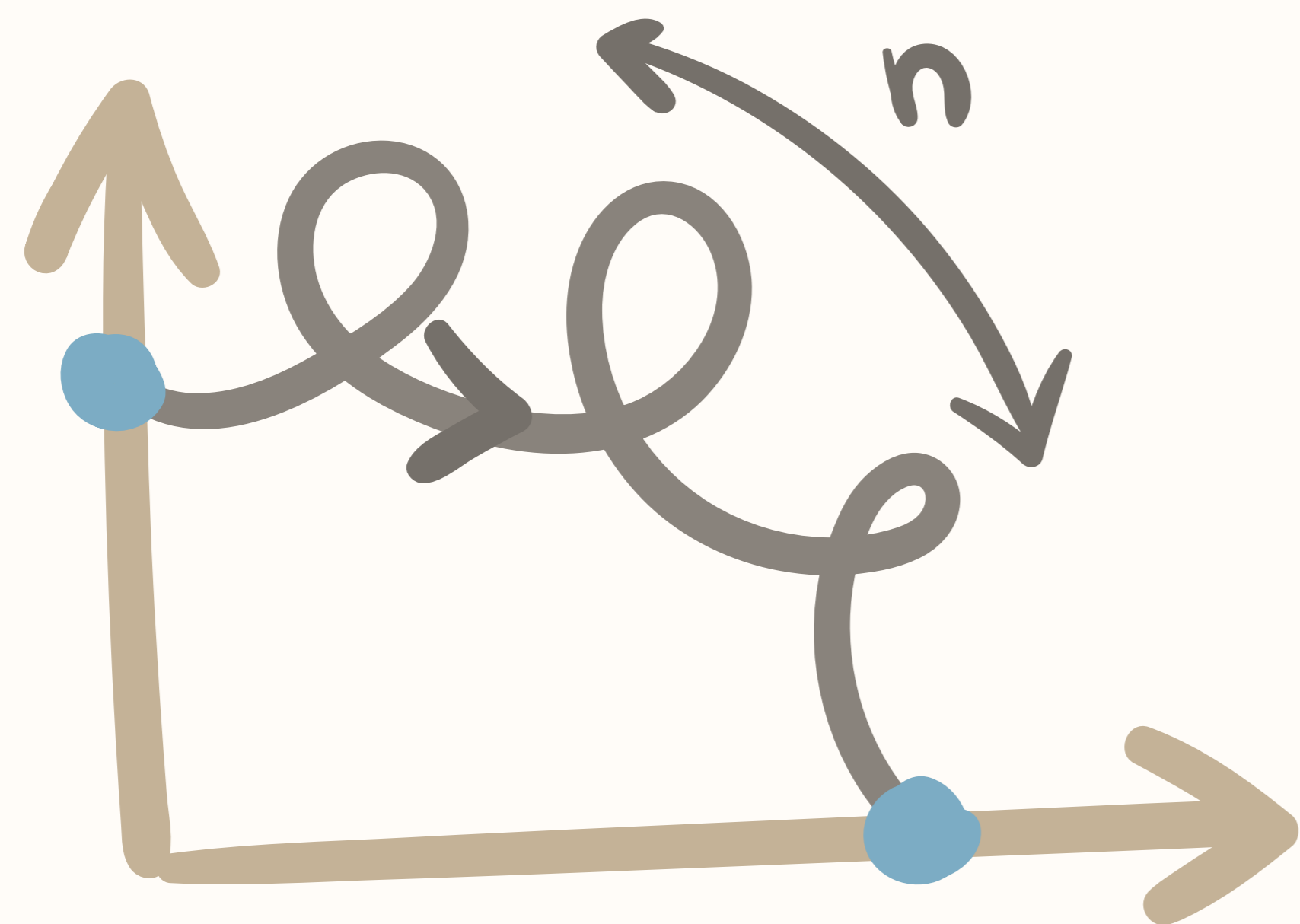
Asymptotic counting results



Asymptotic counting results



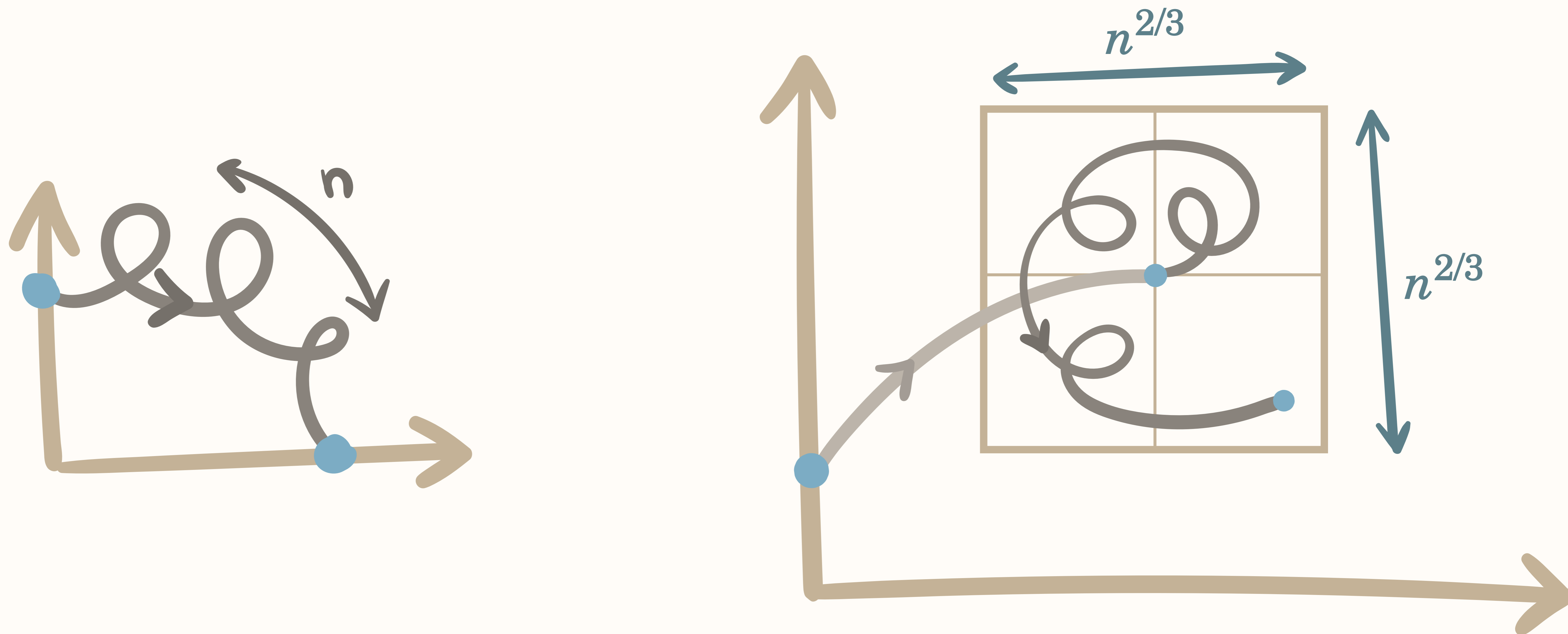
Asymptotic counting results

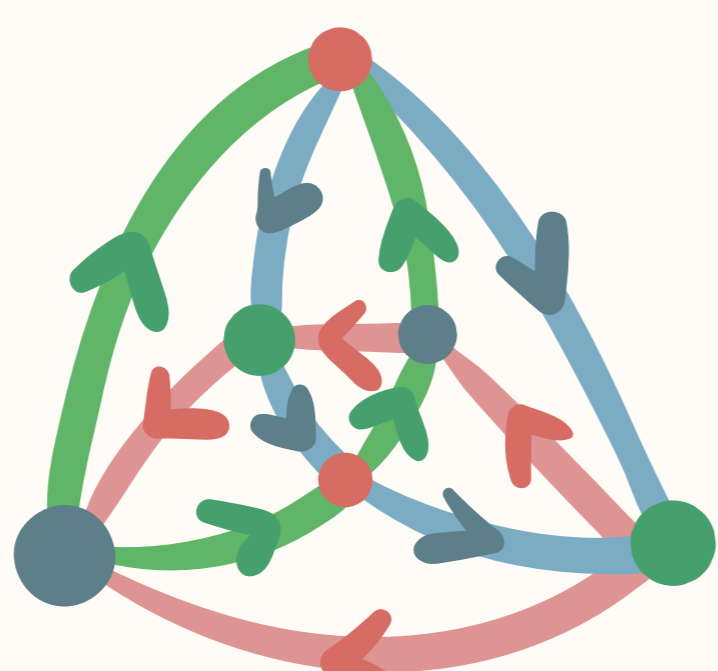


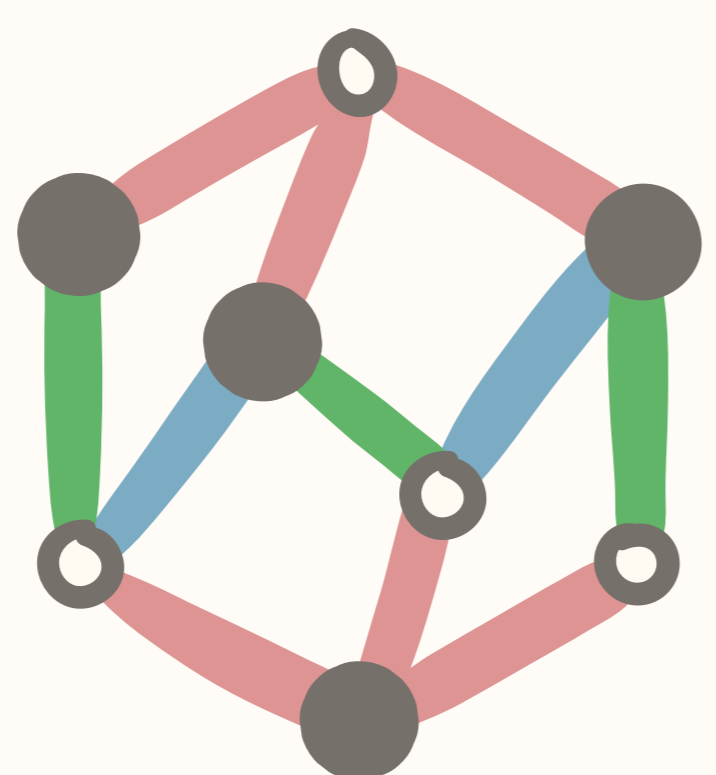
$$\# \text{ paths of length } n \leq \left(\frac{9}{2}\right)^n$$

$$\# \text{ paths of length } n \leq 2 \cdot \left(\frac{16}{3}\right)^n$$

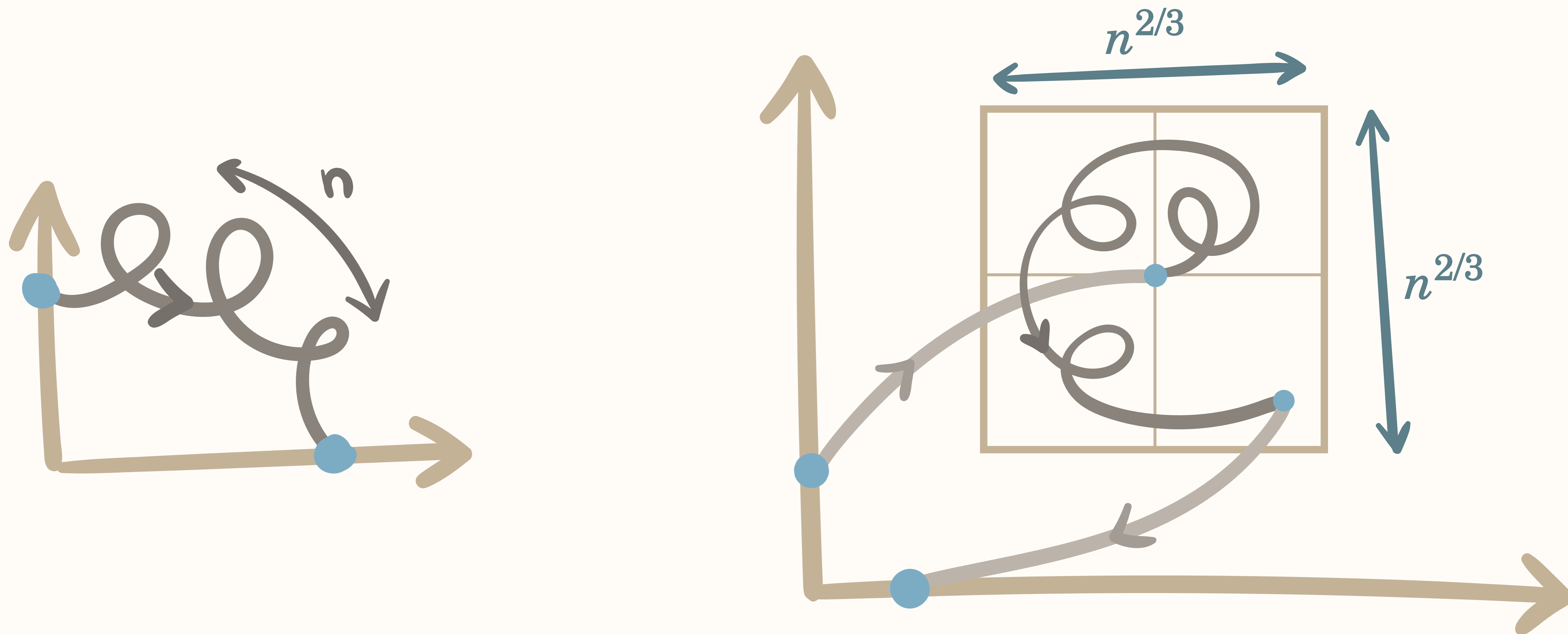
Asymptotic counting results

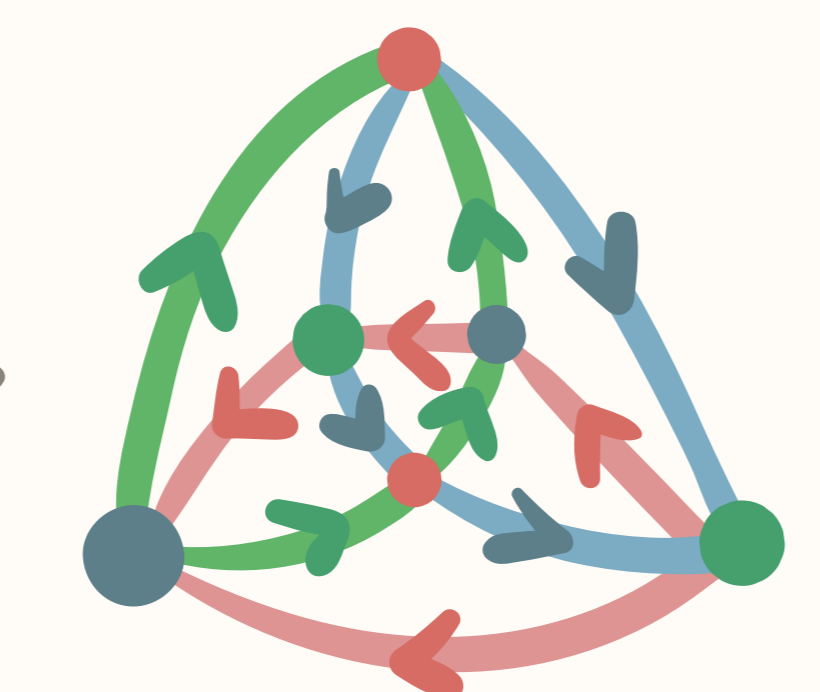


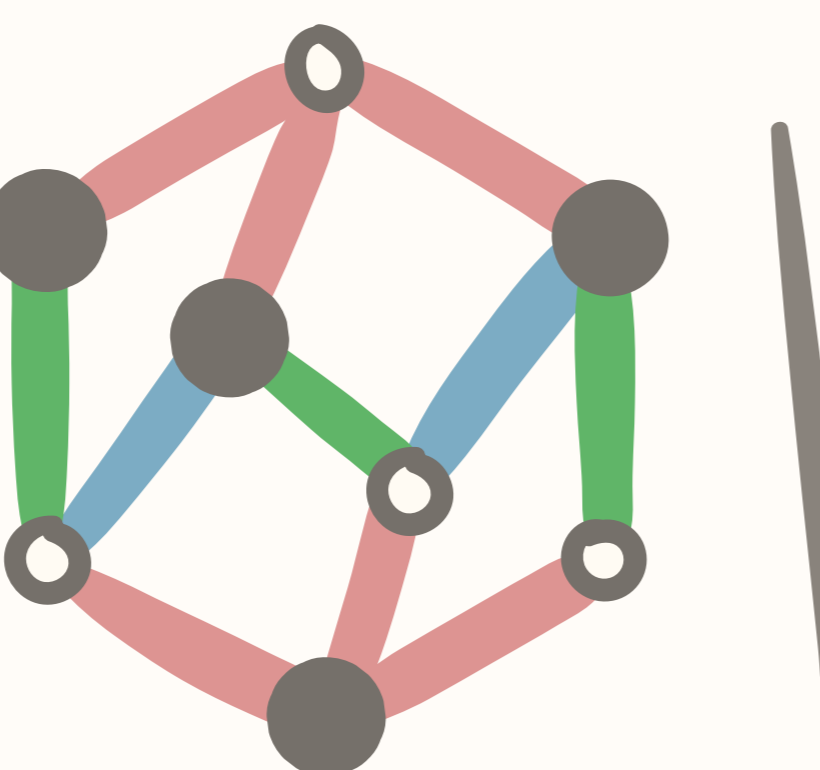
 $|_n \leq \left(\frac{9}{2}\right)^n$

 $|_n \leq 2 \cdot \left(\frac{16}{3}\right)^n$

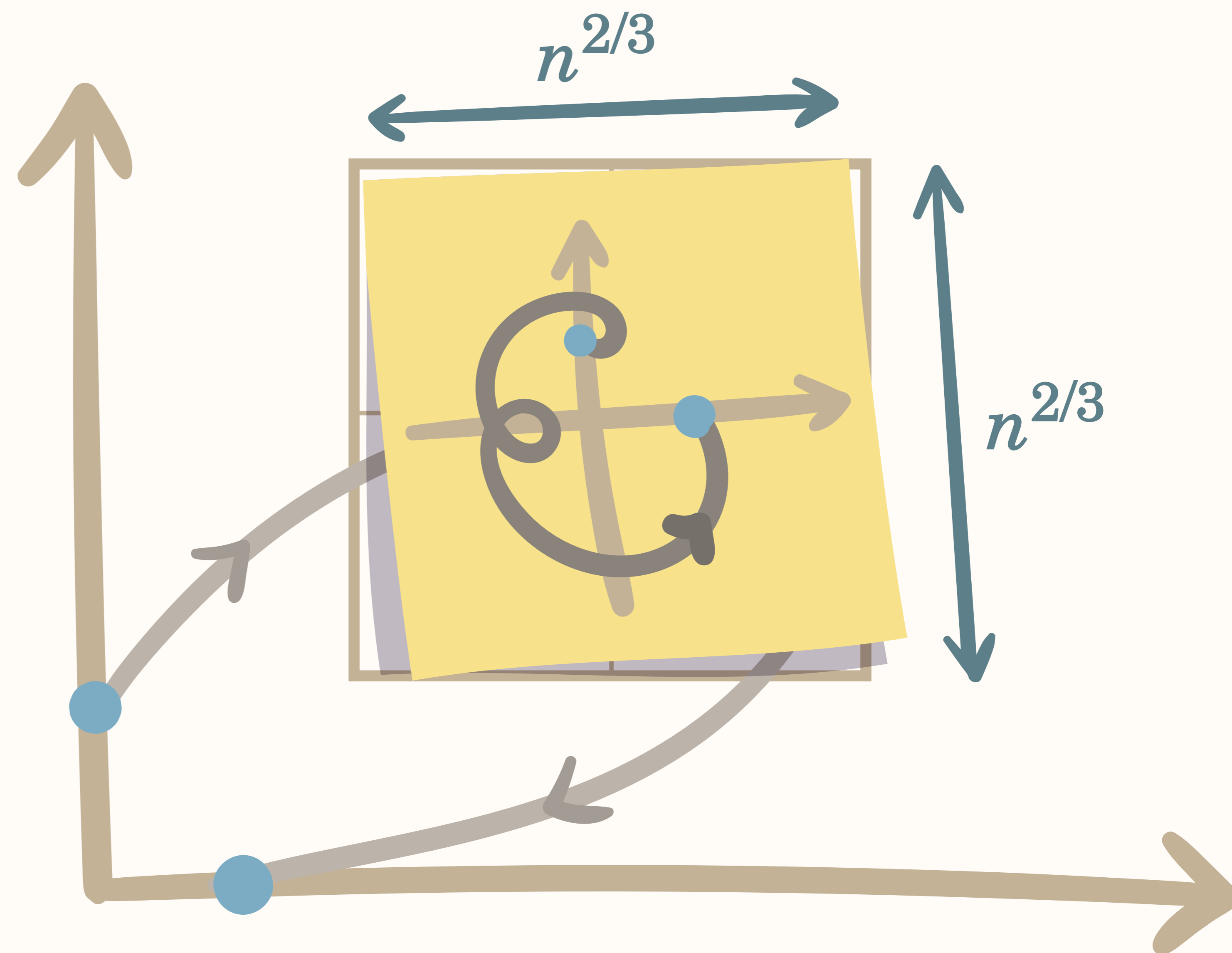
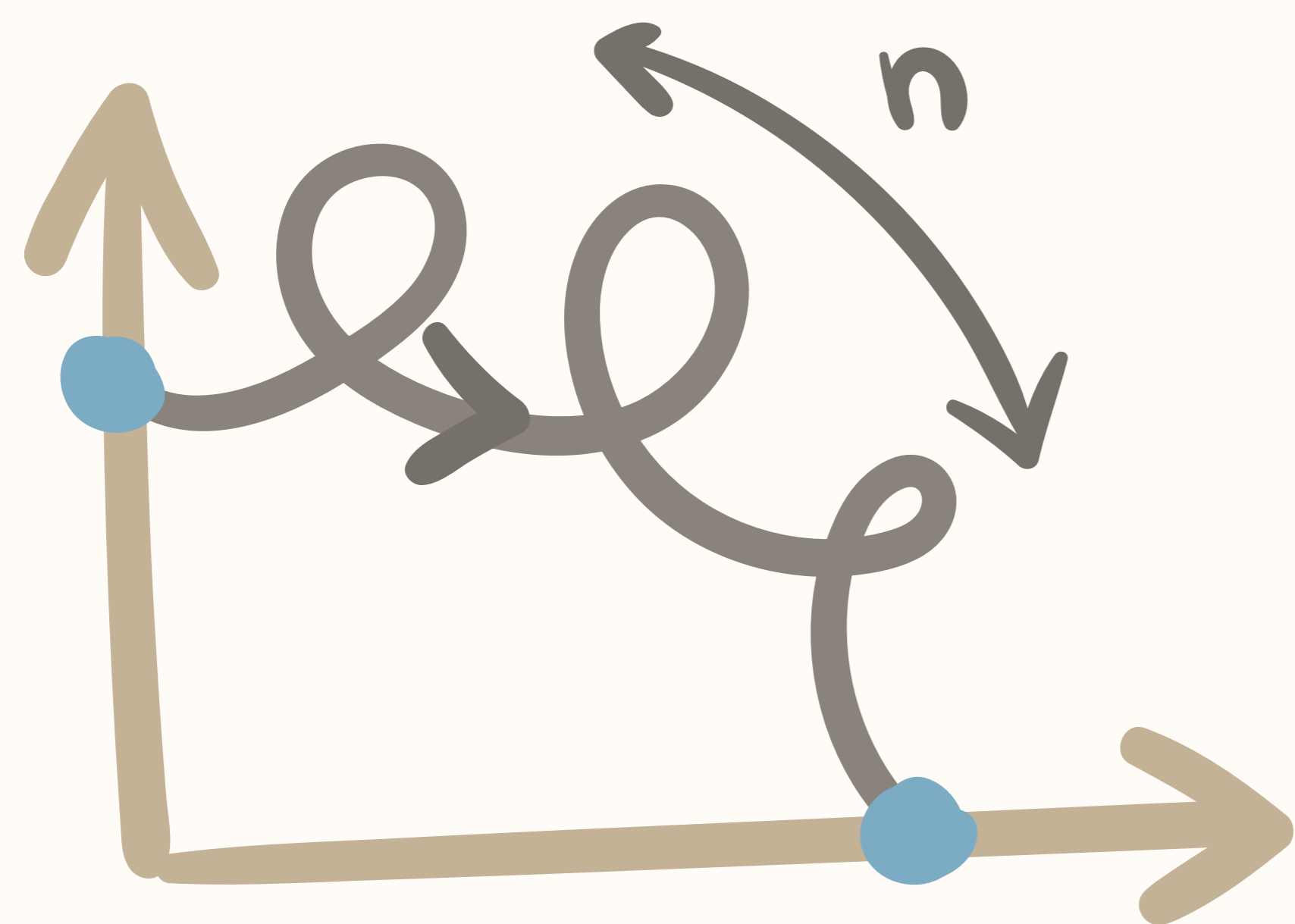
Asymptotic counting results



 $\big|_n \ll \left(\frac{9}{2}\right)^n$

 $\big|_n \ll 2 \cdot \left(\frac{16}{3}\right)^n$

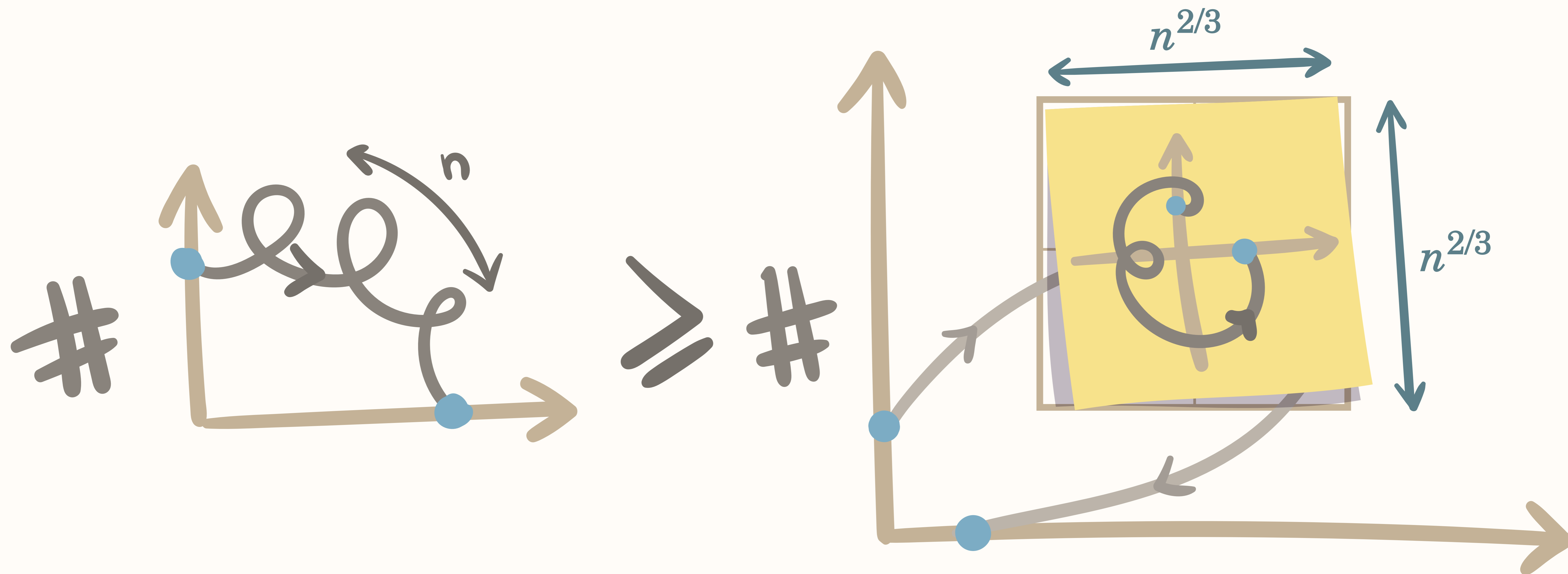
Asymptotic counting results



$$\# \text{ (looped path) } \Big|_n \leq \left(\frac{9}{2}\right)^n$$

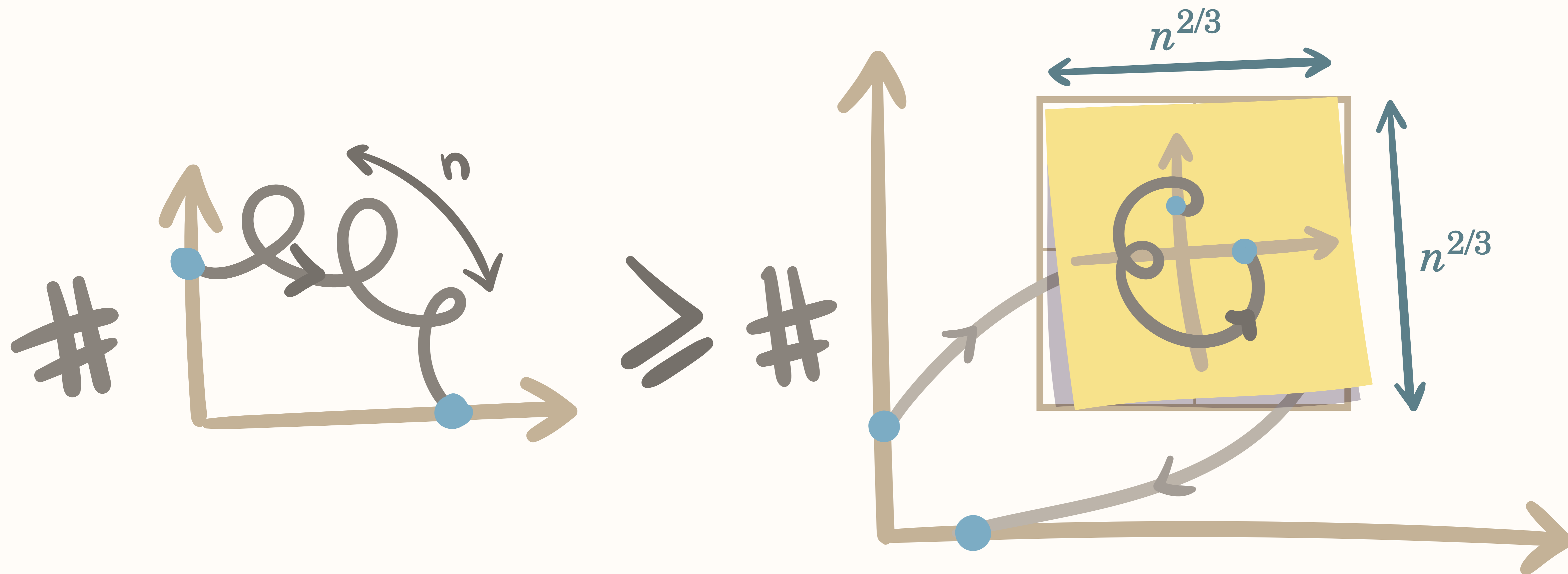
$$\# \text{ (looped path) } \Big|_n \leq 2 \cdot \left(\frac{16}{3}\right)^n$$

Asymptotic counting results



$$\# \text{ paths of length } n \leq \left(\frac{9}{2}\right)^n$$
$$\# \text{ paths of length } n \leq 2 \cdot \left(\frac{16}{3}\right)^n$$

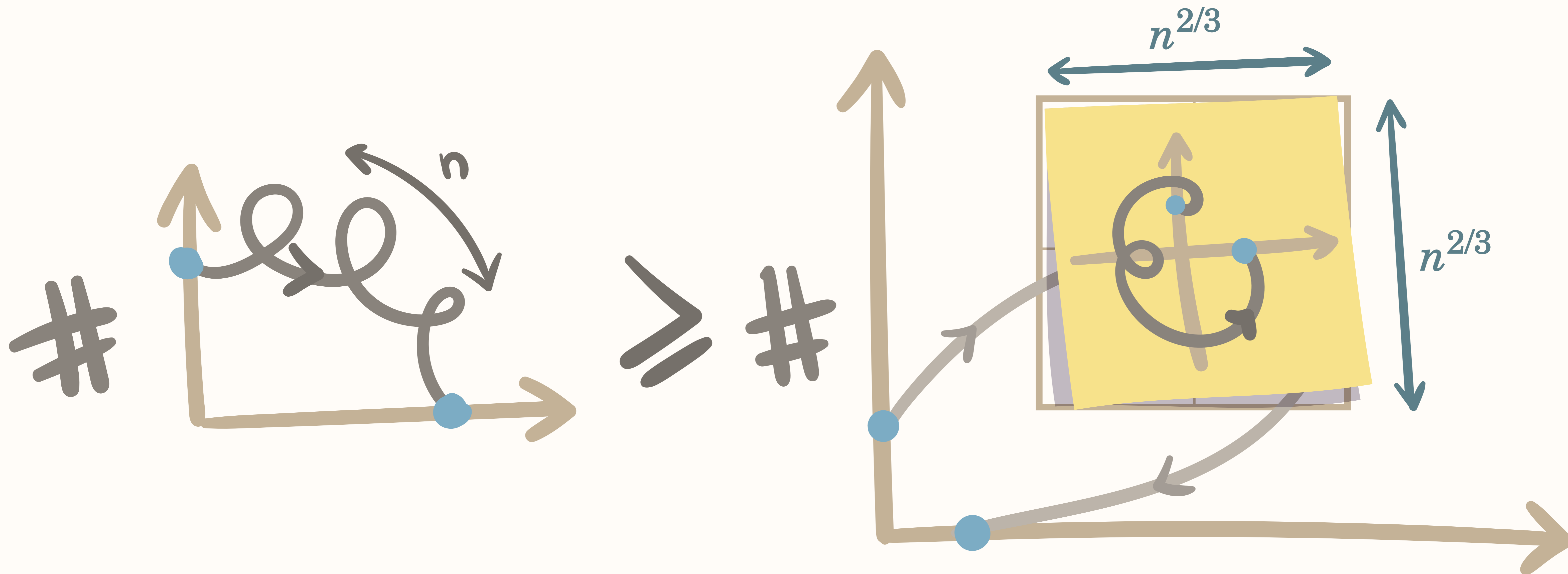
Asymptotic counting results



$$\left(\frac{9}{2}\right)^{n+o(n)} \leq \# \text{ (graph with 6 nodes and 9 edges)} \Big|_n \leq \left(\frac{9}{2}\right)^n$$

$$\left(\frac{16}{3}\right)^{n+o(n)} \leq \# \text{ (graph with 6 nodes and 16 edges)} \Big|_n \leq 2 \cdot \left(\frac{16}{3}\right)^n$$

Asymptotic counting results



$$\left(\frac{9}{2}\right)^{n+o(n)} \leq \# \text{ (cyclic paths) } \Big|_n \leq \left(\frac{9}{2}\right)^n$$

$$\lim p_n^{1/n} = 9/2$$

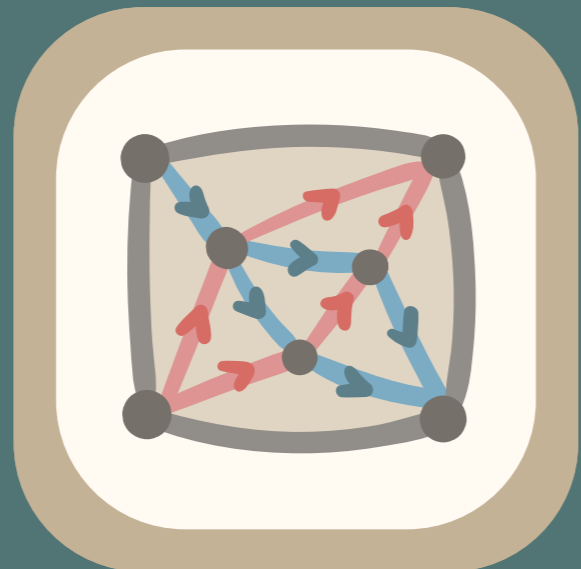
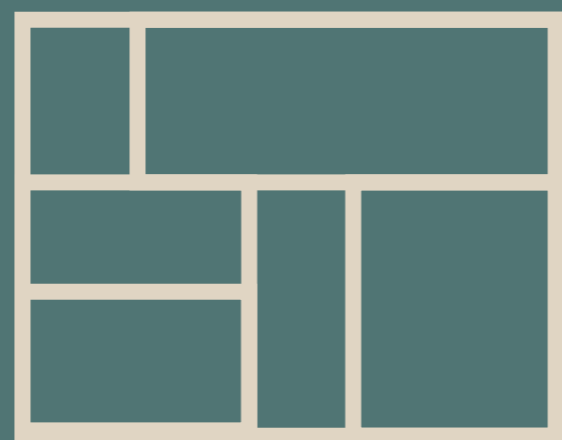


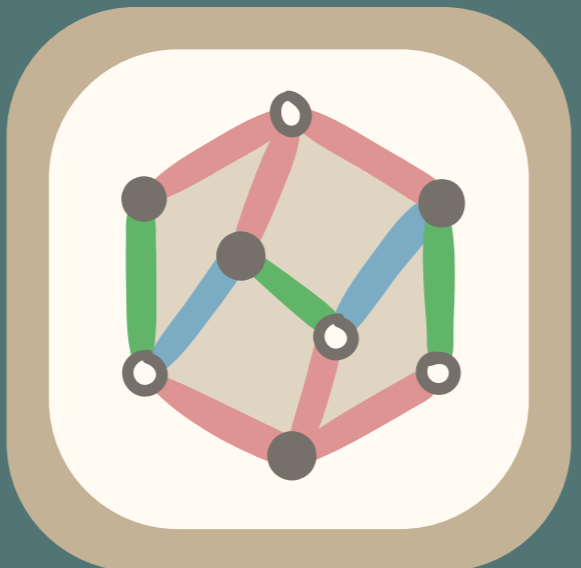
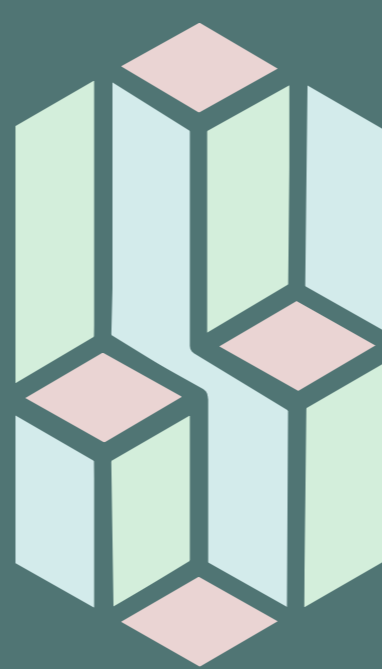


$$\left(\frac{16}{3}\right)^{n+o(n)} \leq \# \text{ (paths with a central vertex) } \Big|_n \leq 2 \cdot \left(\frac{16}{3}\right)^n$$

$$\lim s_n^{1/n} = 16/3$$

Summary

$$a_n \sim \kappa \cdot \gamma^n n^\alpha$$

$$\alpha = 1 + \pi / \arccos(\xi)$$

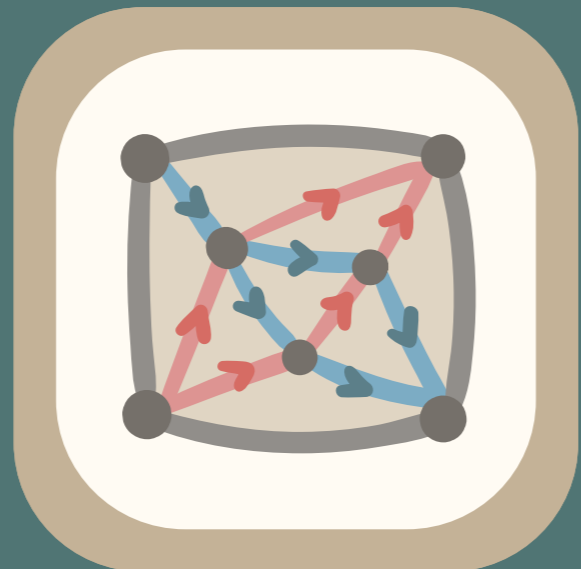
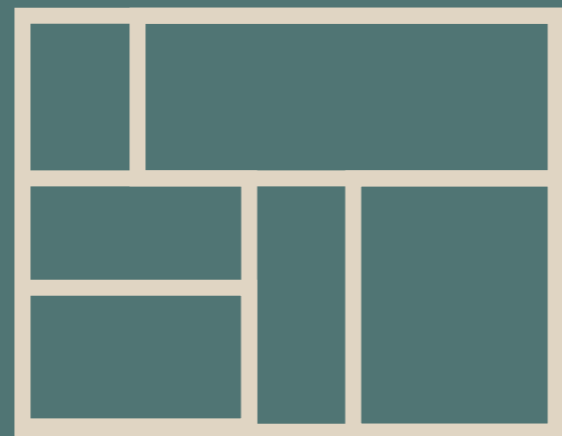


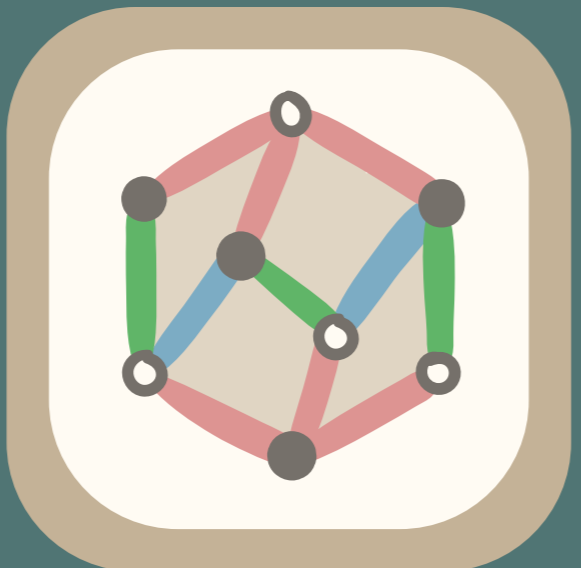
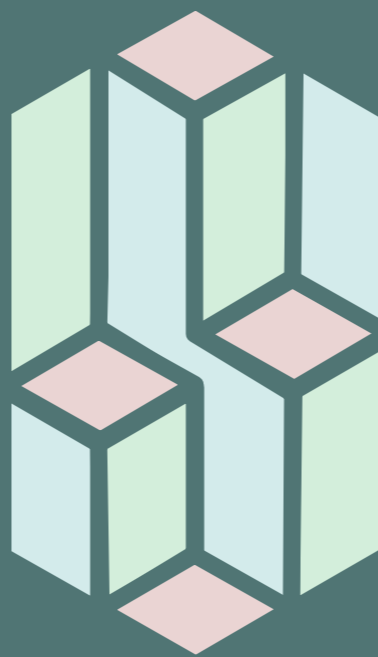


Model			Asymptotics	
Transversal structures <i>n blue edges</i>			$\gamma = 27/2$	non D-Finite
			$\alpha \approx 7.21$	$\xi = 7/8$
Polyhedral orientations <i>n inner faces</i>				
Schnyder colorings <i>n inner faces</i>				
Posets <i>n vertices</i>			$\gamma = (11 + 5\sqrt{5})/2$	D-Finite
			$\alpha = 6$	$\xi = (1 + \sqrt{5})/4$
Posets <i>n+2 edges</i>			$\gamma \approx 4.80$	non D-Finite
			$\alpha \approx 5.14$	$\xi \approx 0.73$

Summary

$$\lim a_n^{1/n} = \gamma$$

$$a_n \sim \kappa \cdot \gamma^n n^\alpha$$

$$\alpha = 1 + \pi / \arccos(\xi)$$

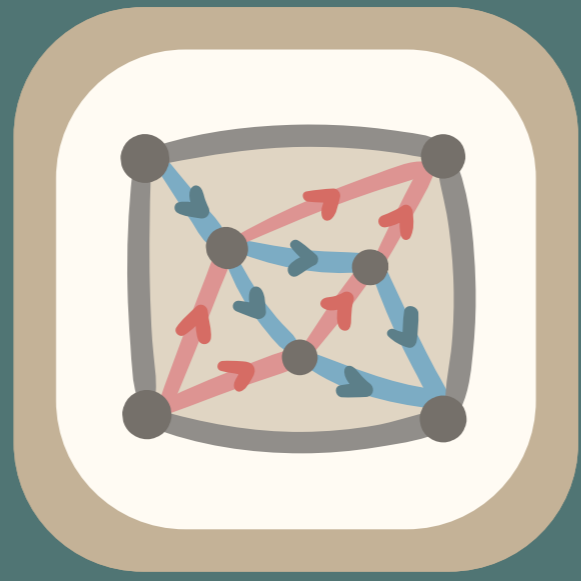
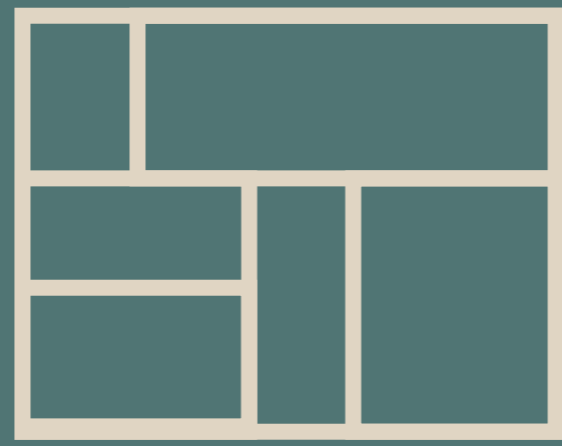



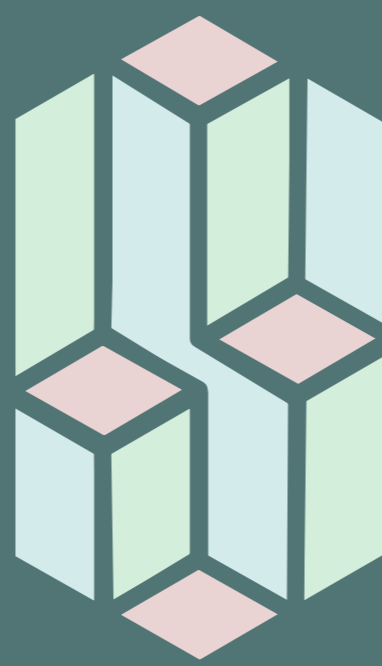


Model		Asymptotics		
Transversal structures <i>n blue edges</i>			$\gamma = 27/2$	non D-Finite
			$\alpha \approx 7.21$	$\xi = 7/8$
Polyhedral orientations <i>n inner faces</i>			$\gamma = 9/2$	
Schnyder colorings <i>n inner faces</i>			$\gamma = 16/3$	
Posets <i>n vertices</i>			$\gamma = (11 + 5\sqrt{5})/2$	D-Finite
			$\alpha = 6$	$\xi = (1 + \sqrt{5})/4$
Posets <i>n+2 edges</i>			$\gamma \approx 4.80$	non D-Finite
			$\alpha \approx 5.14$	$\xi \approx 0.73$

Summary

$$\lim a_n^{1/n} = \gamma$$

$$a_n \sim \kappa \cdot \gamma^n n^\alpha$$

$$\alpha = 1 + \pi / \arccos(\xi)$$

Model		Asymptotics	
Transversal structures <i>n blue edges</i> 		$\gamma = 27/2$	non D-Finite
		$\alpha \approx 7.21$	$\xi = 7/8$
Polyhedral orientations <i>n inner faces</i> 		$\gamma = 9/2$	(non D-Finite)
		$(\alpha \approx 4.23)$	$(\xi = 9/16)$
Schnyder colorings <i>n inner faces</i> 		$\gamma = 16/3$	(non D-Finite)
		$(\alpha \approx 6.08)$	$(\xi = 22/27)$
Posets <i>n vertices</i> 		$\gamma = (11 + 5\sqrt{5})/2$	D-Finite
		$\alpha = 6$	$\xi = (1 + \sqrt{5})/4$
Posets <i>n+2 edges</i> 		$\gamma \approx 4.80$	non D-Finite
		$\alpha \approx 5.14$	$\xi \approx 0.73$

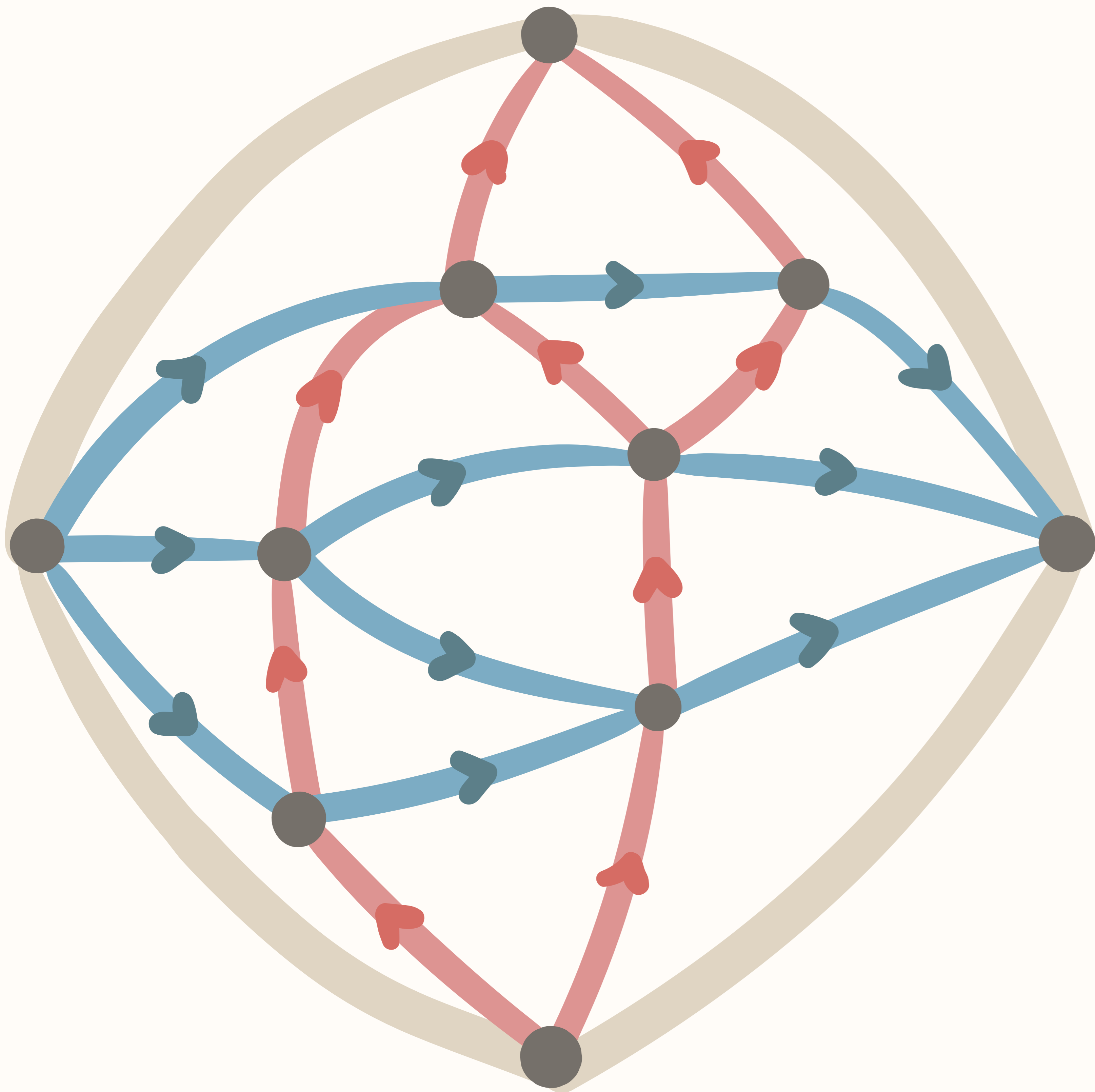
ANNEXES

A. Generalization for transversal structures

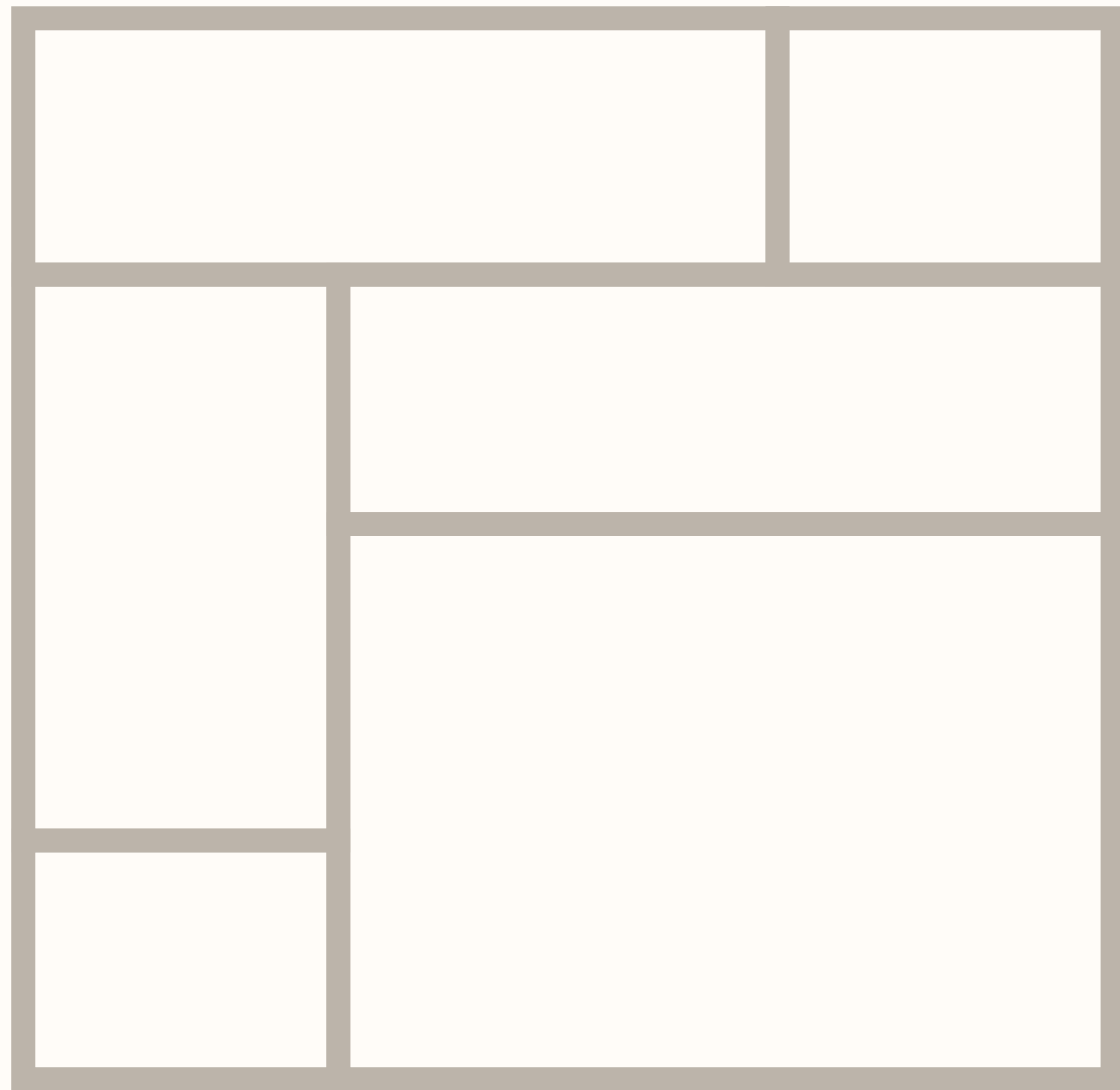
B. Schnyder labelings

C. Bijection for plane permutations

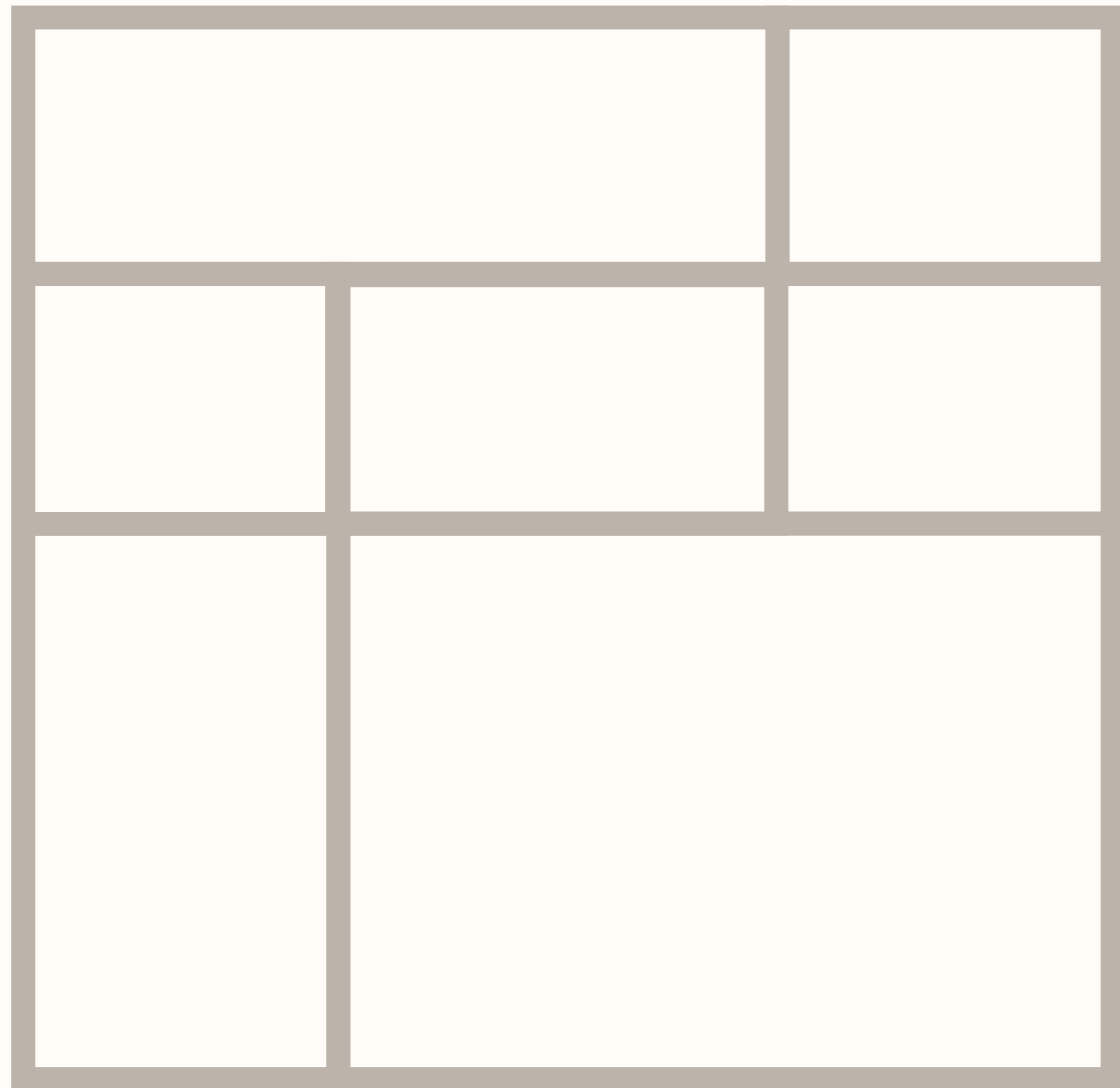
Generic transversal structures



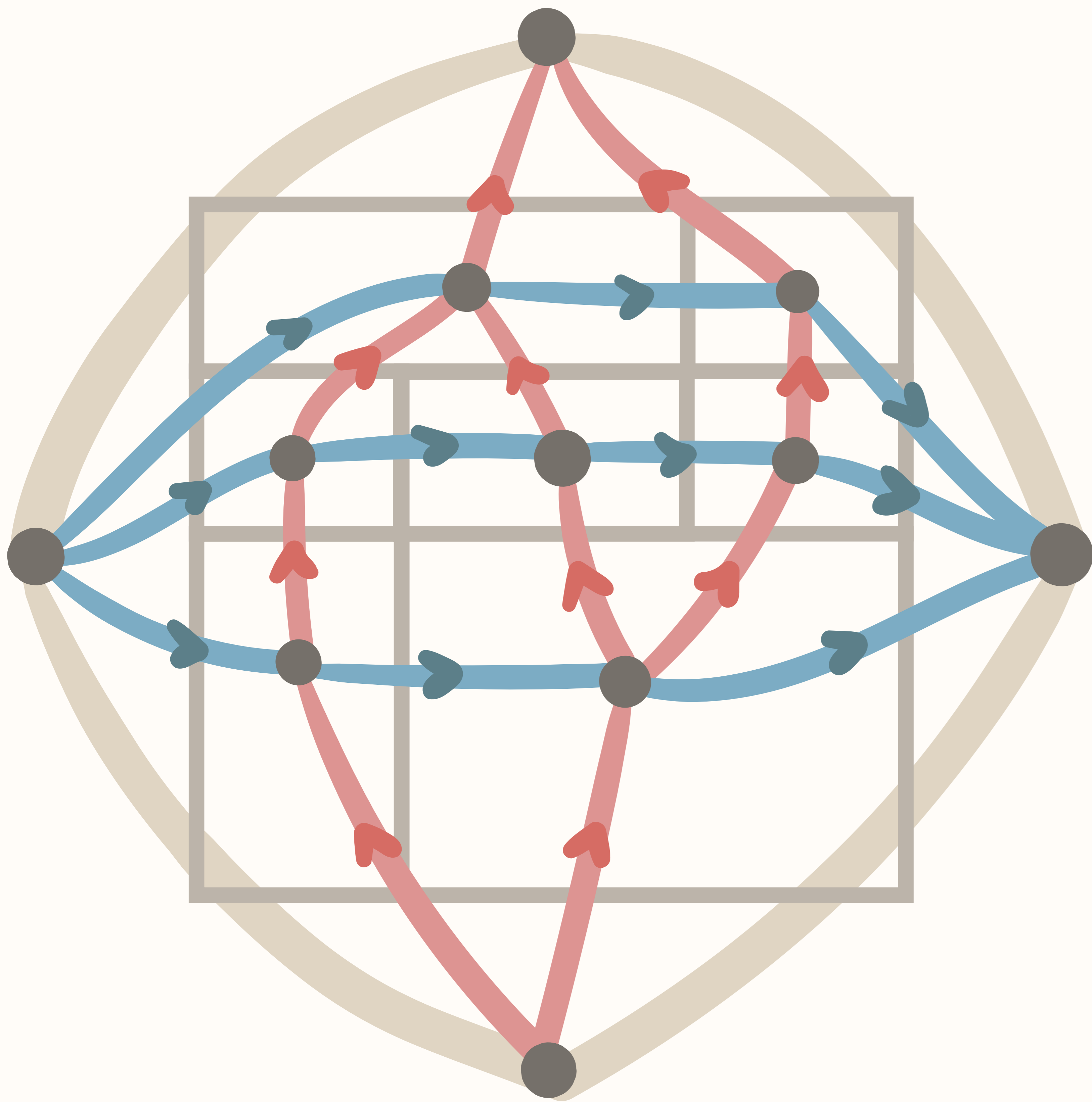
Generic transversal structures



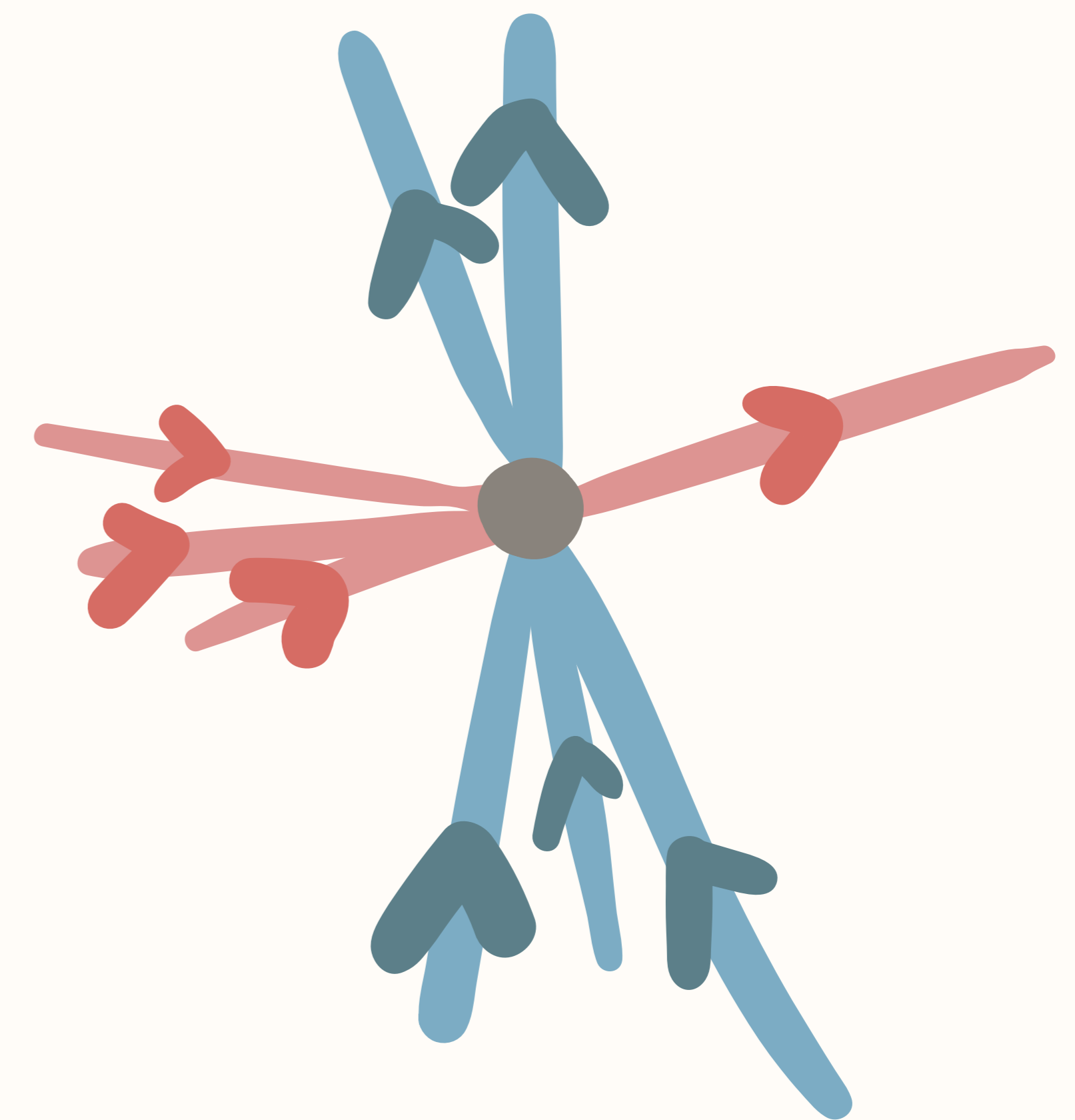
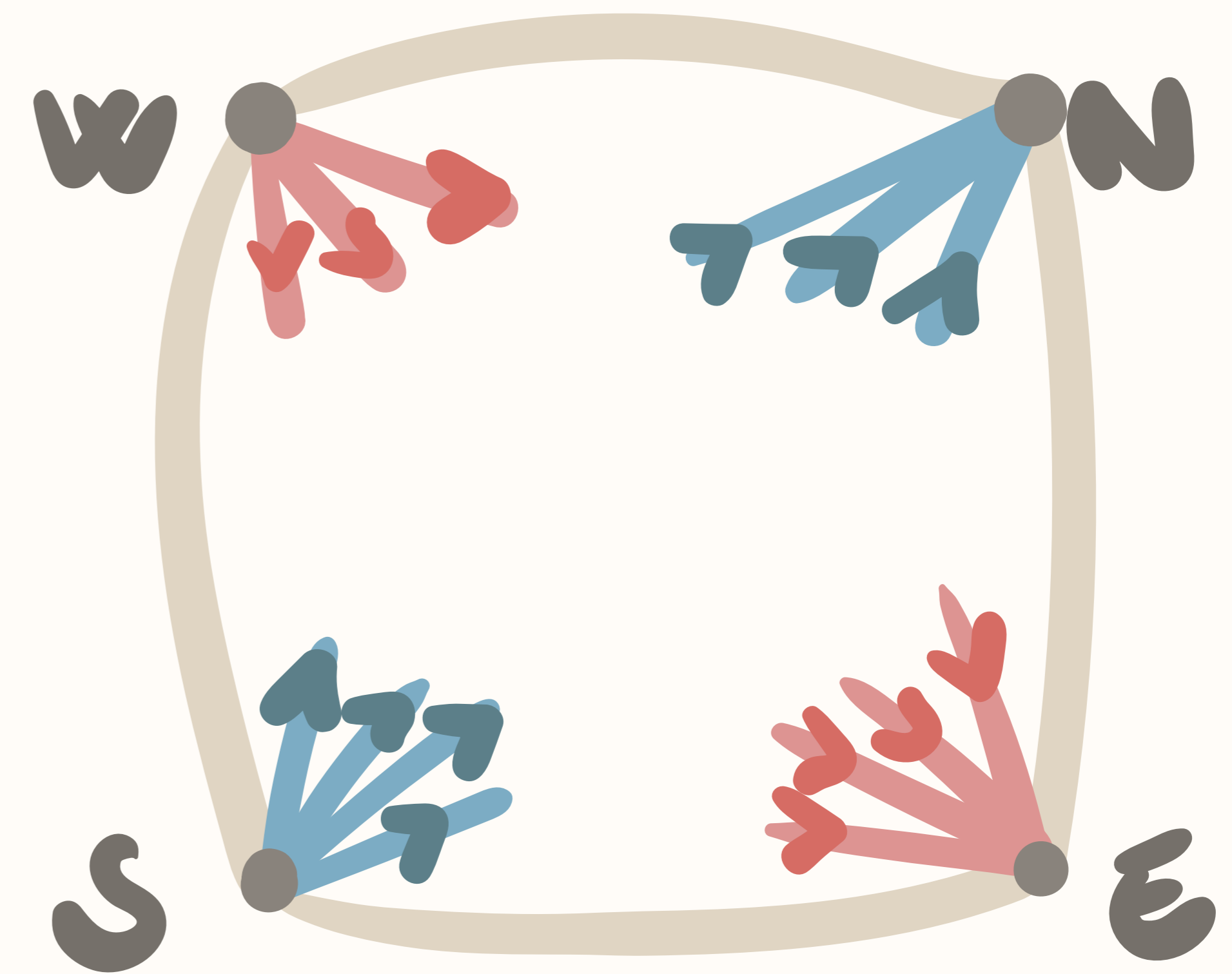
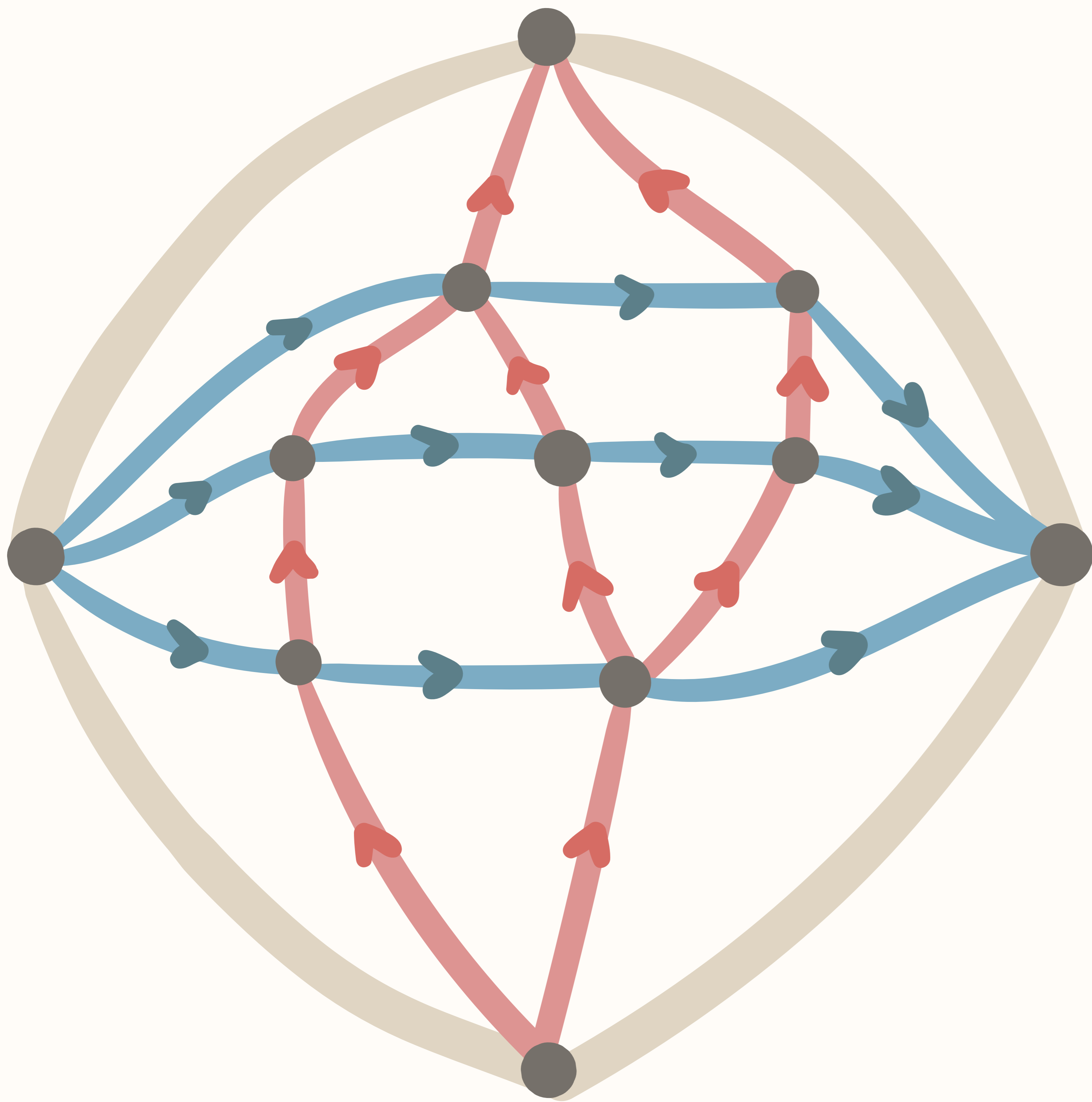
Generic transversal structures



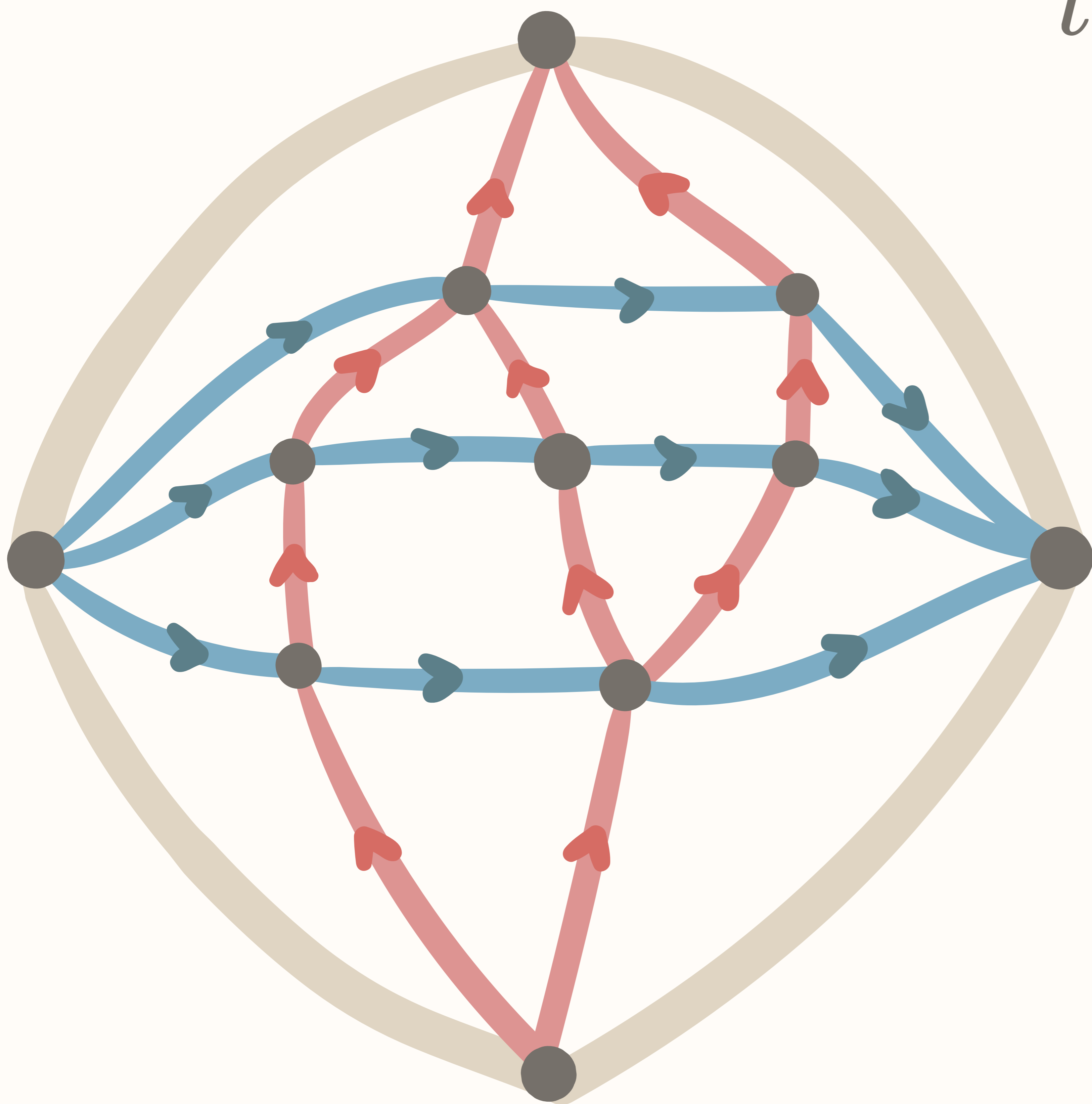
Generic transversal structures



Generic transversal structures



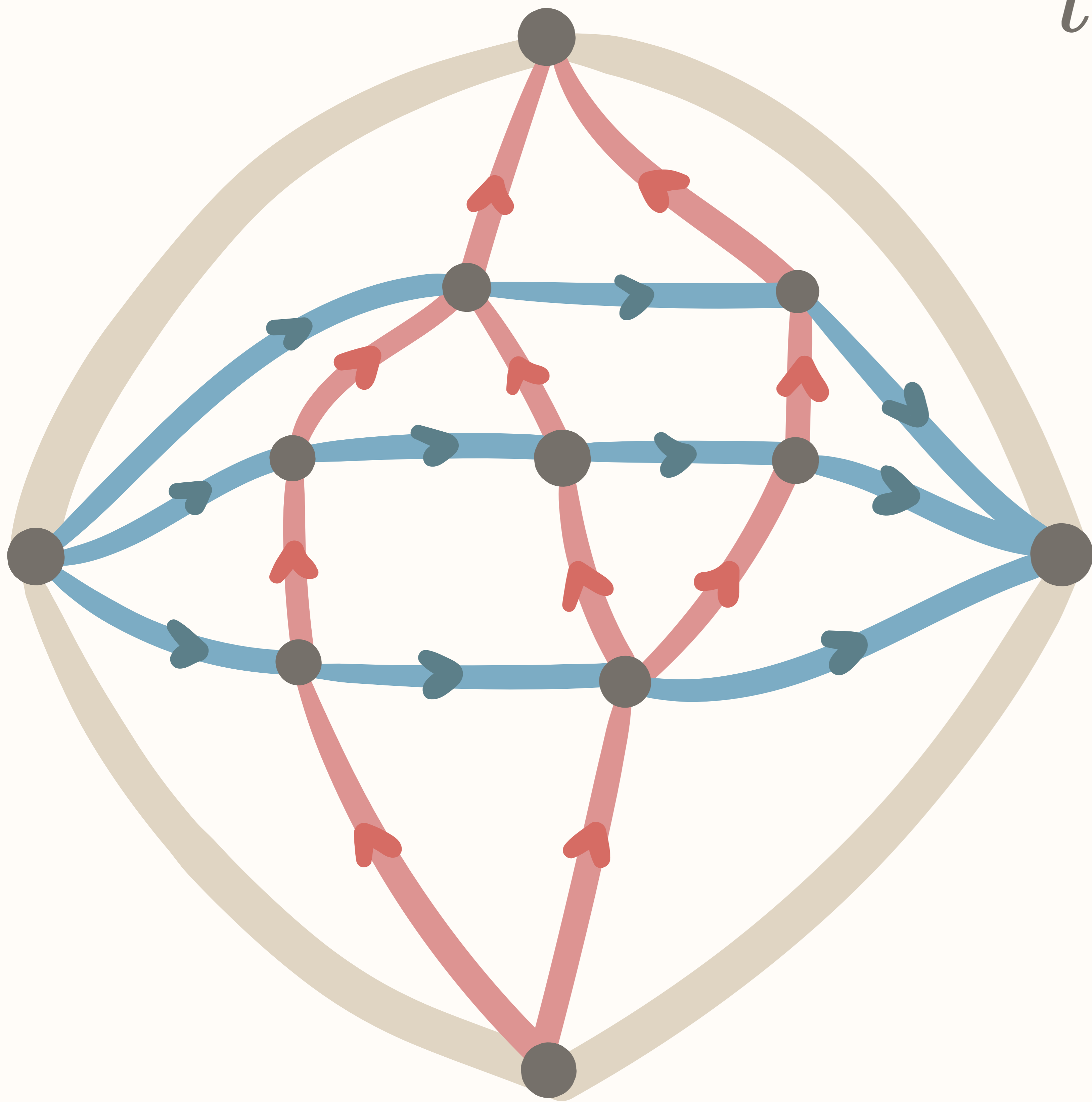
Generic transversal structures



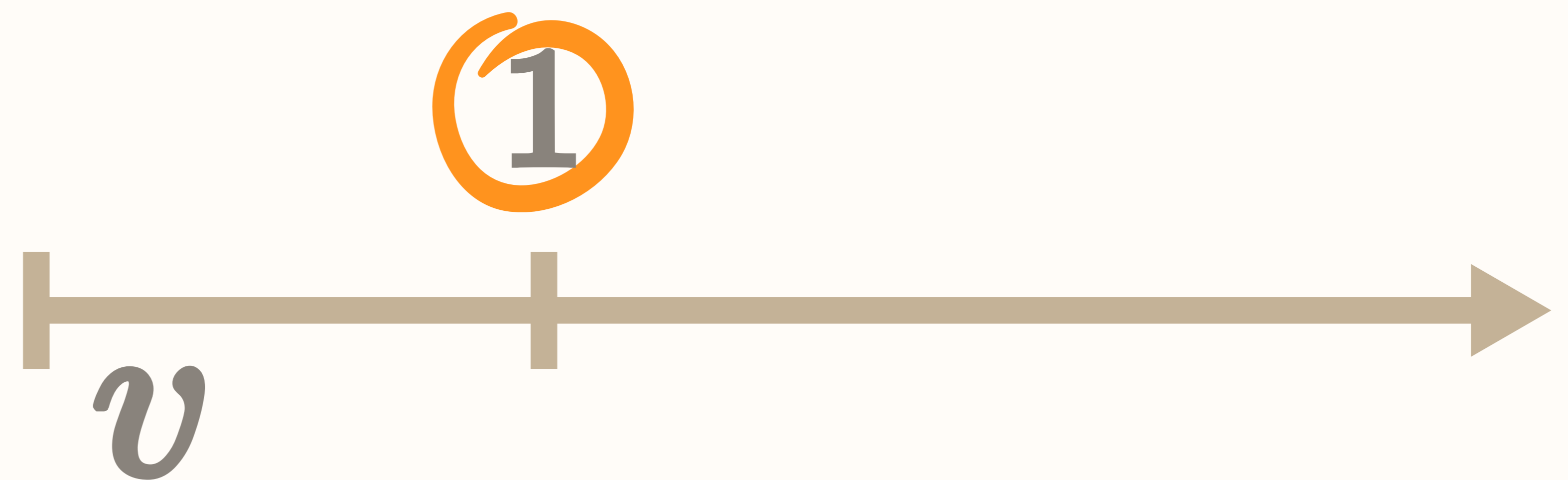
$$t_n \sim \kappa \underline{\gamma(v)}^n n^{-1 - \frac{\pi}{\underline{\arccos(\xi(v))}}}$$



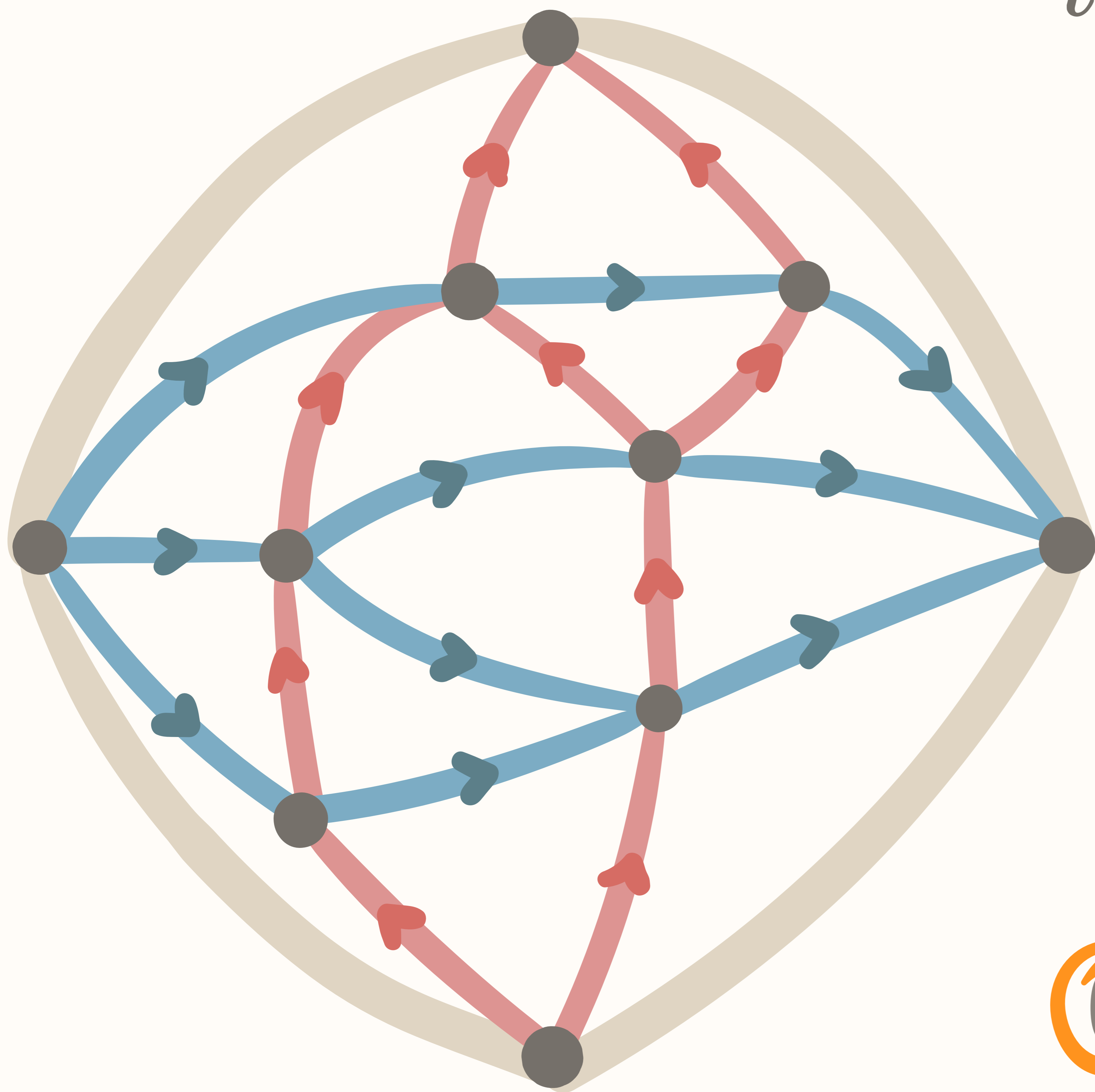
Generic transversal structures



$$t_n \sim \kappa \underline{\gamma(v)}^n n^{-1 - \frac{\pi}{\arccos(\underline{\xi(v)})}}$$



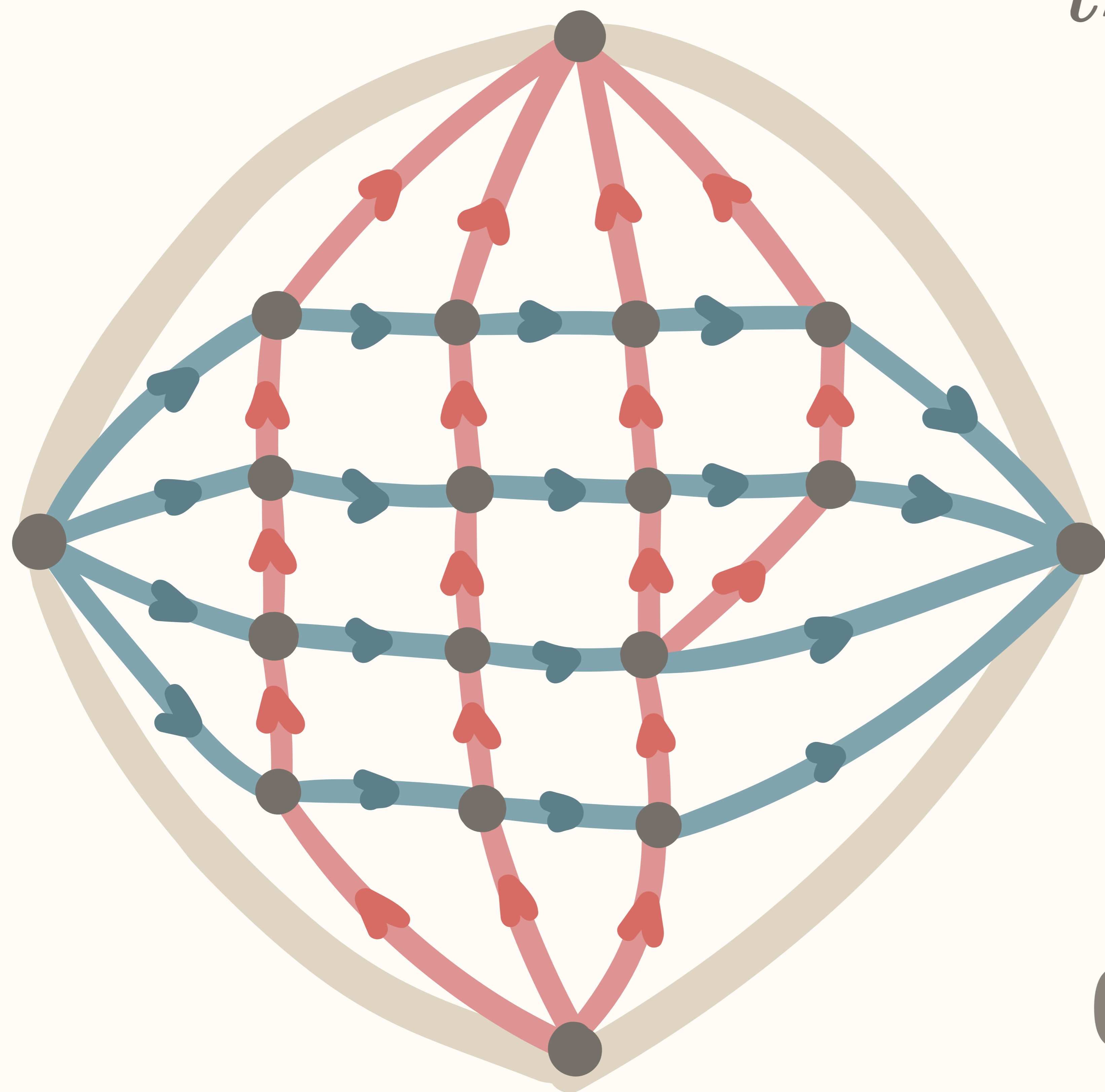
Generic transversal structures



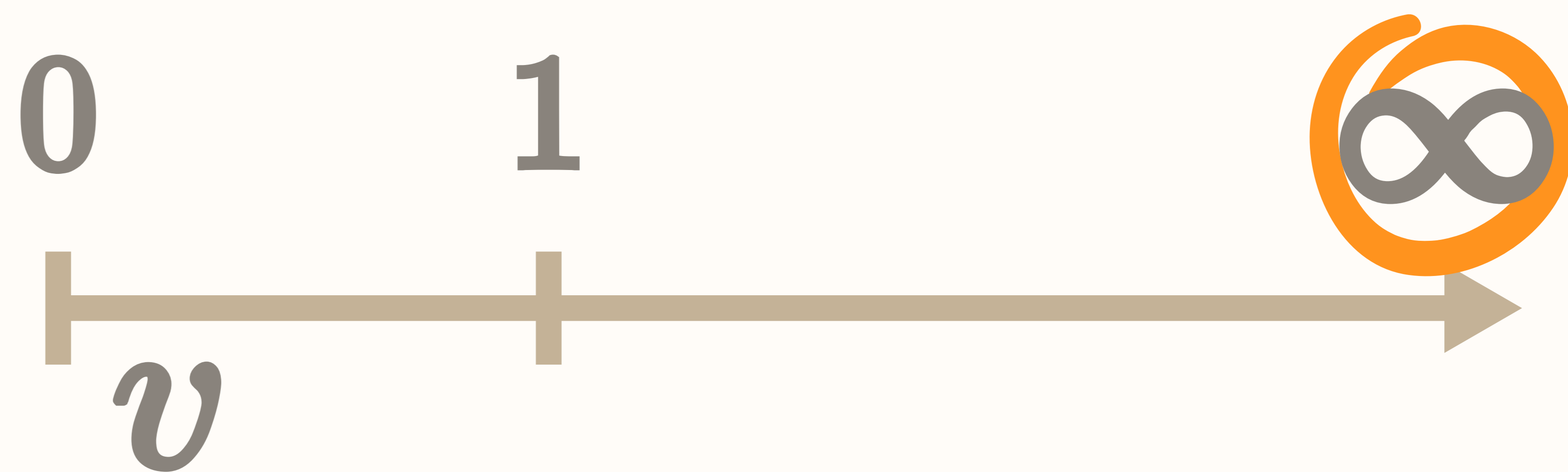
$$t_n \sim \kappa \left(\frac{27}{2} \right)^n n^{-1 - \frac{\pi}{\arccos(7/8)}}$$



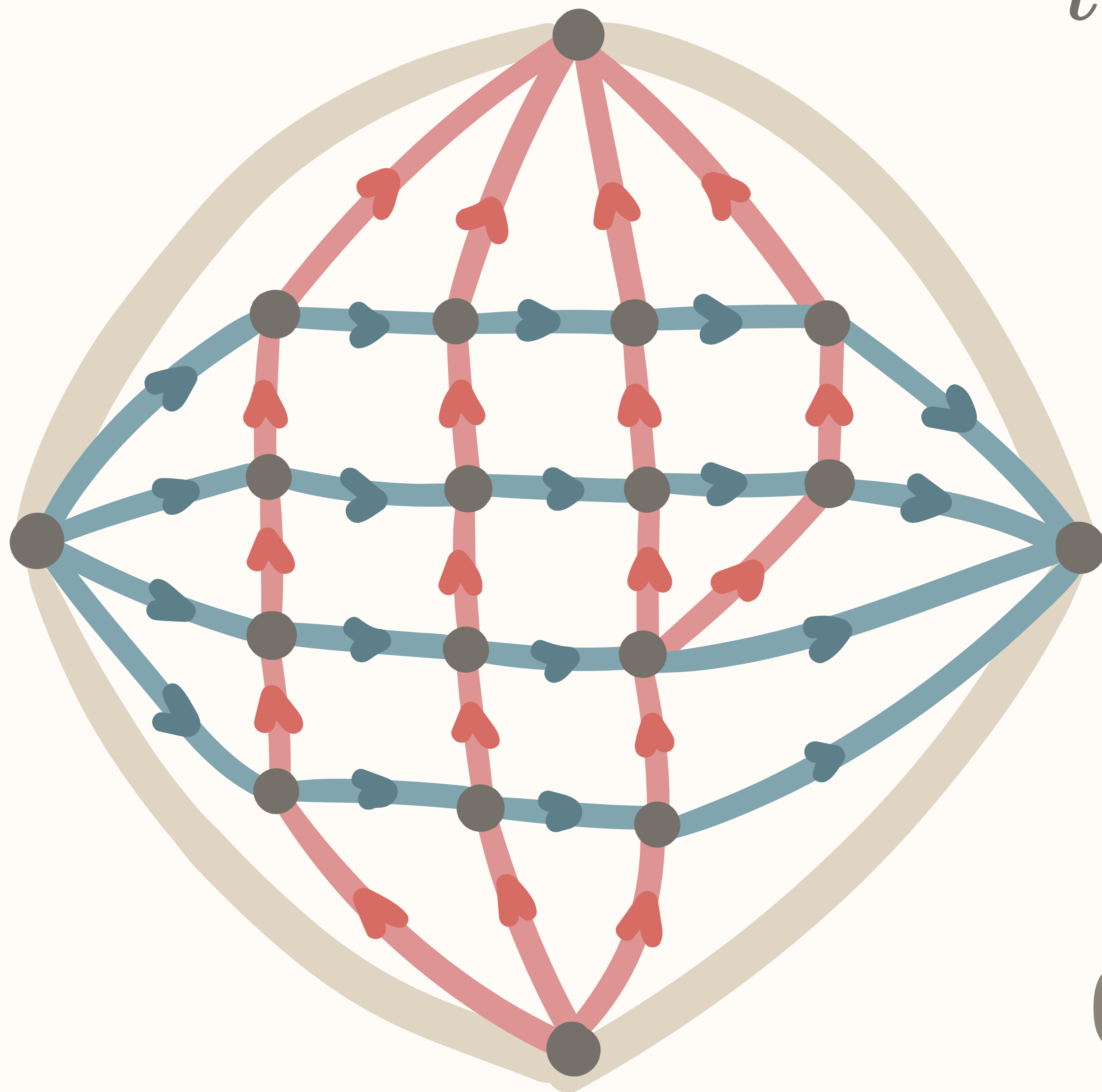
Generic transversal structures



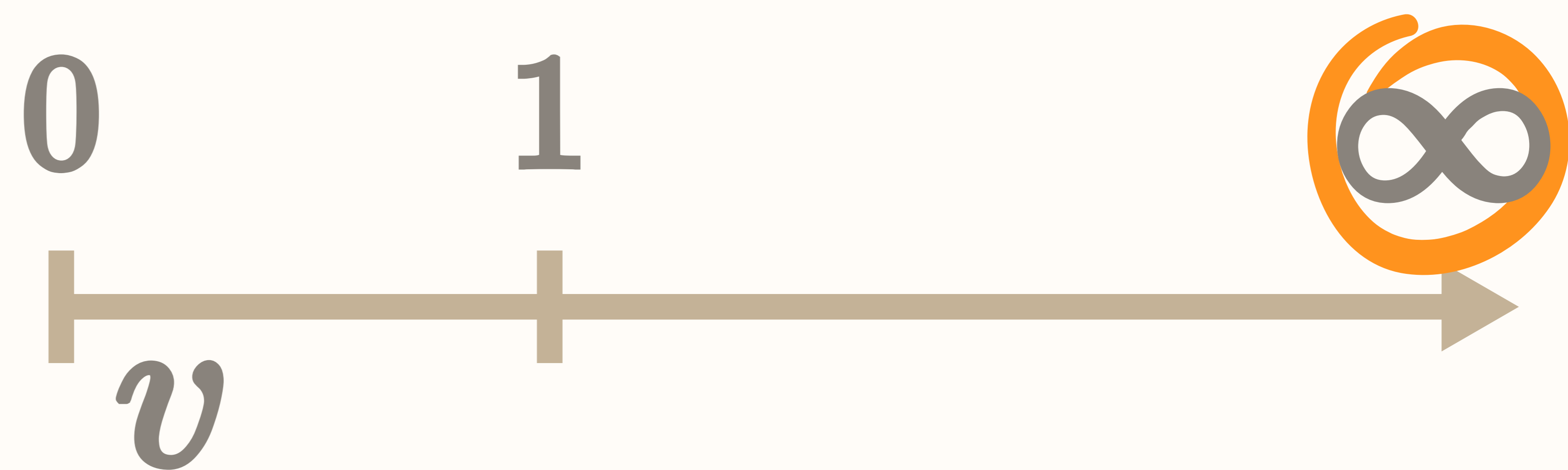
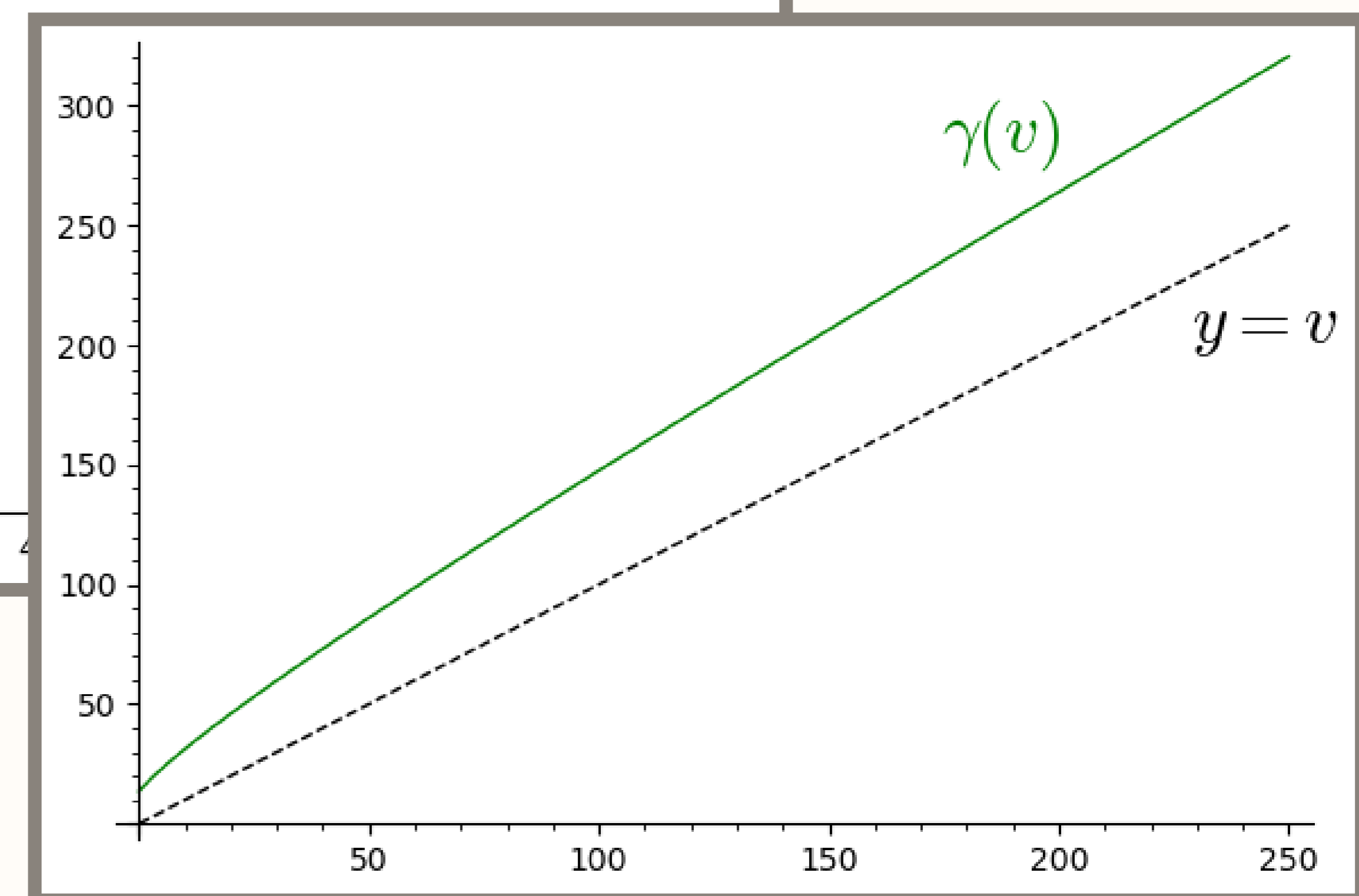
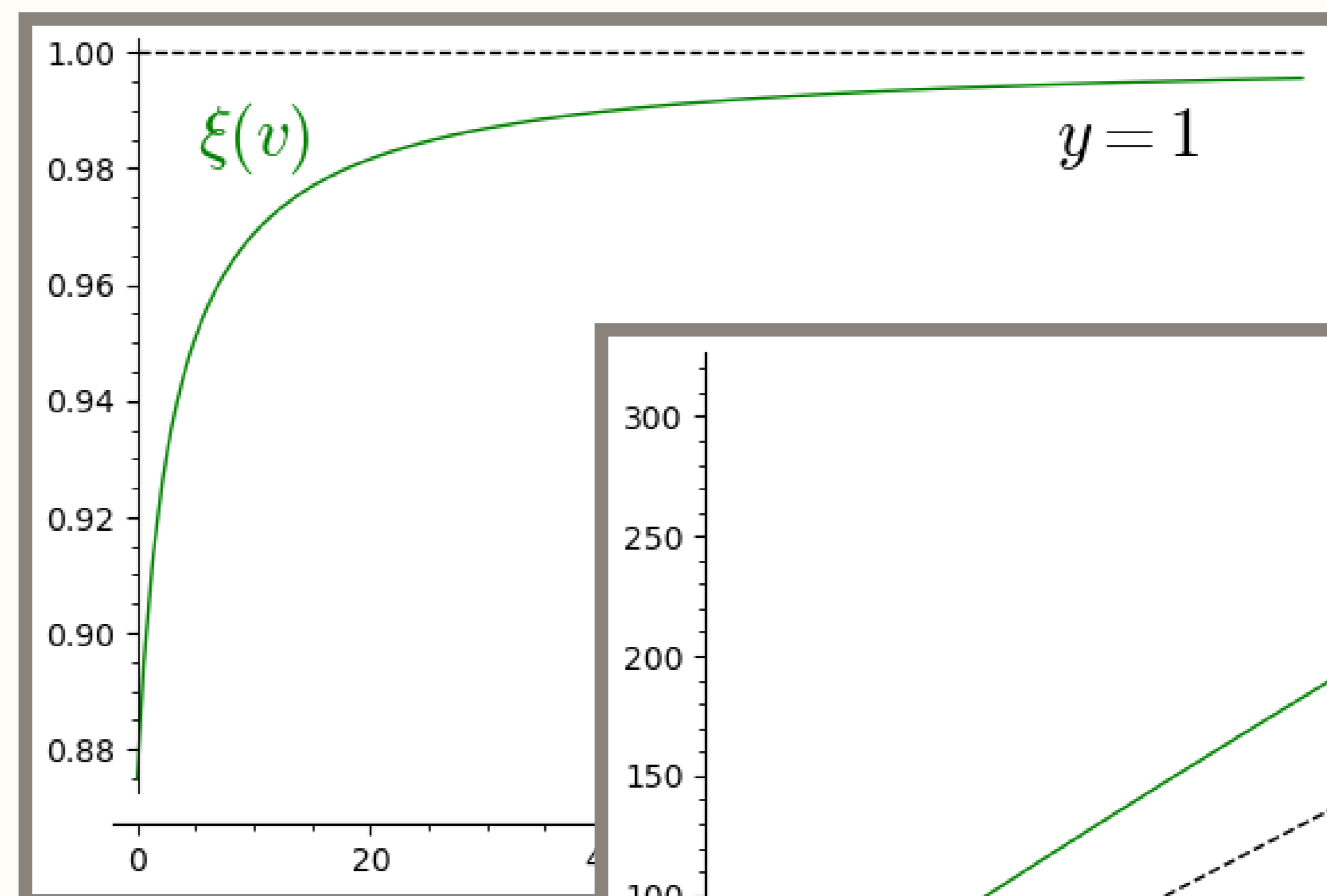
$$t_n \sim \kappa \underline{\gamma(v)}^n n^{-1 - \frac{\pi}{\arccos(\underline{\xi(v)})}}$$



Generic transversal structures



$$t_n \sim \kappa \underline{\gamma(v)}^n n^{-1 - \frac{\pi}{\arccos(\underline{\xi(v)})}}$$



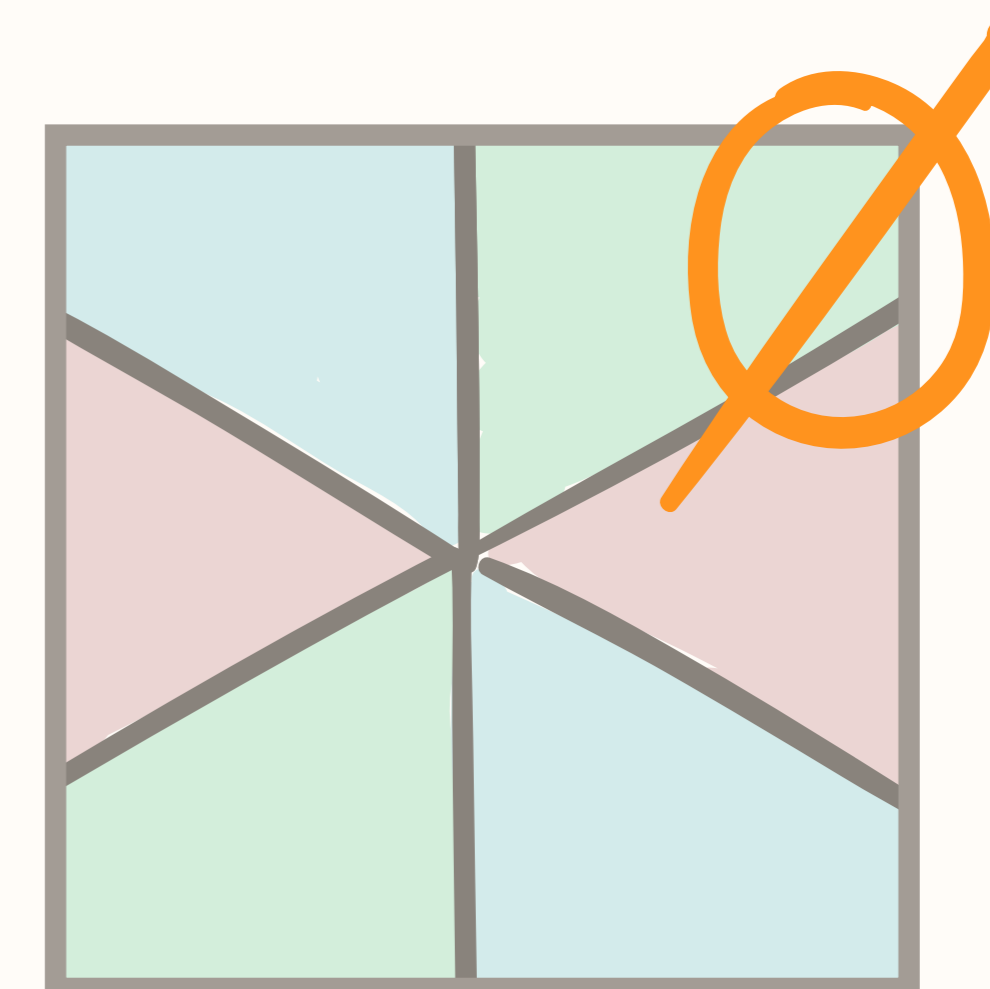
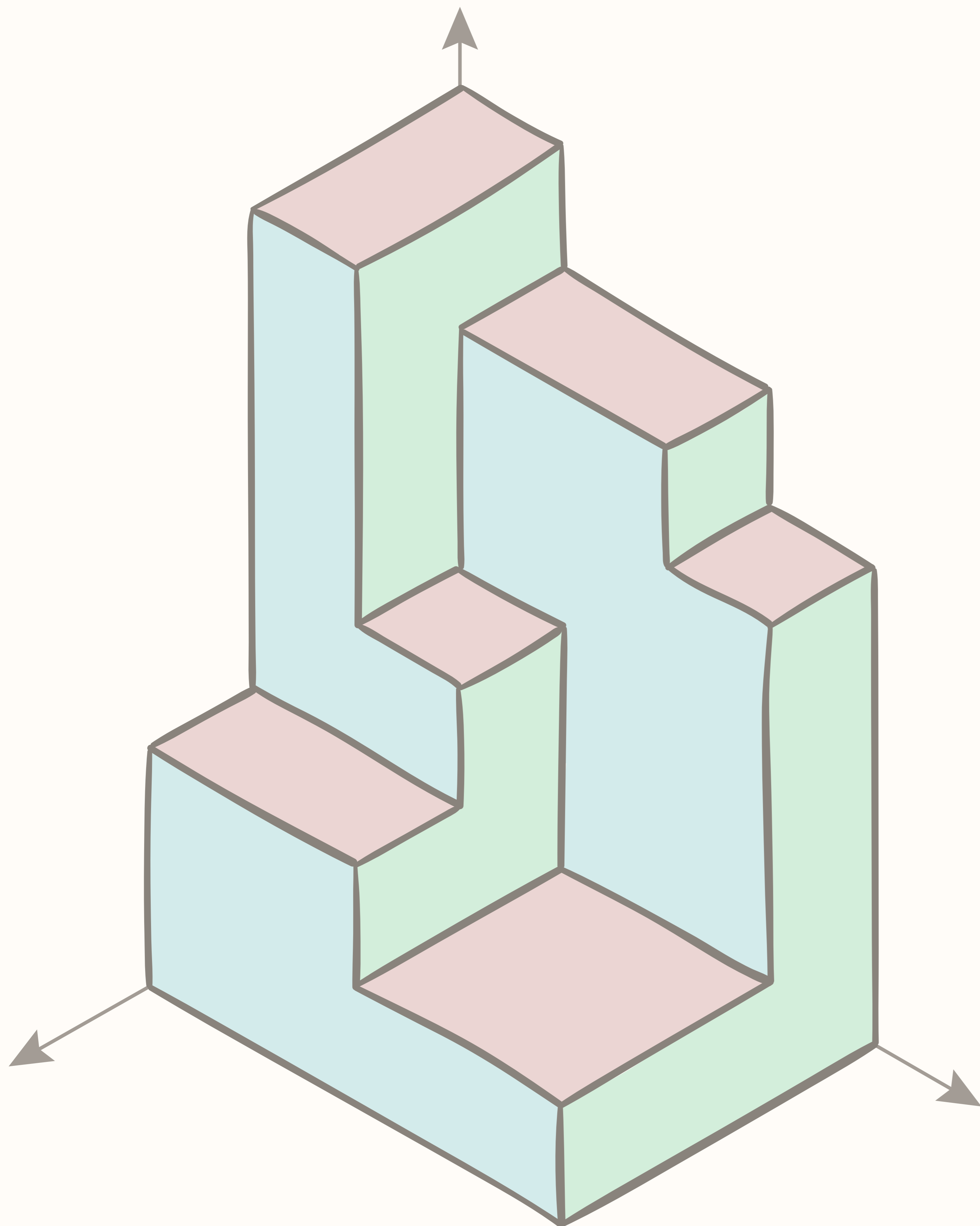
ANNEXES

A. Generalization for transversal structures

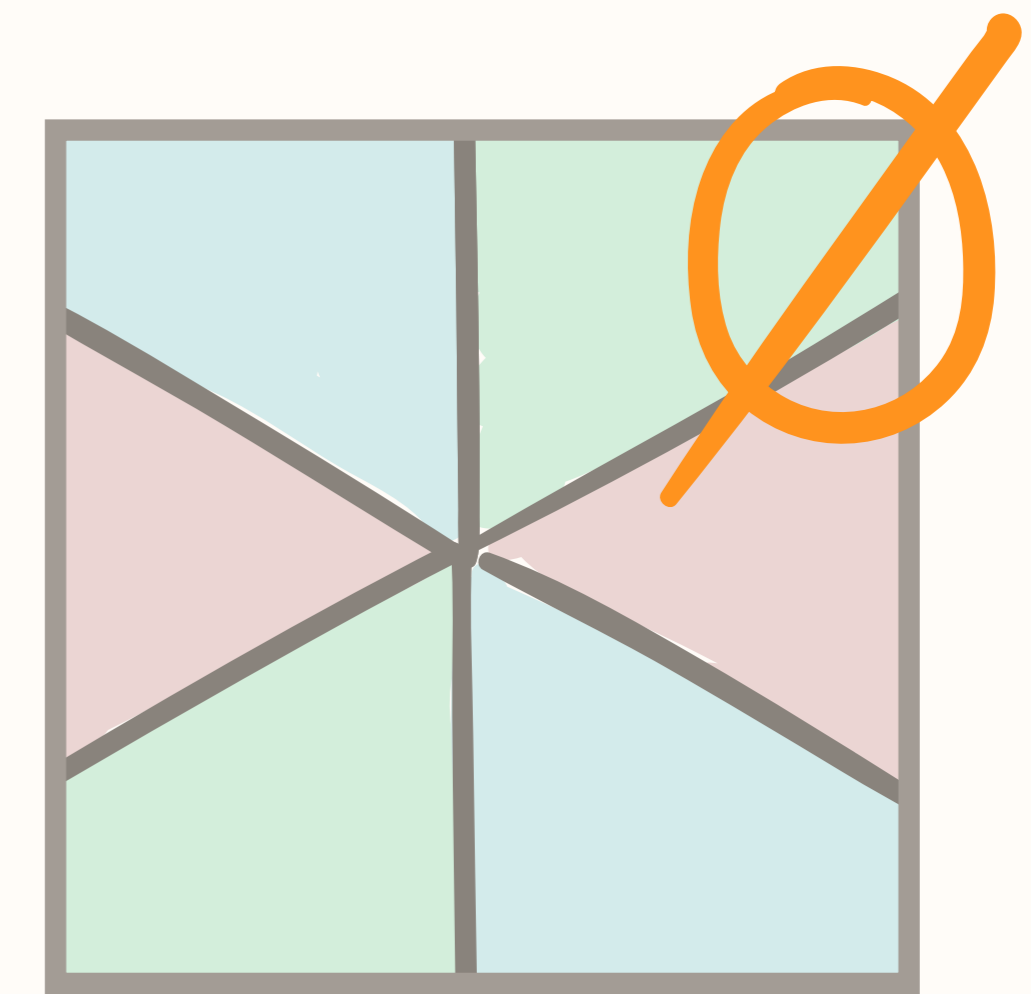
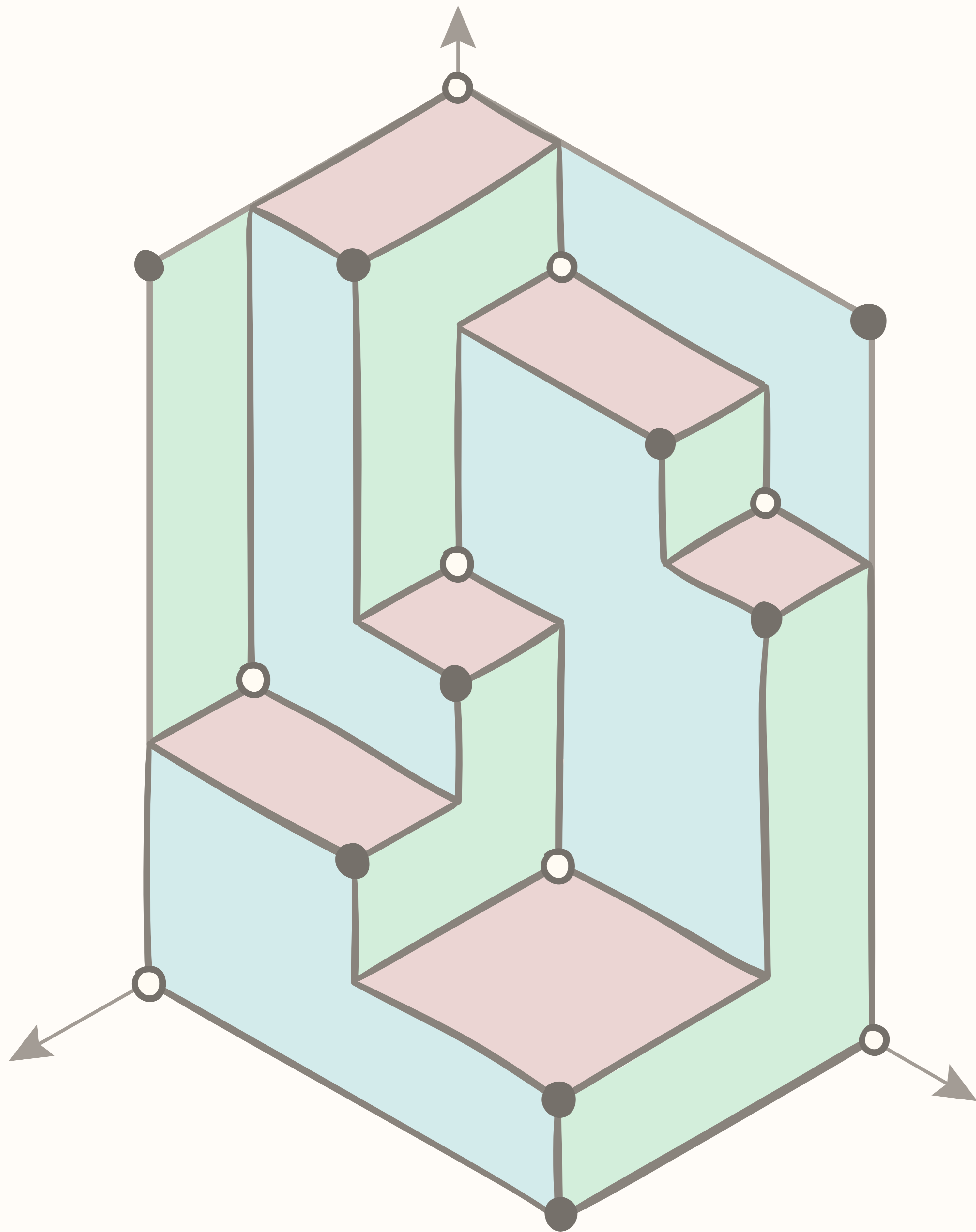
B. Schnyder labelings

C. Bijection for plane permutations

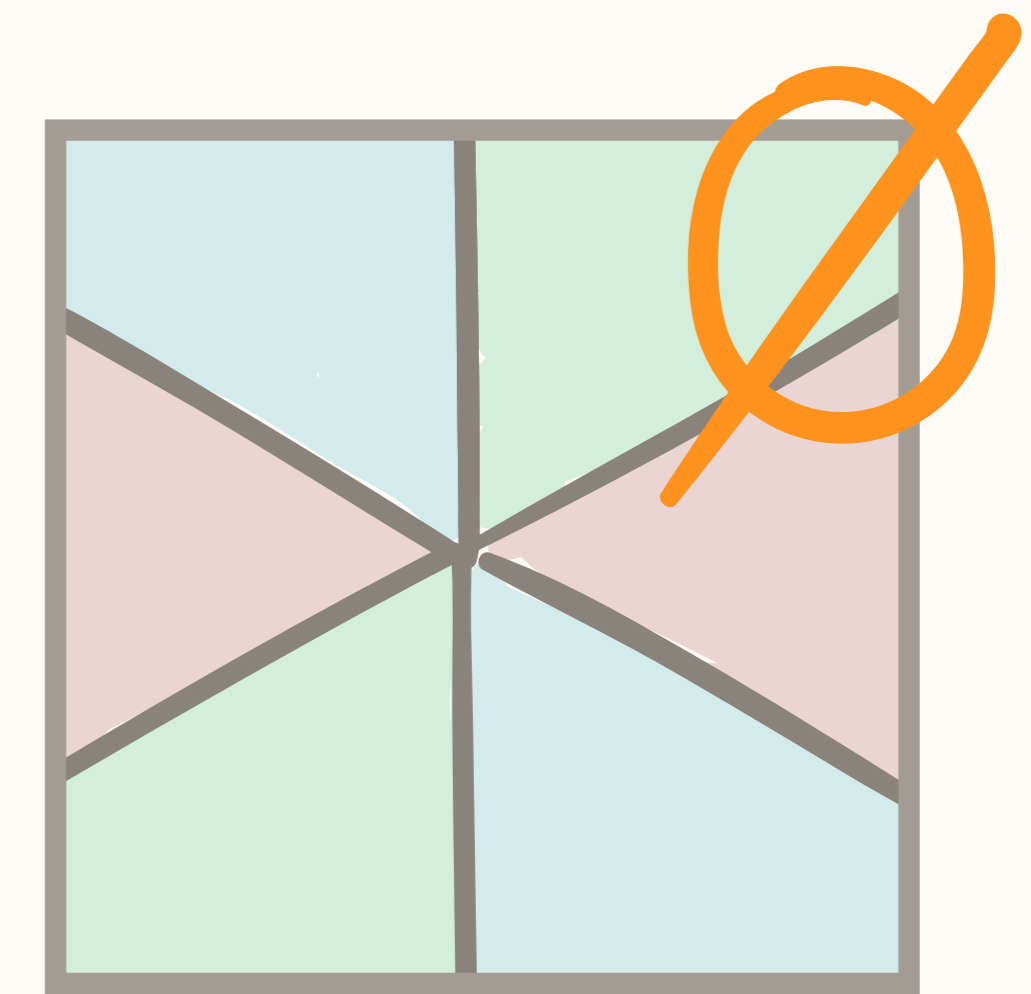
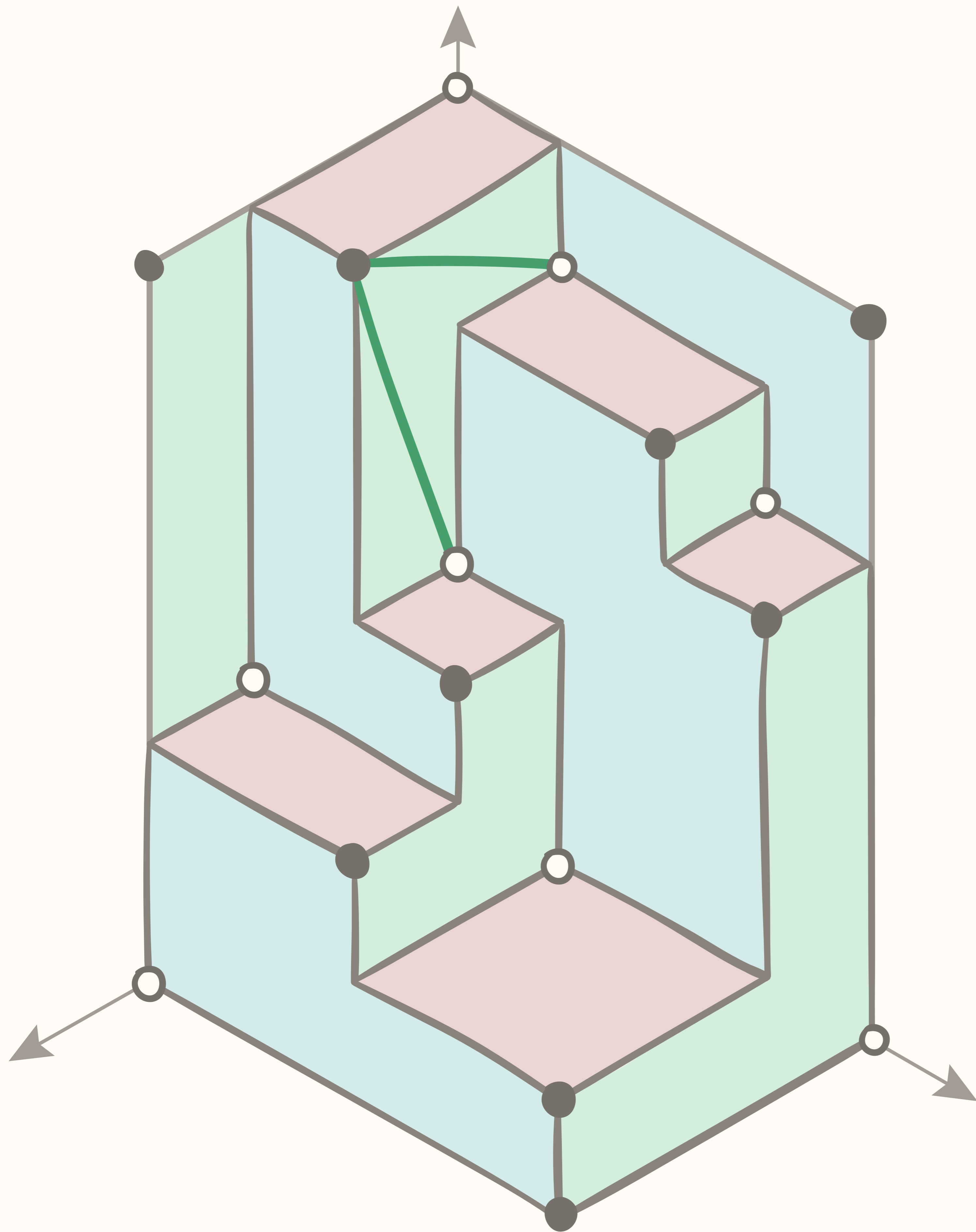
Rigid corner polyhedra



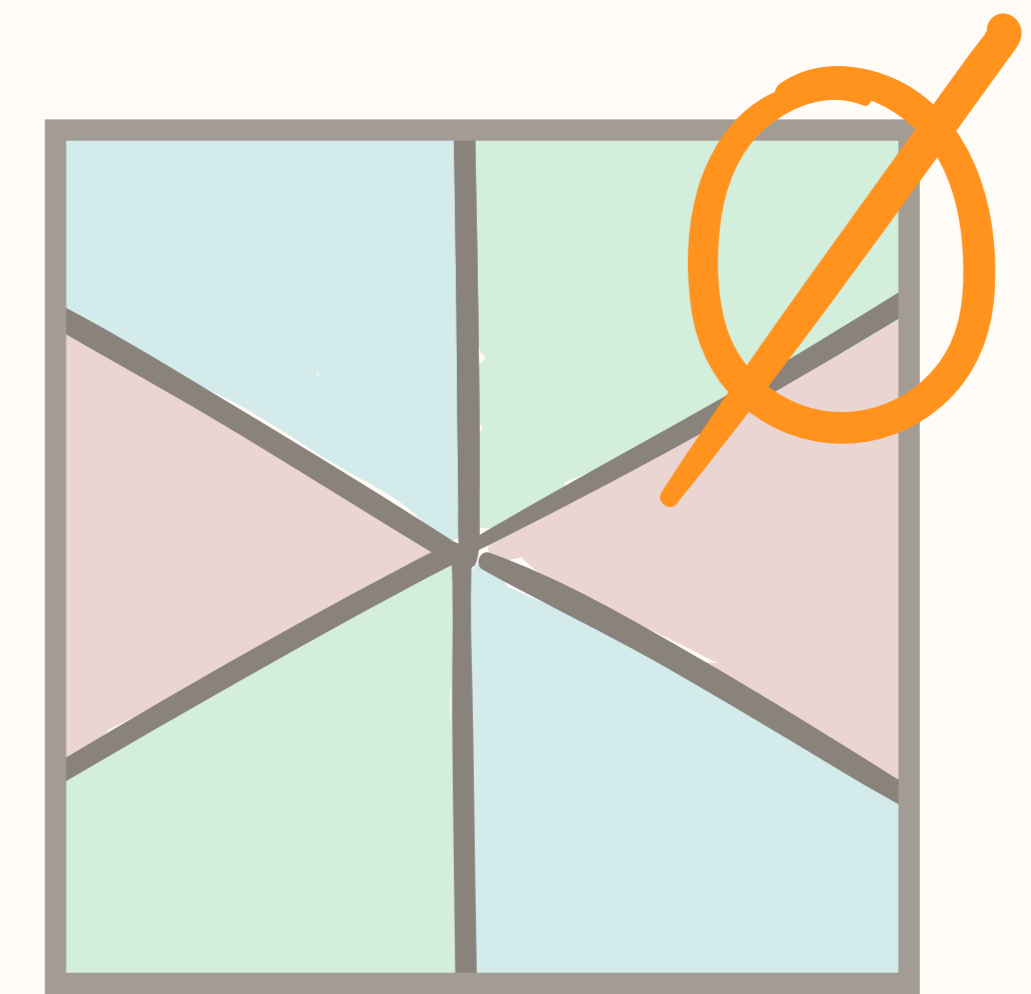
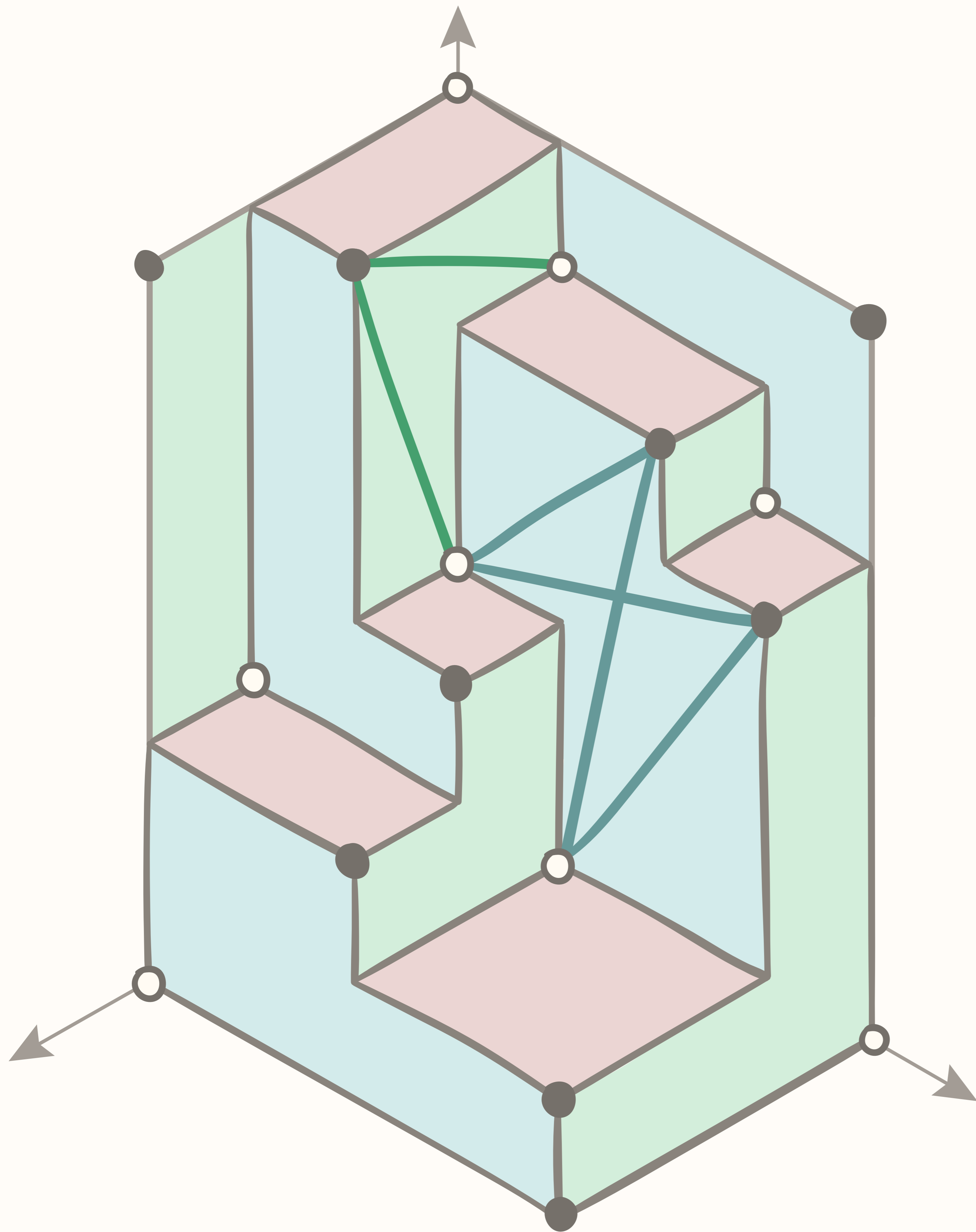
Rigid corner polyhedra



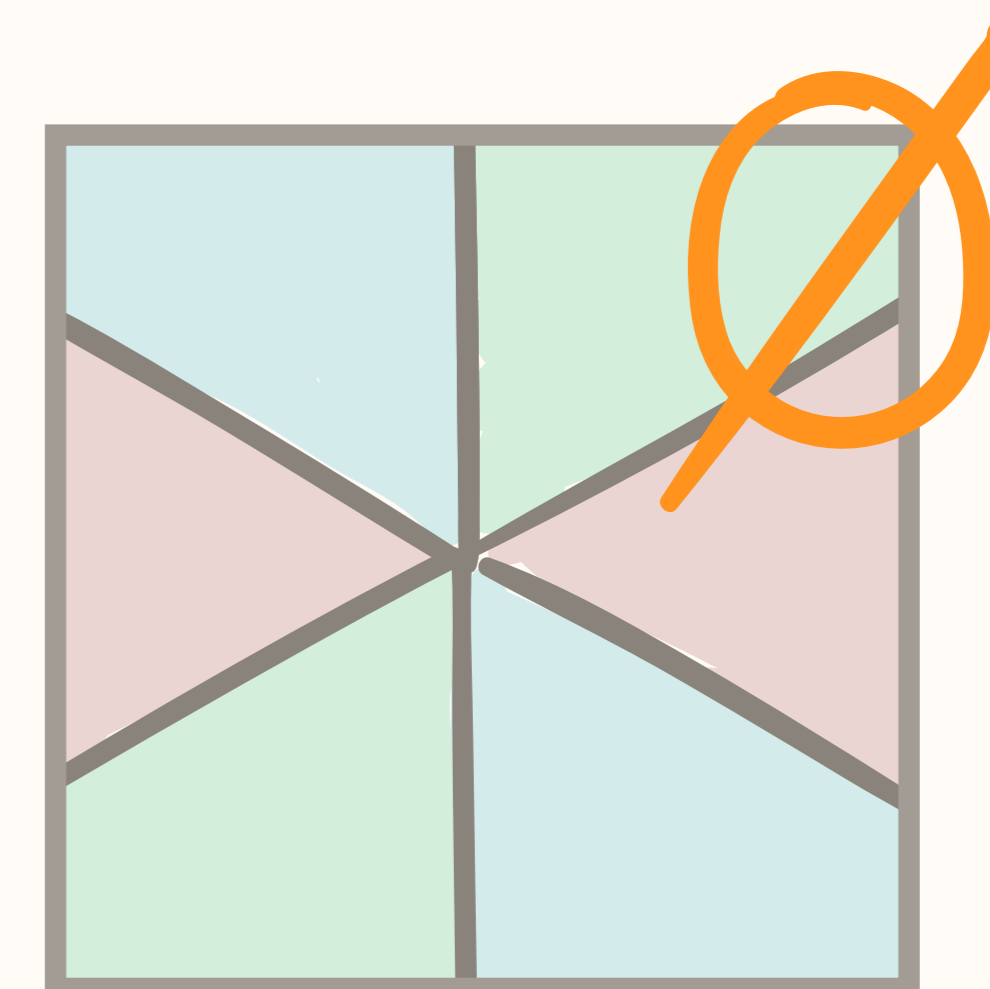
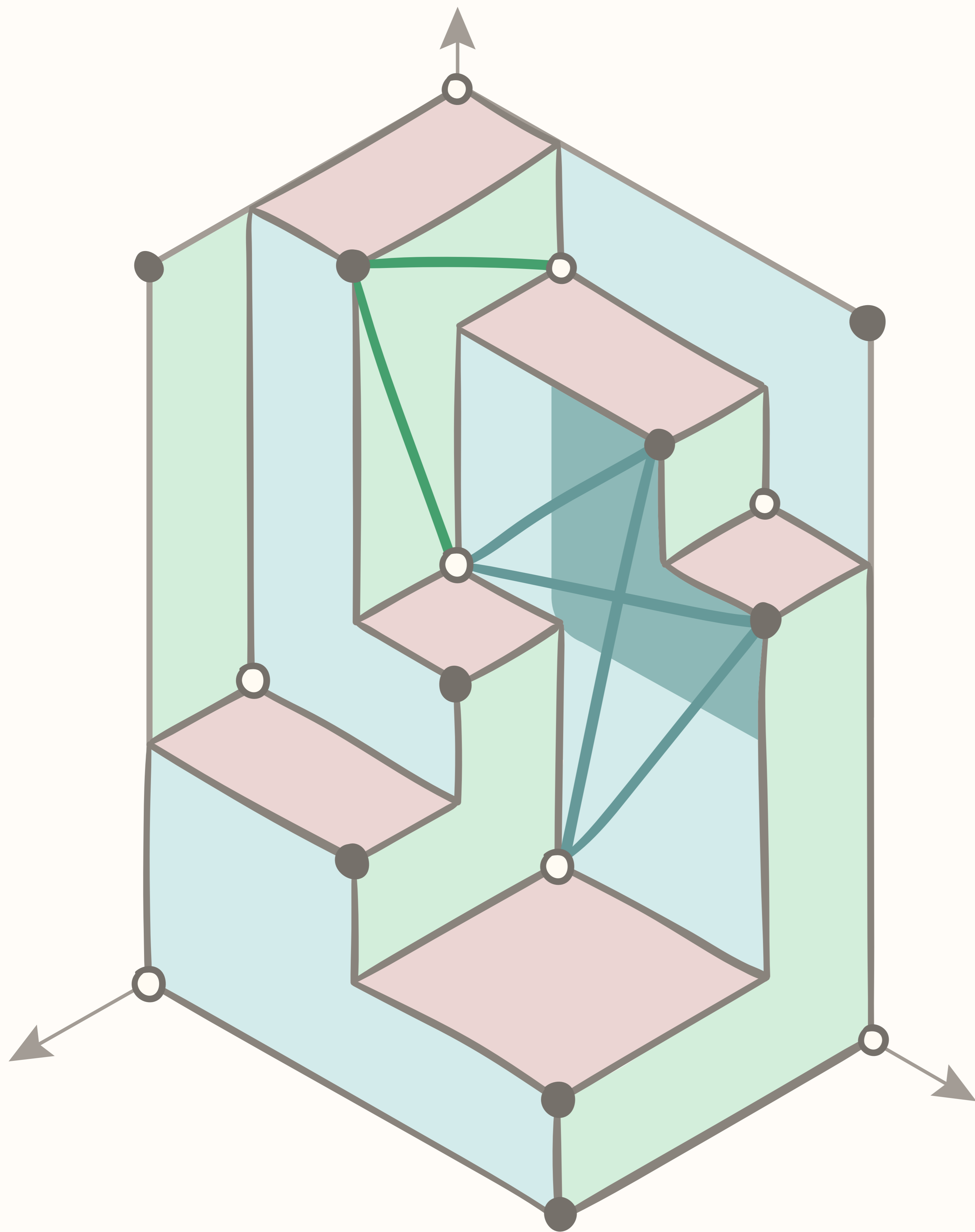
Rigid corner polyhedra



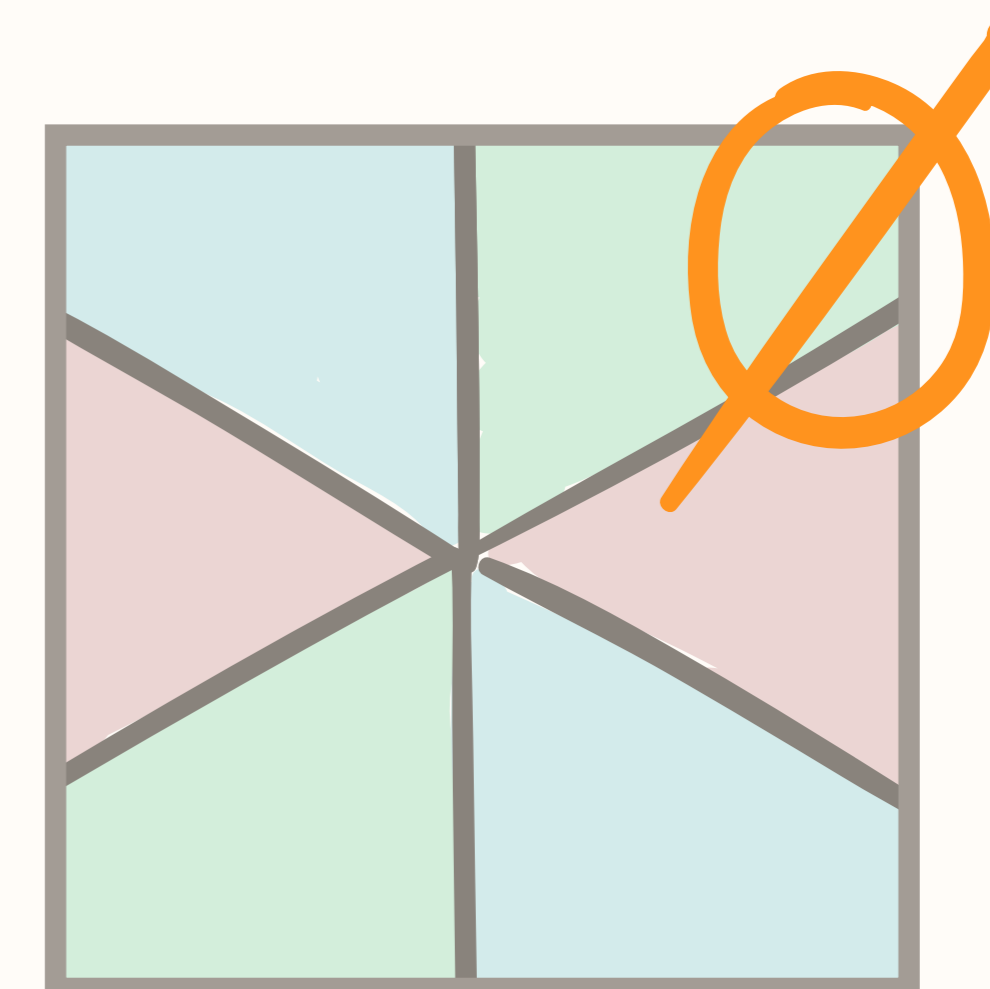
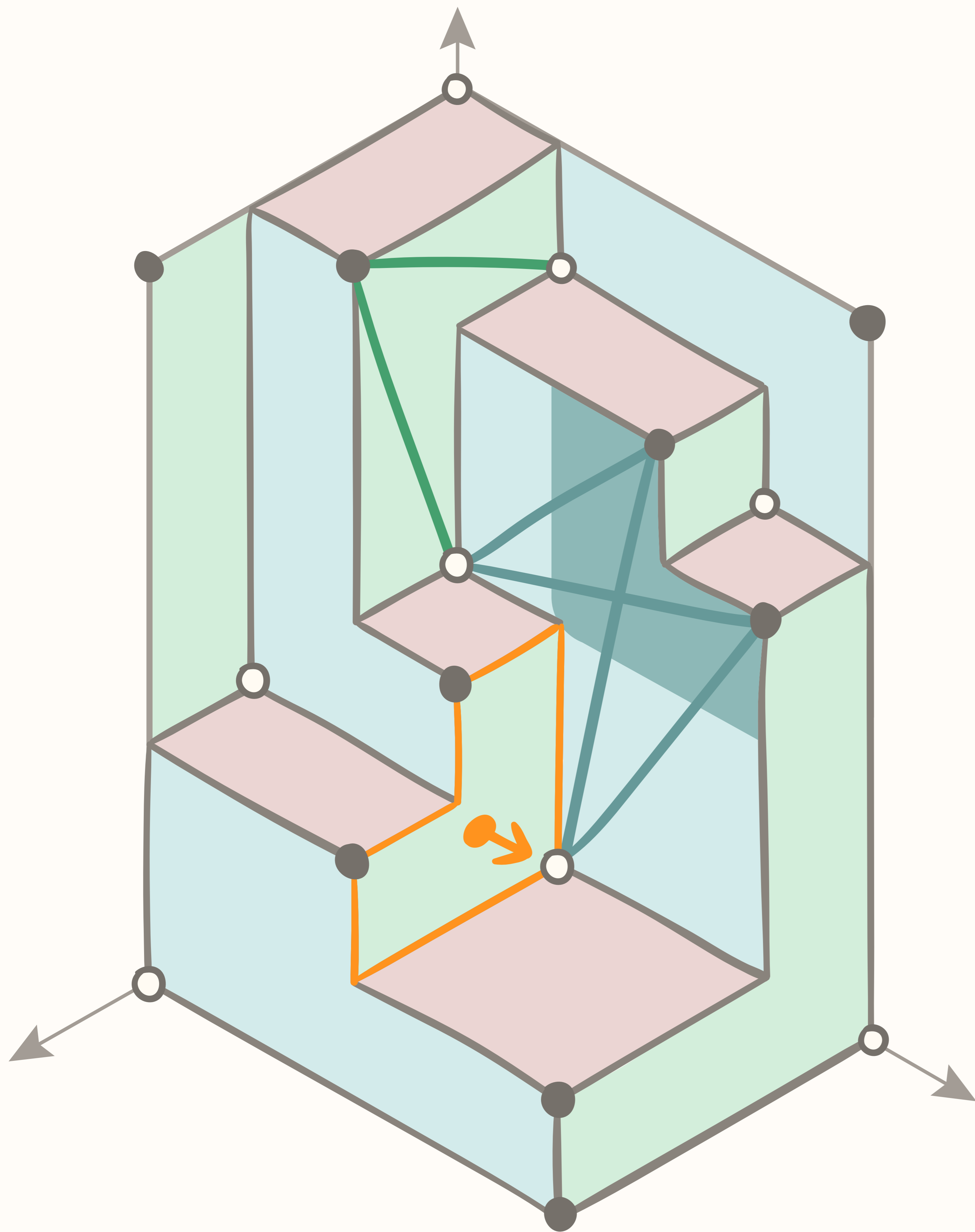
Rigid corner polyhedra



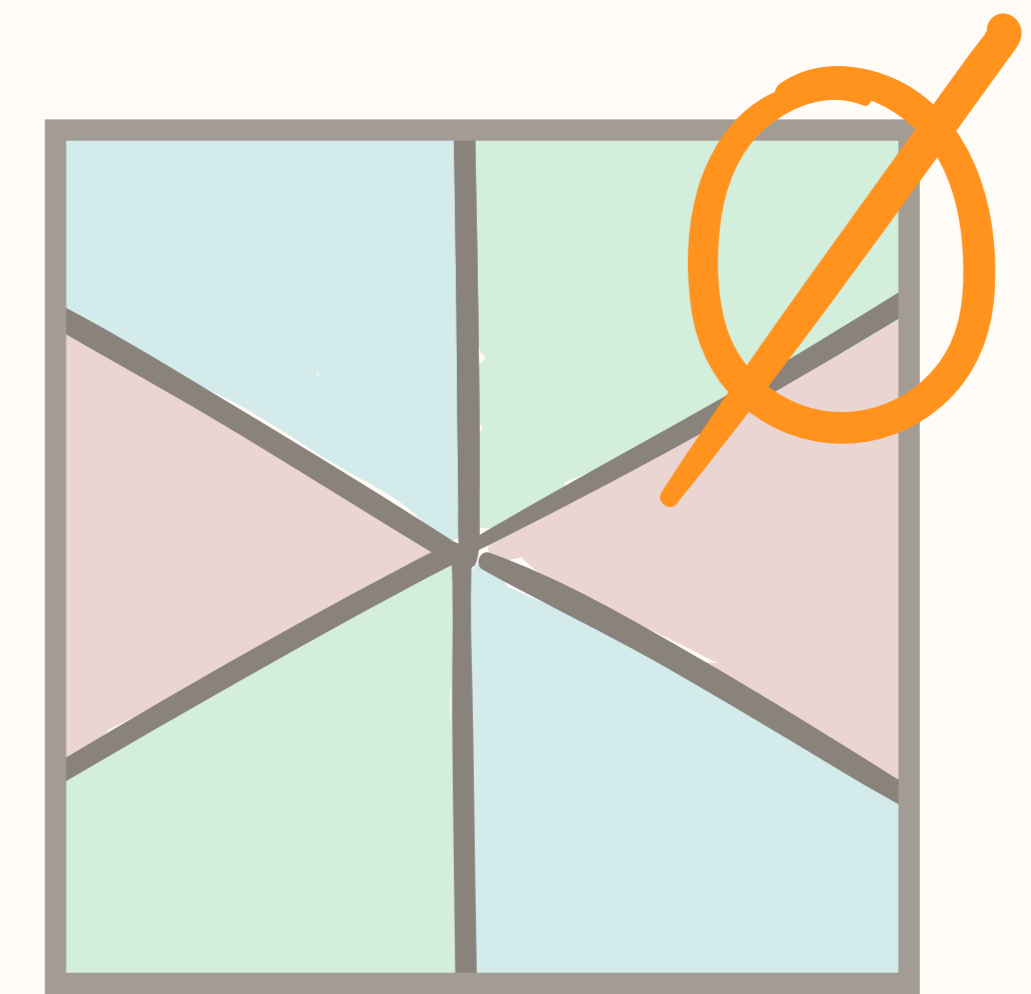
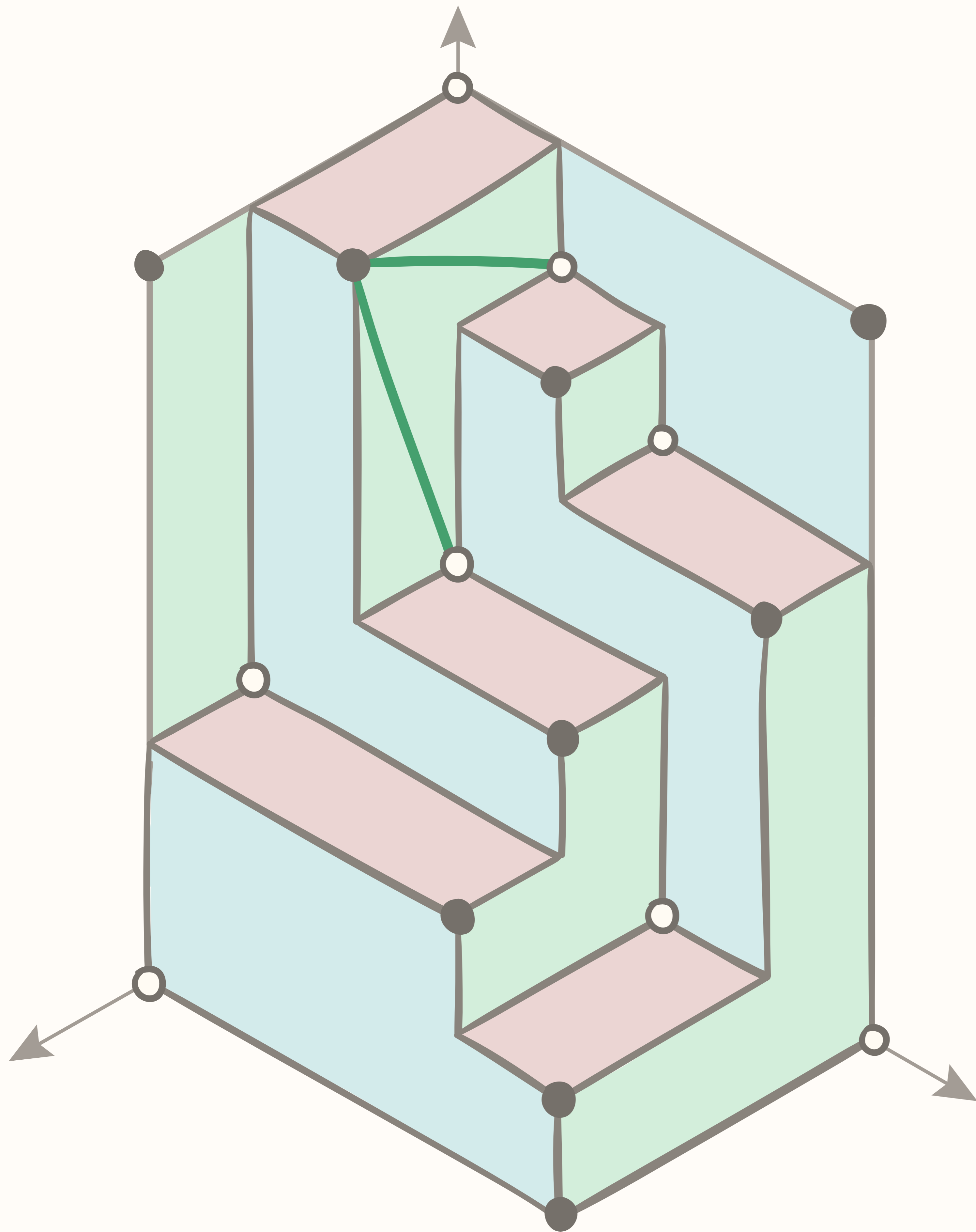
Rigid corner polyhedra



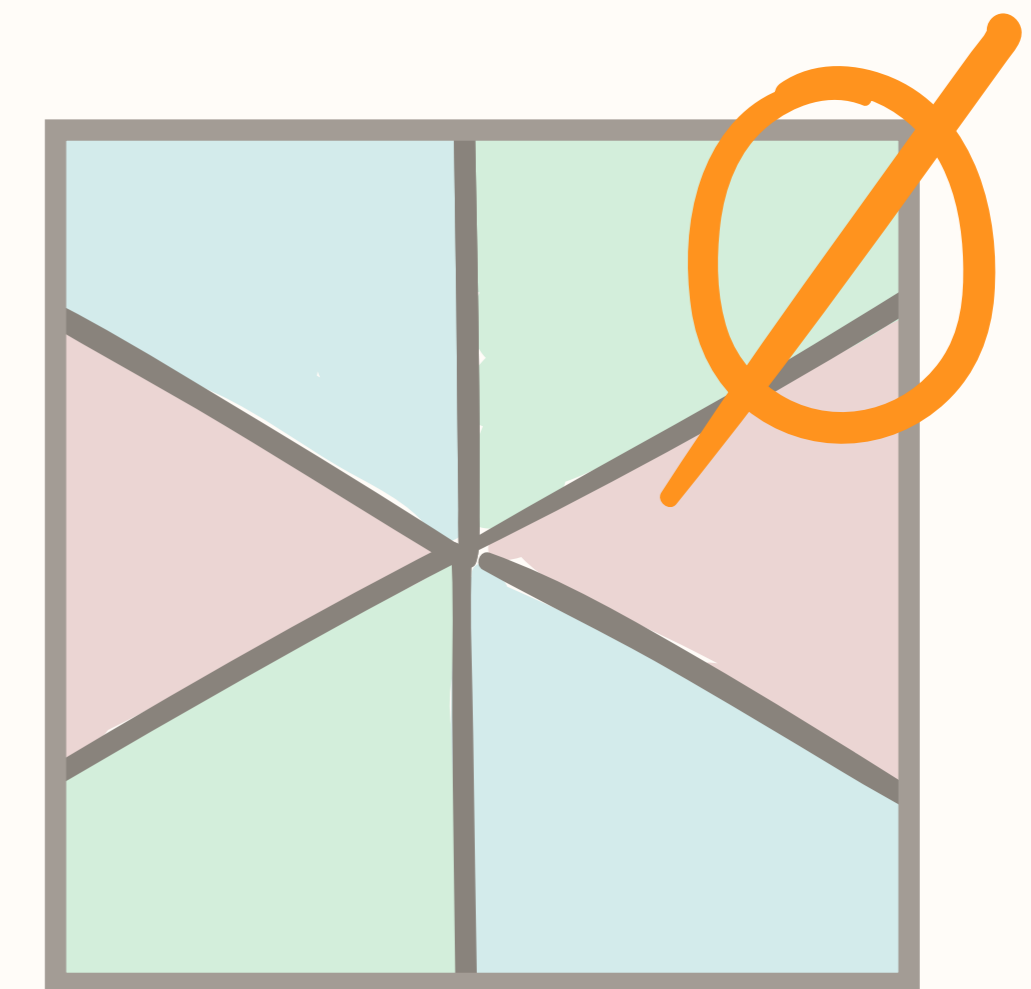
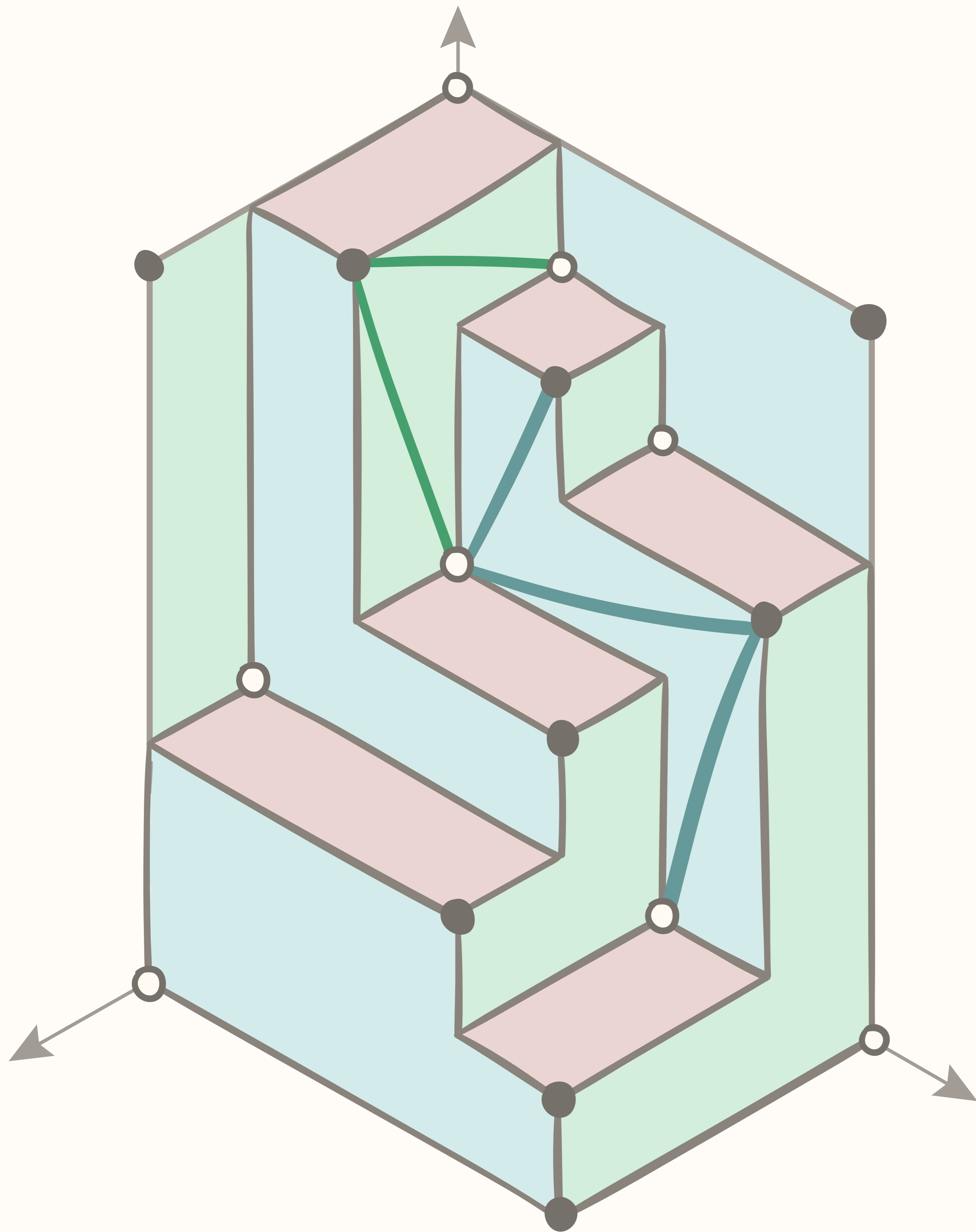
Rigid corner polyhedra



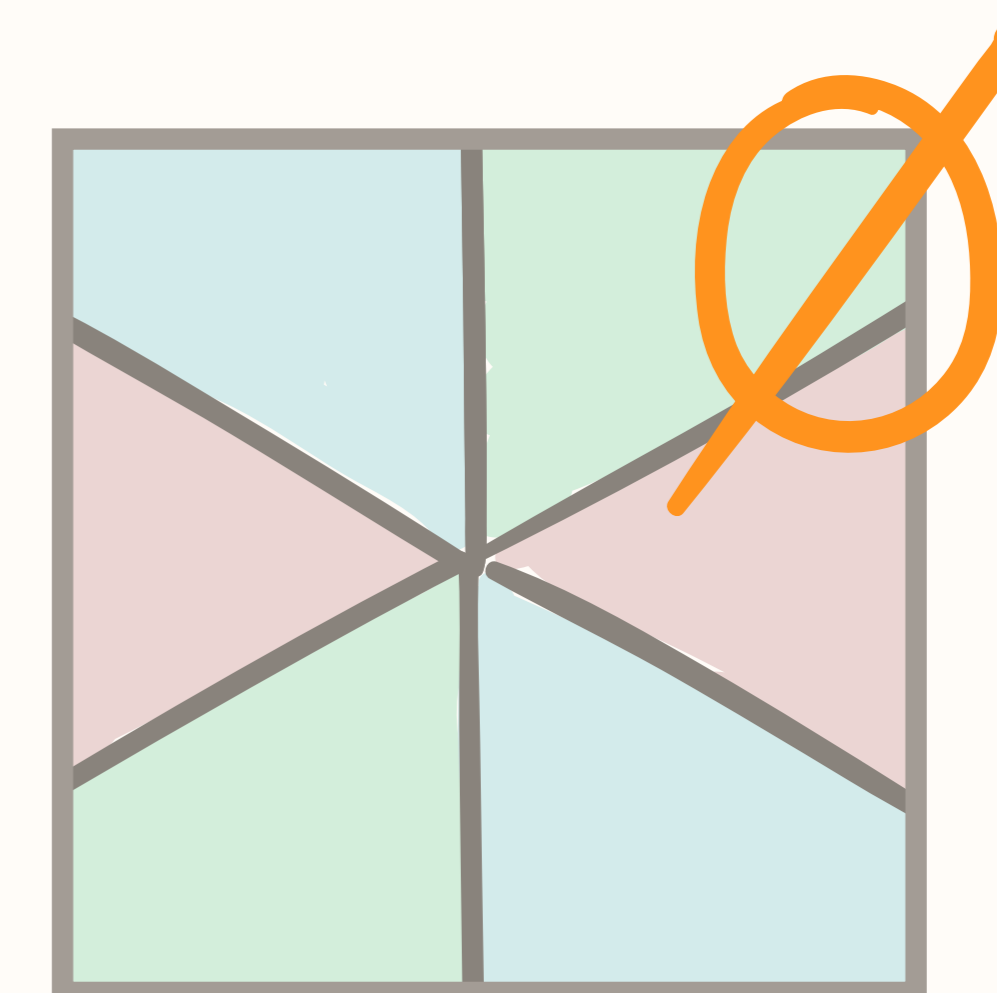
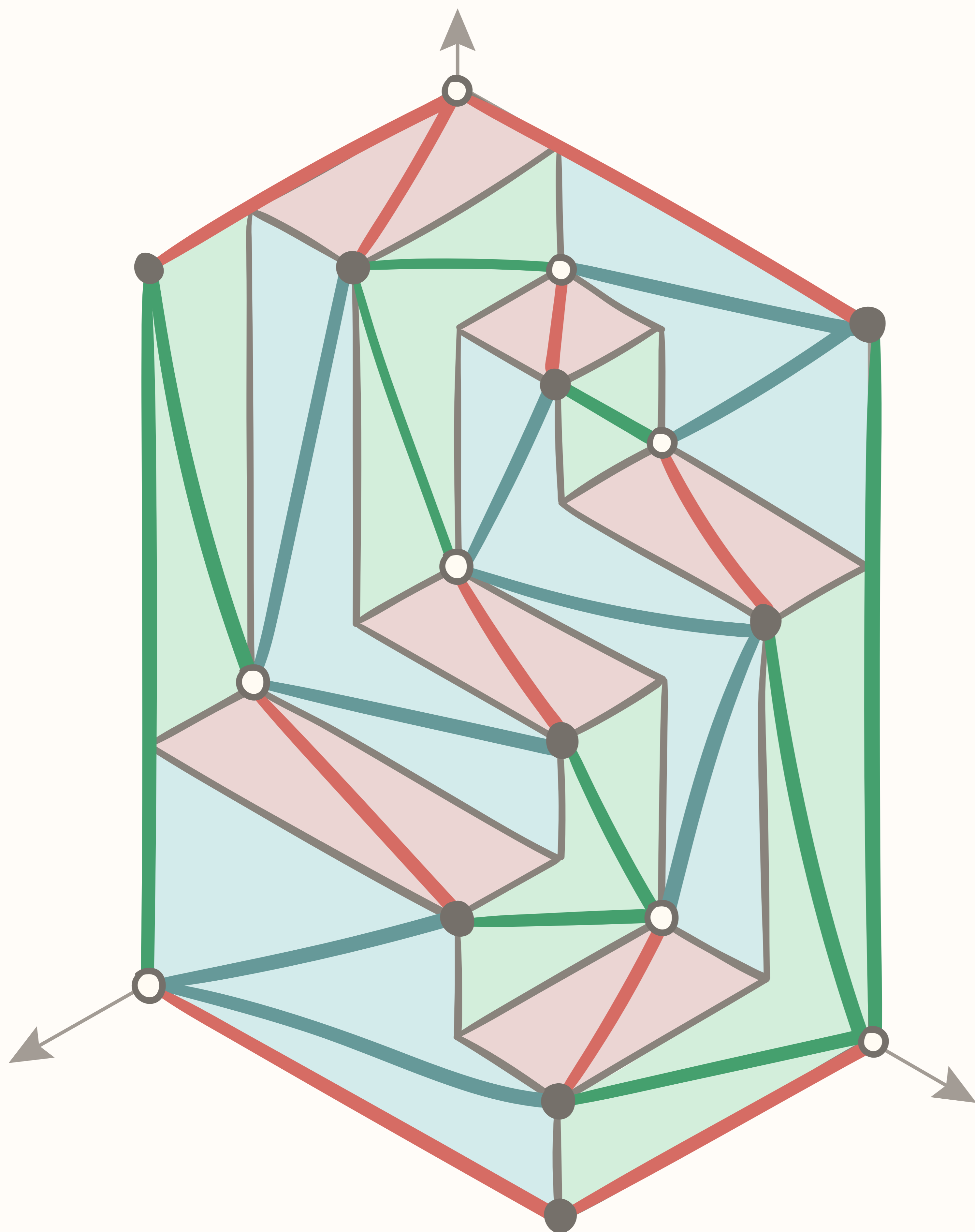
Rigid corner polyhedra



Rigid corner polyhedra



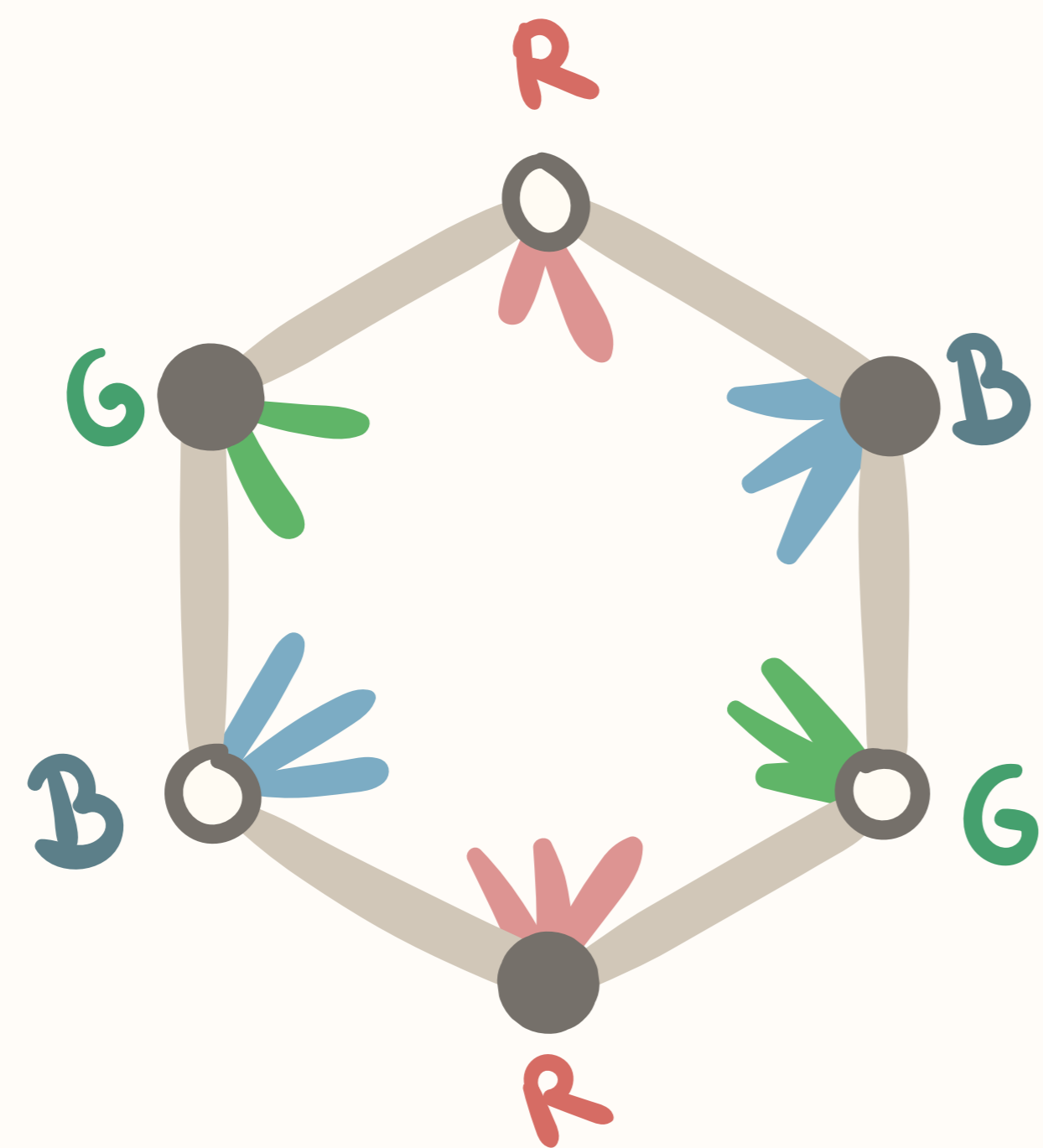
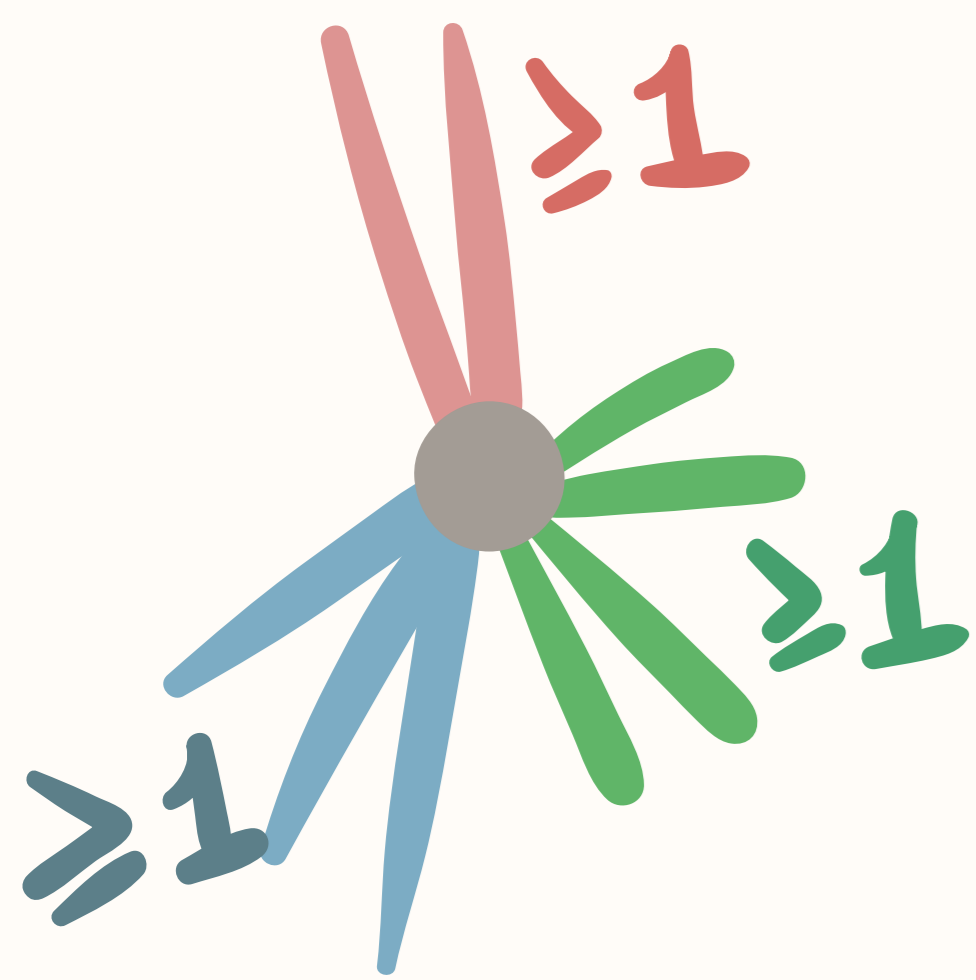
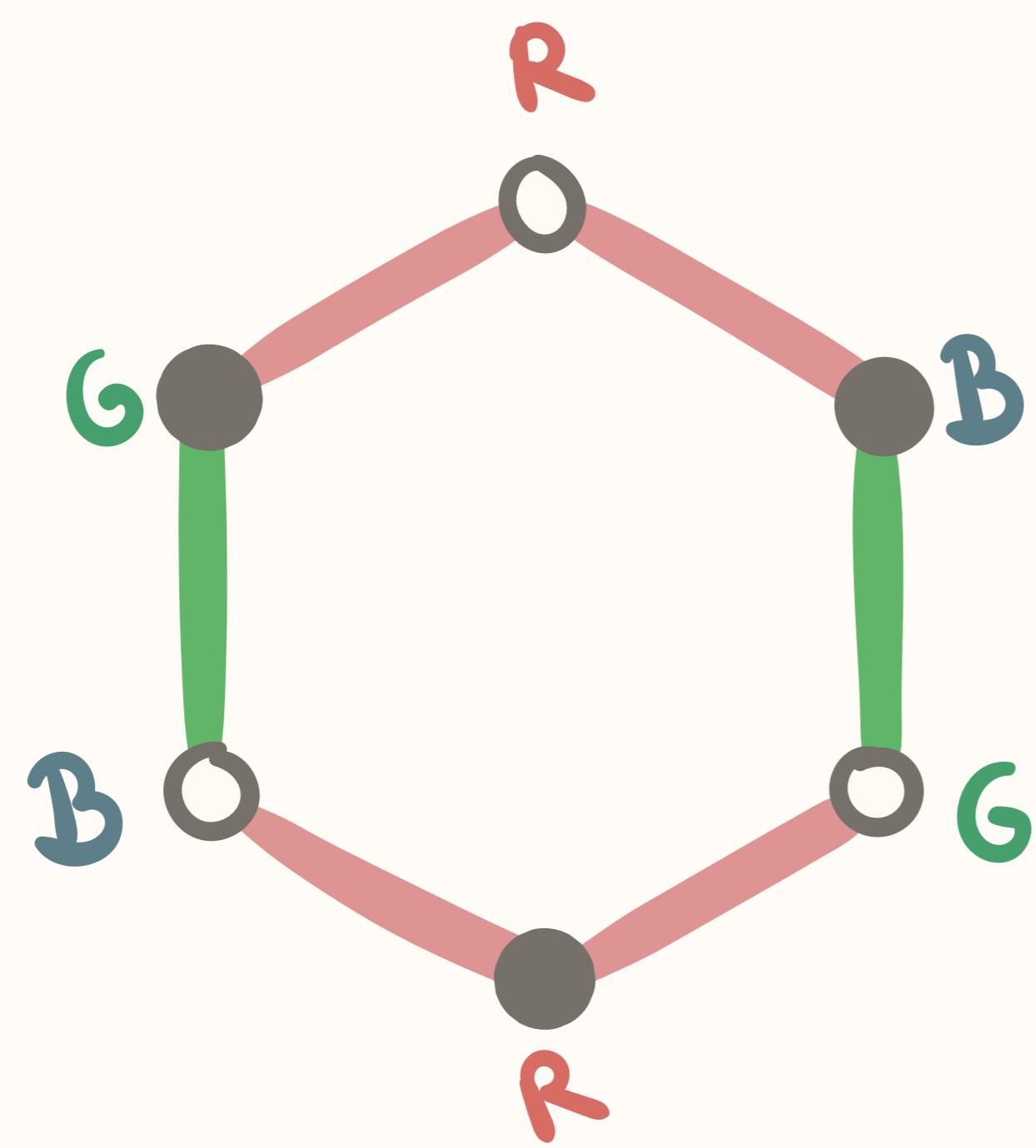
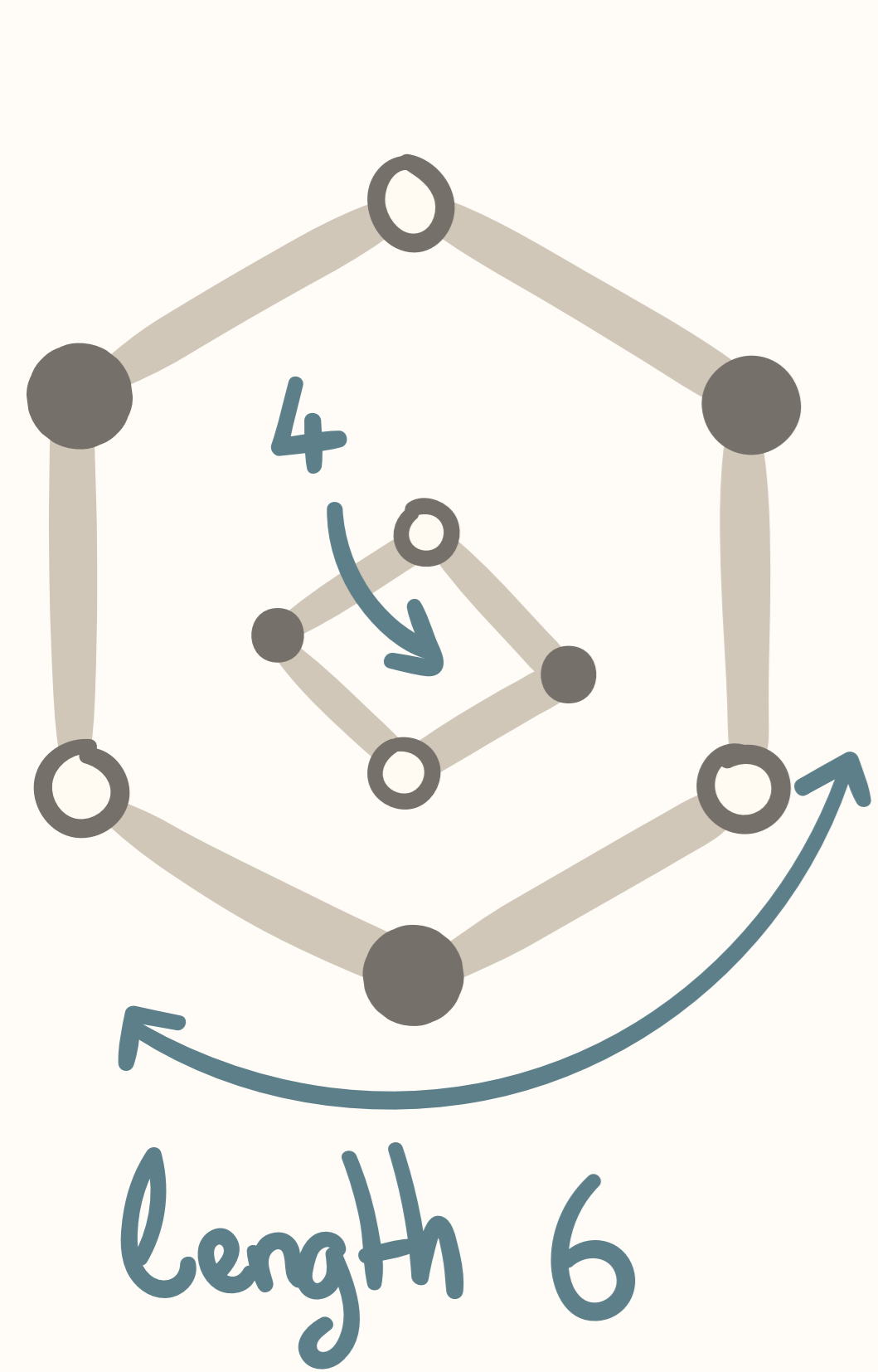
Rigid corner polyhedra



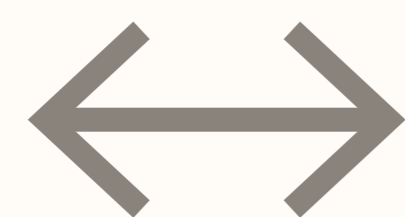
rigid corner polyhedra \longleftrightarrow *Schnyder labellings*

\Rightarrow *Geodesic embeddings and planar graphs,*
S. Felsner (2003)

Schnyder labellings

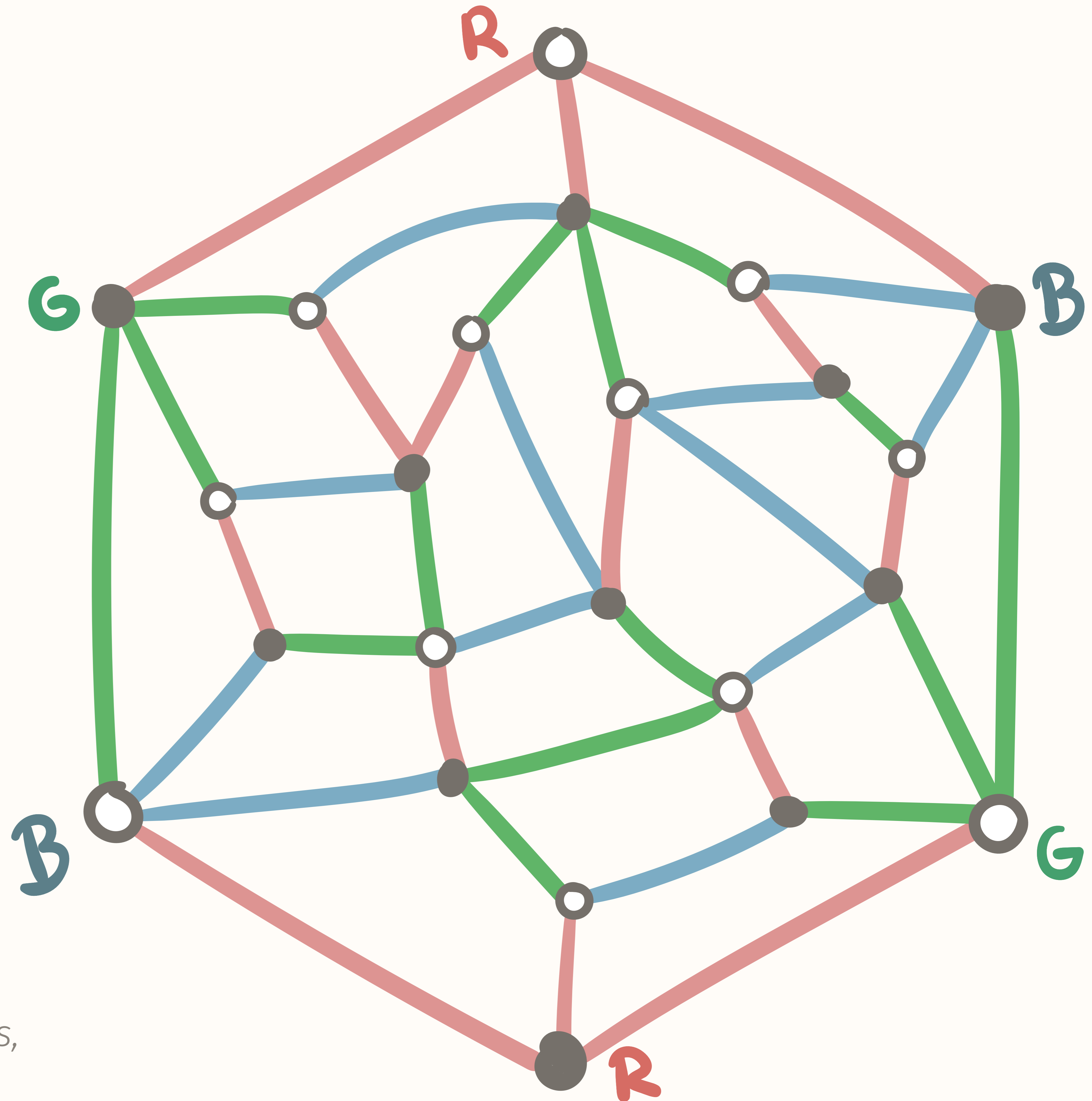


rigid corner
polyhedra

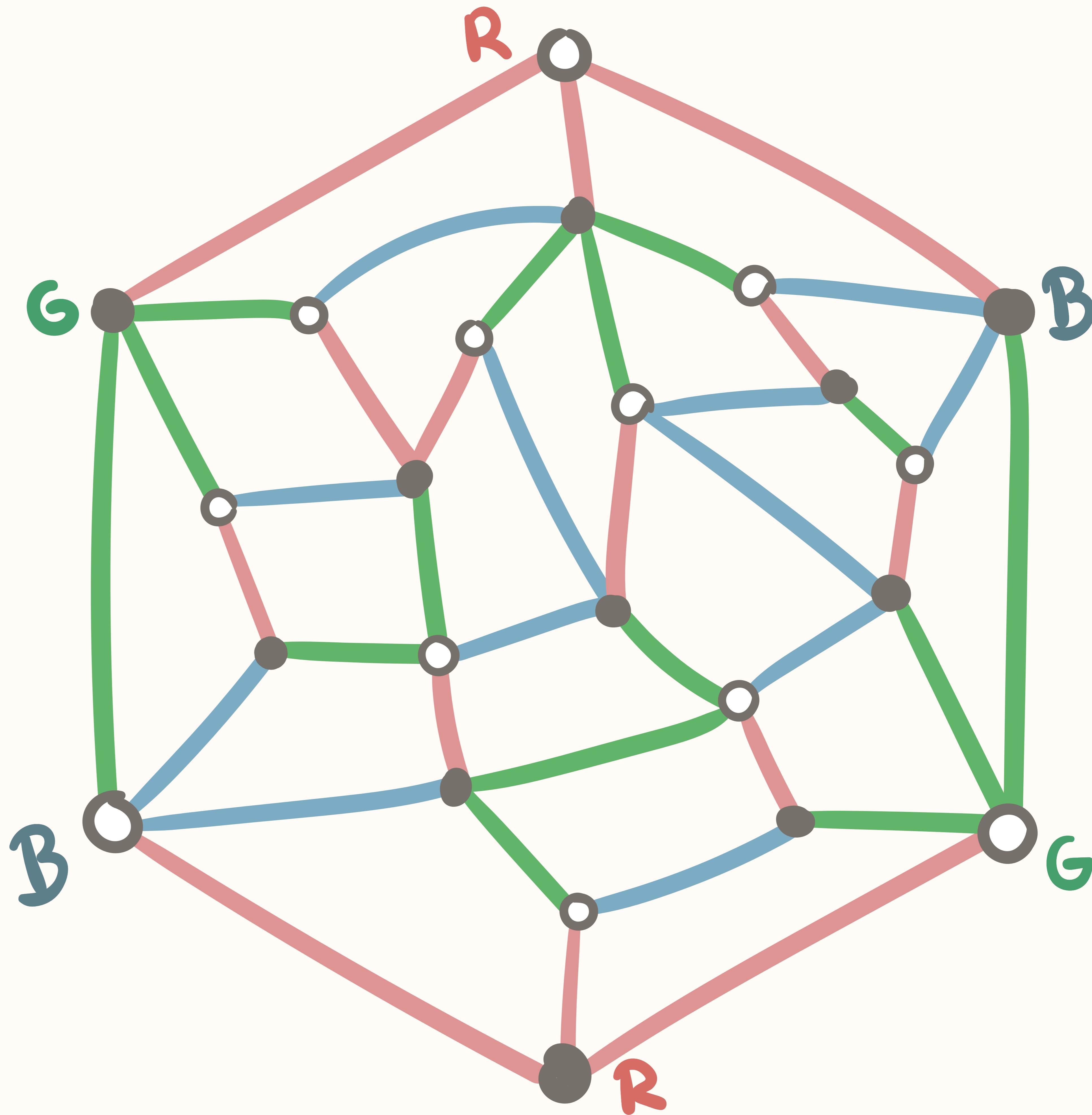
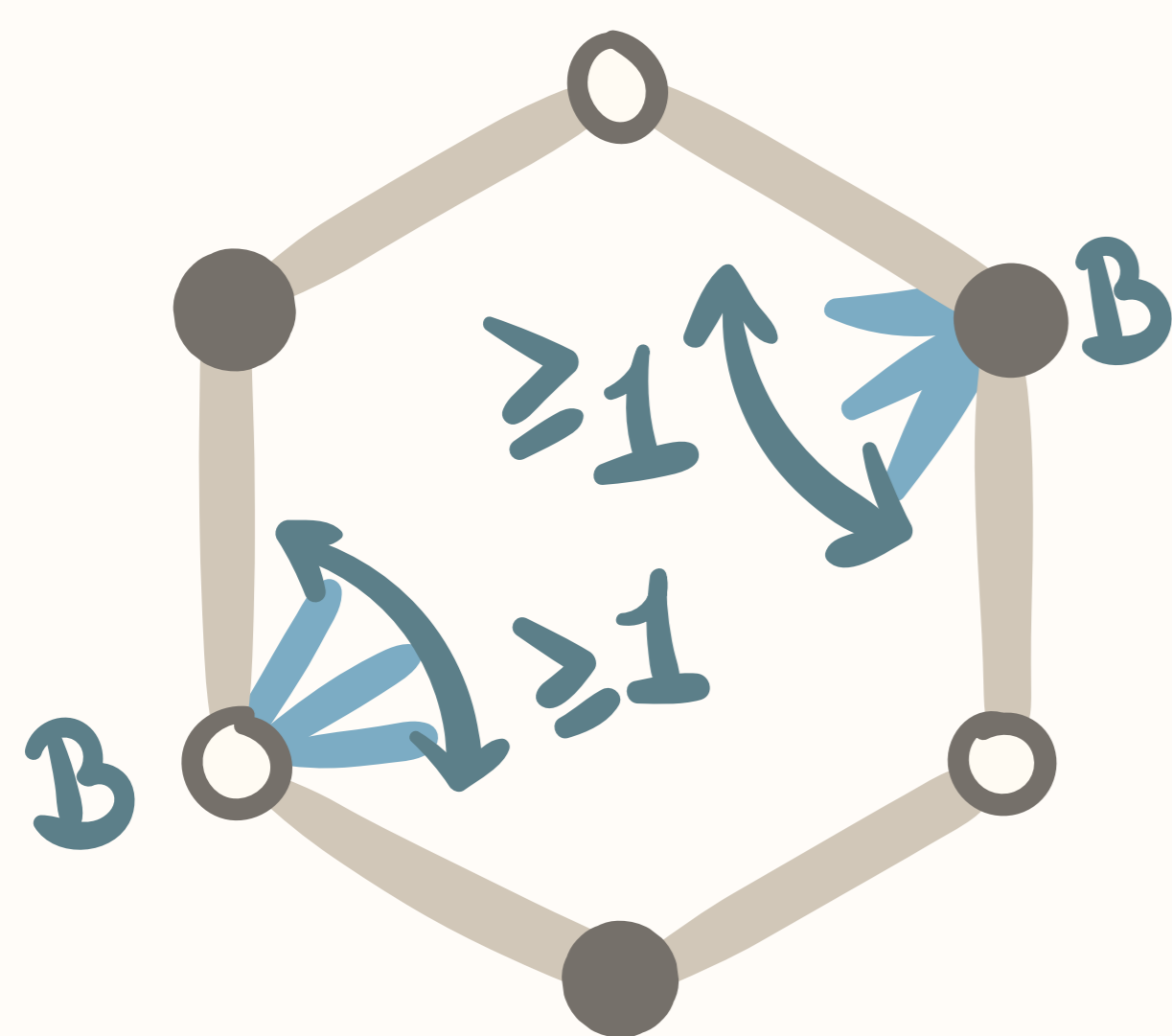
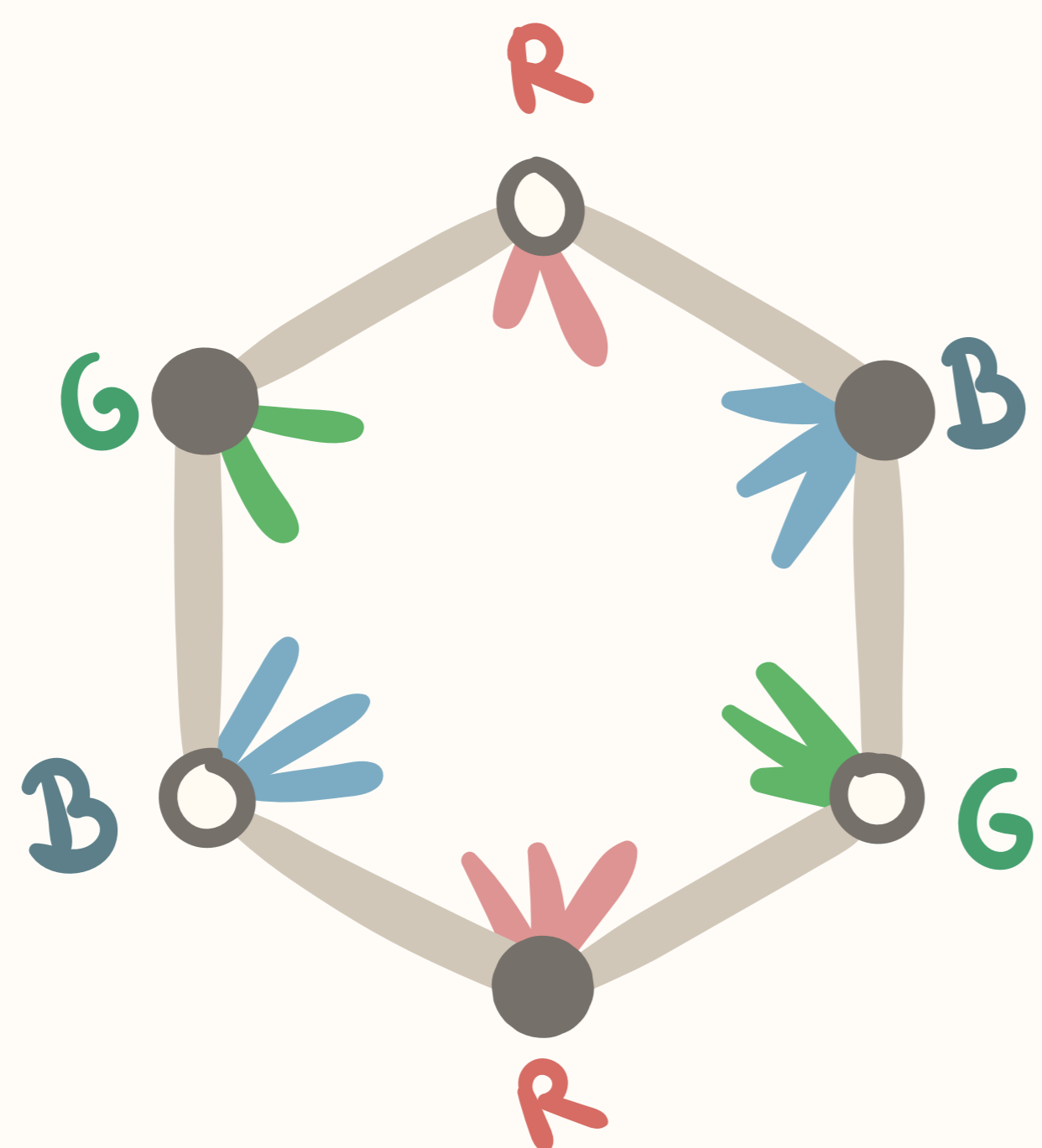
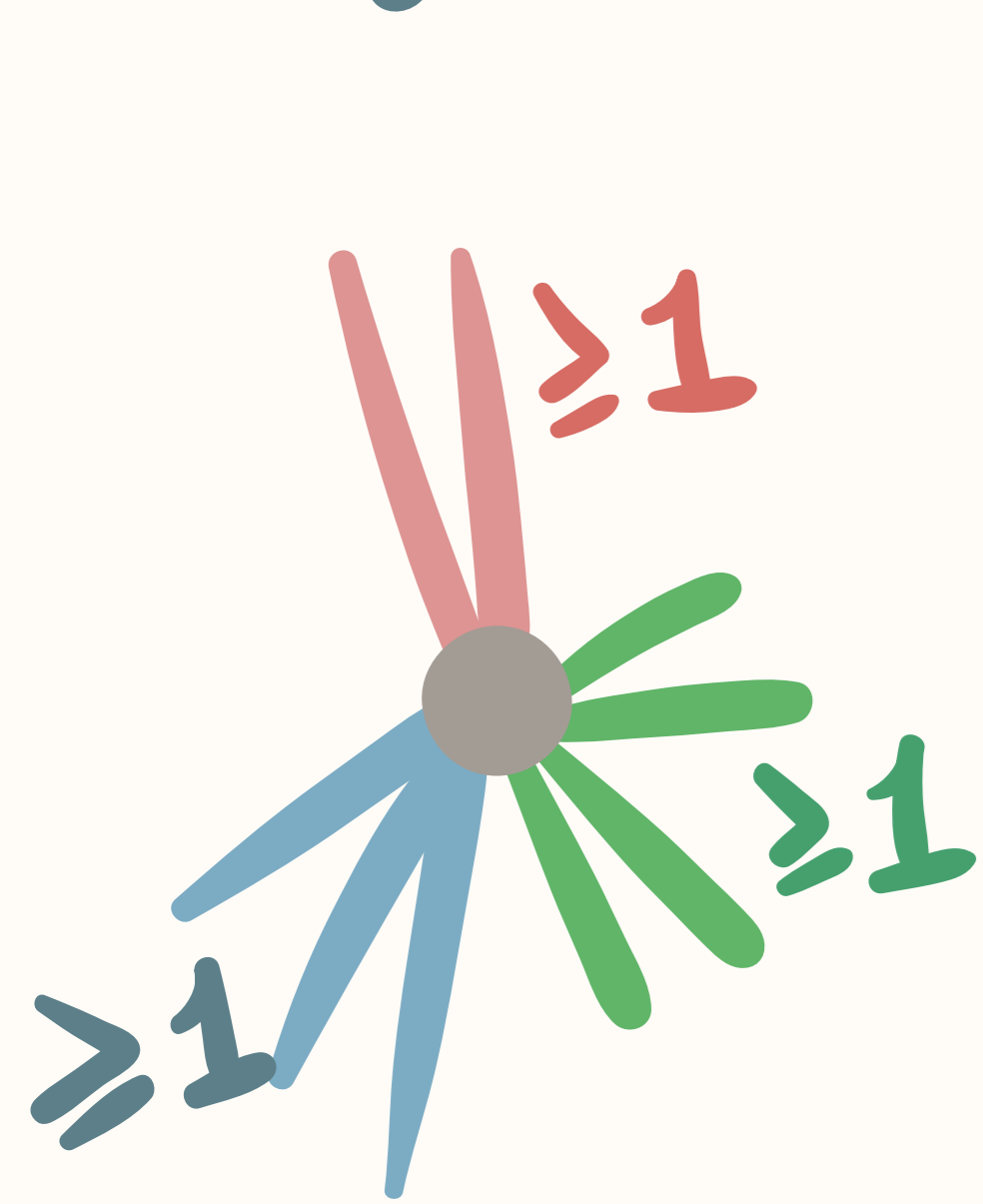
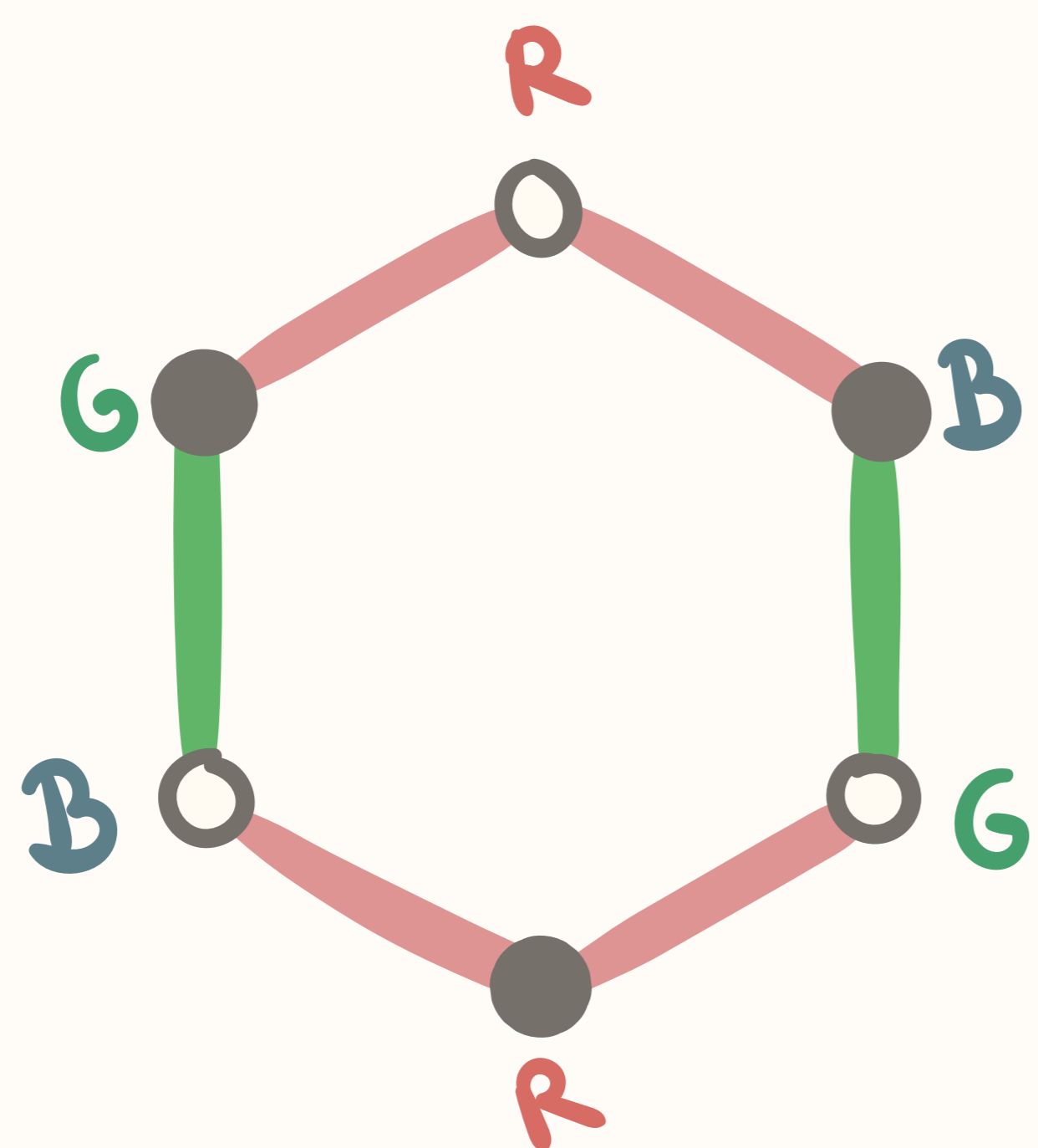
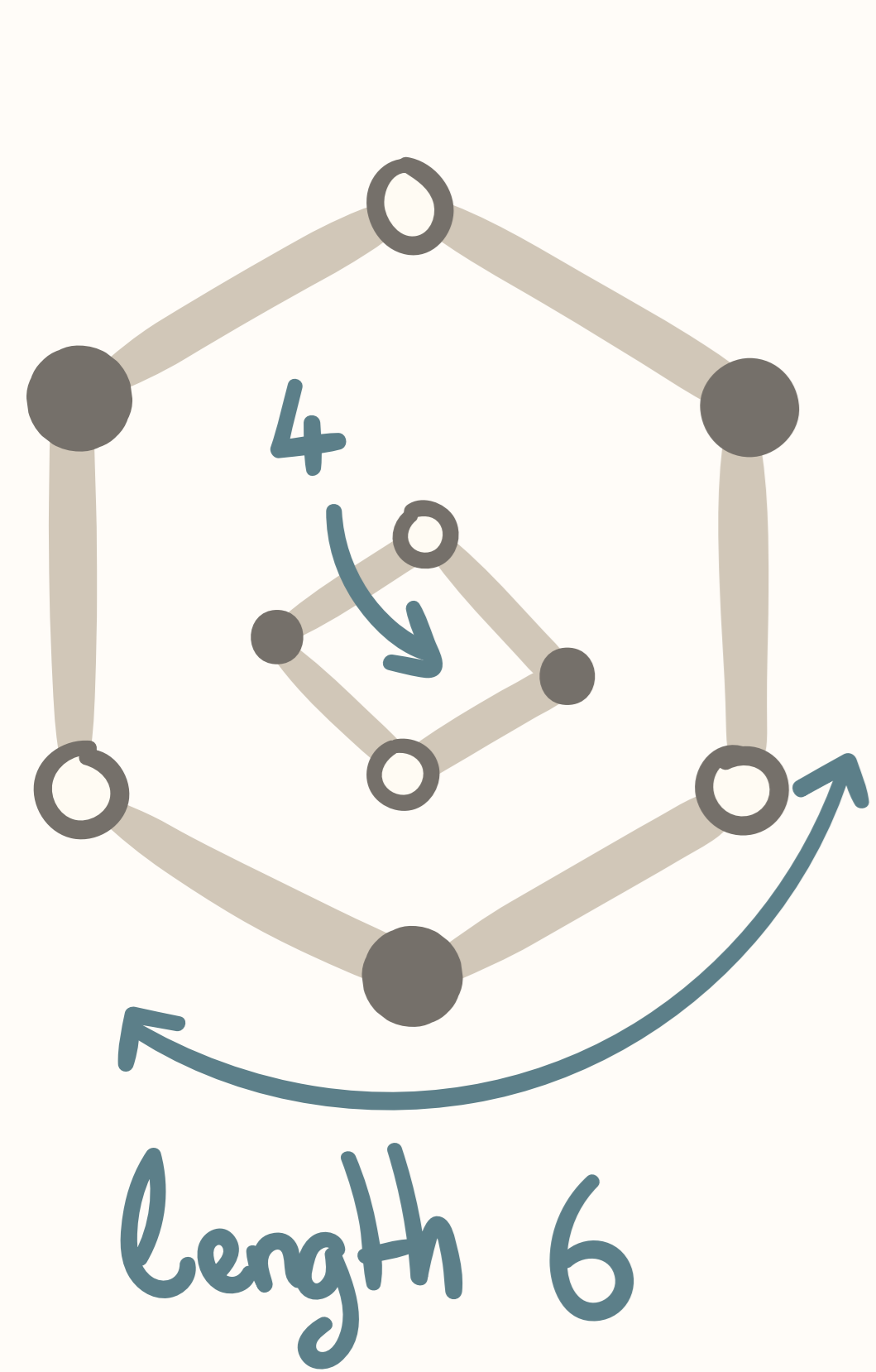


Schnyder
labellings

⇒ Geodesic embeddings and planar graphs,
S. Felsner (2003)

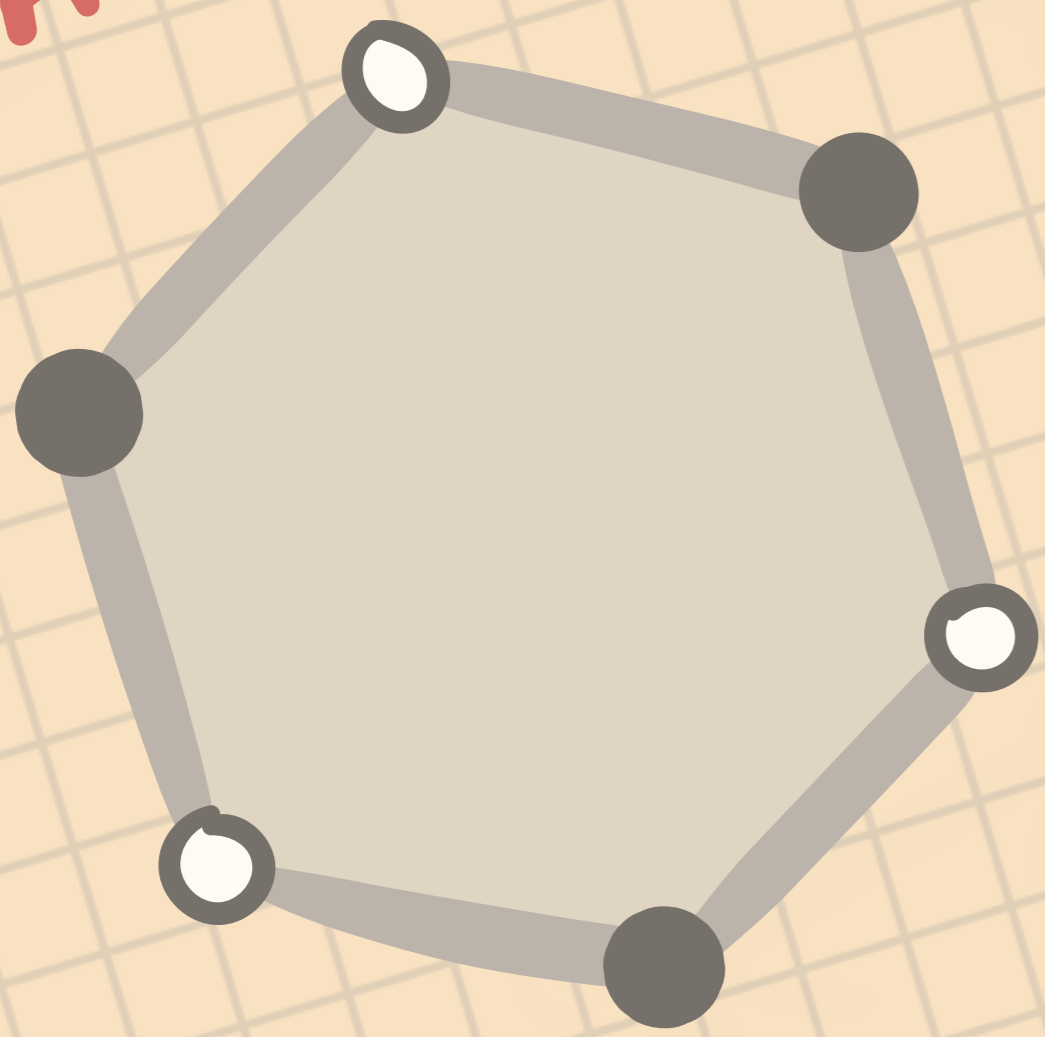


Schnyder labellings

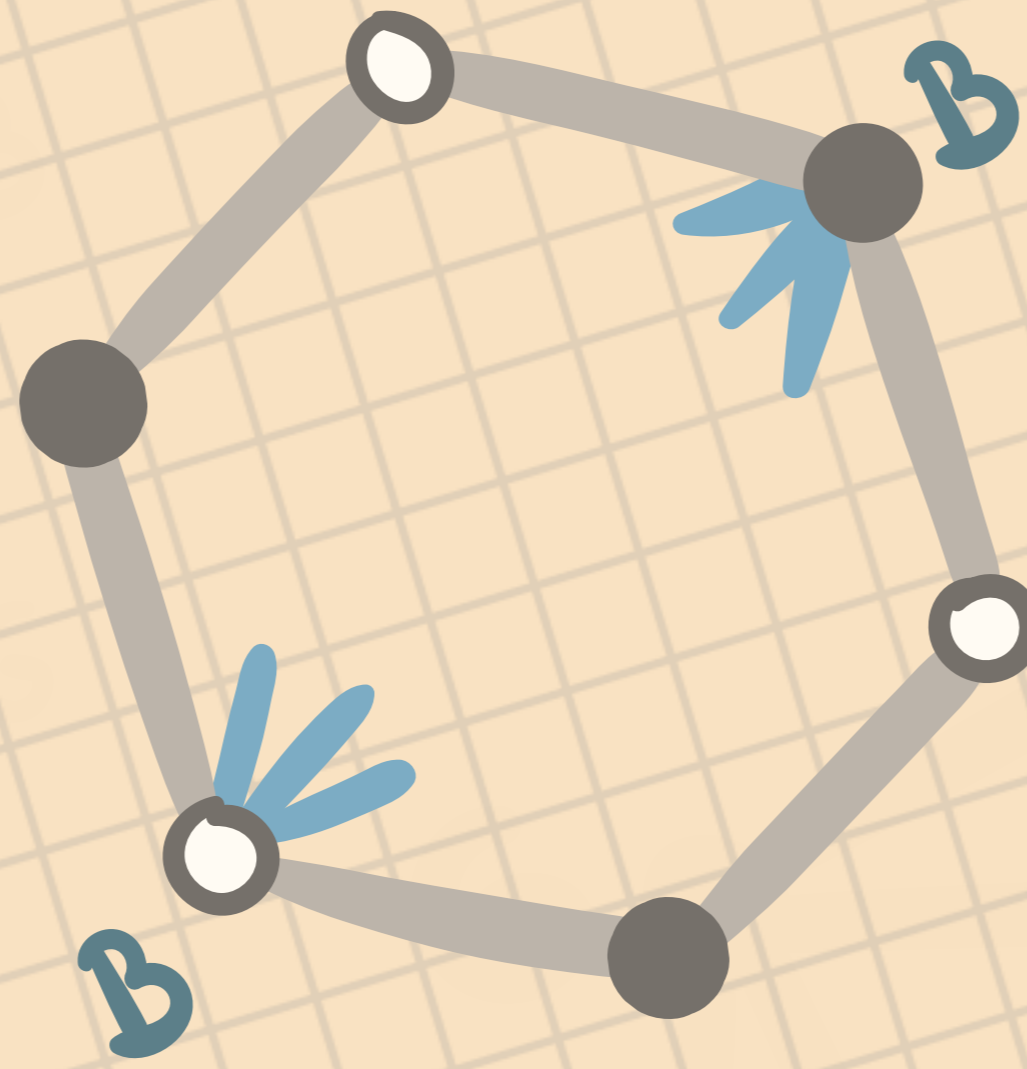


Schnyder labelling

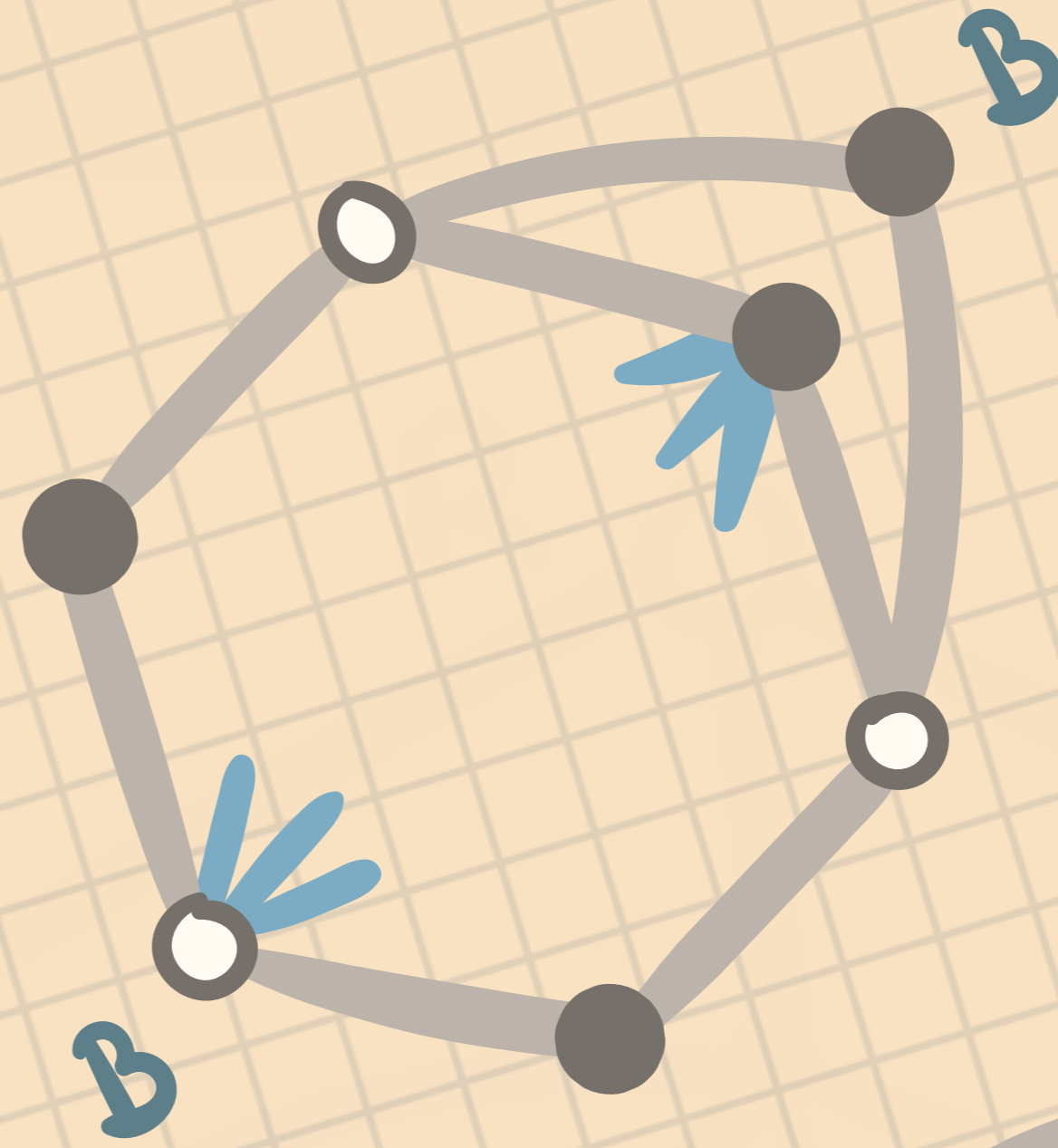
ALL SCHNYDER LABELLINGS



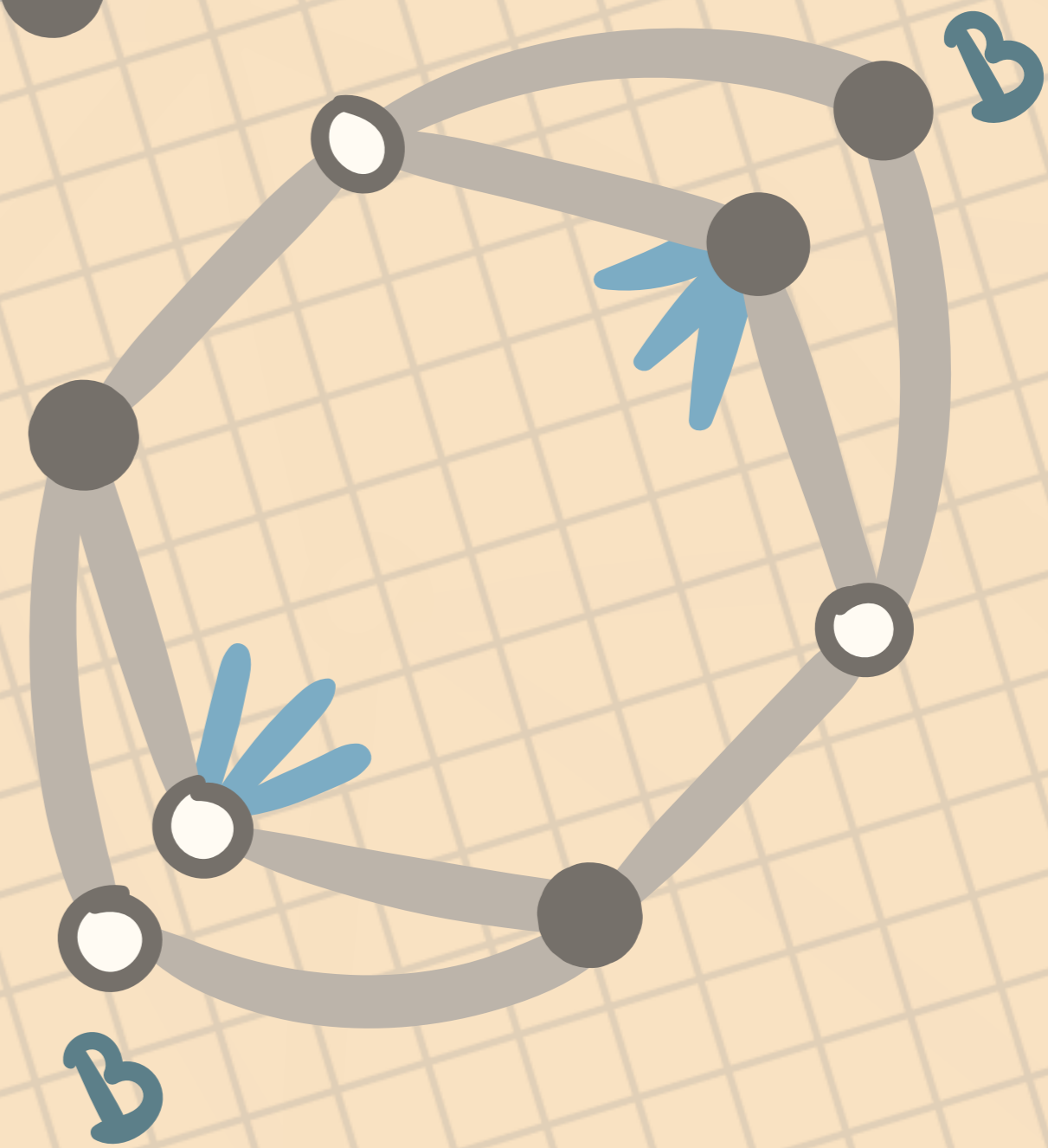
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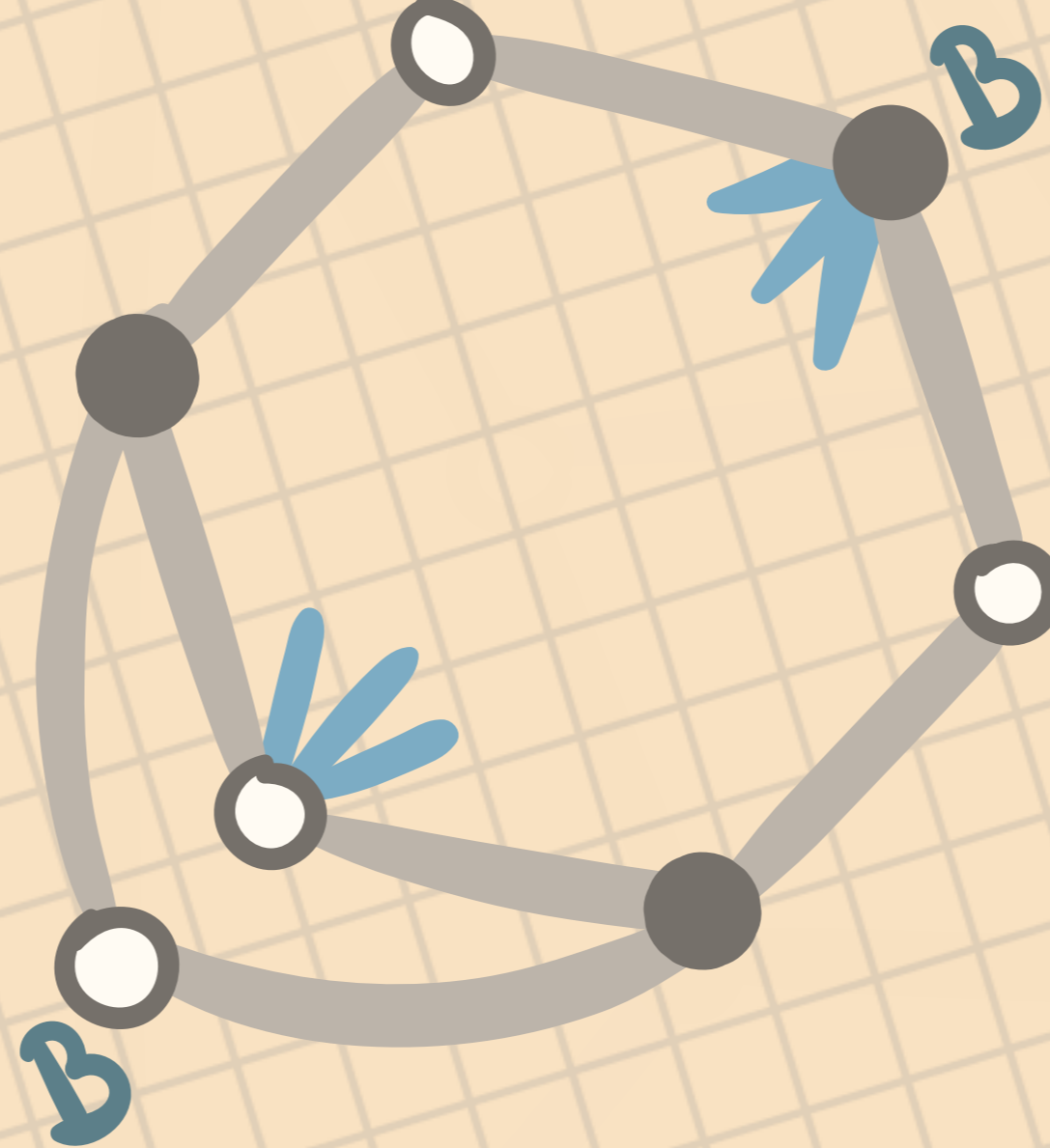
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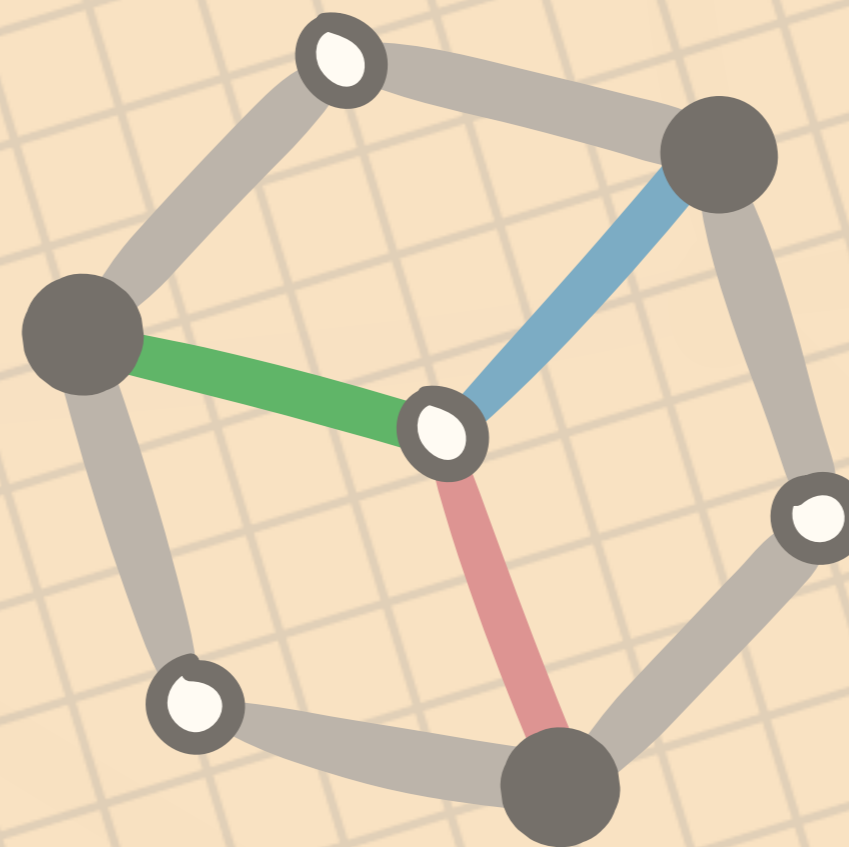
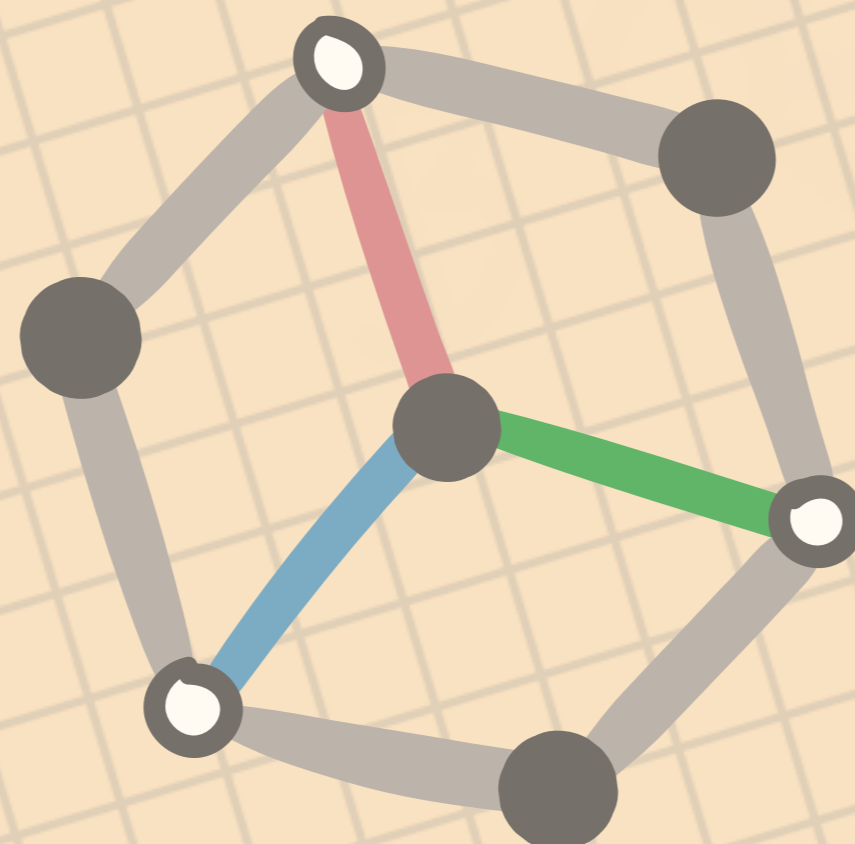
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+



EXEPT:



G

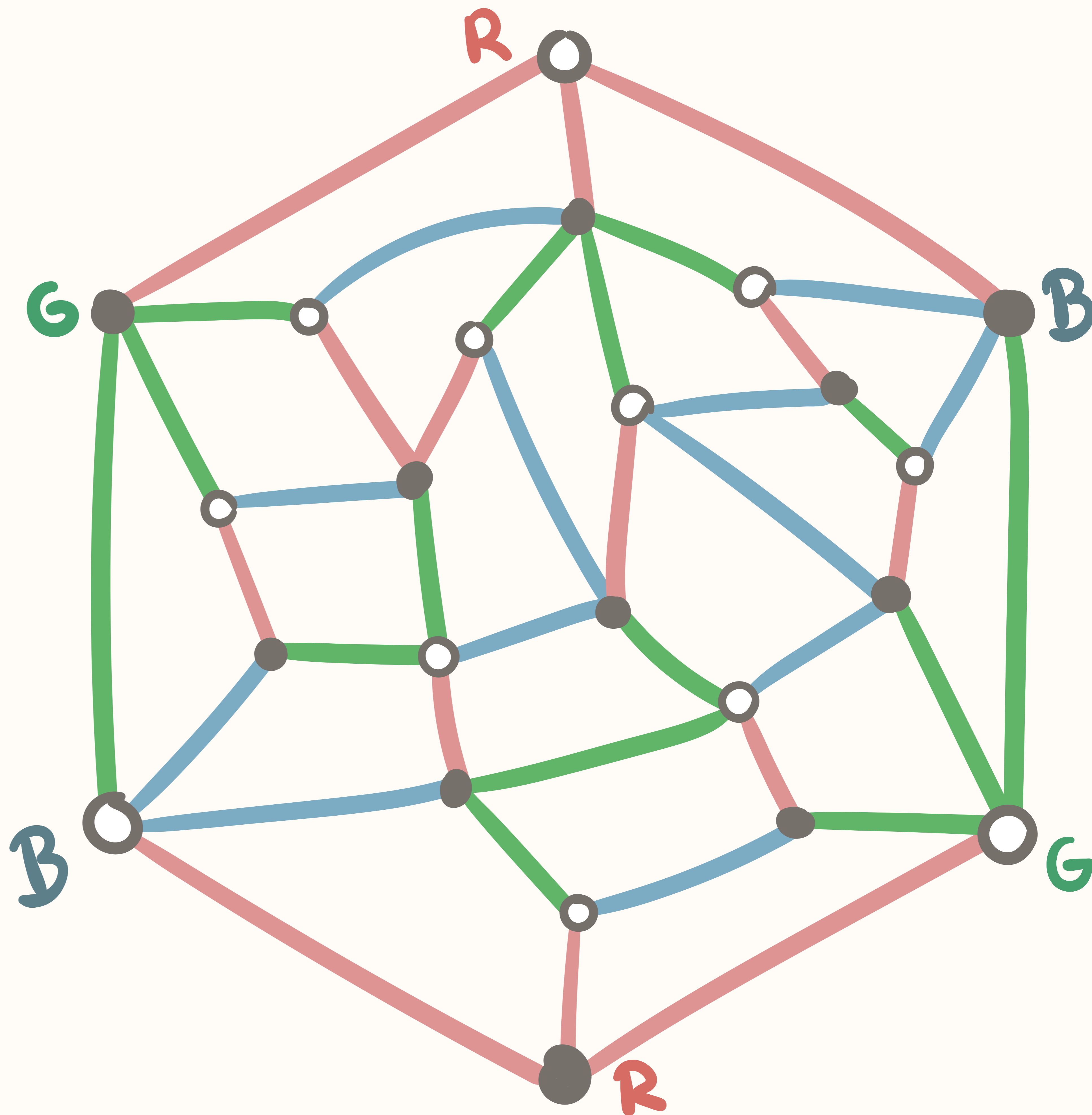
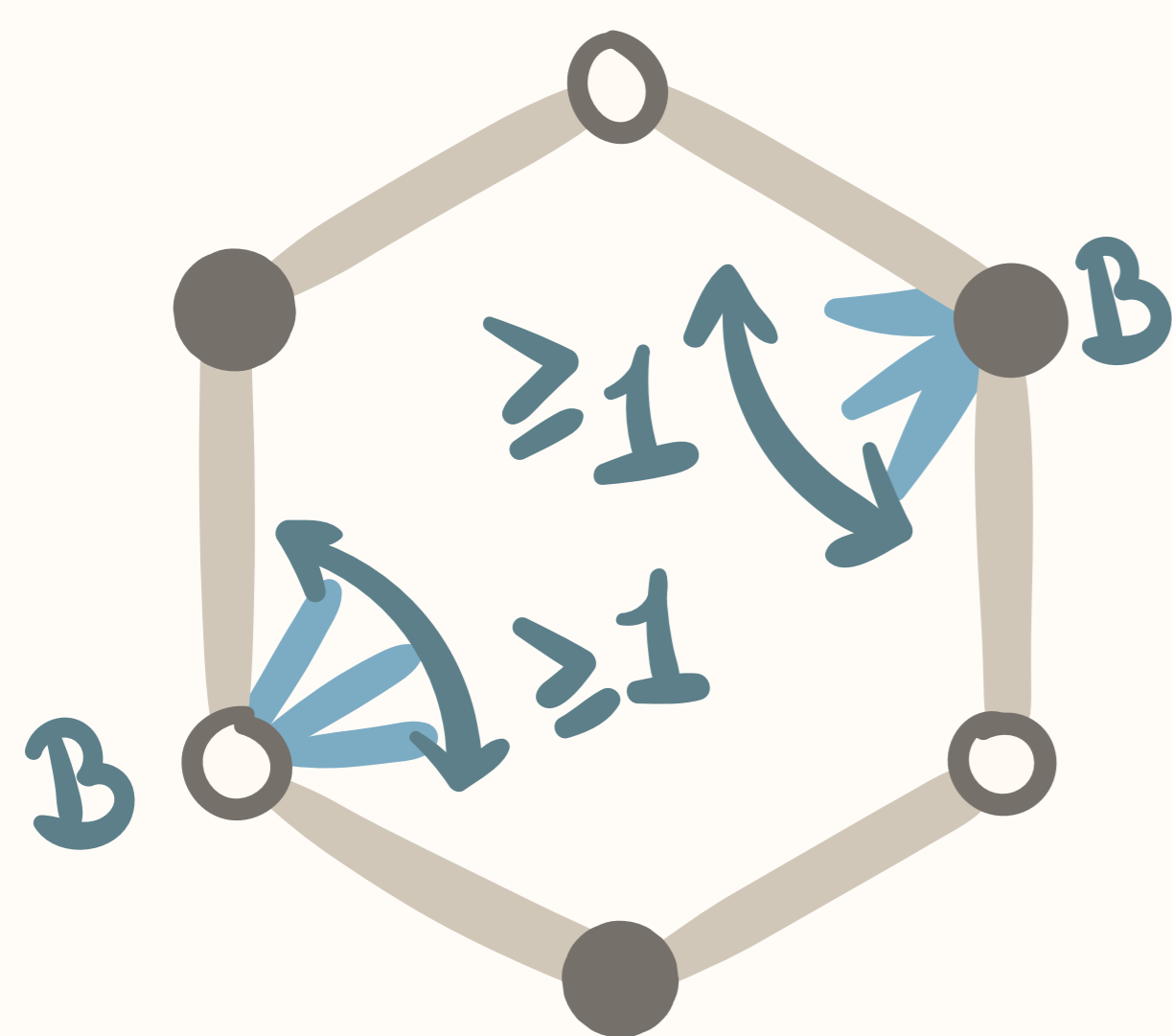
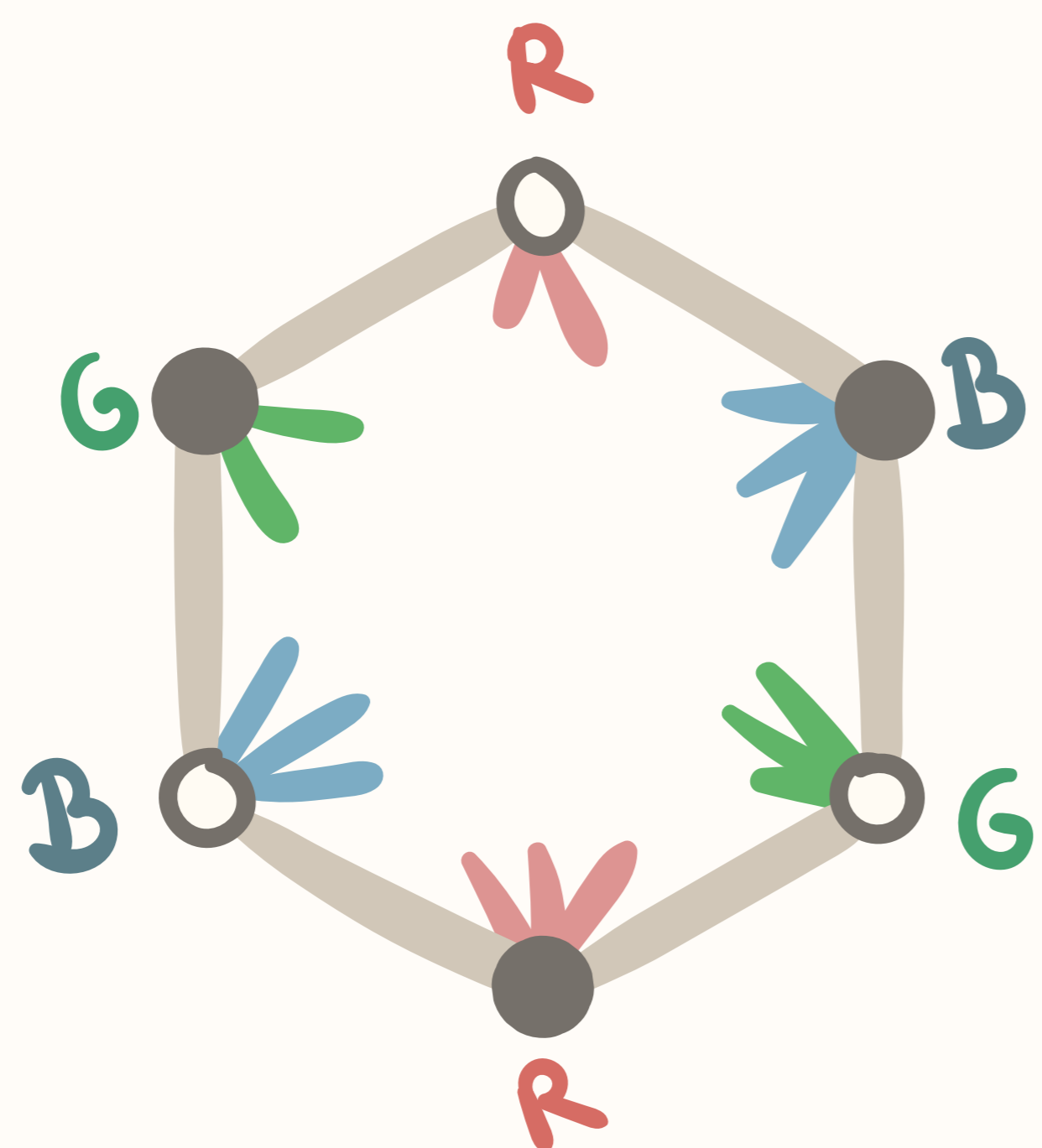
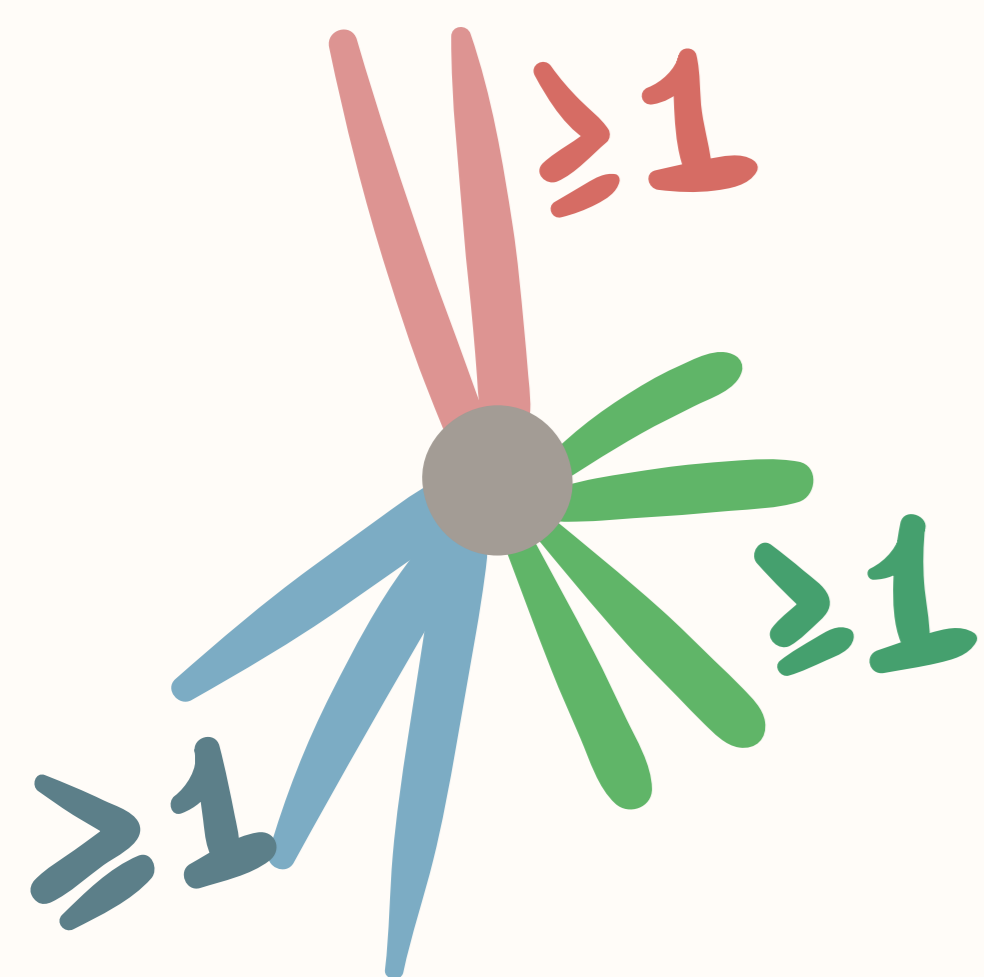
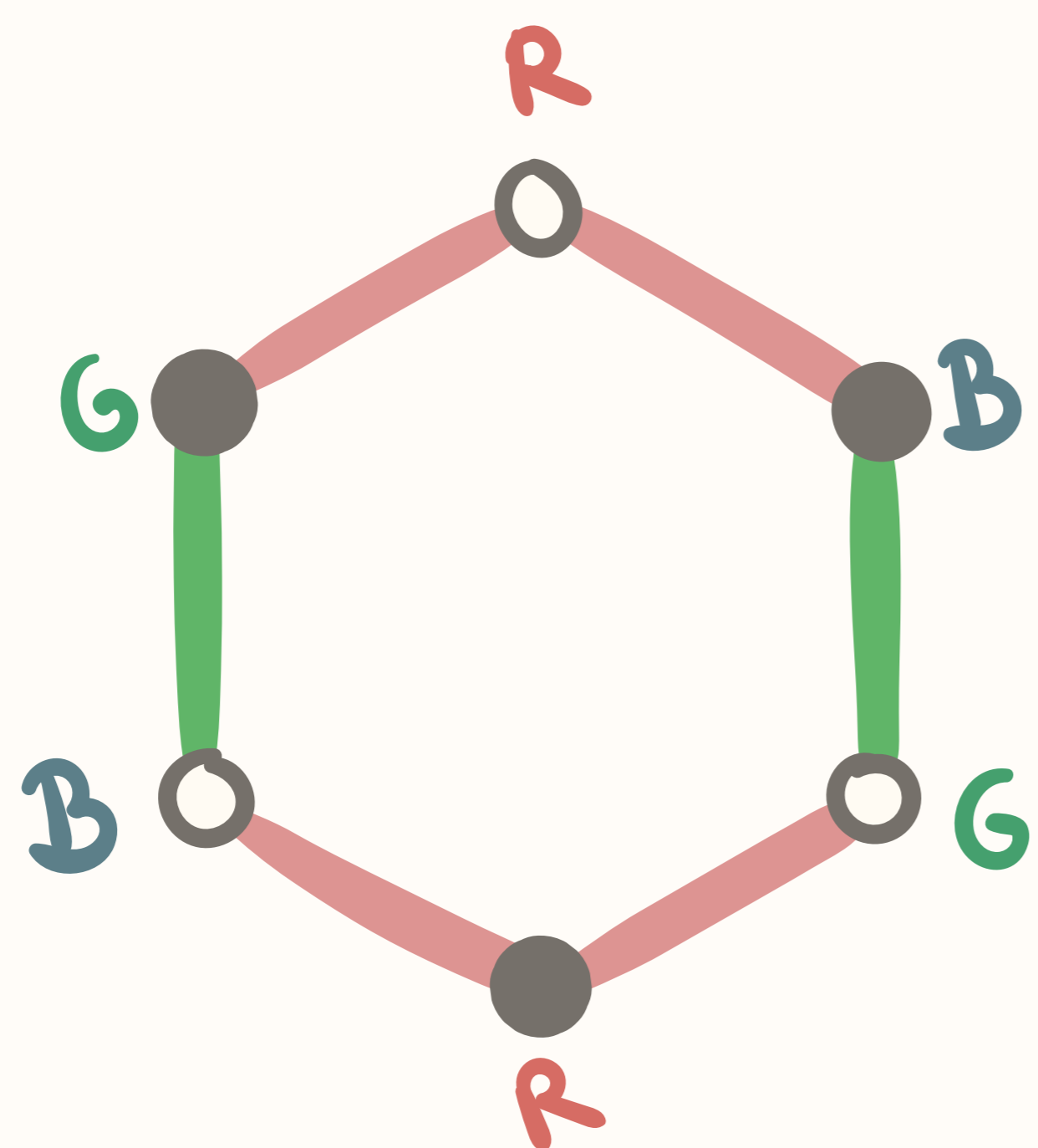
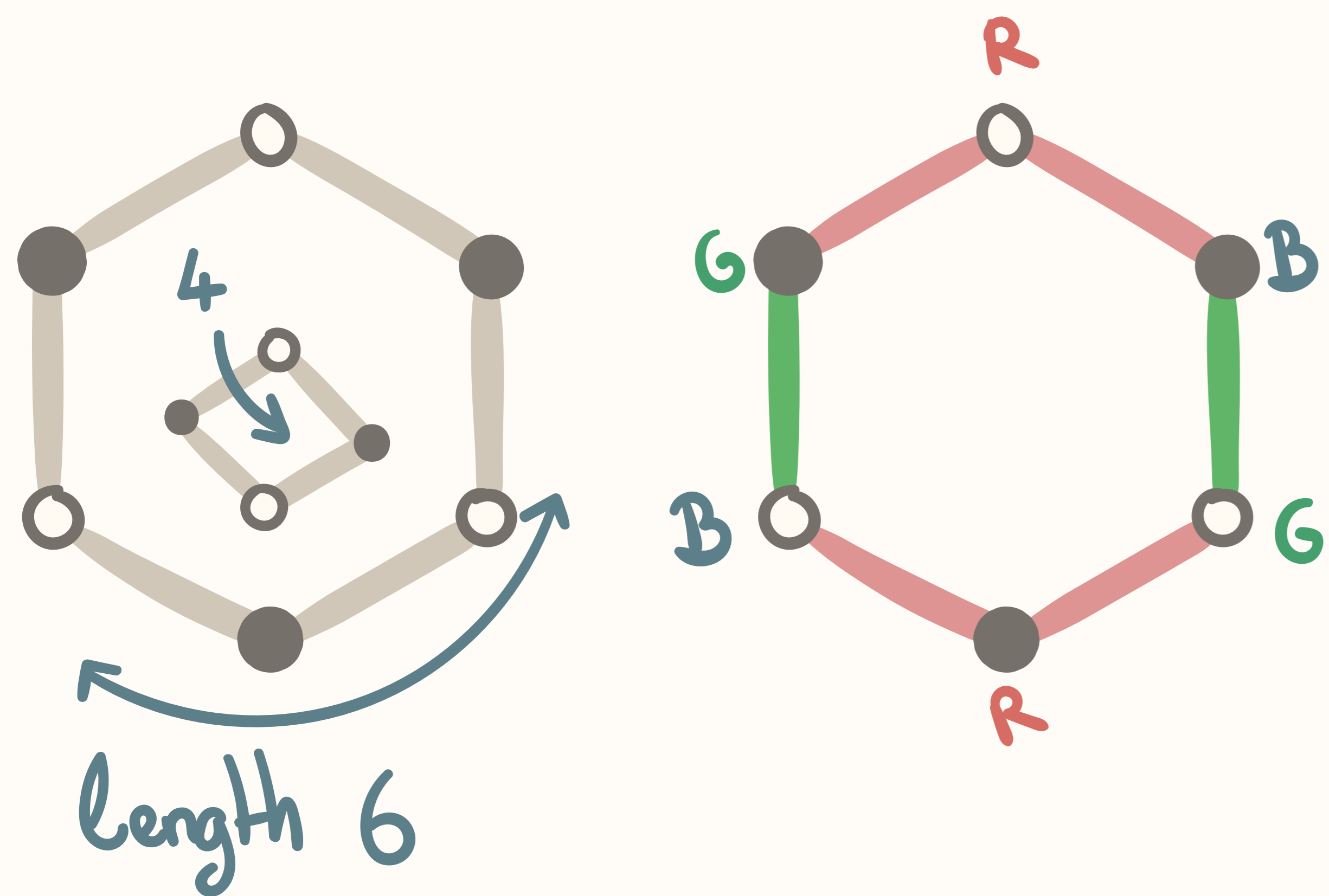
R

B

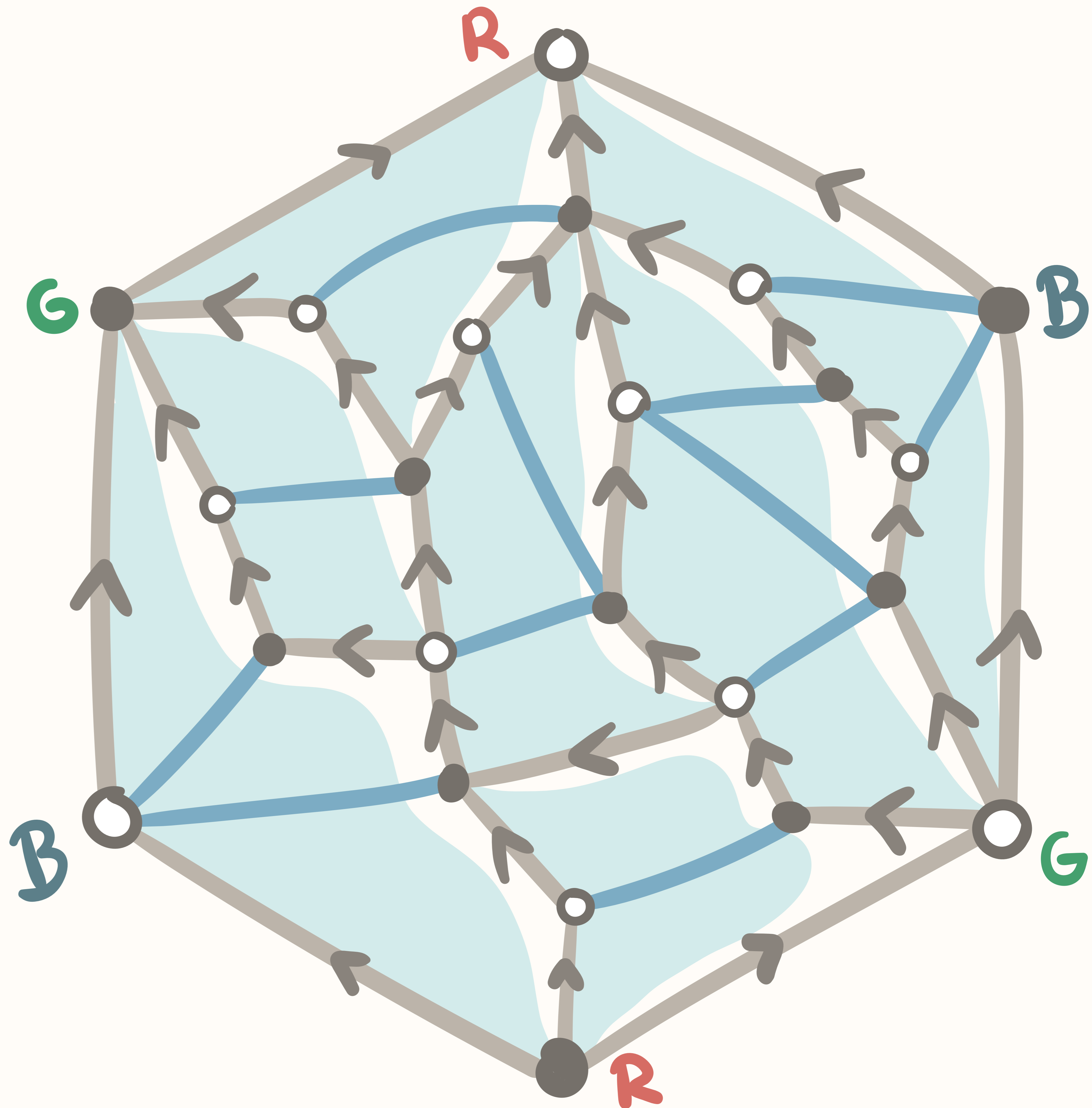
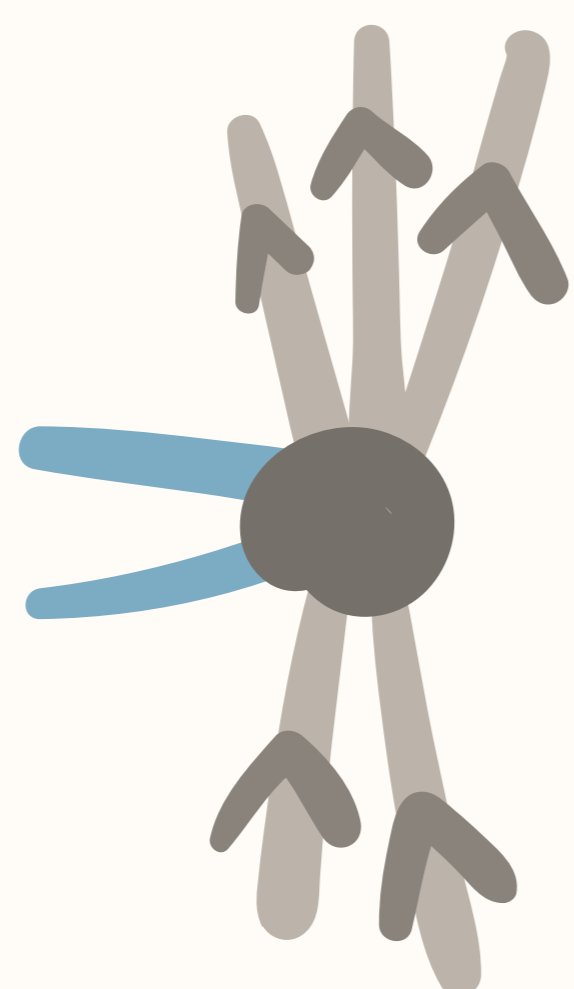
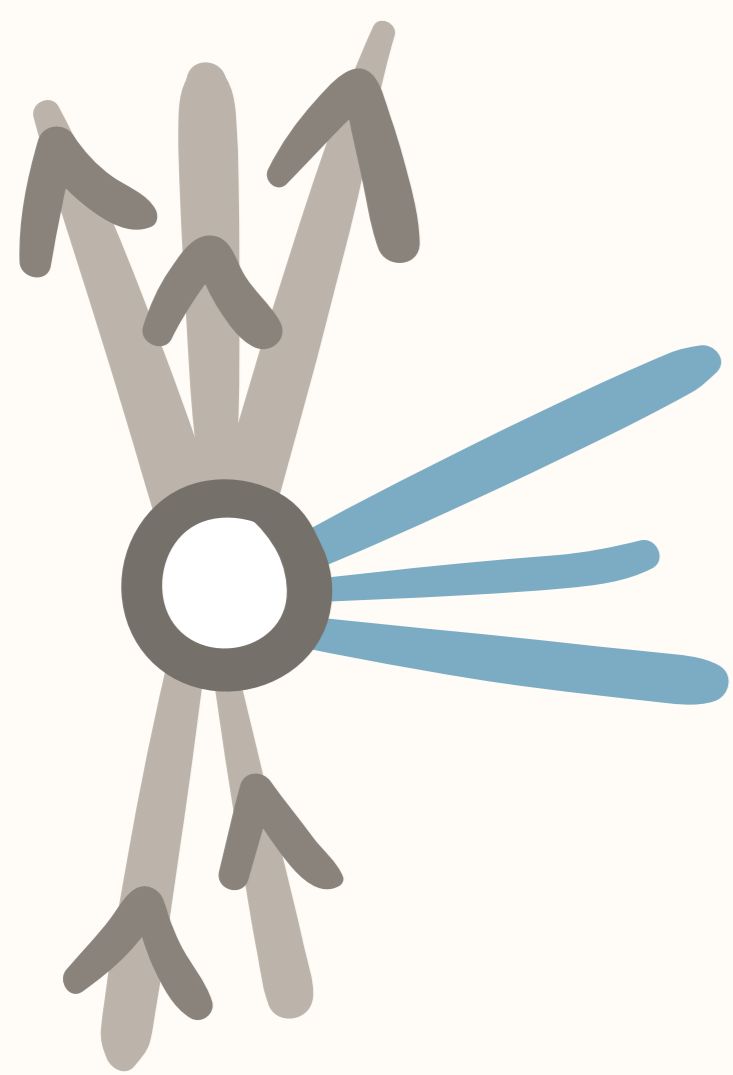
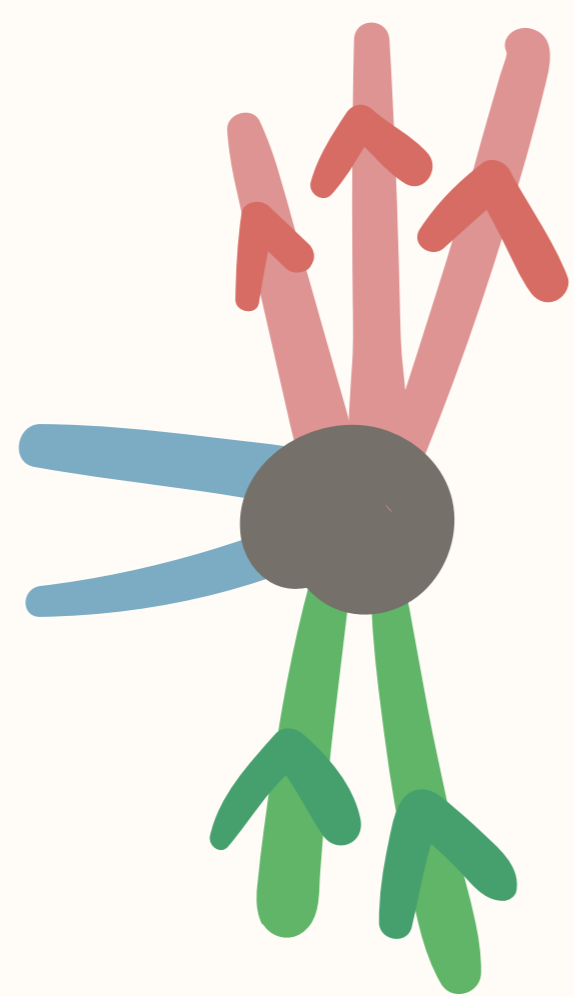
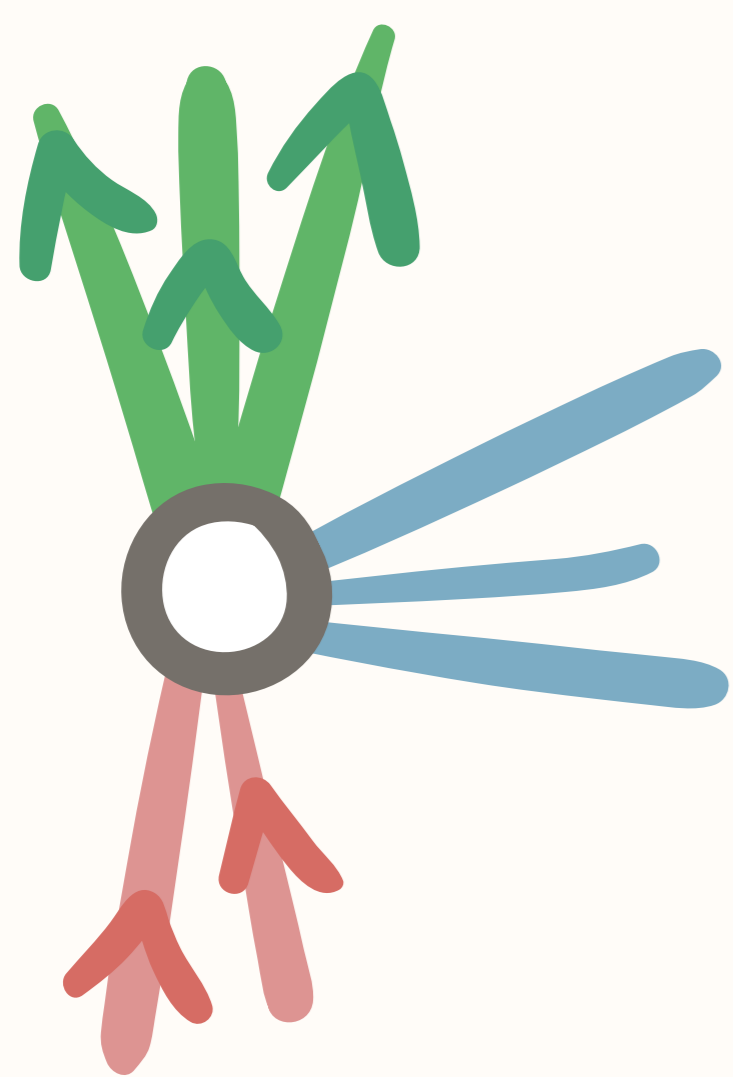
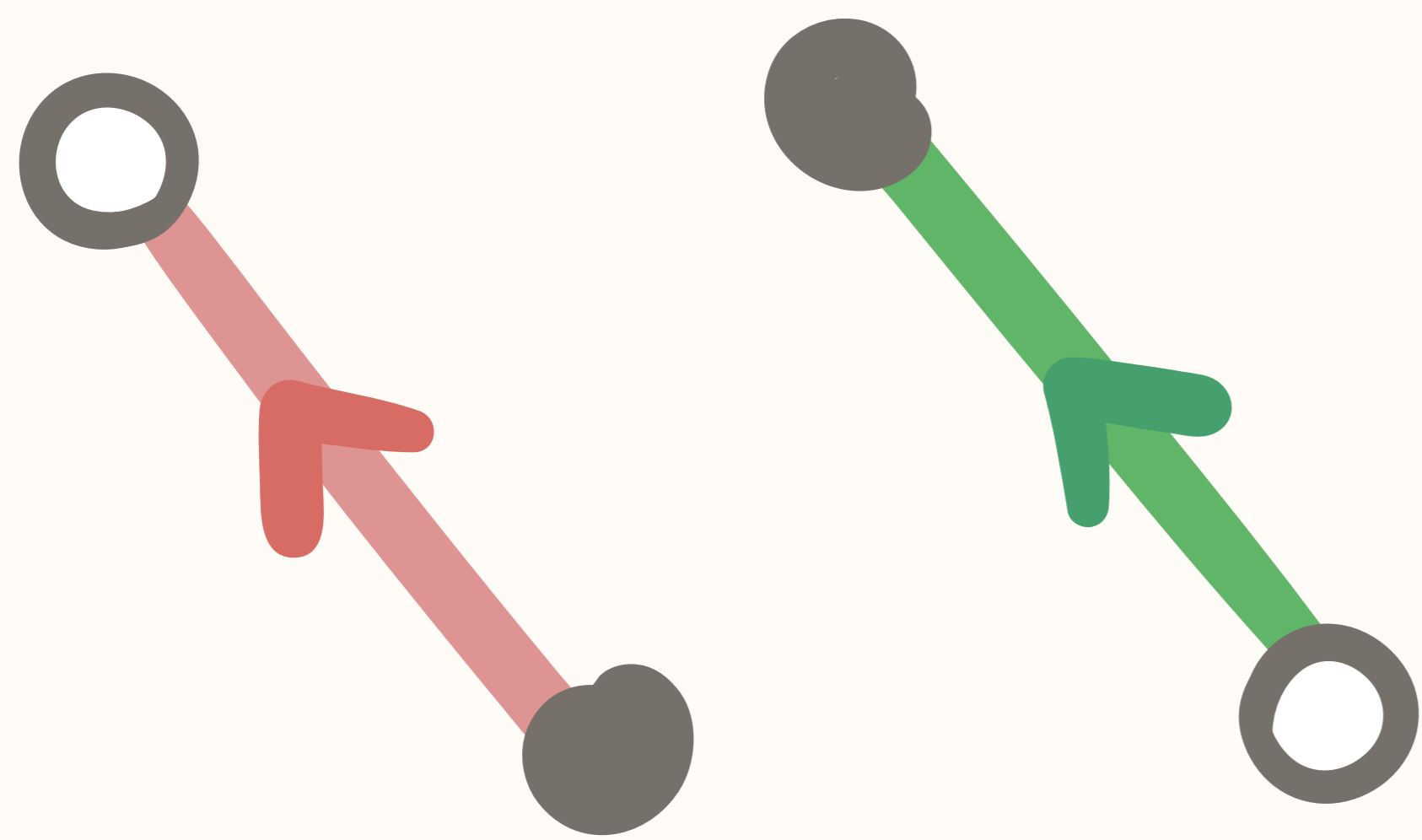
B

≥ 1

Schnyder labellings



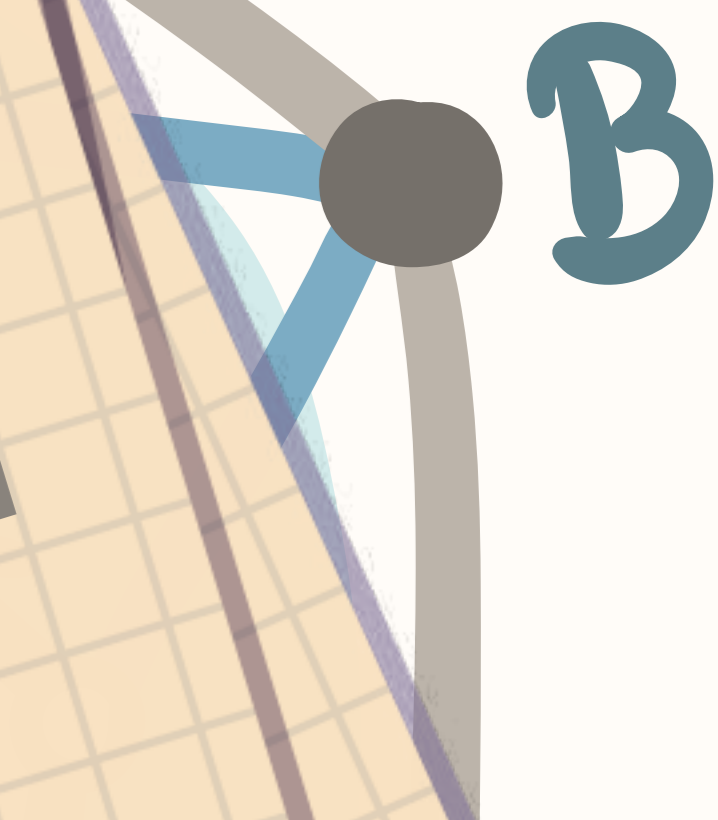
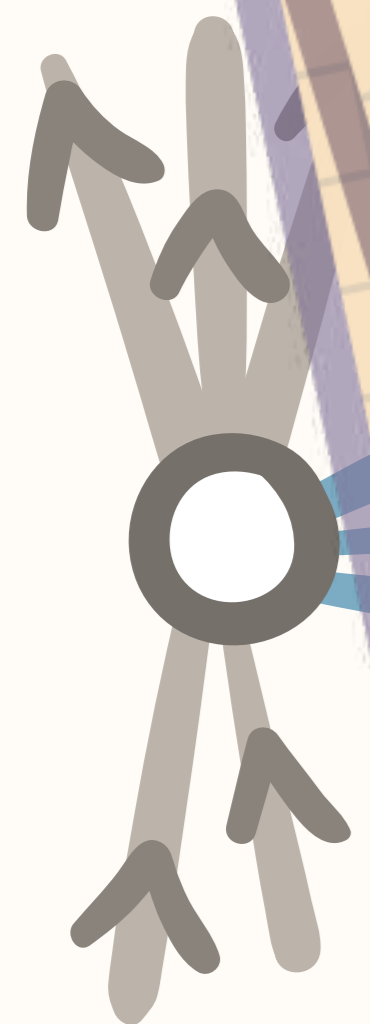
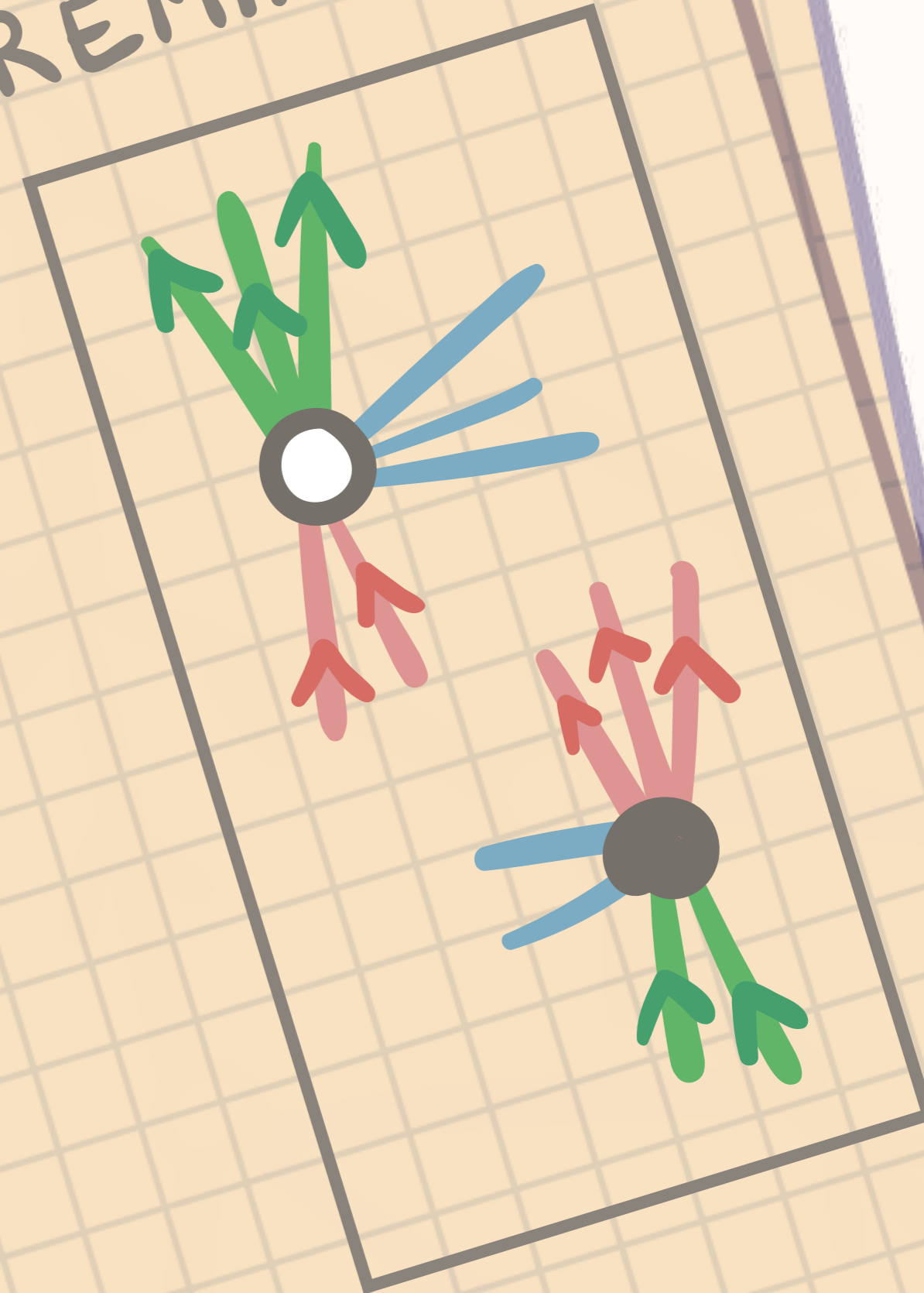
Schnyder labellings



Schnyder labelling

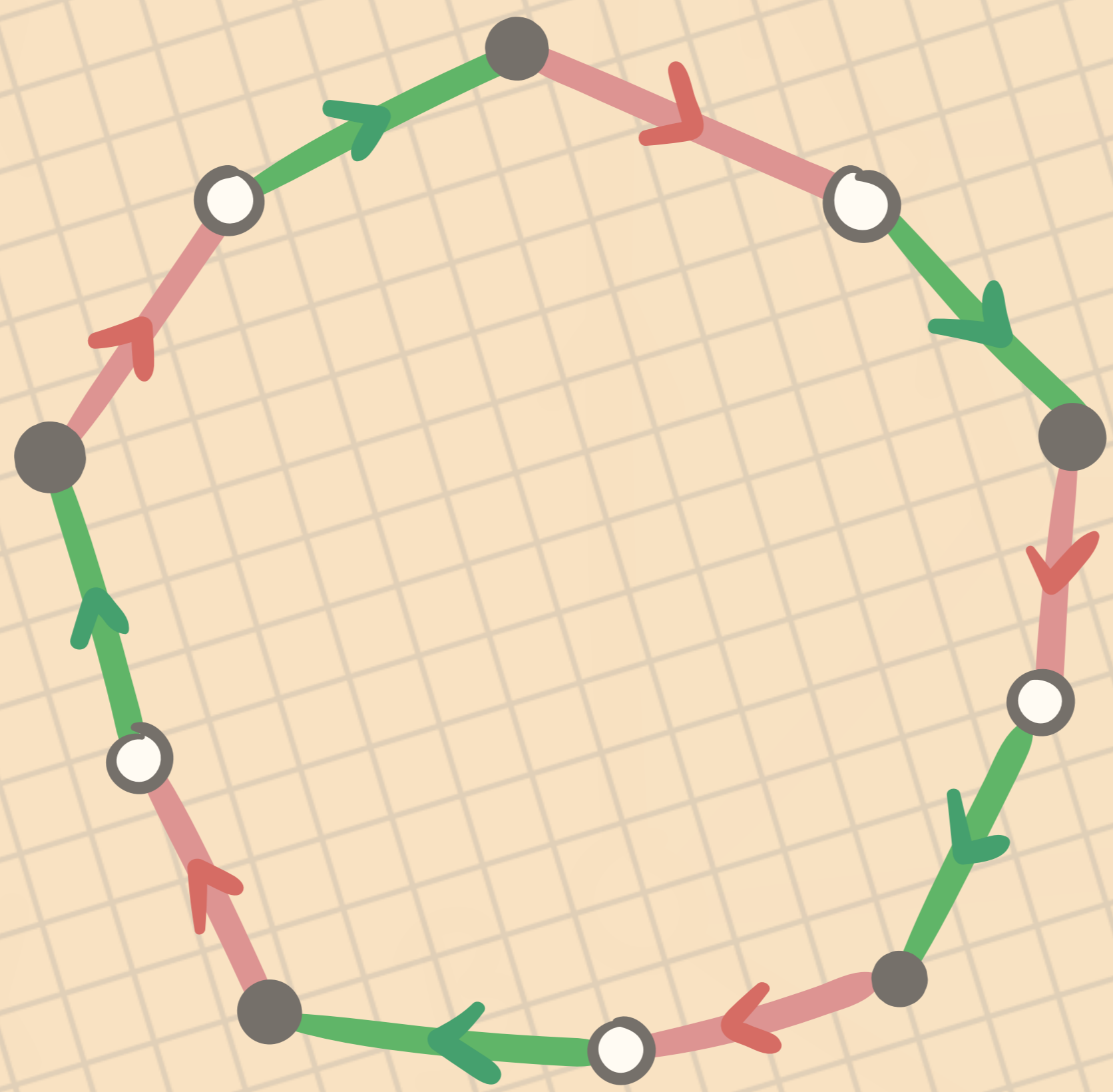
IT IS A BIPOLAR ORIENTATION :
1) ACYCLIC

REMINDER

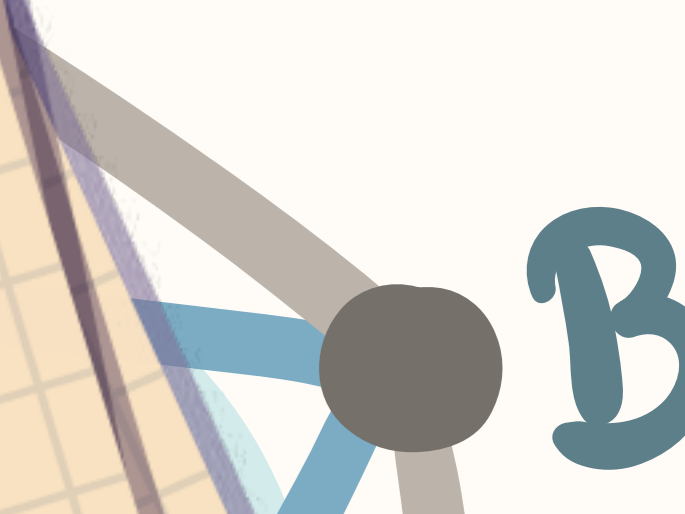
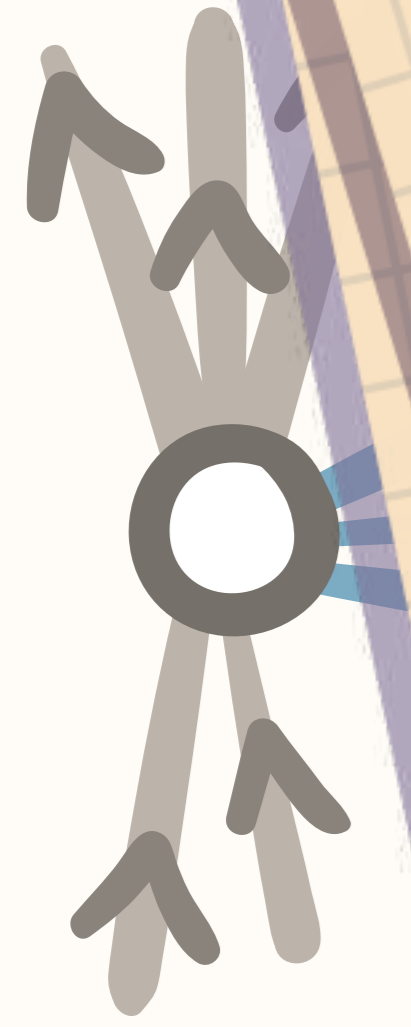
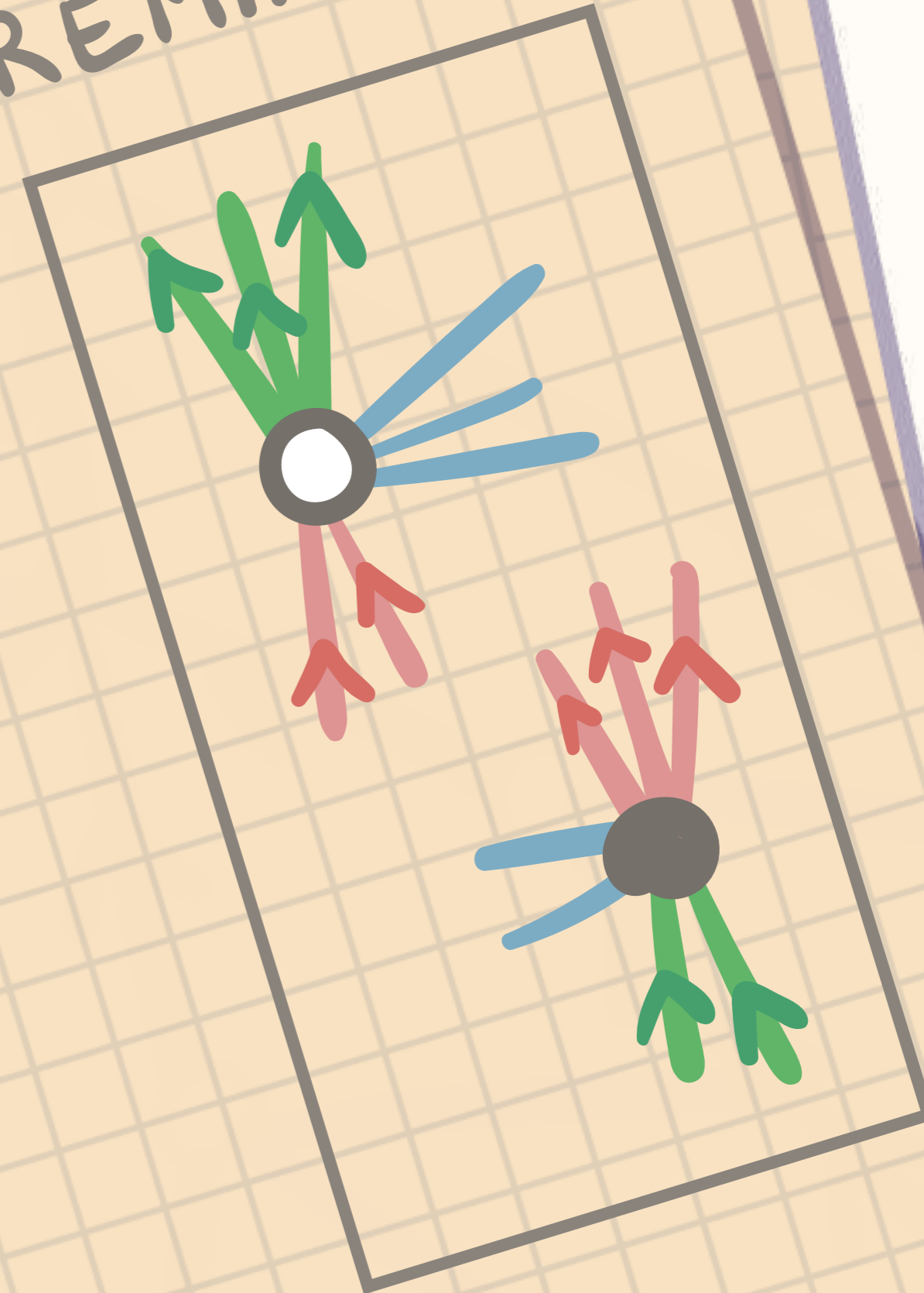


Schnyder labelling

IT IS A BIPOLAR ORIENTATION :
1) ACYCLIC
ASSUME \exists CYCLE, TAKEN MINIMAL

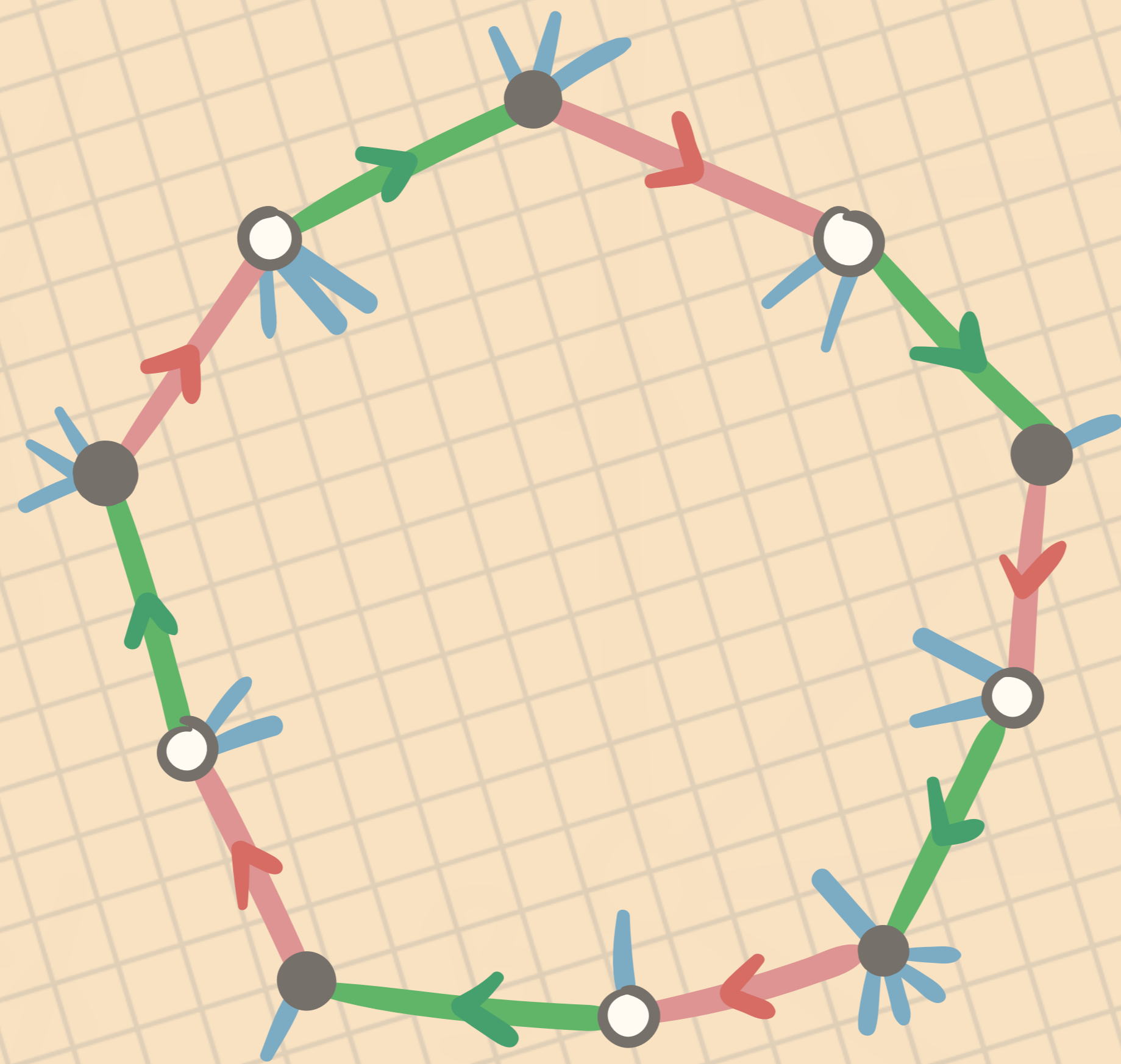


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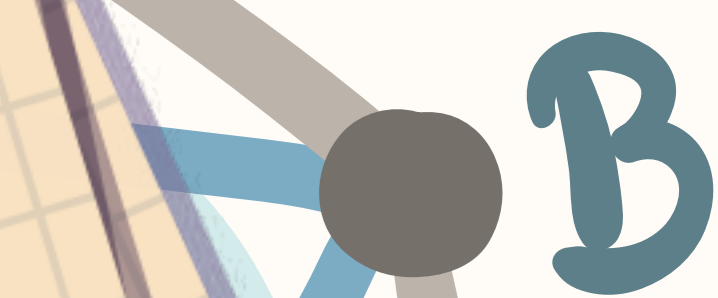
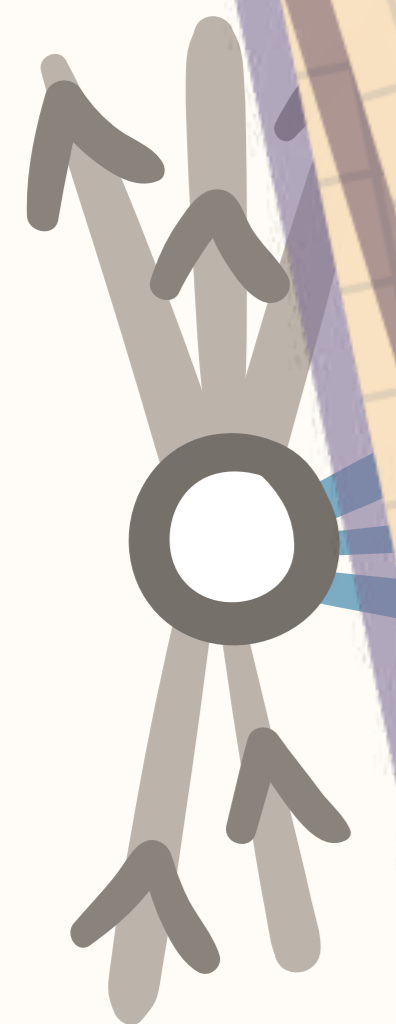
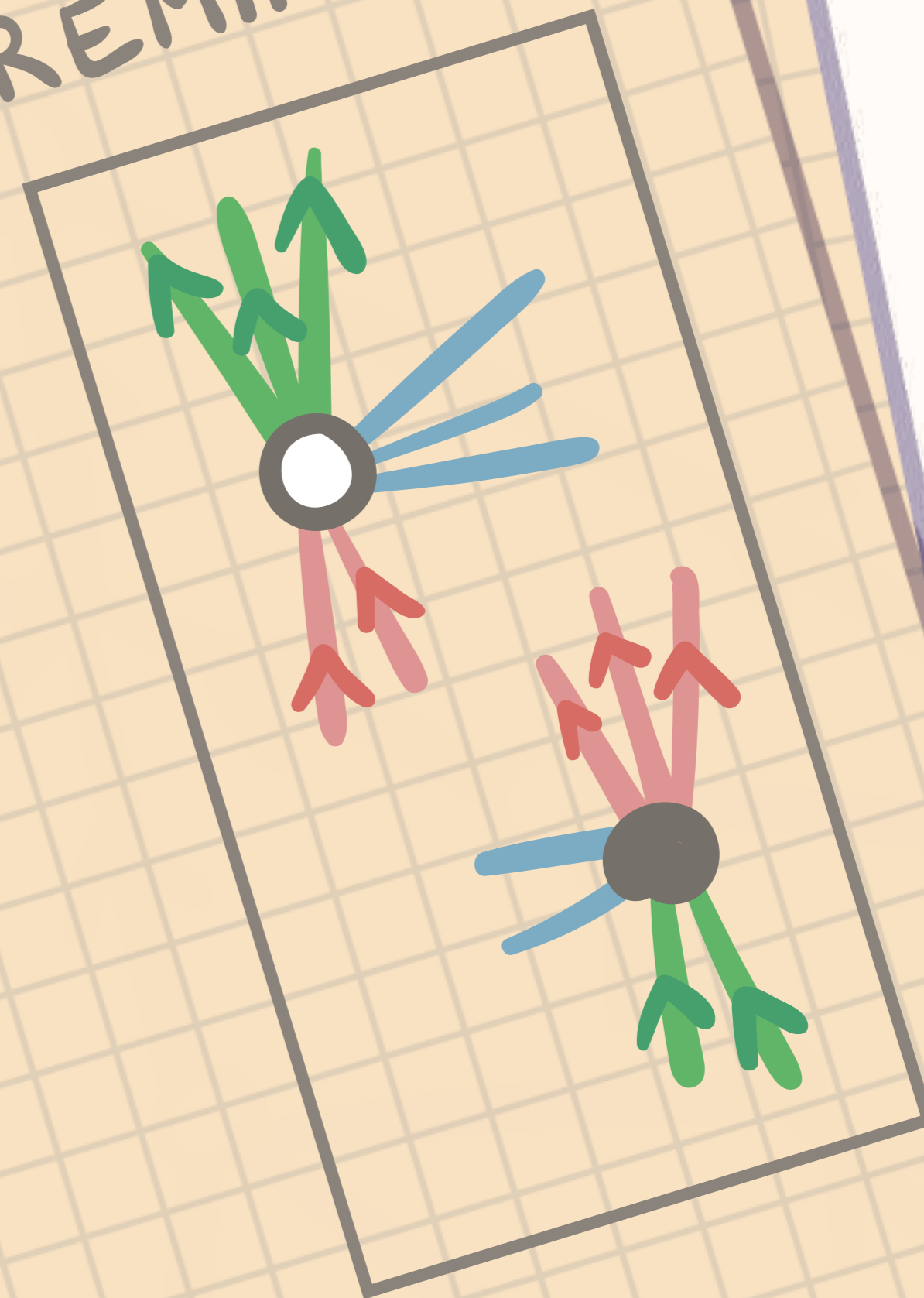


Schnyder labelling

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ASSUME \exists CYCLE, TAKEN MINIMAL

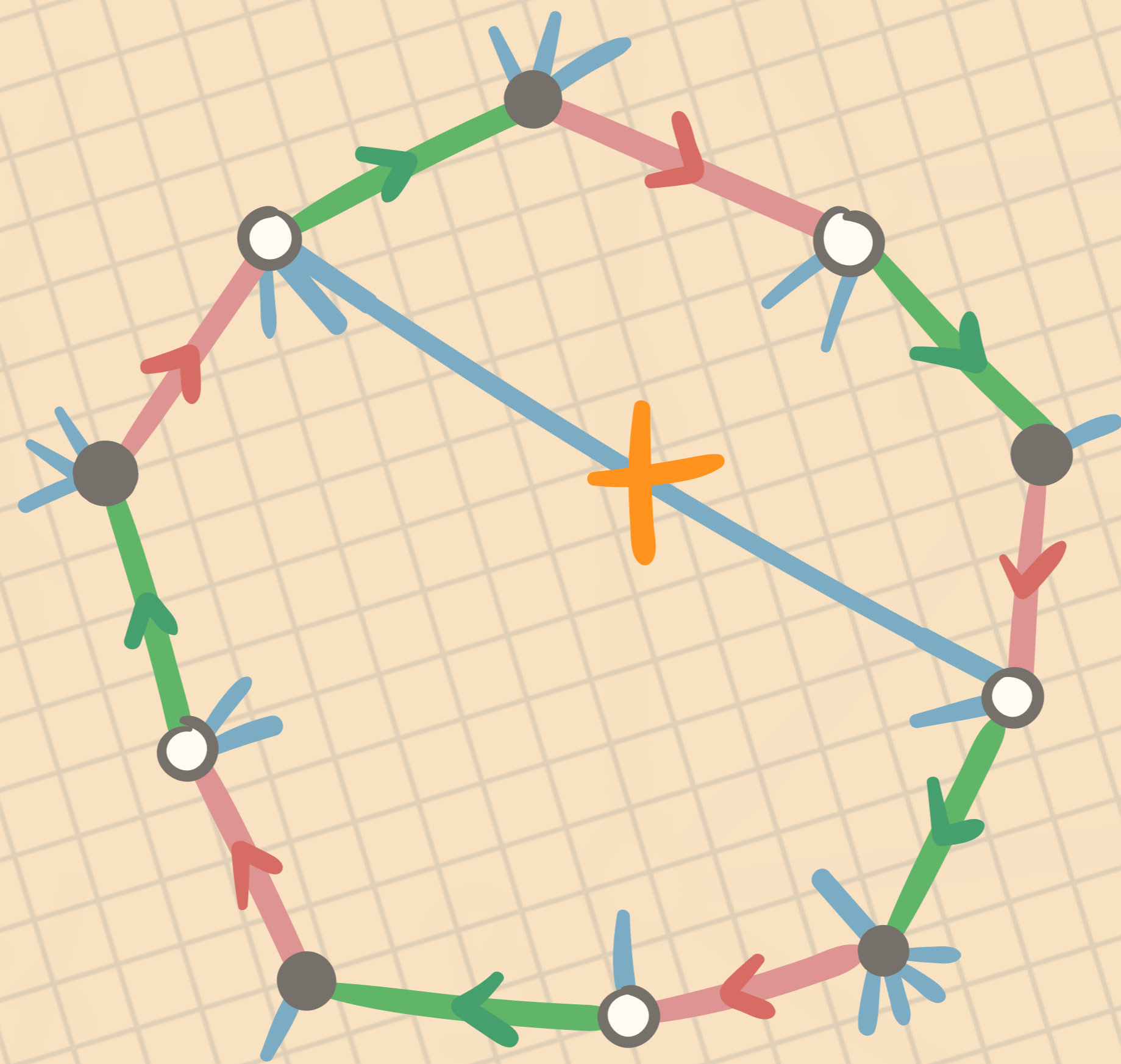


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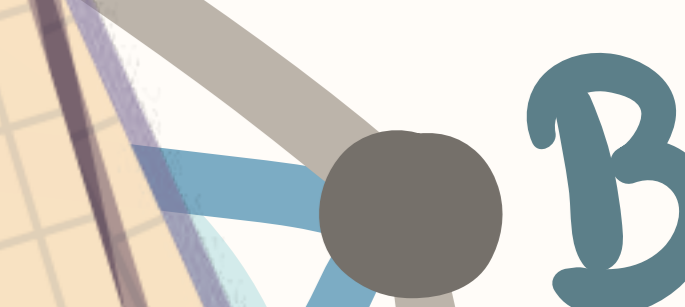
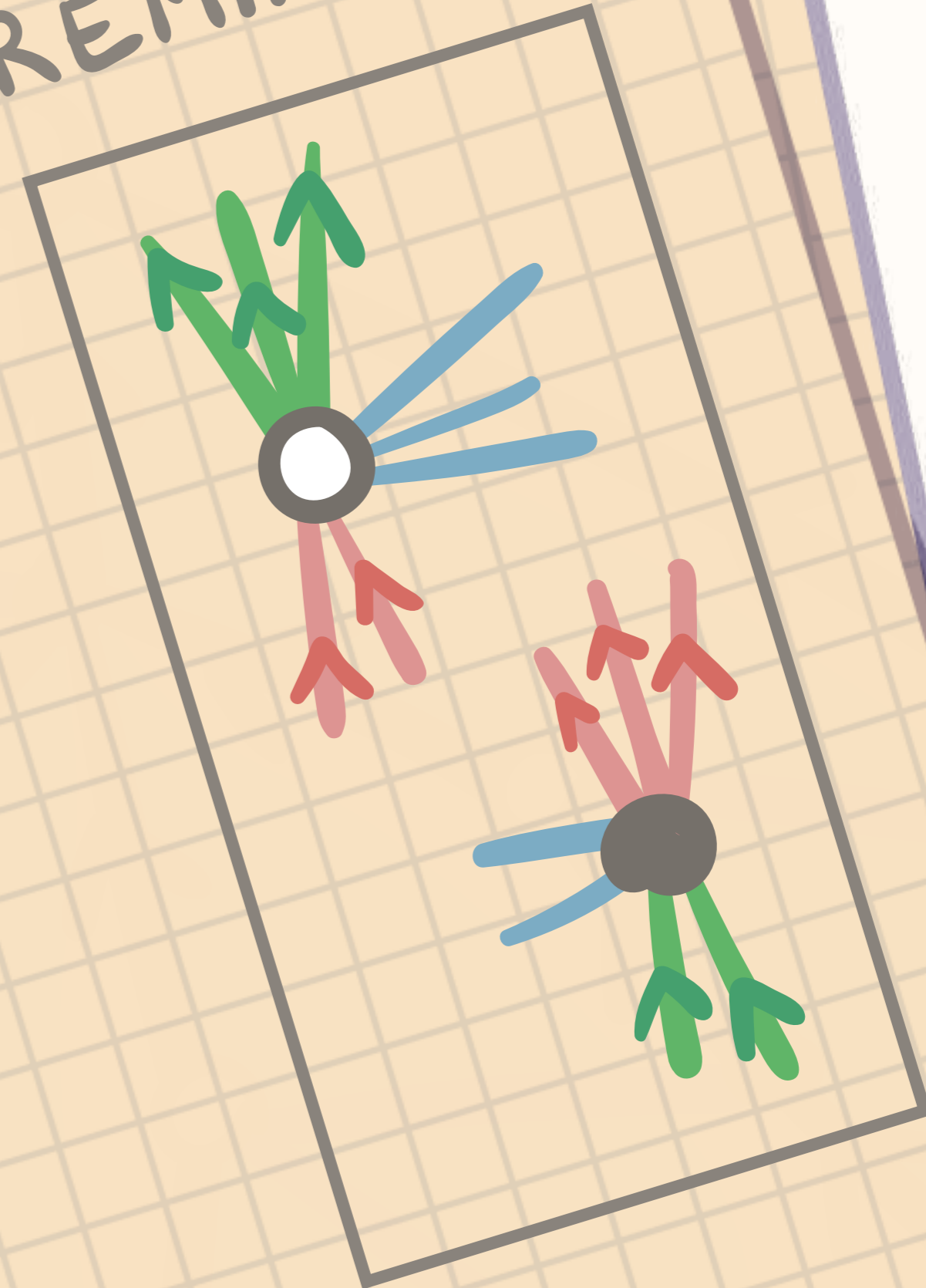


Schnyder labelling

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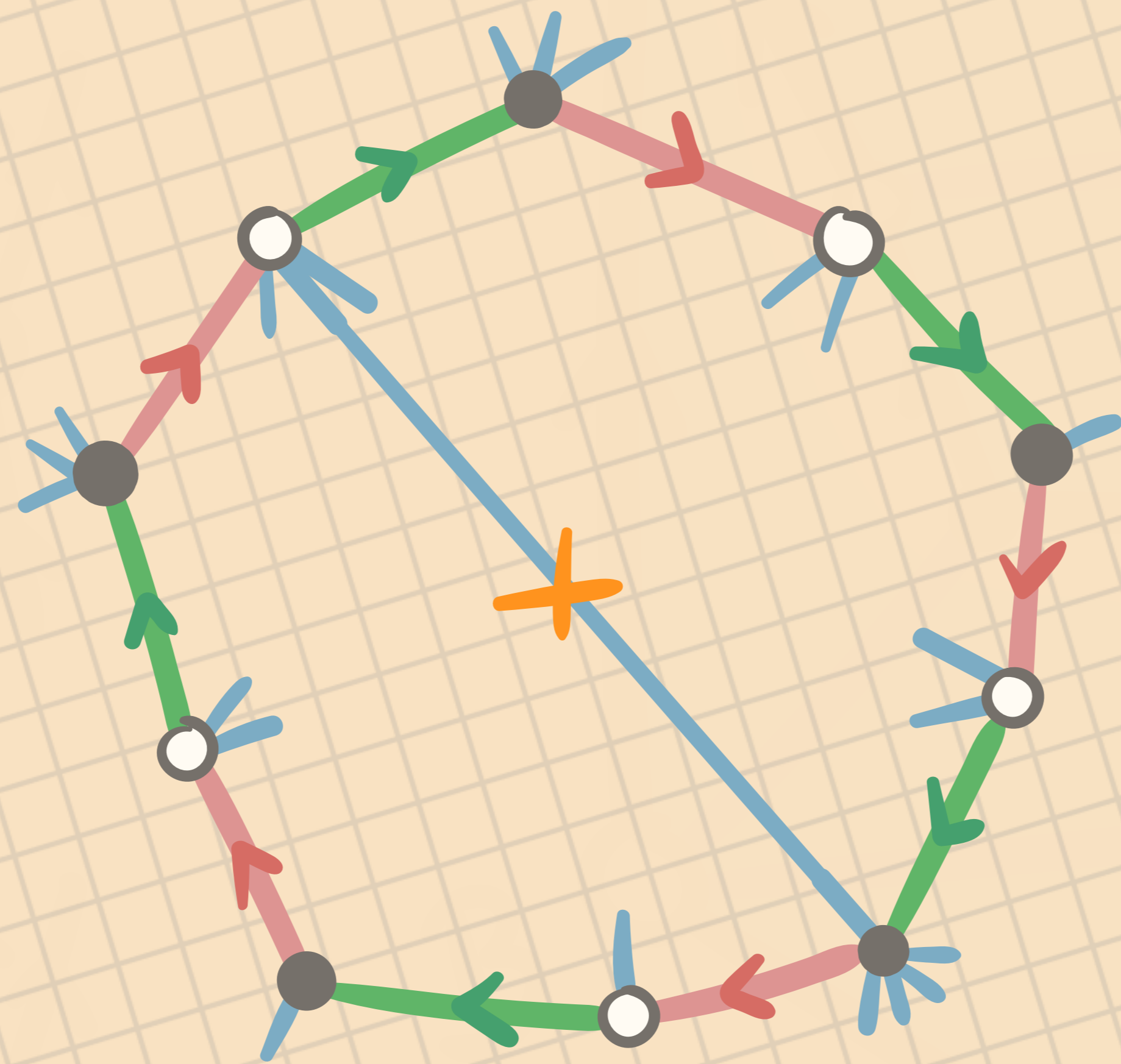


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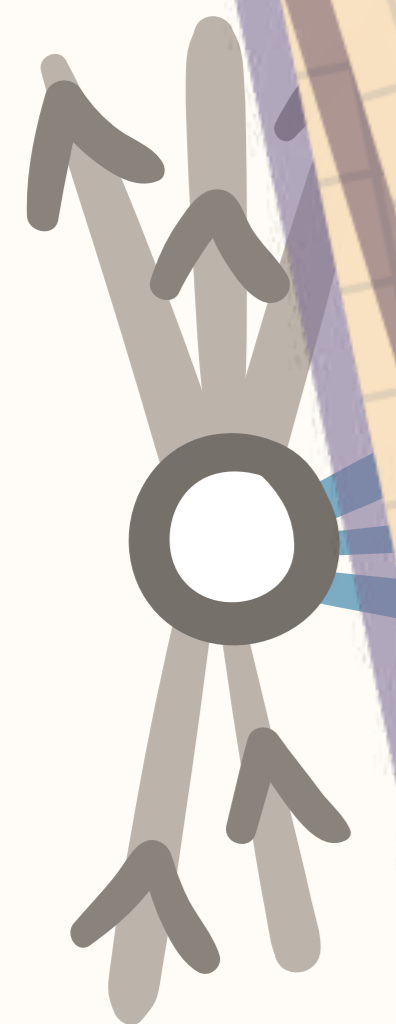
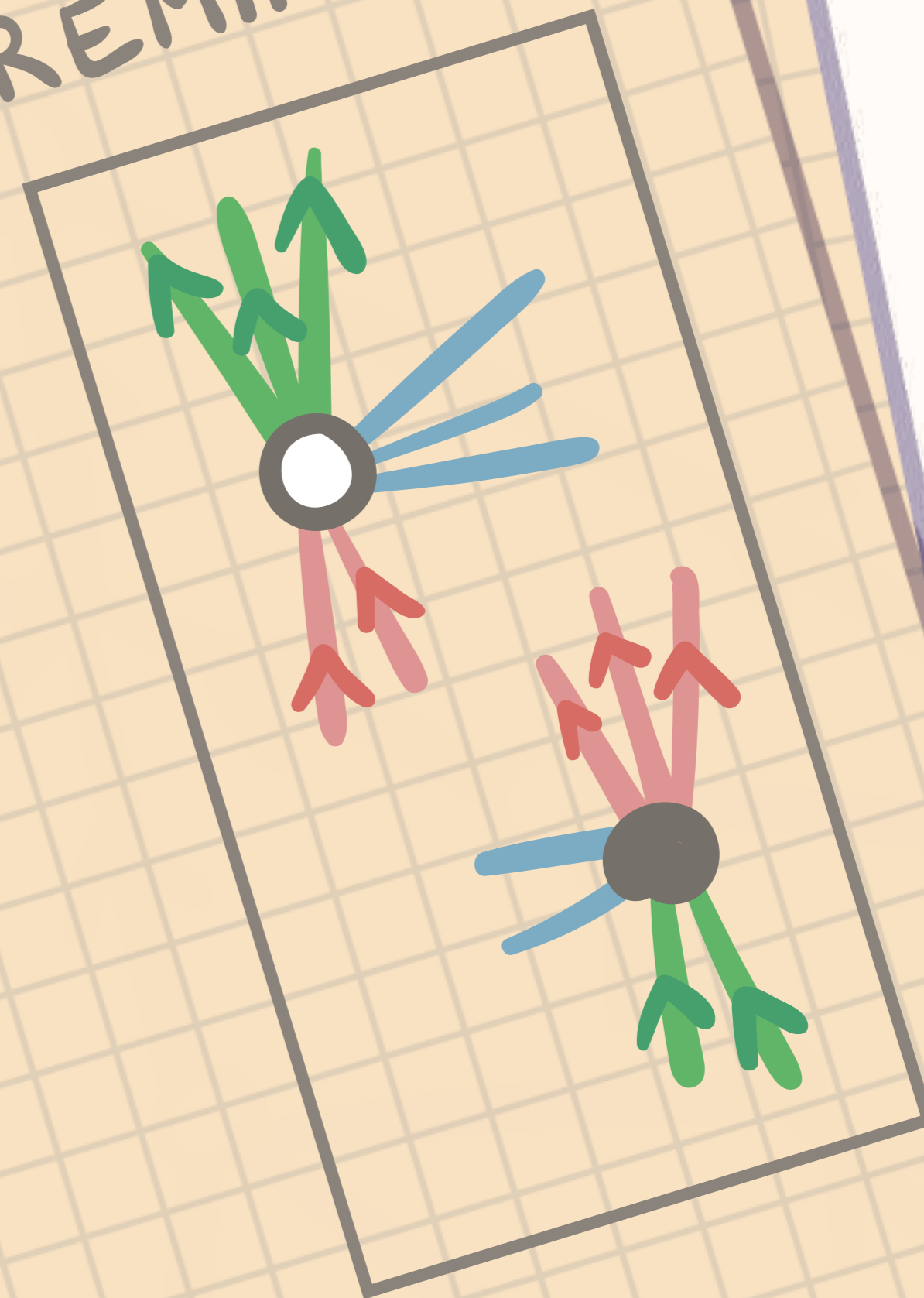


Schnyder labelling

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ASSUME \exists CYCLE, TAKEN MINIMAL



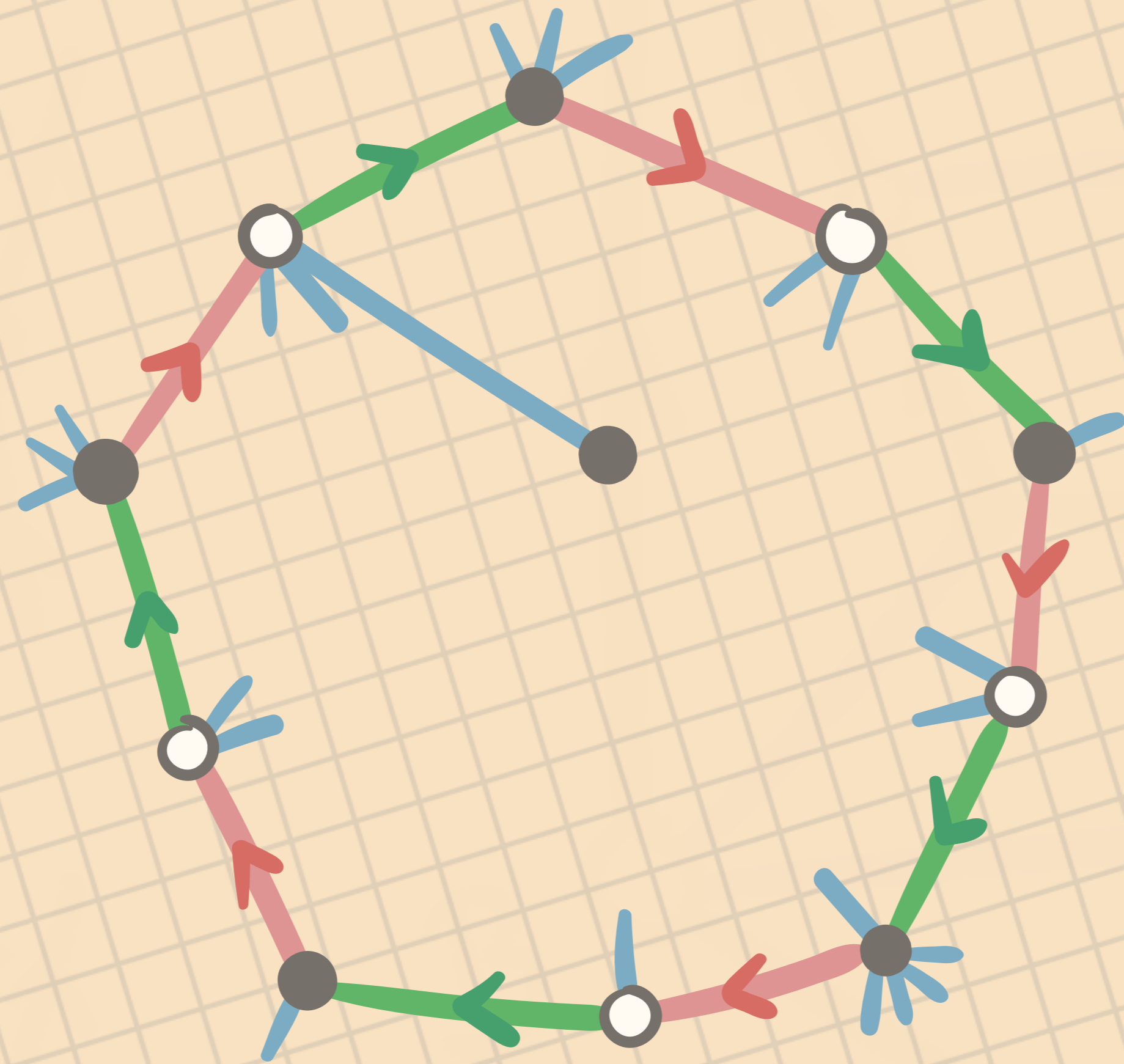
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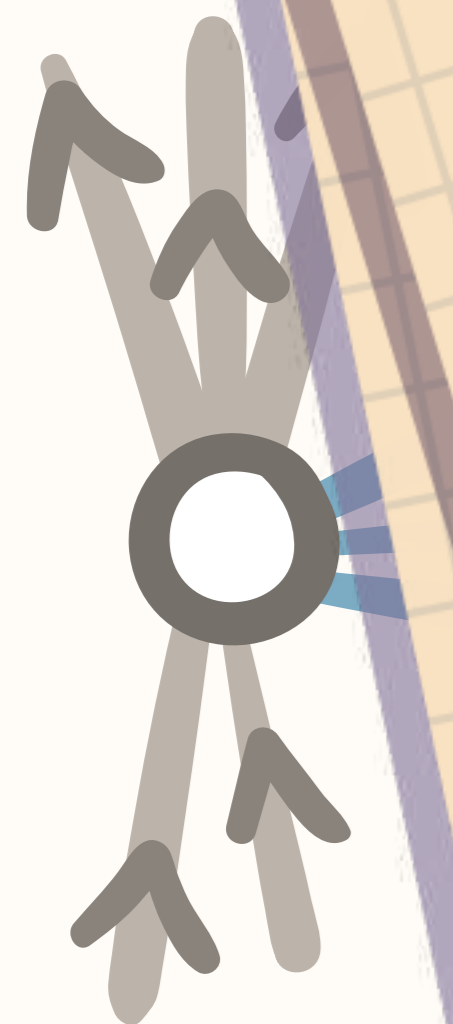
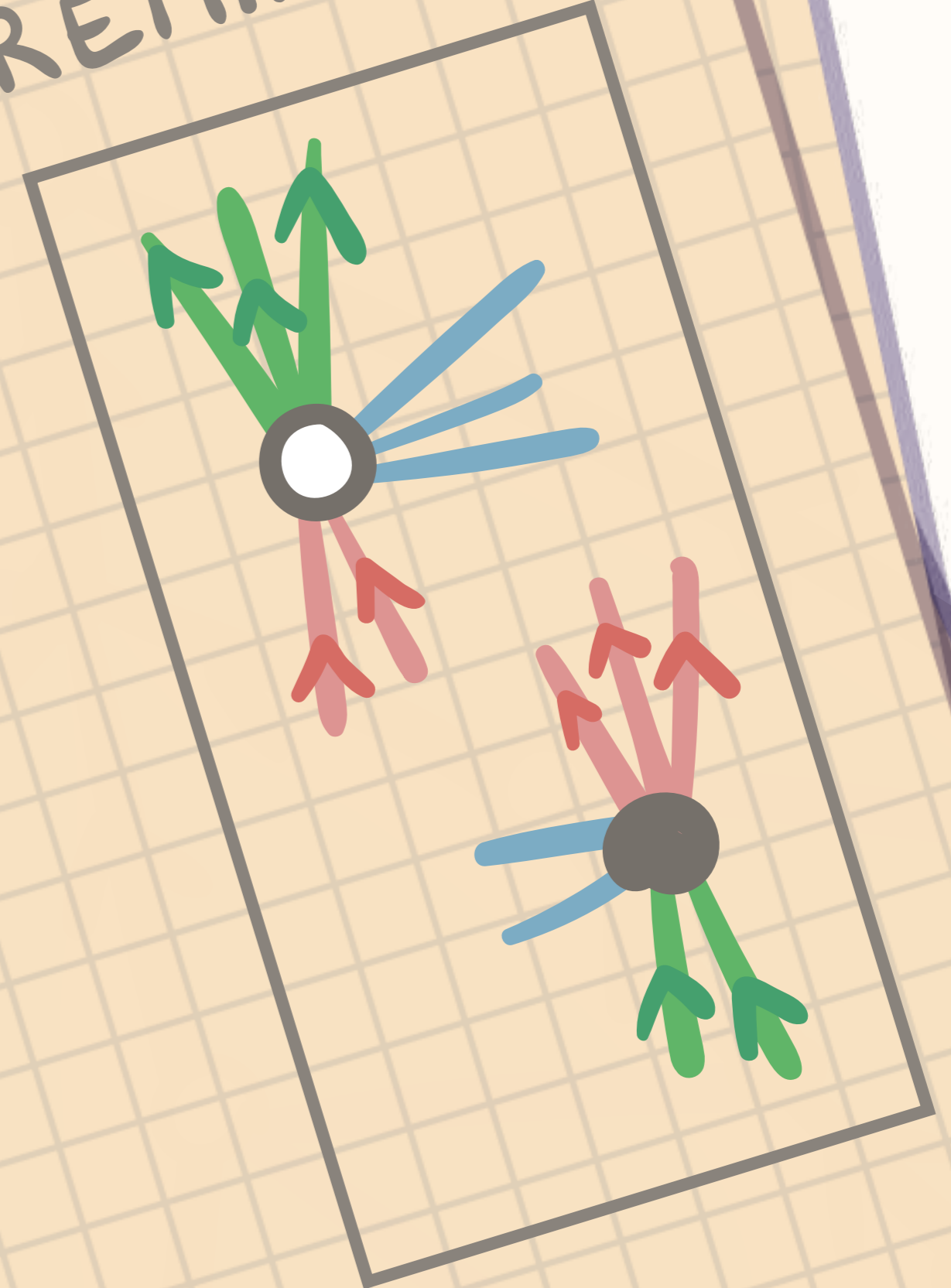
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Schnyder labelling

IT IS A BIPOLAR ORIENTATION :
1) ACYCLIC
ASSUME \exists CYCLE, TAKEN MINIMAL



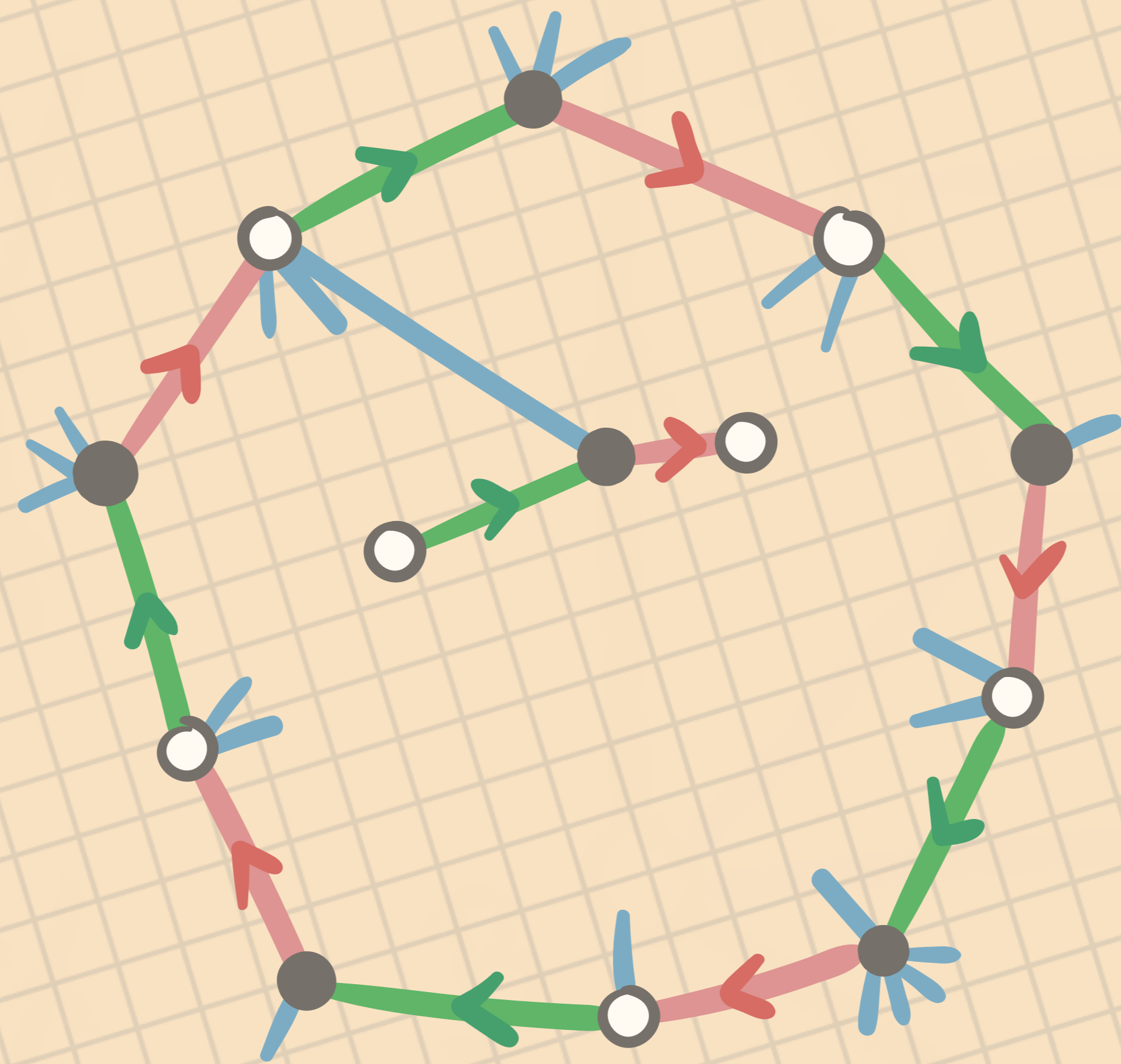
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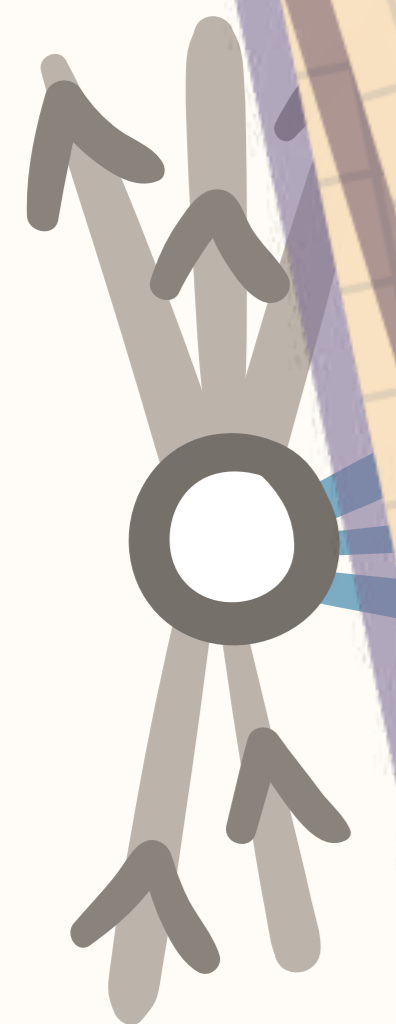
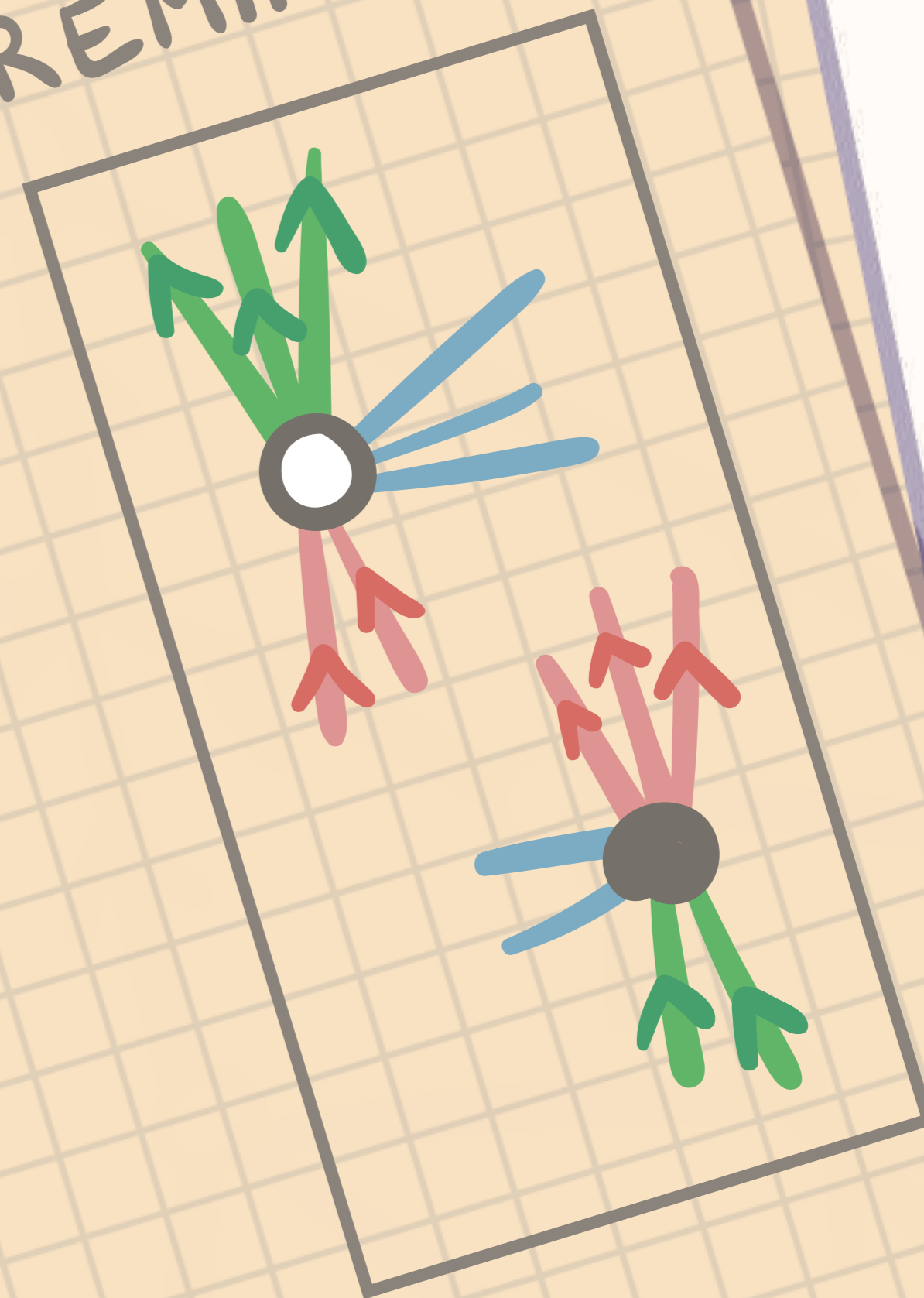
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Schnyder labelling

IT IS A BIPOLAR ORIENTATION :
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ASSUME \exists CYCLE, TAKEN MINIMAL



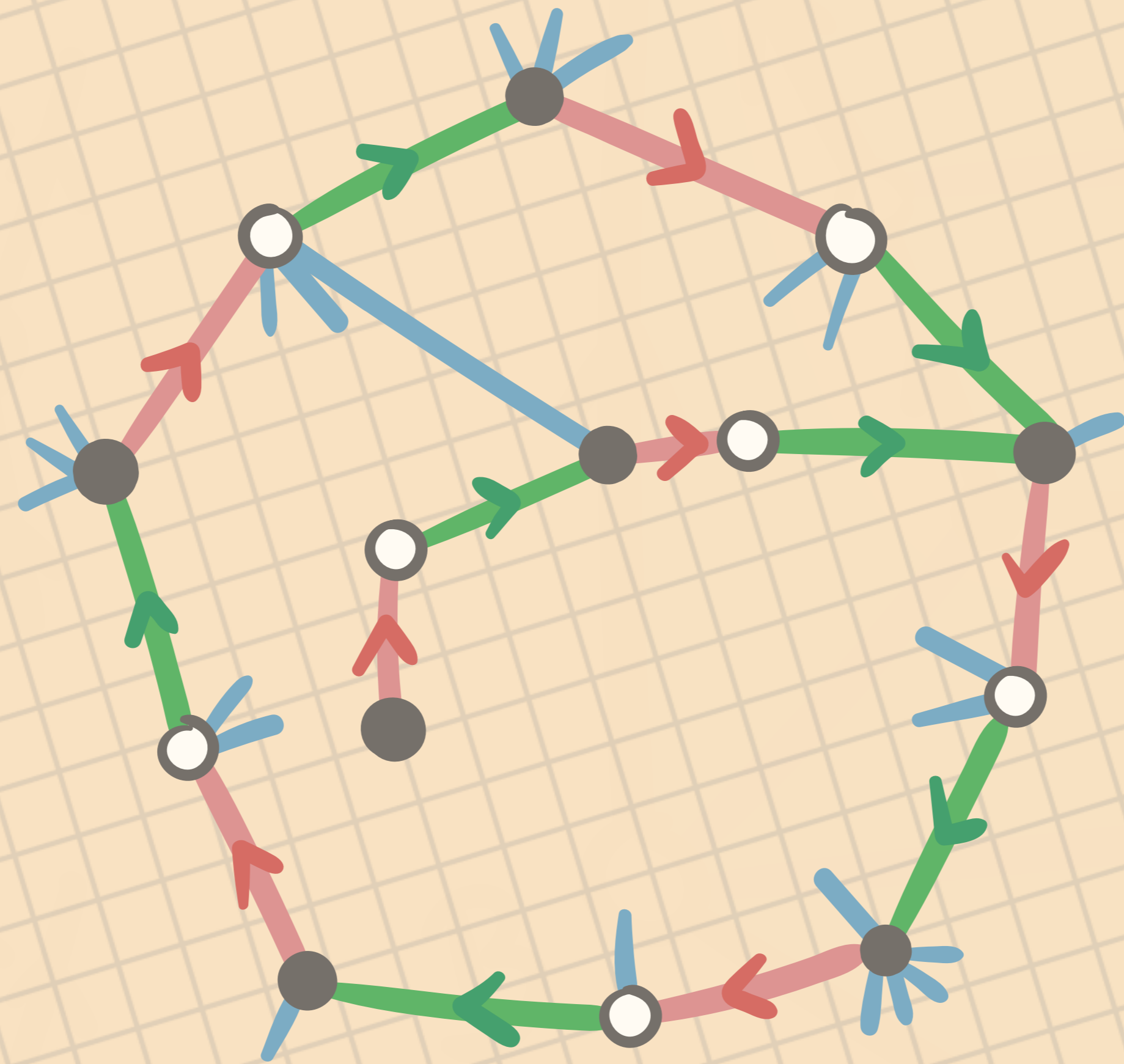
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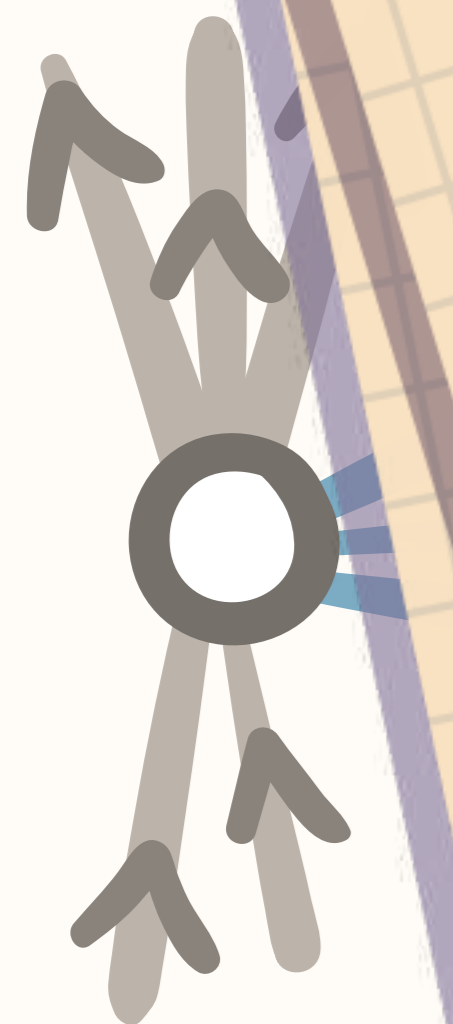
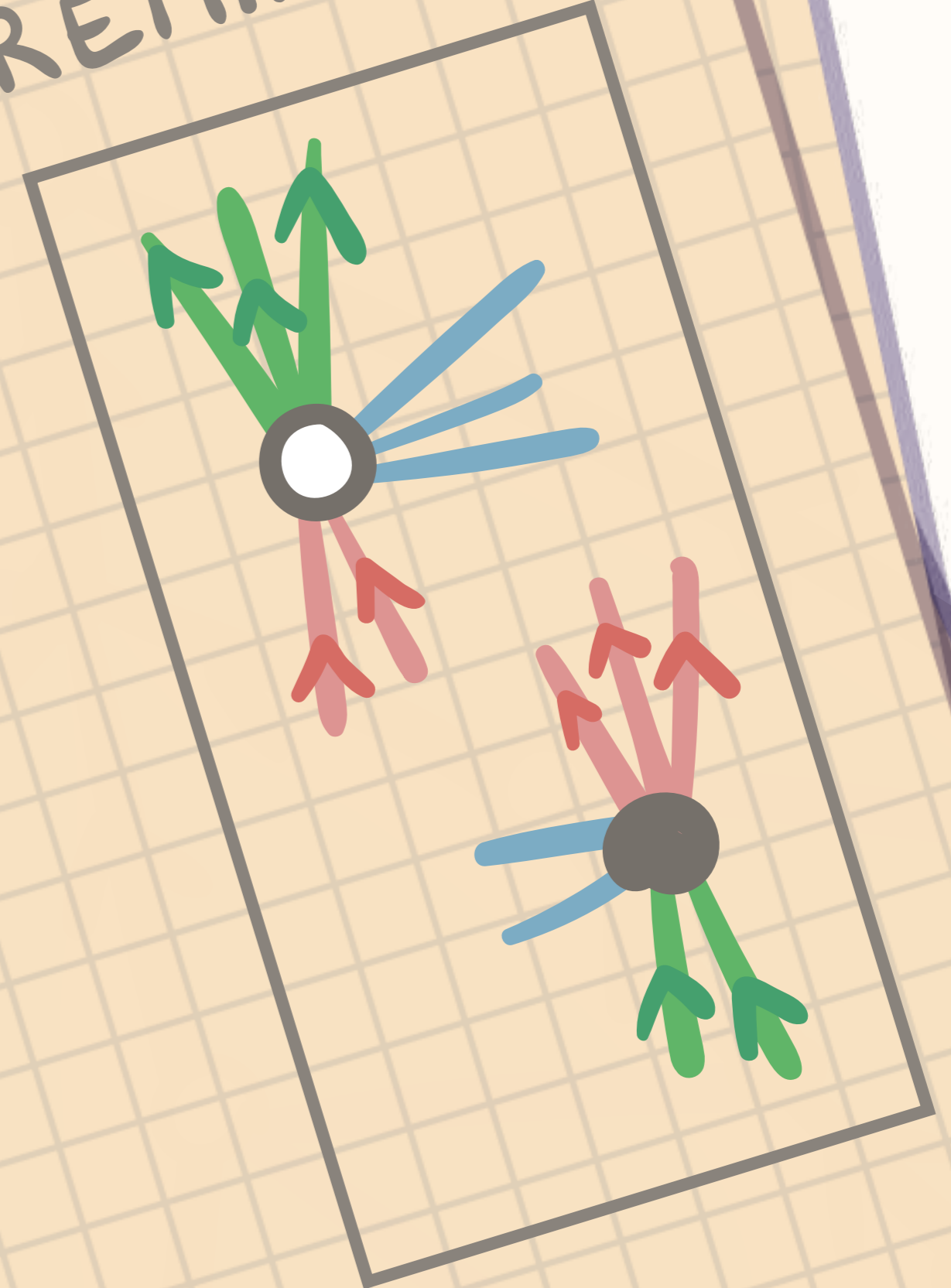
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Schnyder labelling

IT IS A BIPOLAR ORIENTATION :
1) ACYCLIC
ASSUME \exists CYCLE, TAKEN MINIMAL



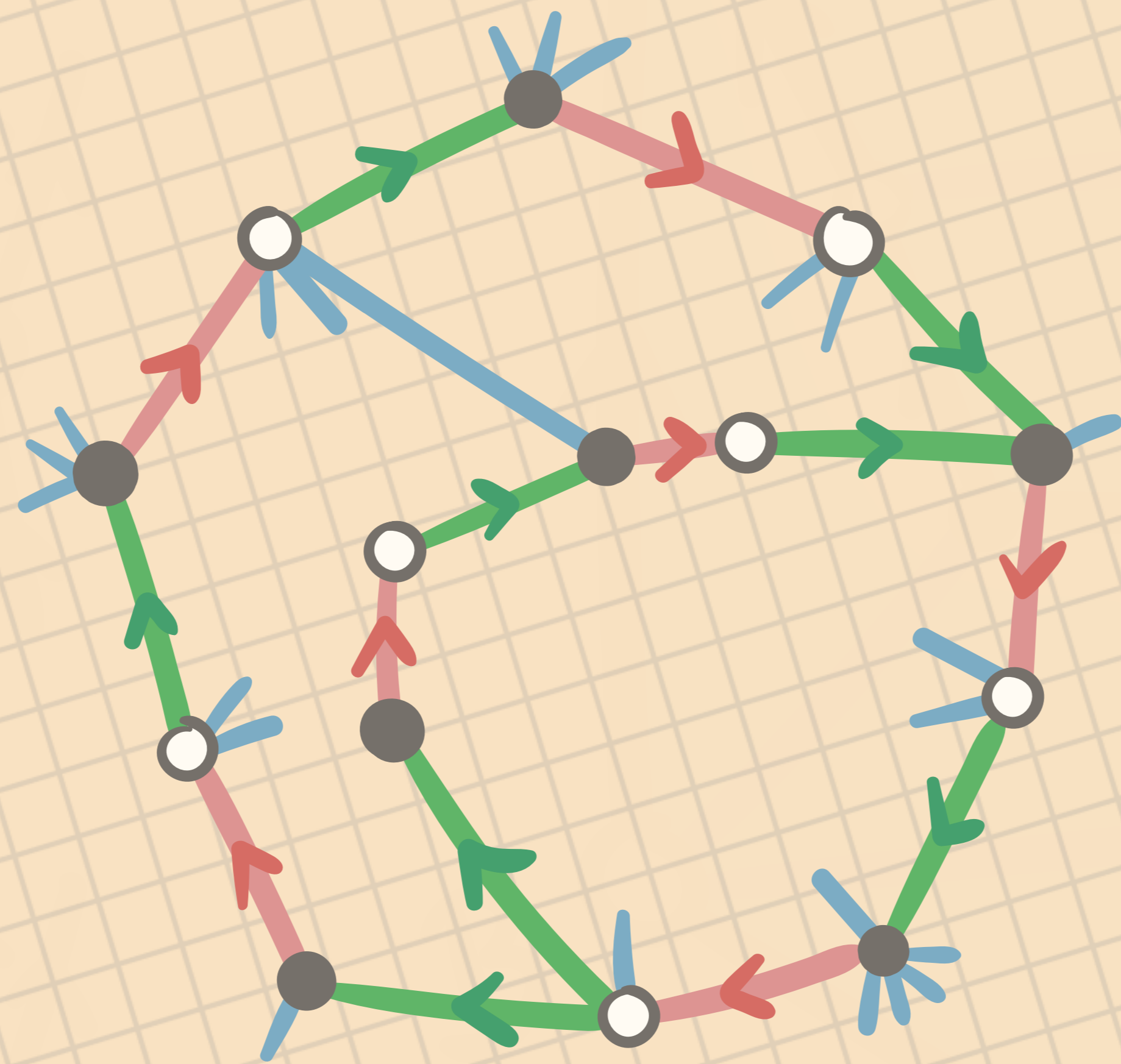
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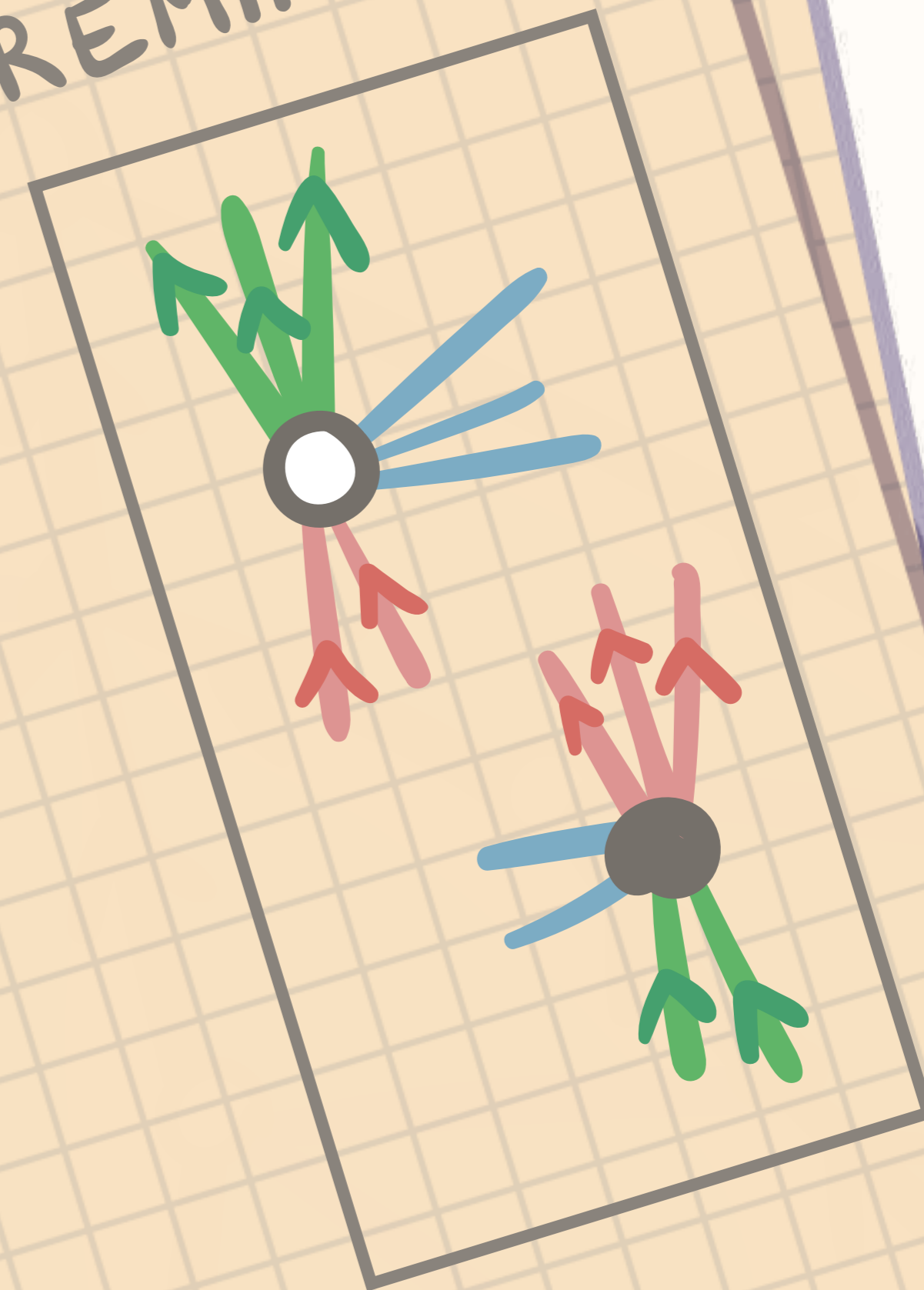
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Schnyder labelling

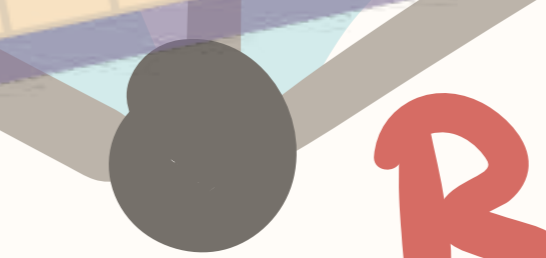
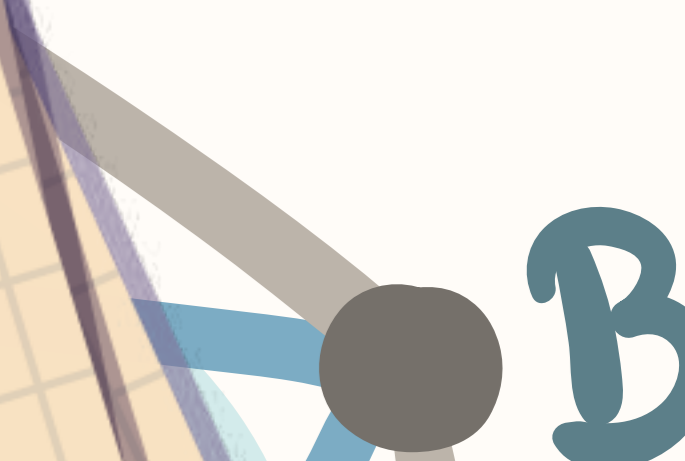
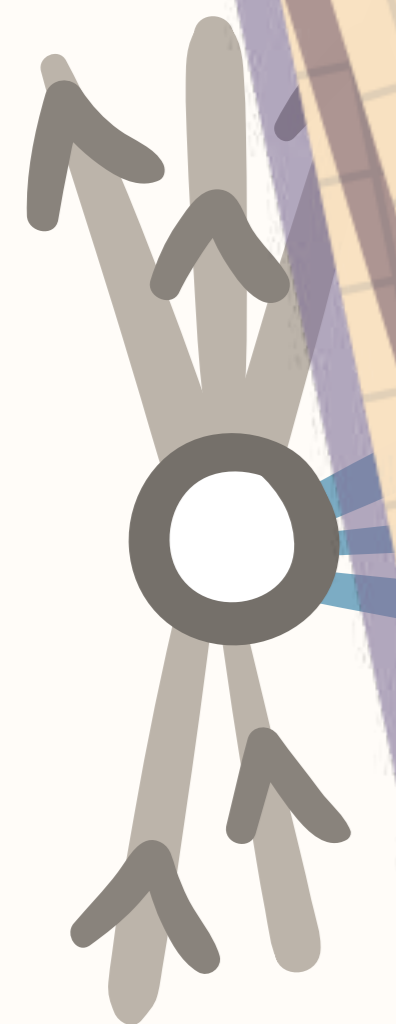
IT IS A BIPOLAR ORIENTATION :
1) ACYCLIC
ASSUME \exists CYCLE, TAKEN MINIMAL



REMINDER



NOT MINIMAL



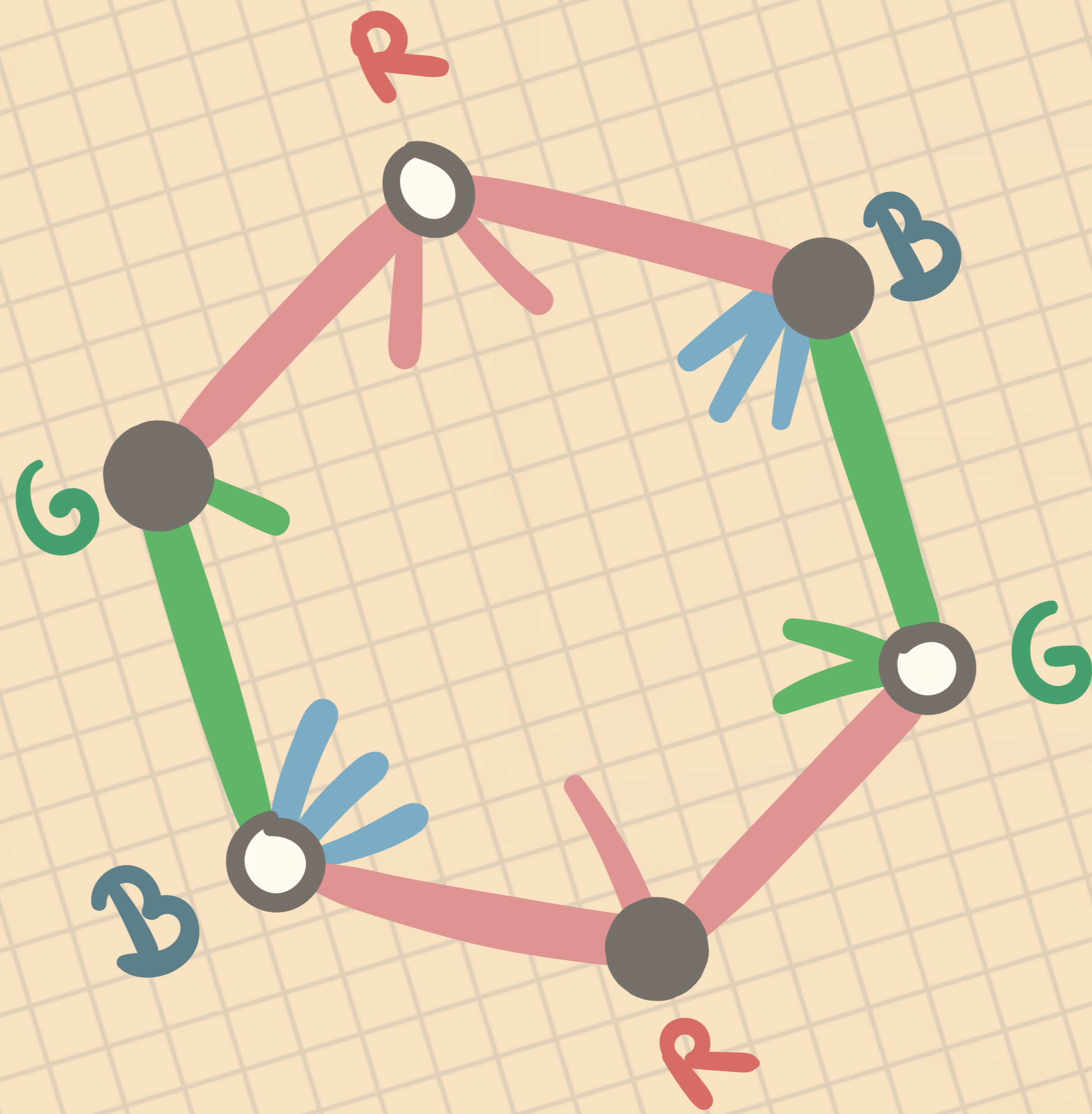
G

B

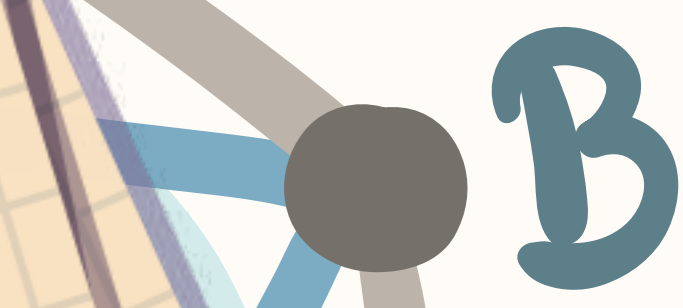
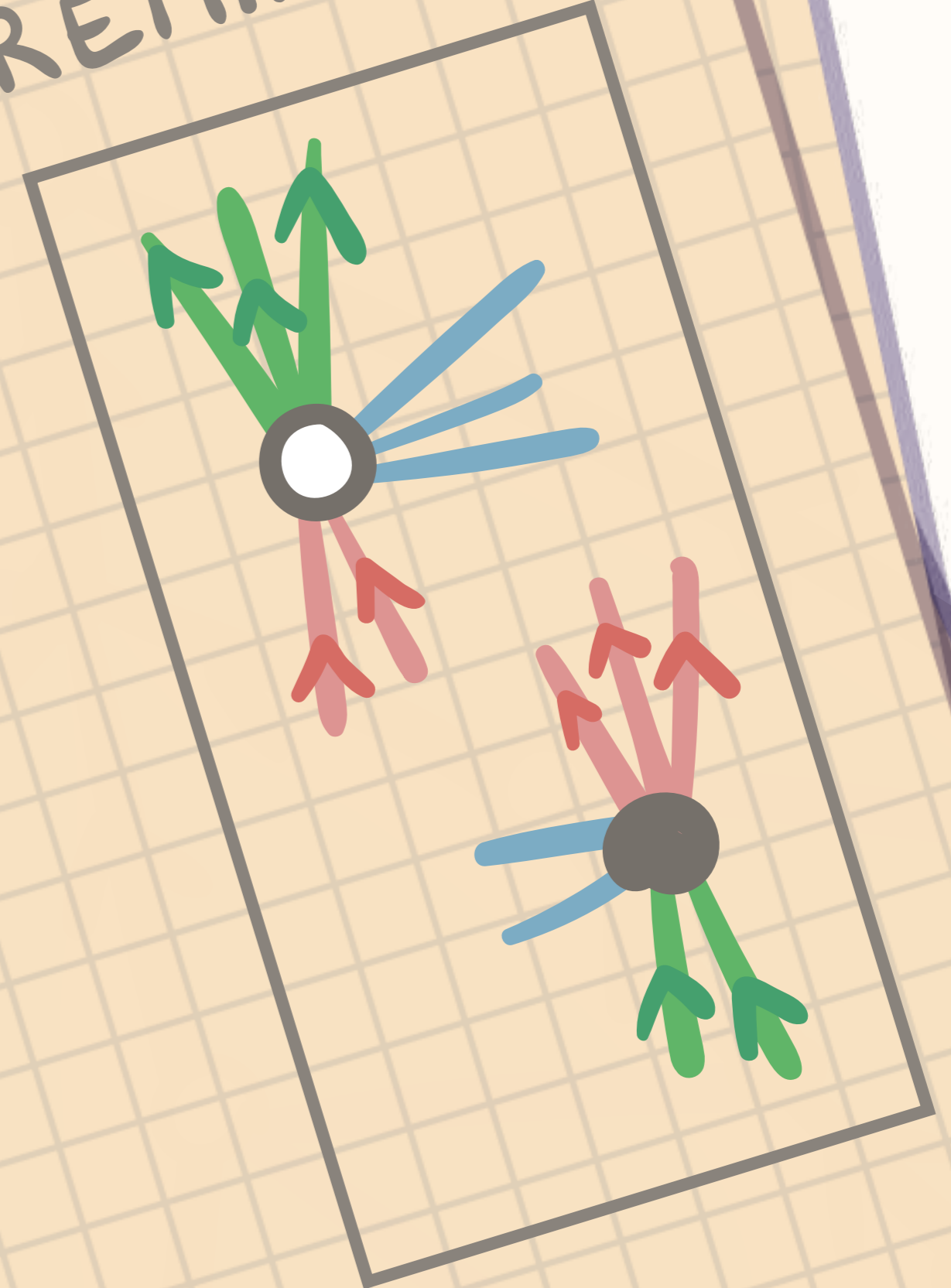
R

Schnyder labelling

- IT IS A BIPOLAR ORIENTATION :
- 1) ACYCLIC
 - 2) SINGLE SINK / SOURCE



REMINDER



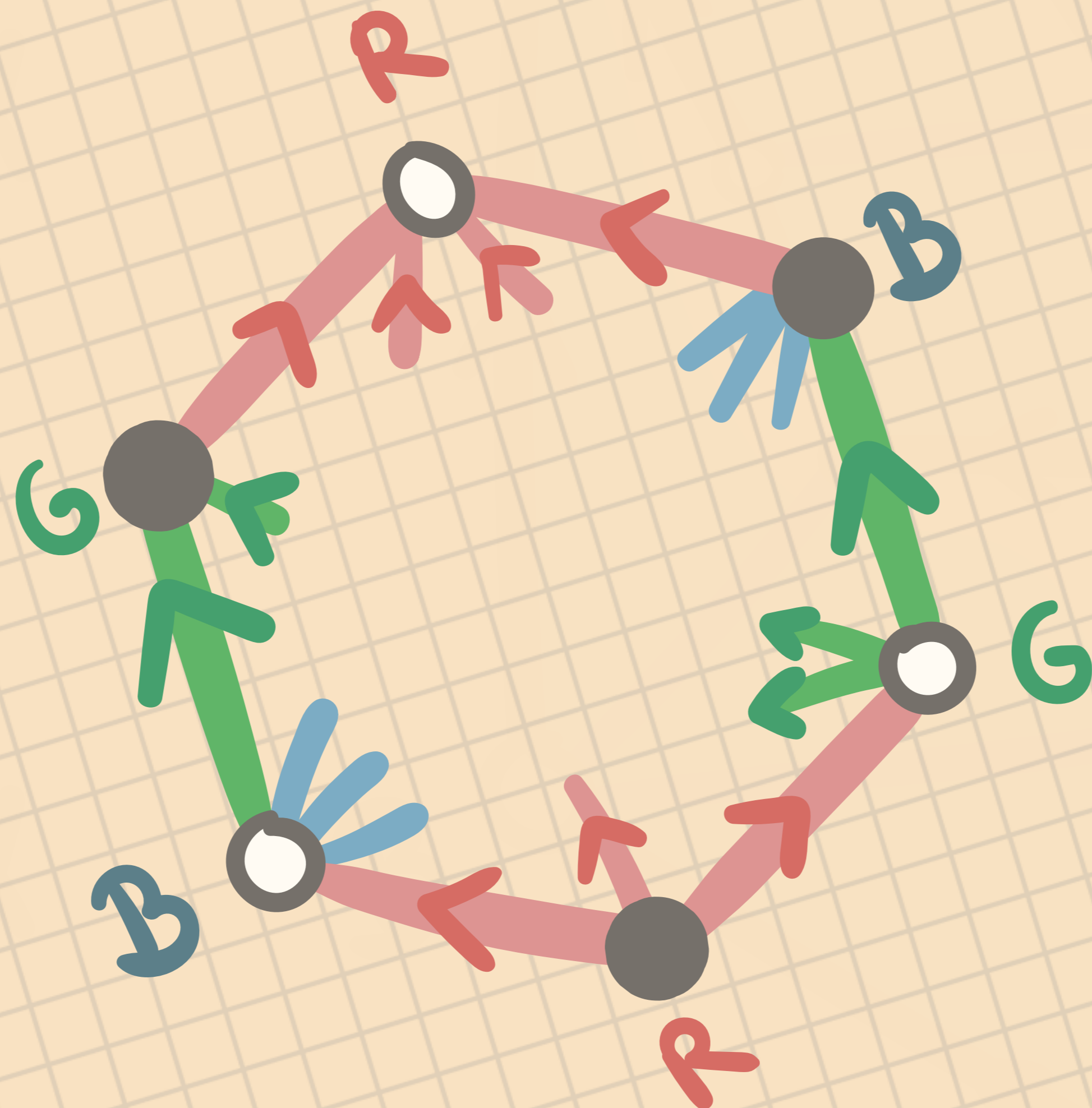
G

R

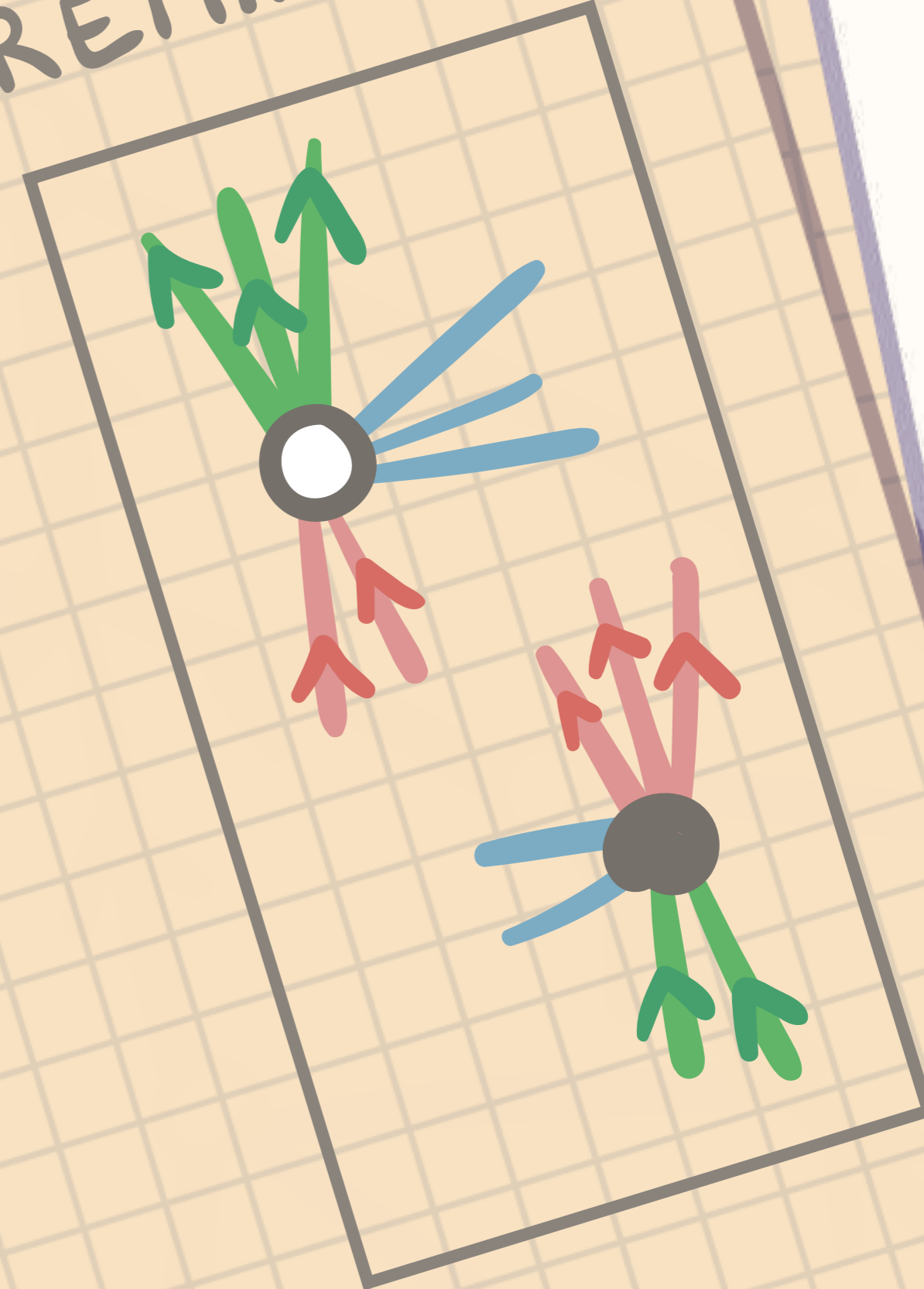
Schnyder labelling

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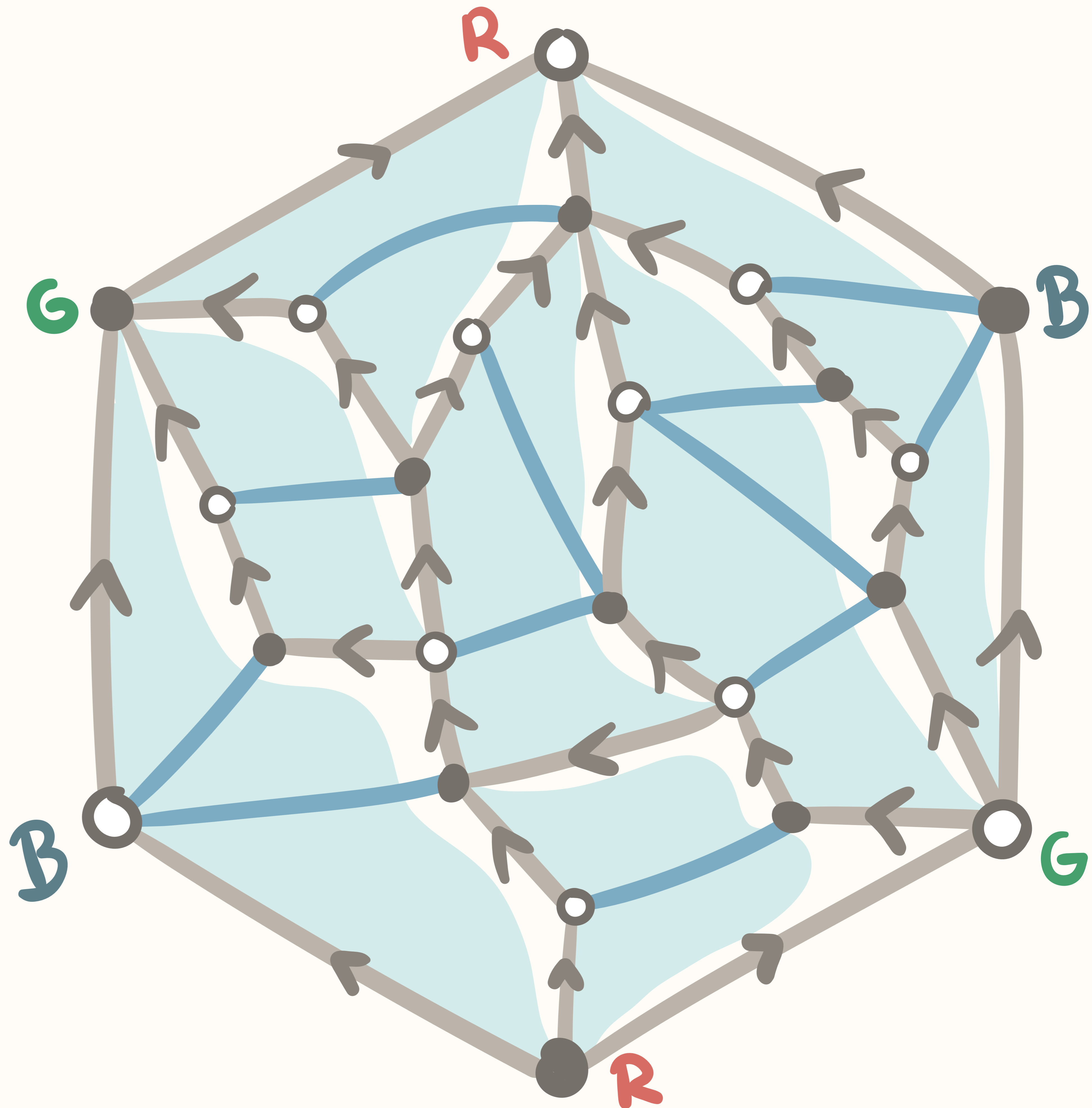
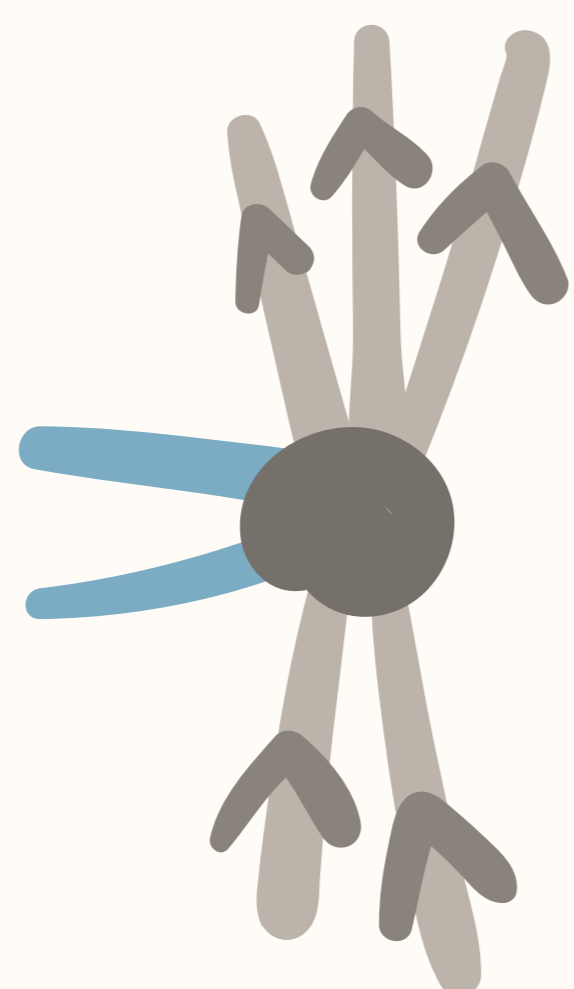
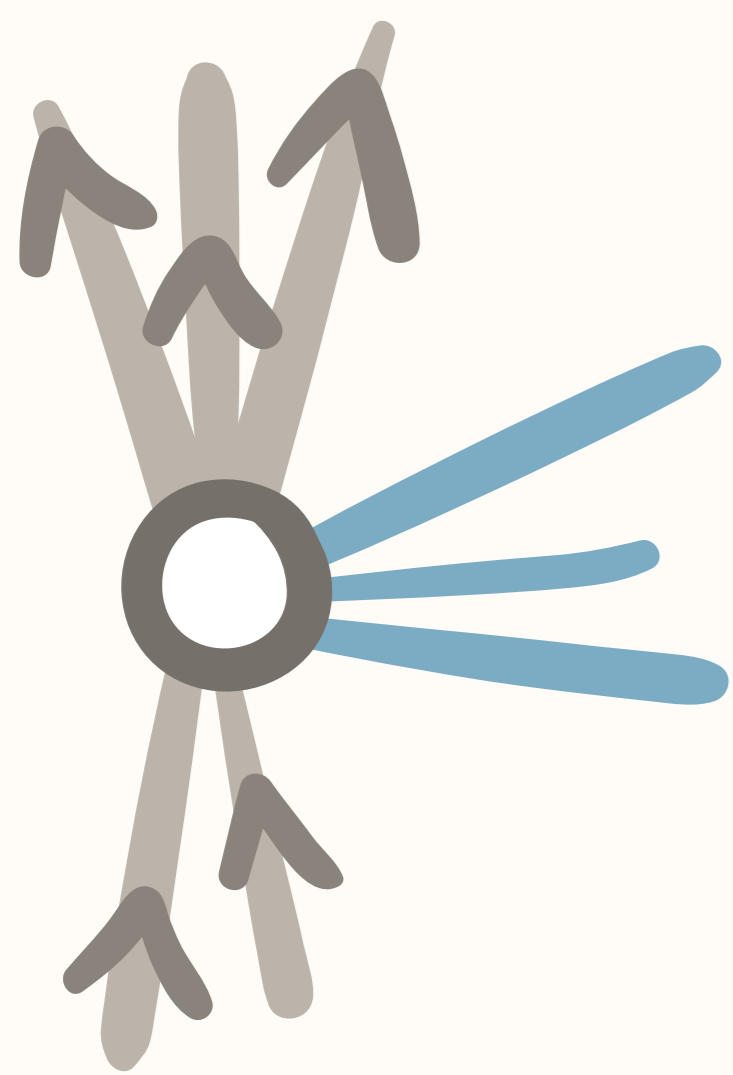
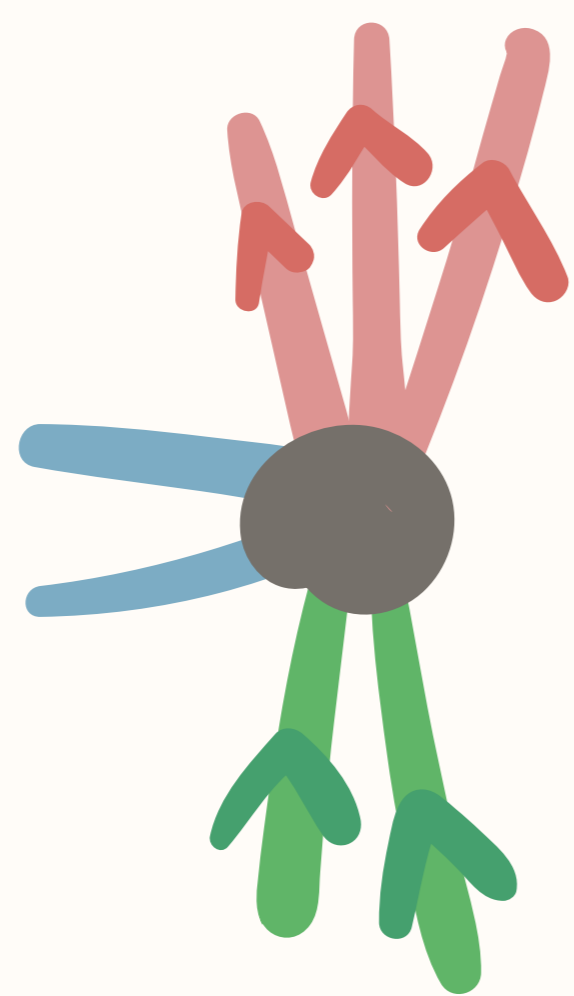
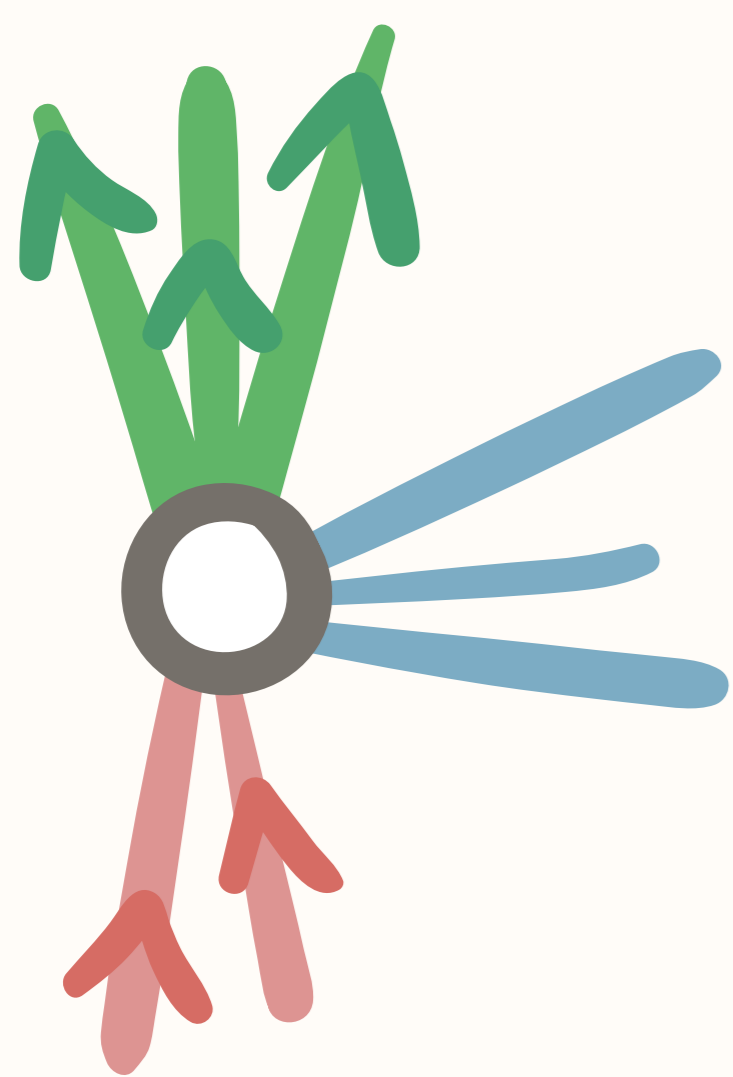
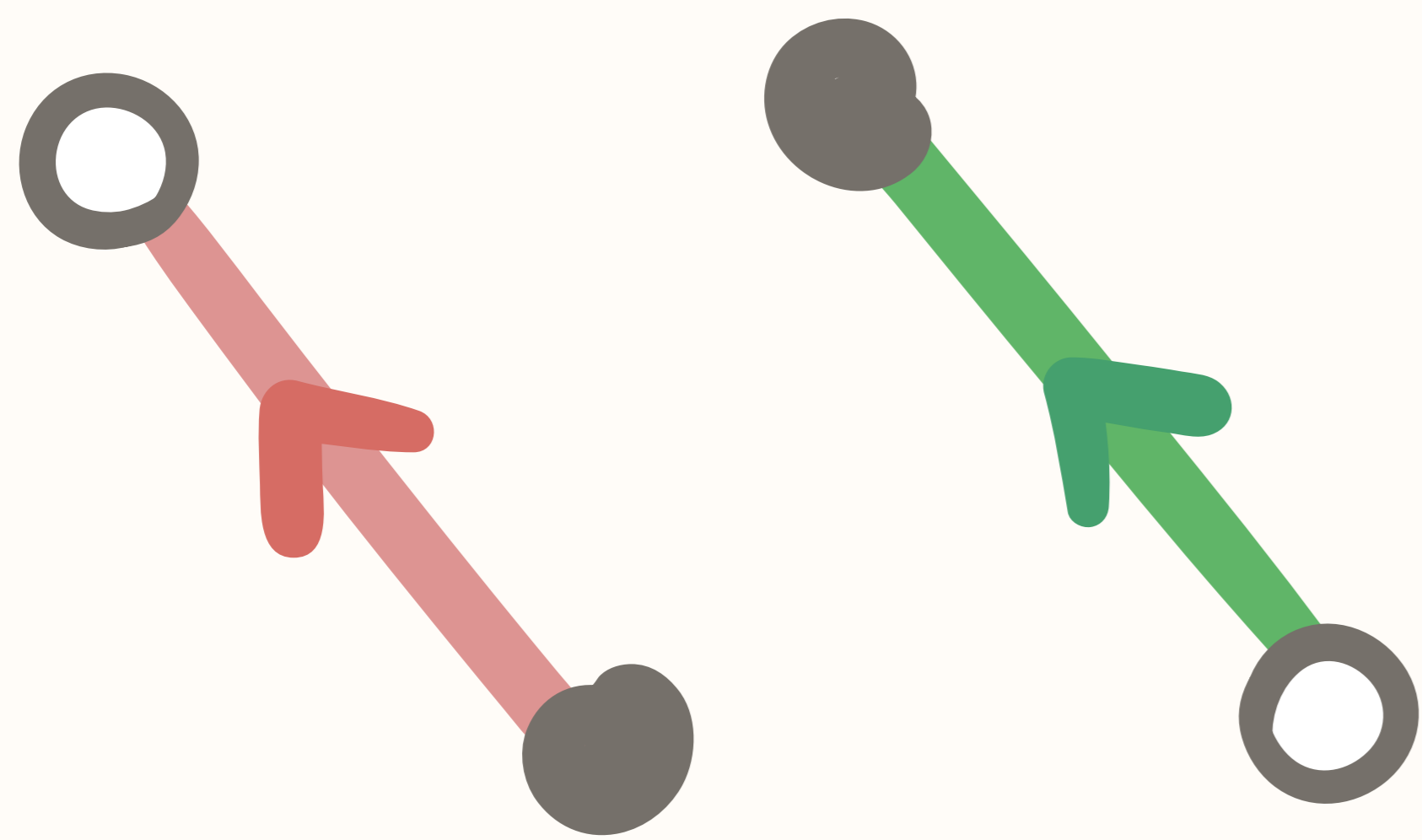
- 1) ACYCLIC
- 2) SINGLE SINK / SOURCE



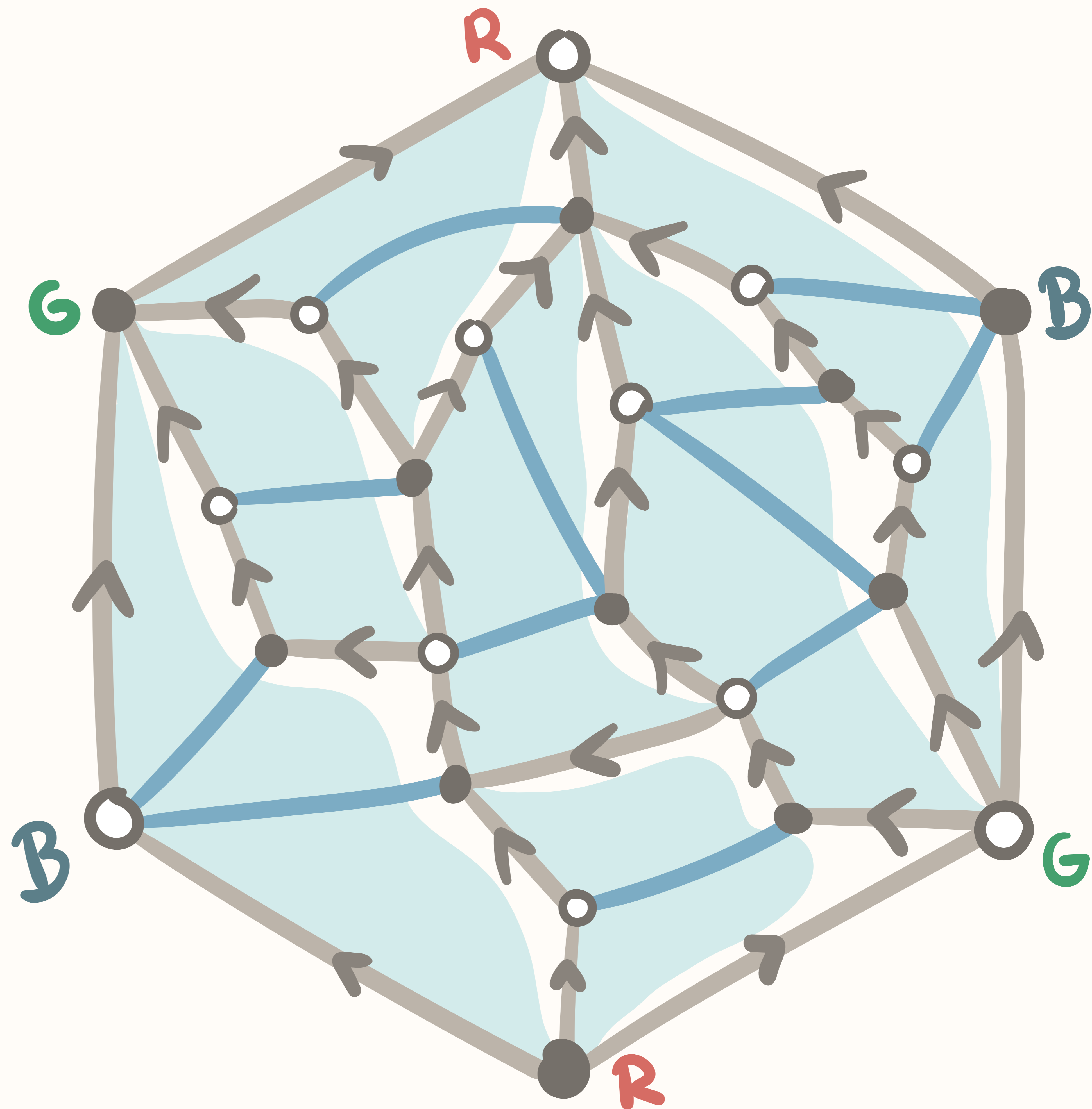
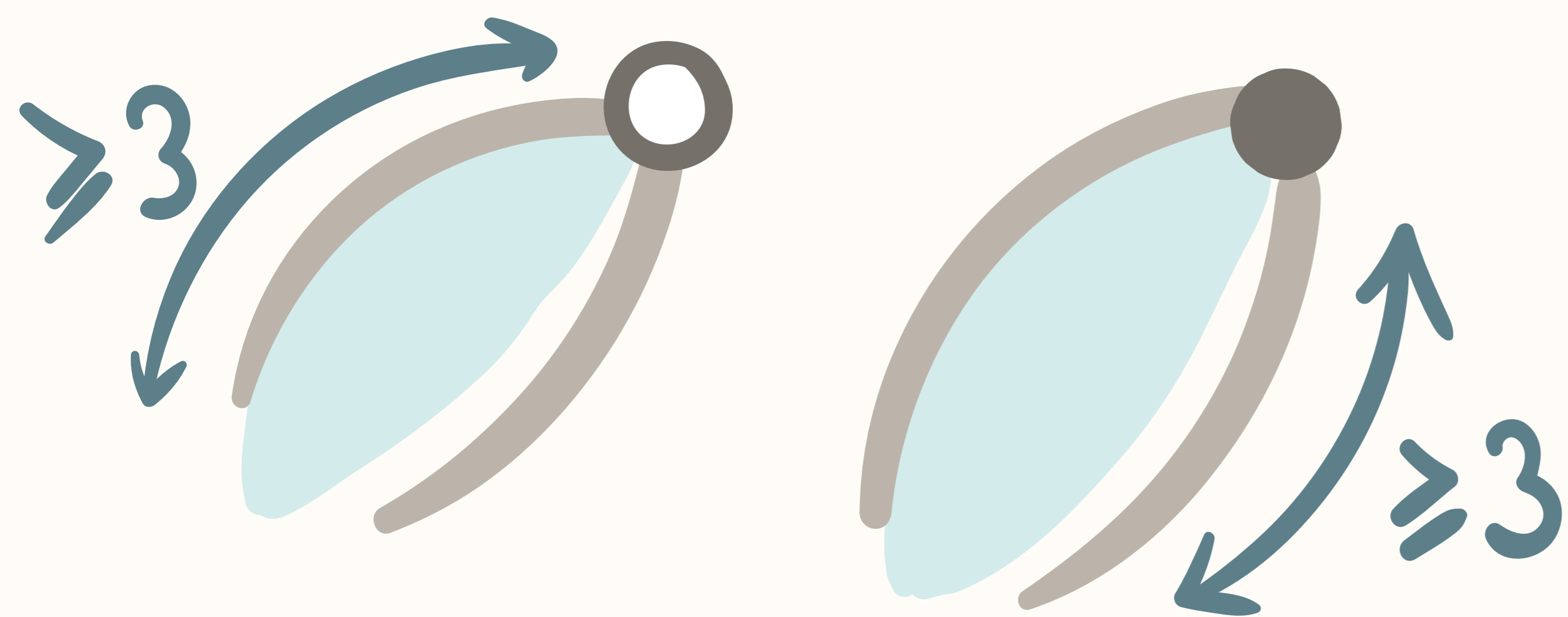
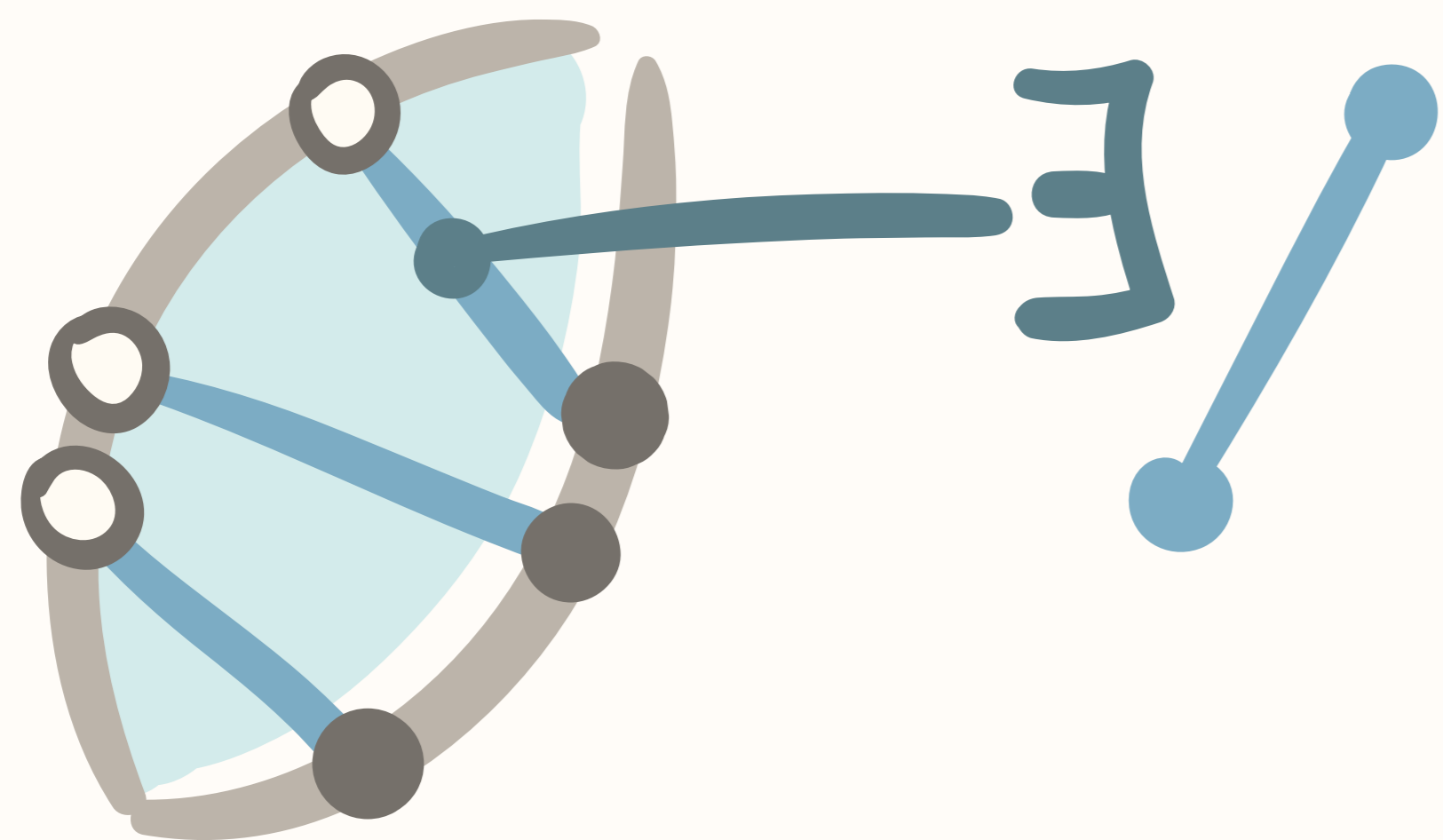
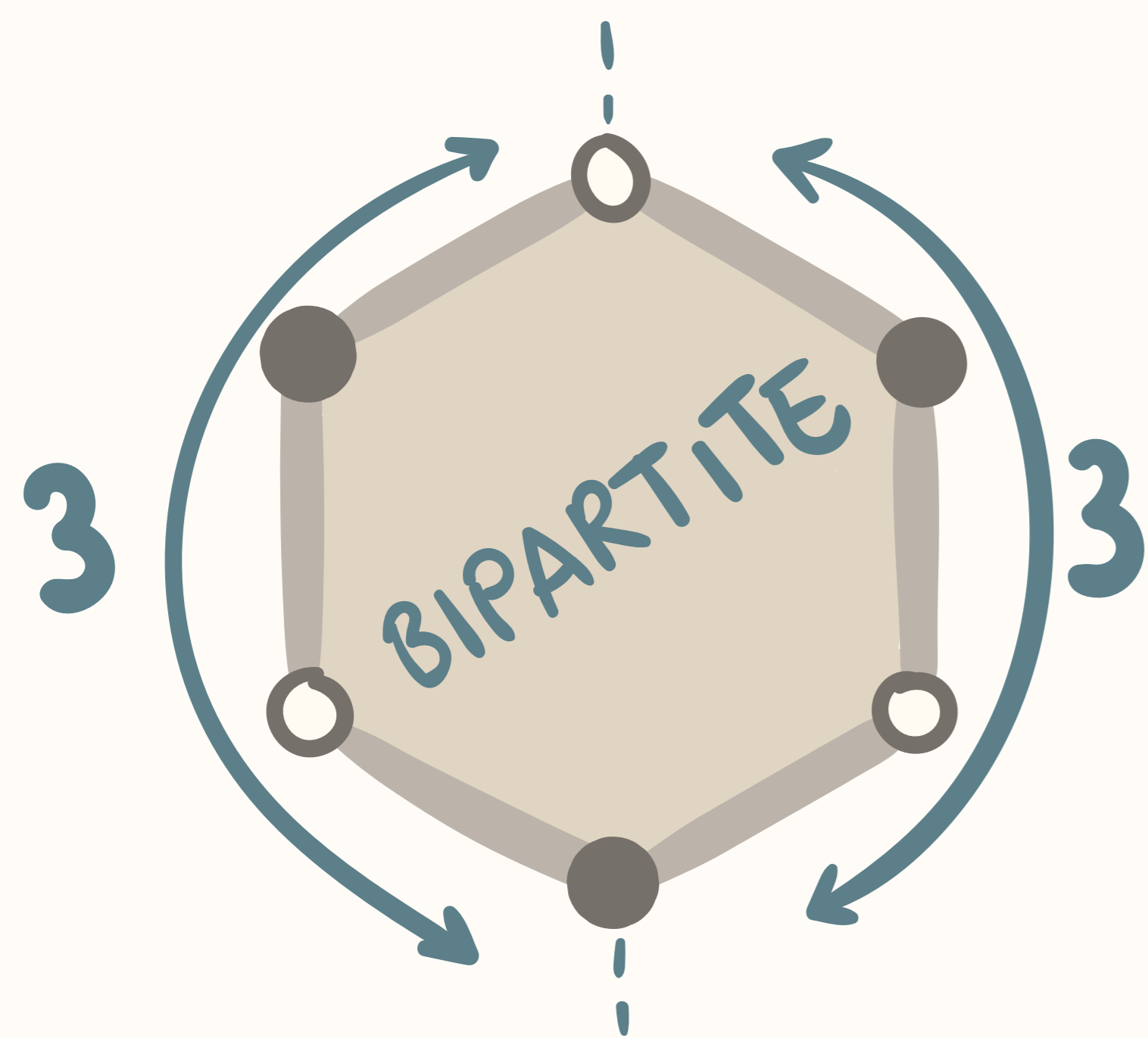
REMINDER



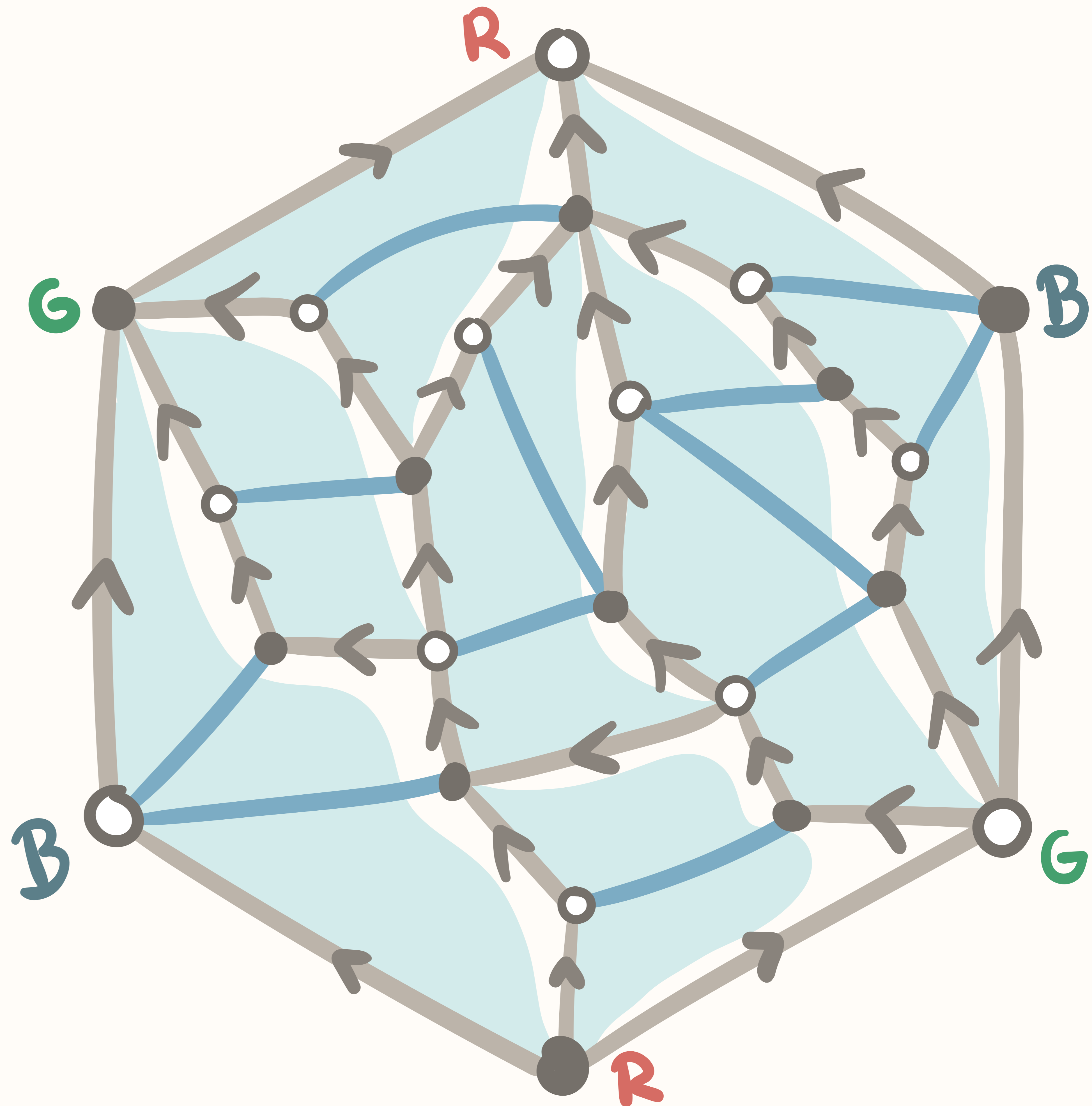
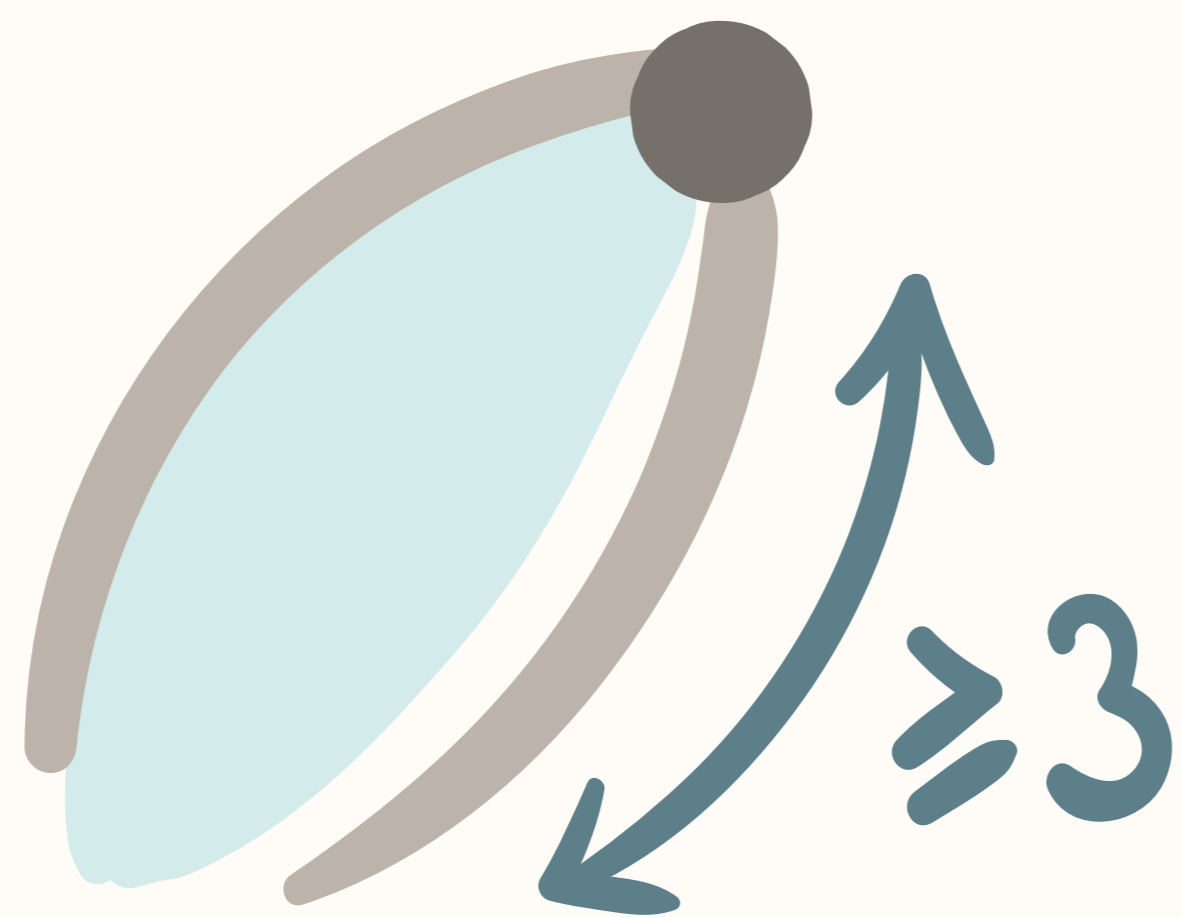
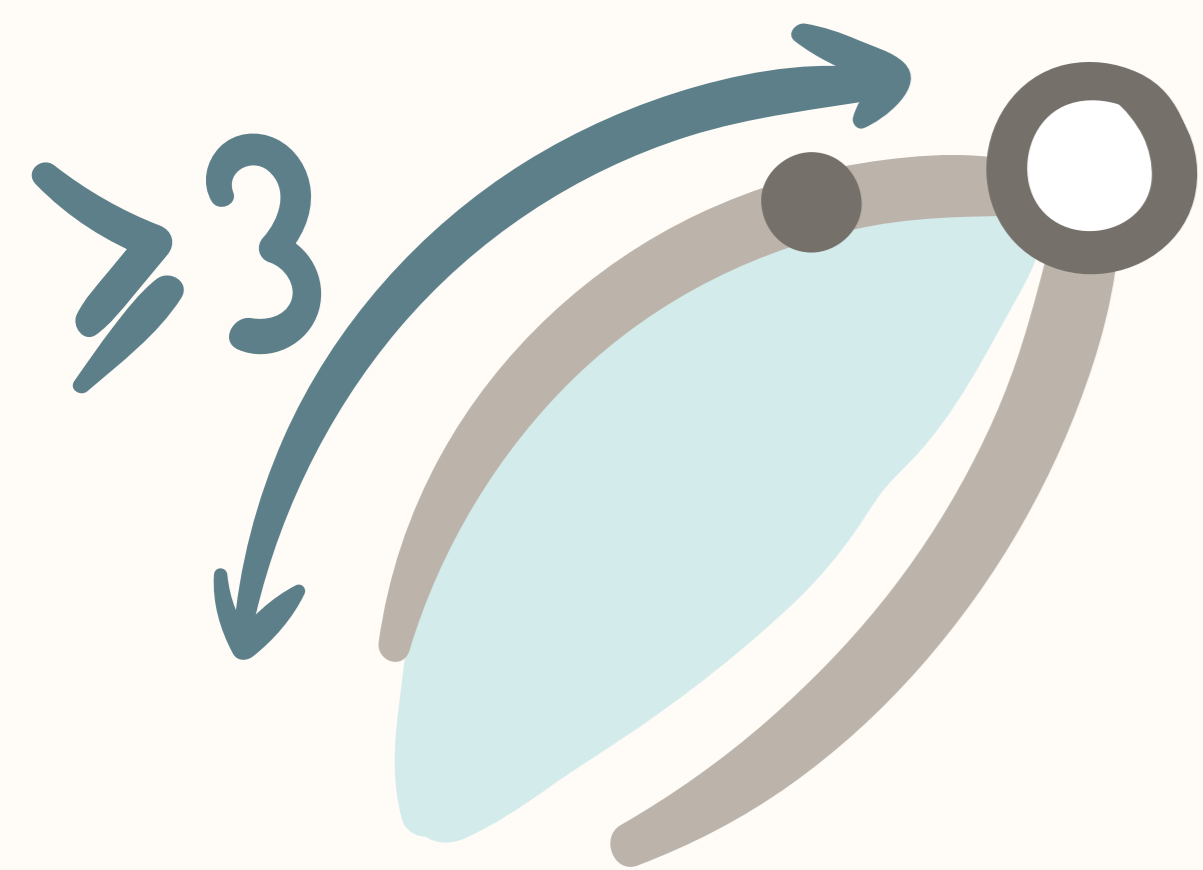
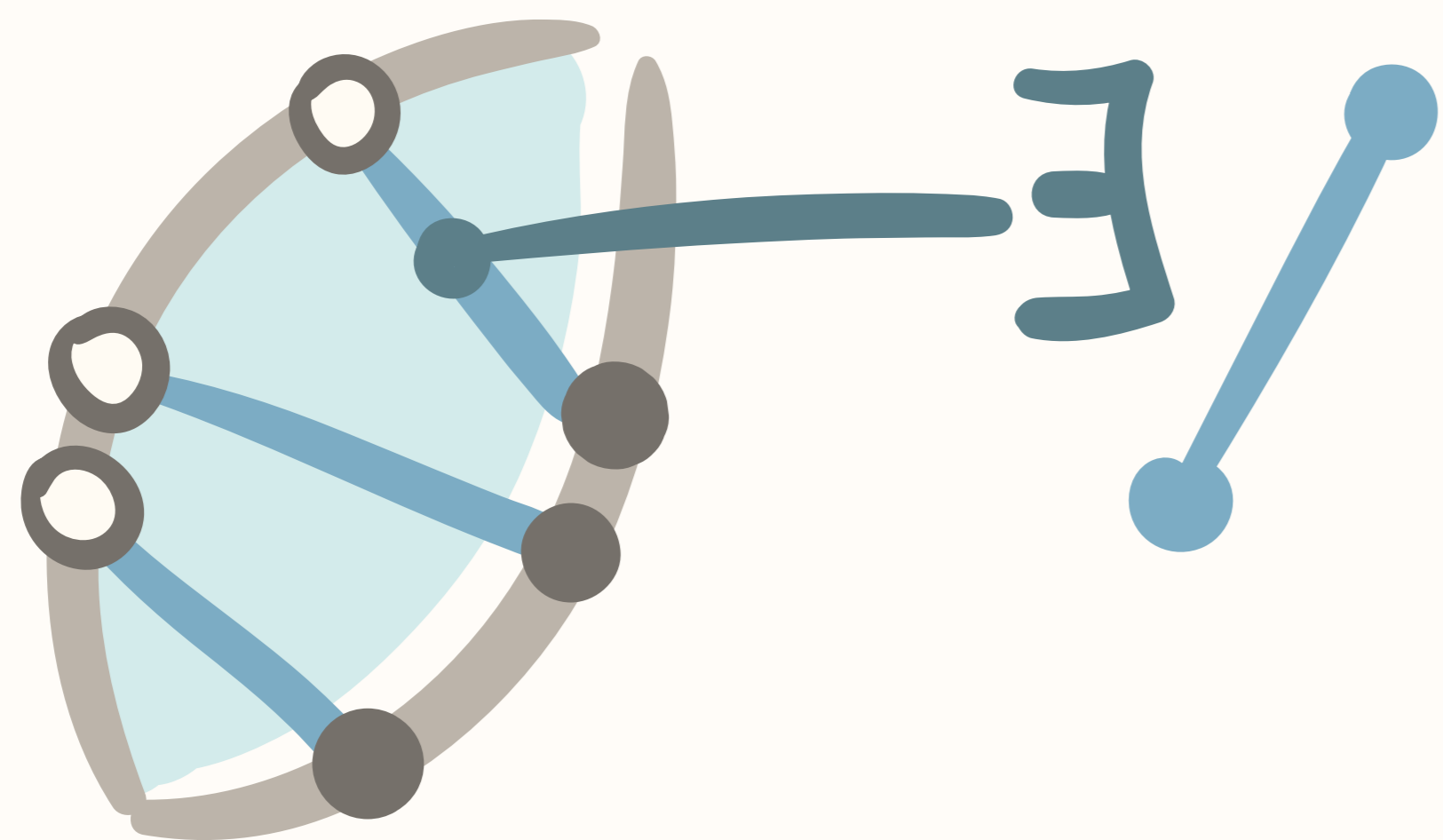
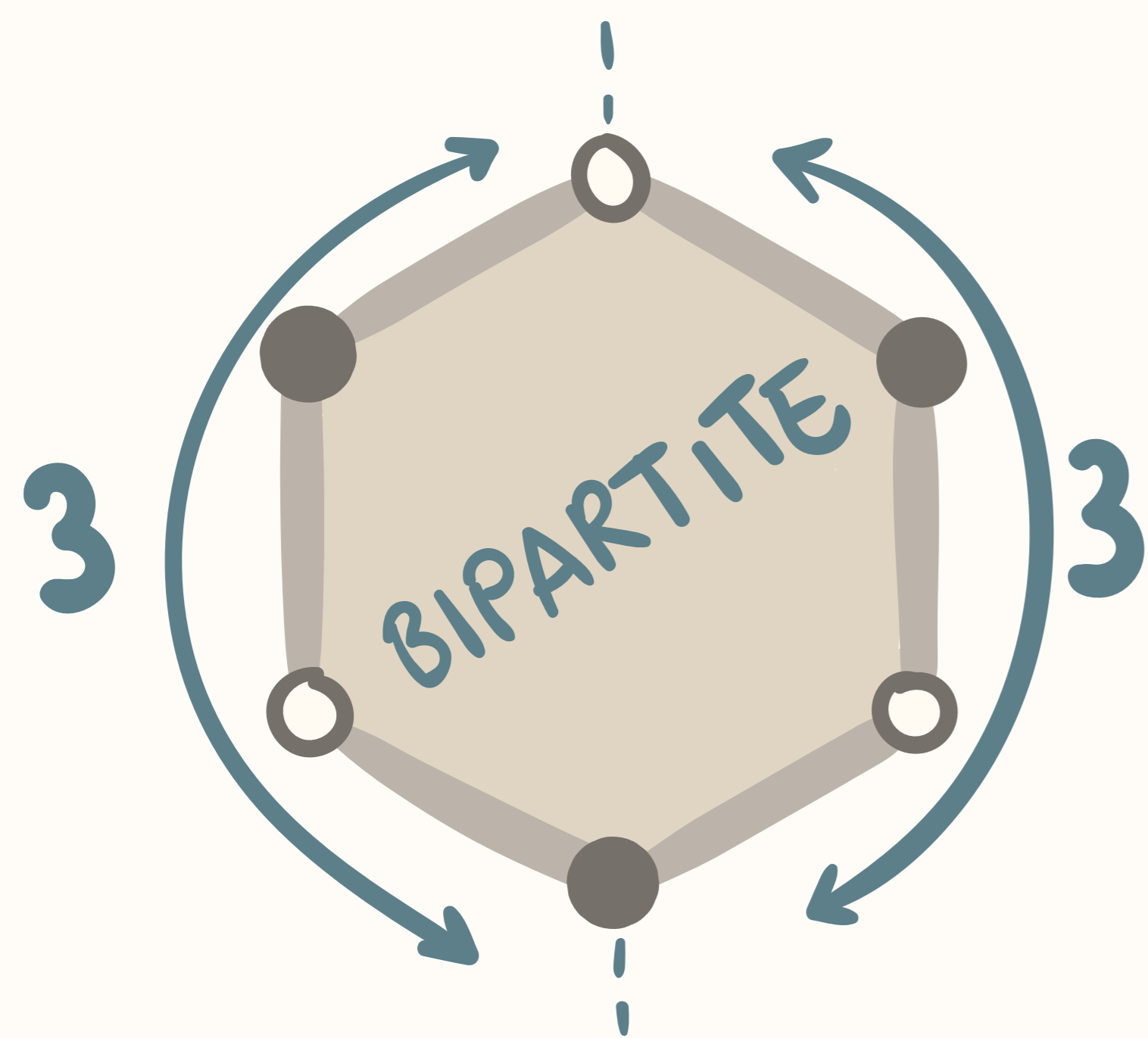
Schnyder labellings



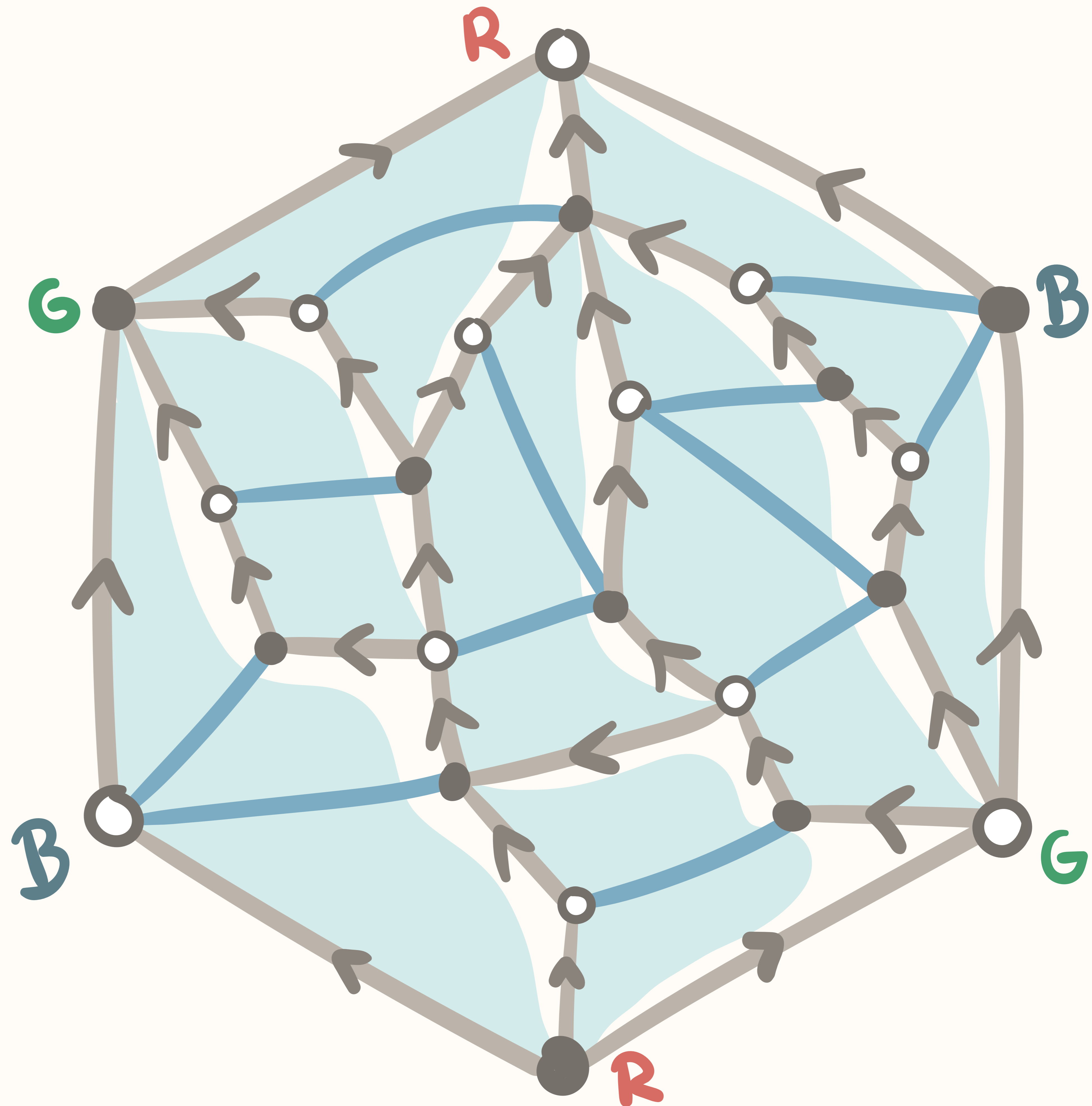
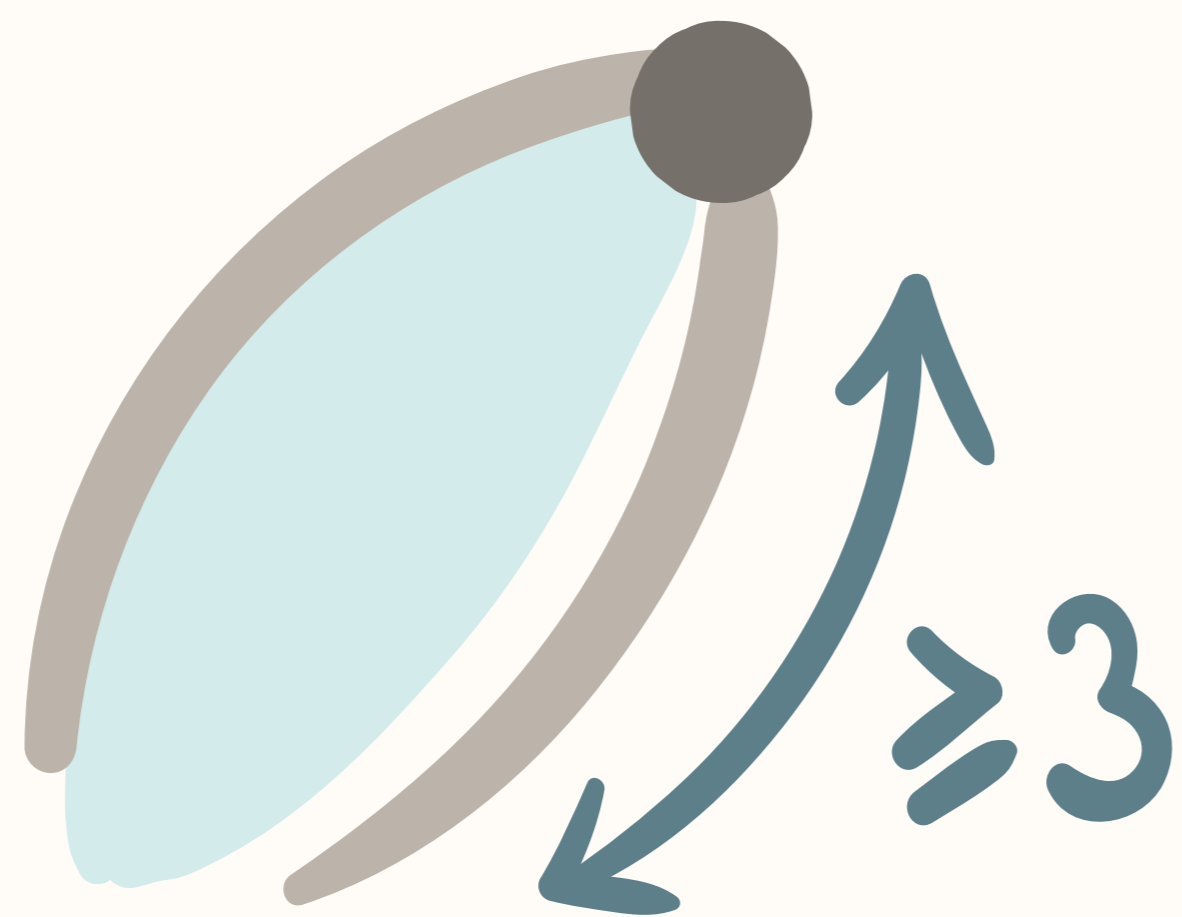
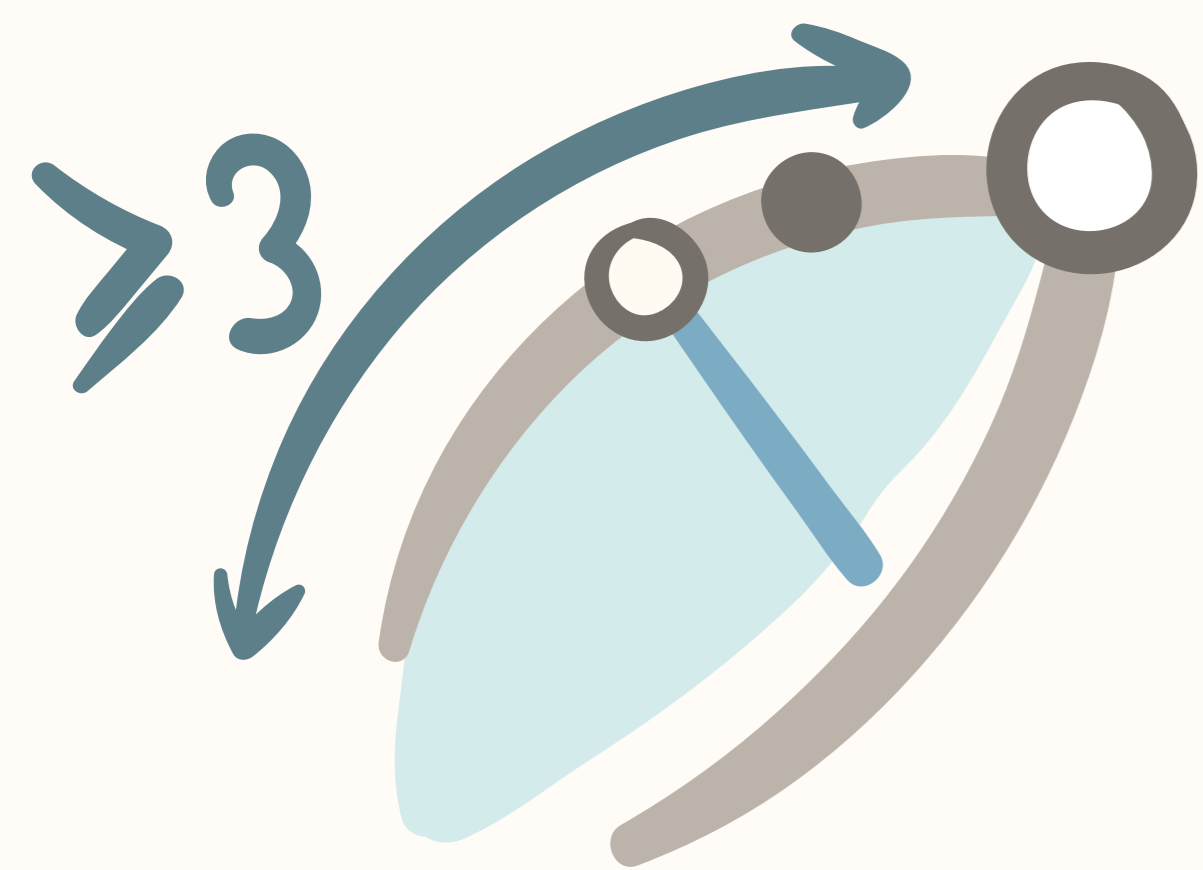
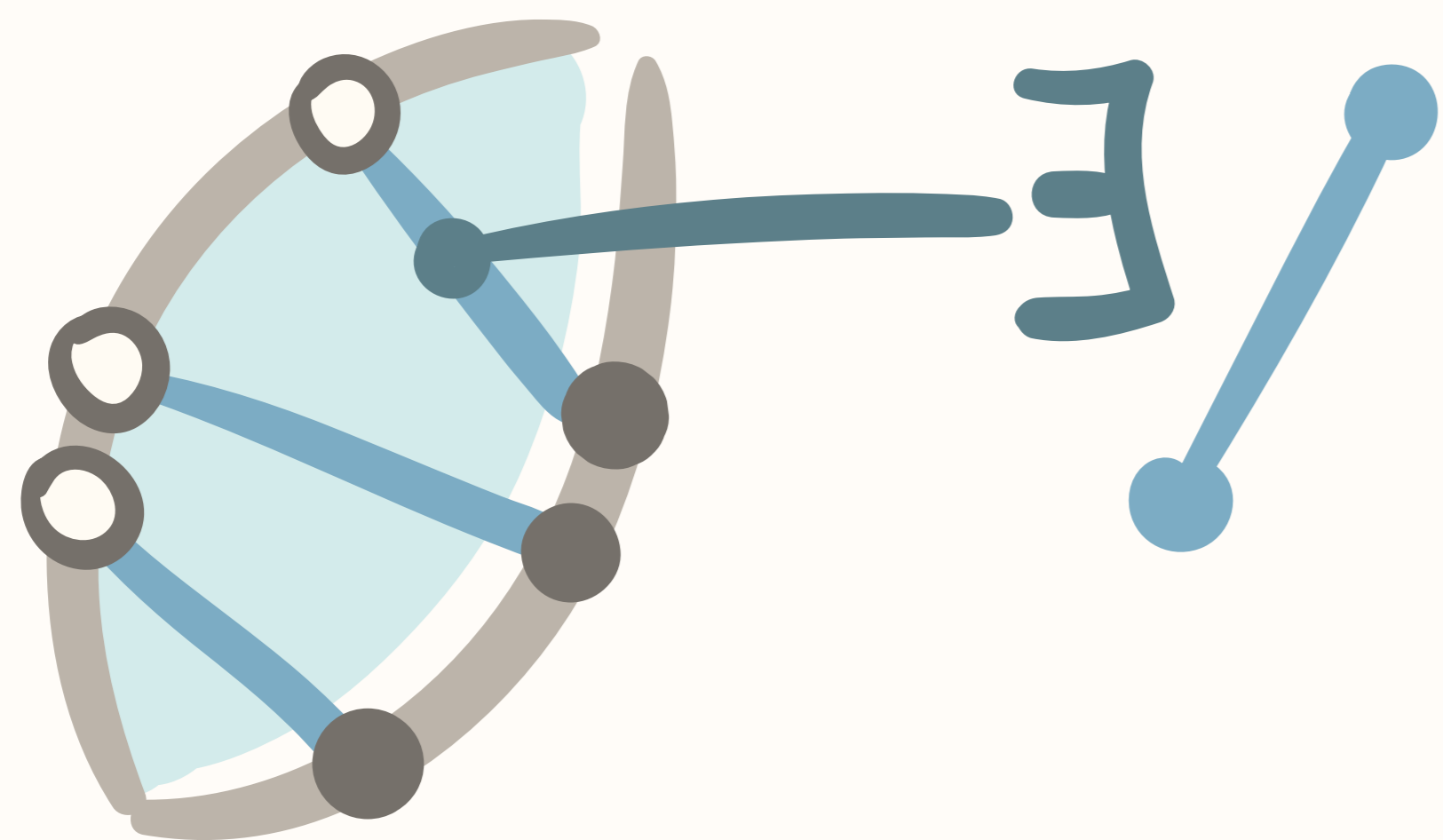
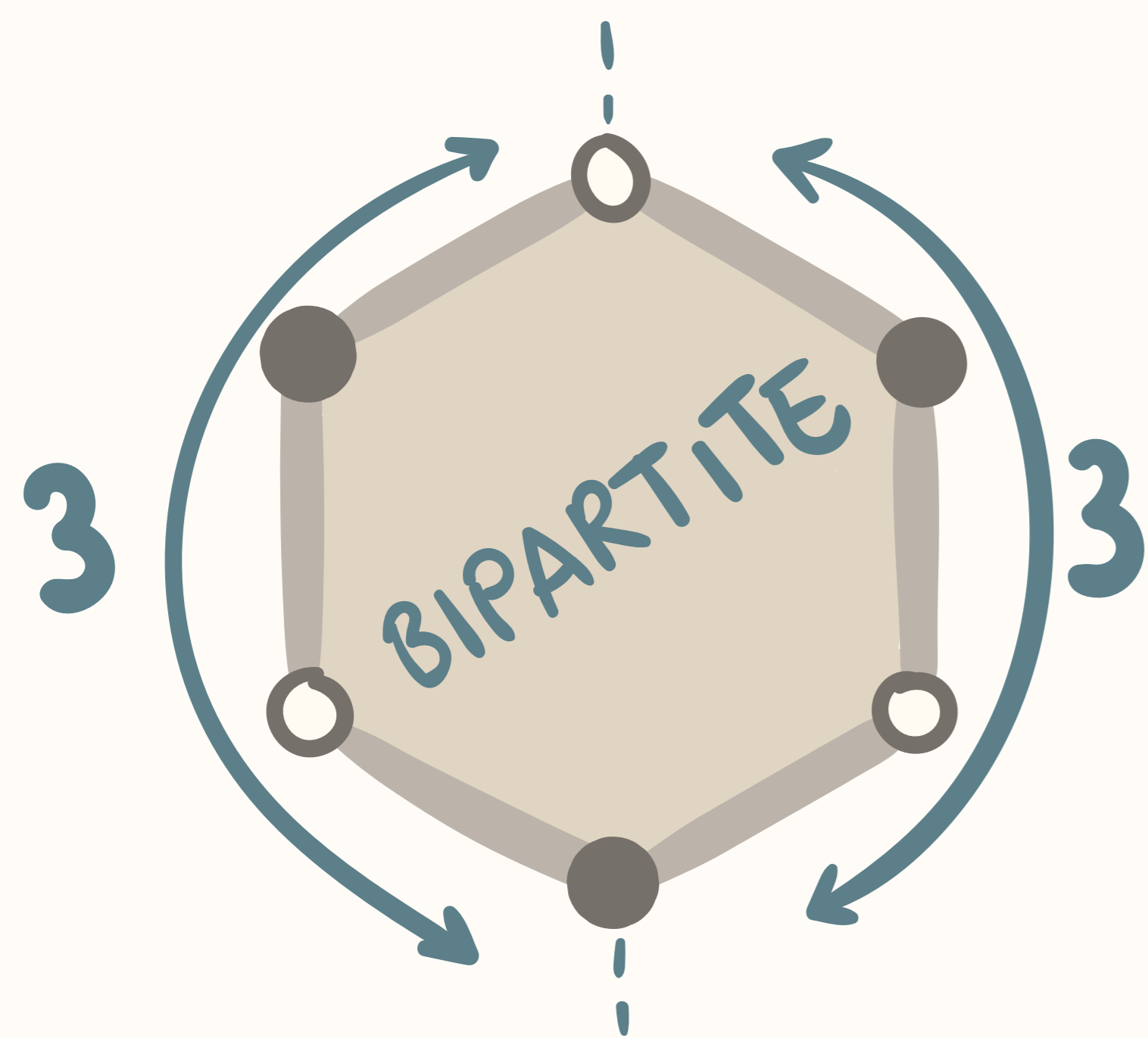
Schnyder labellings



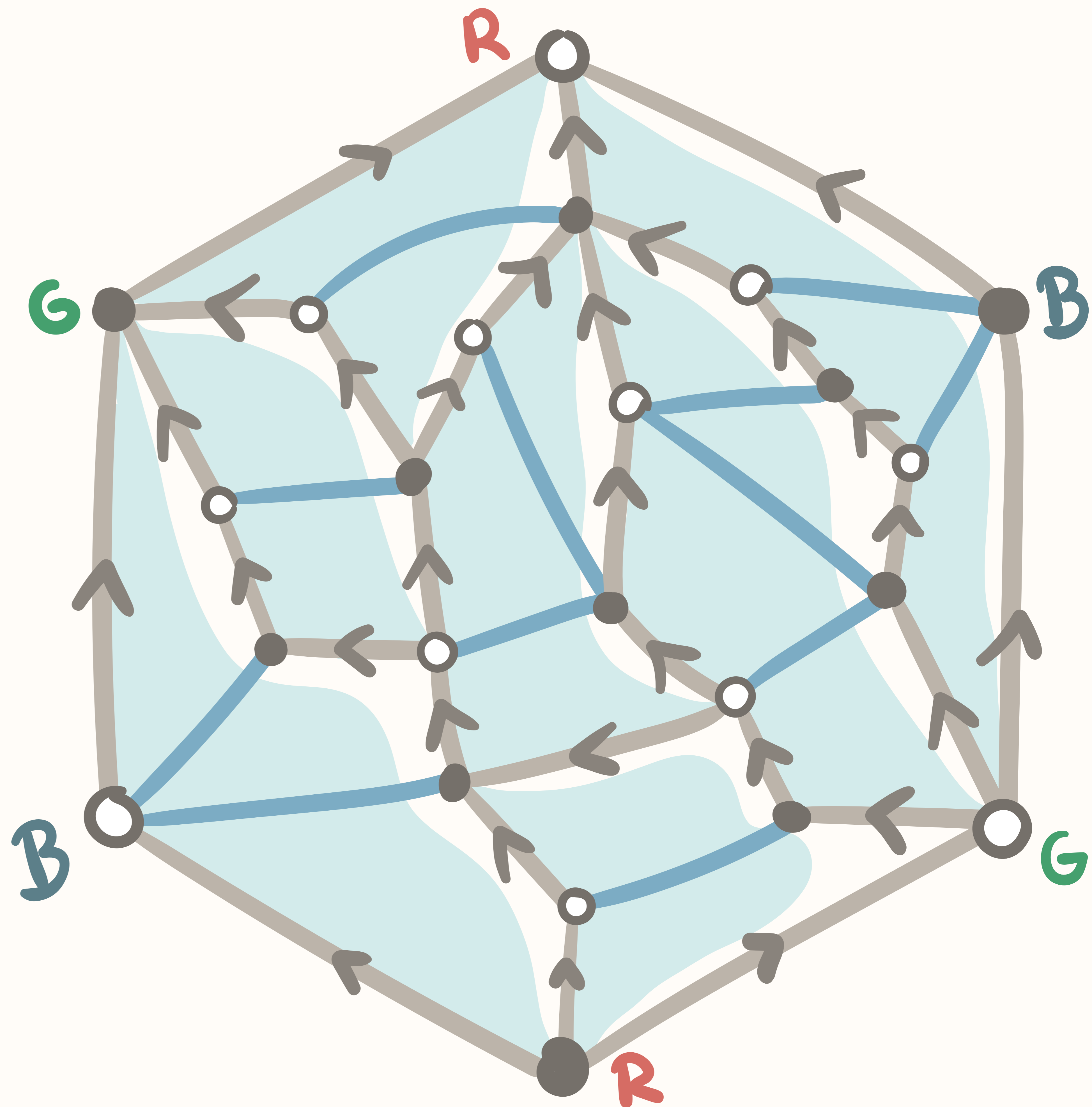
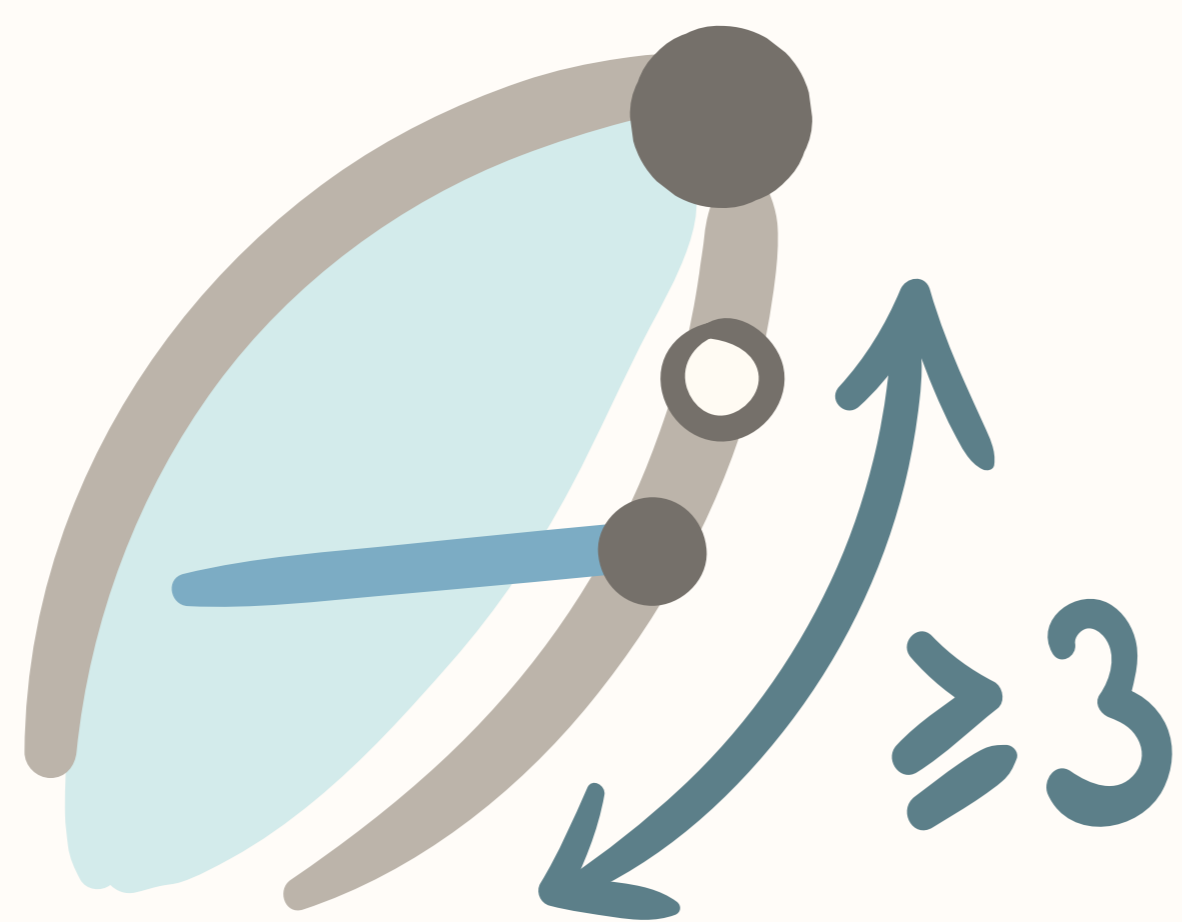
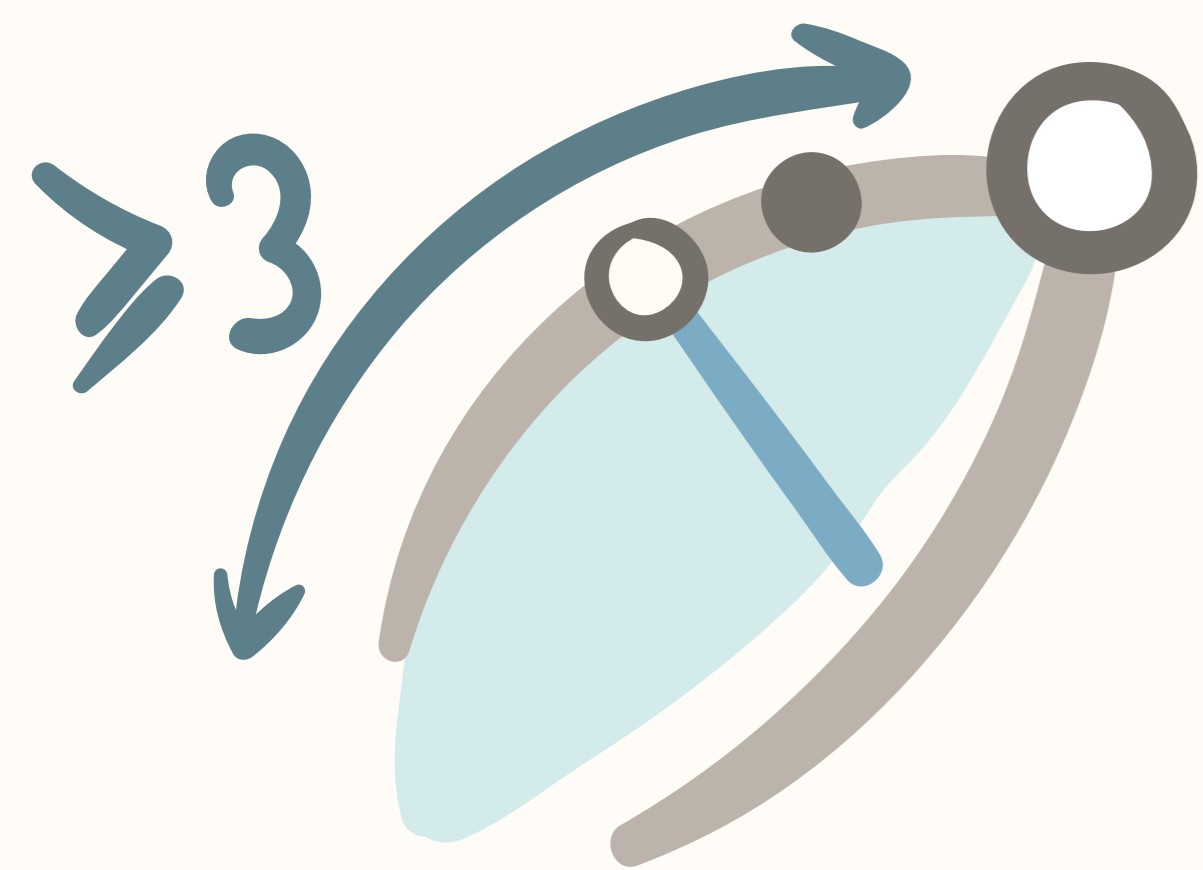
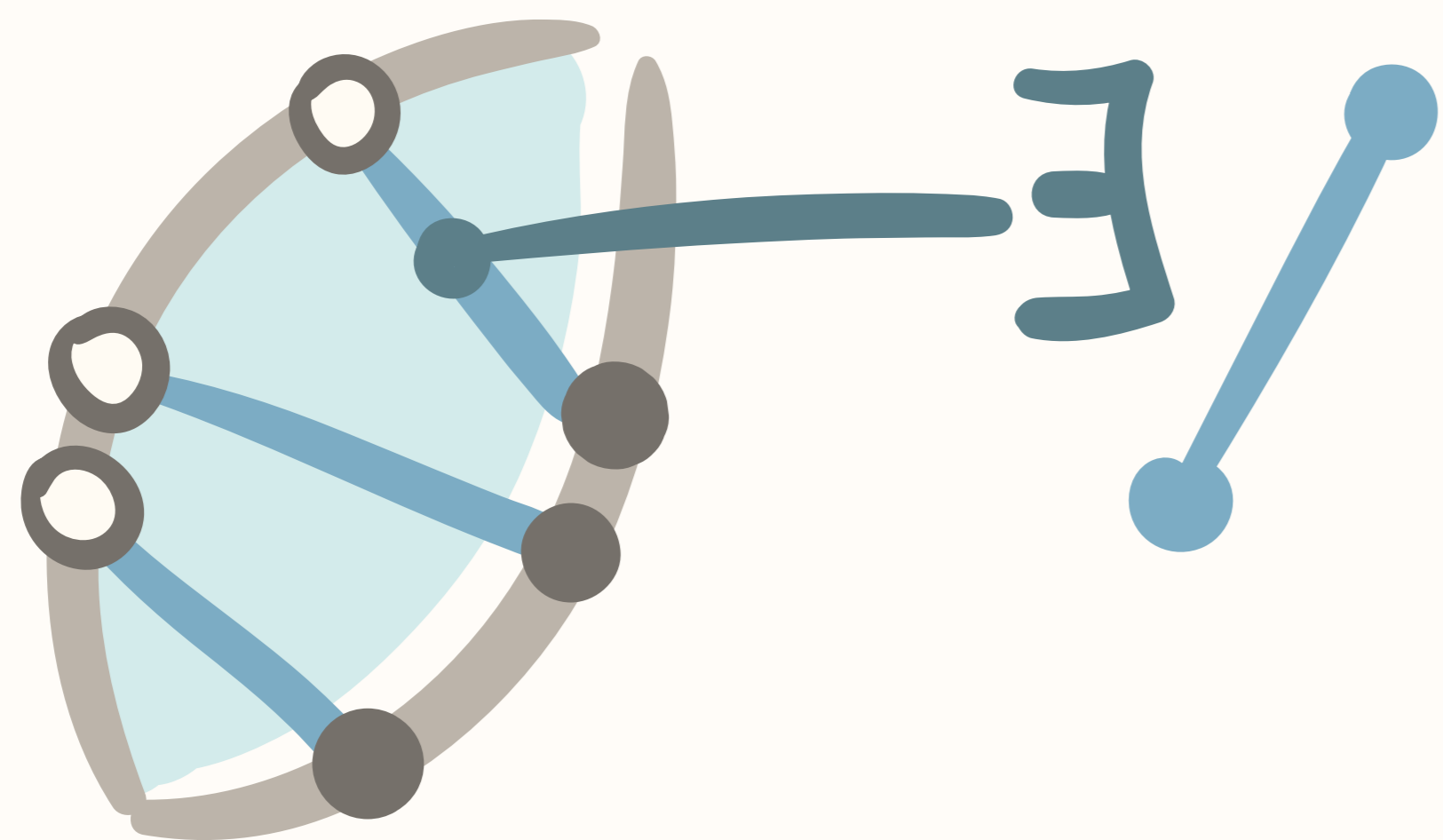
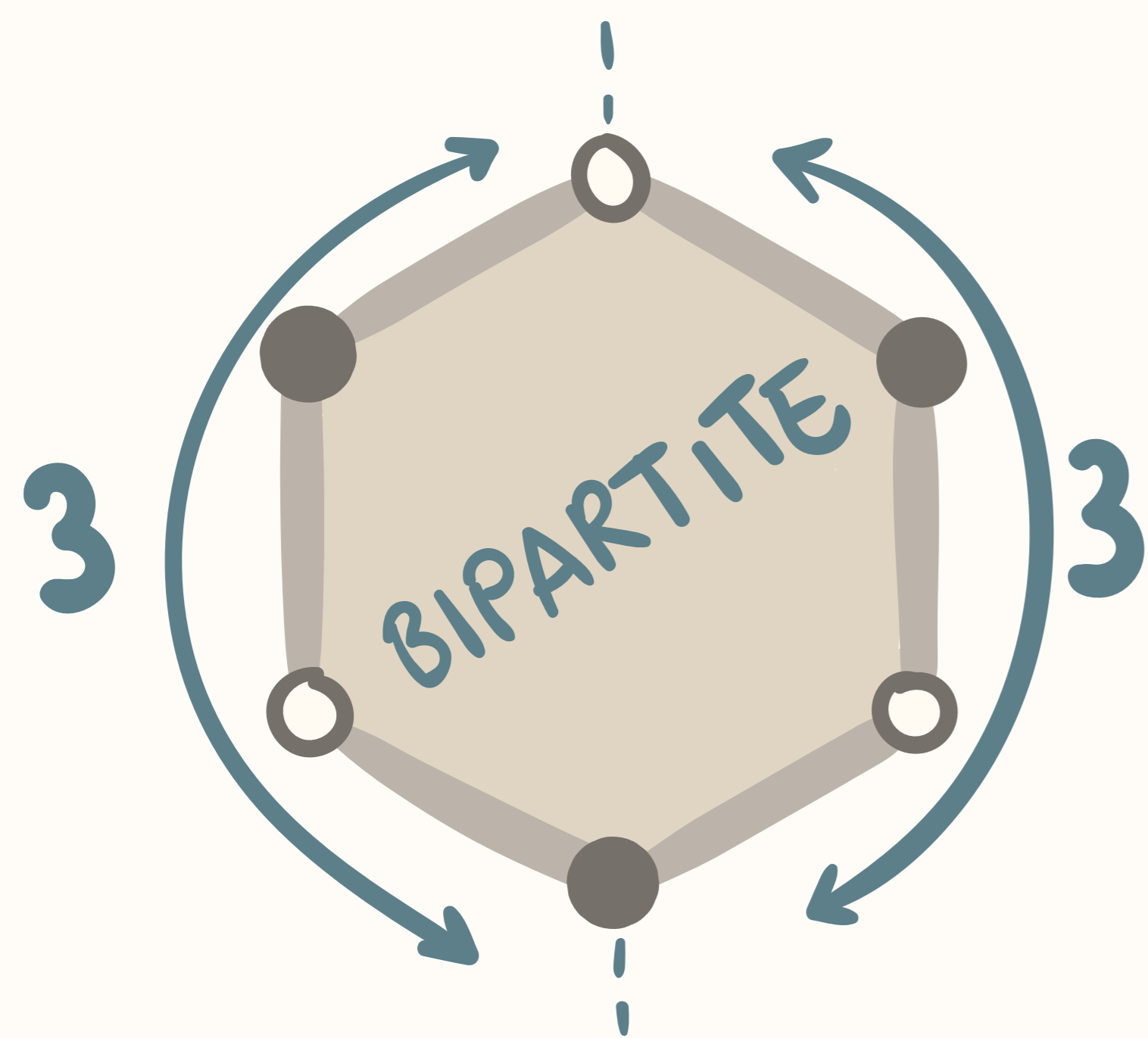
Schnyder labellings



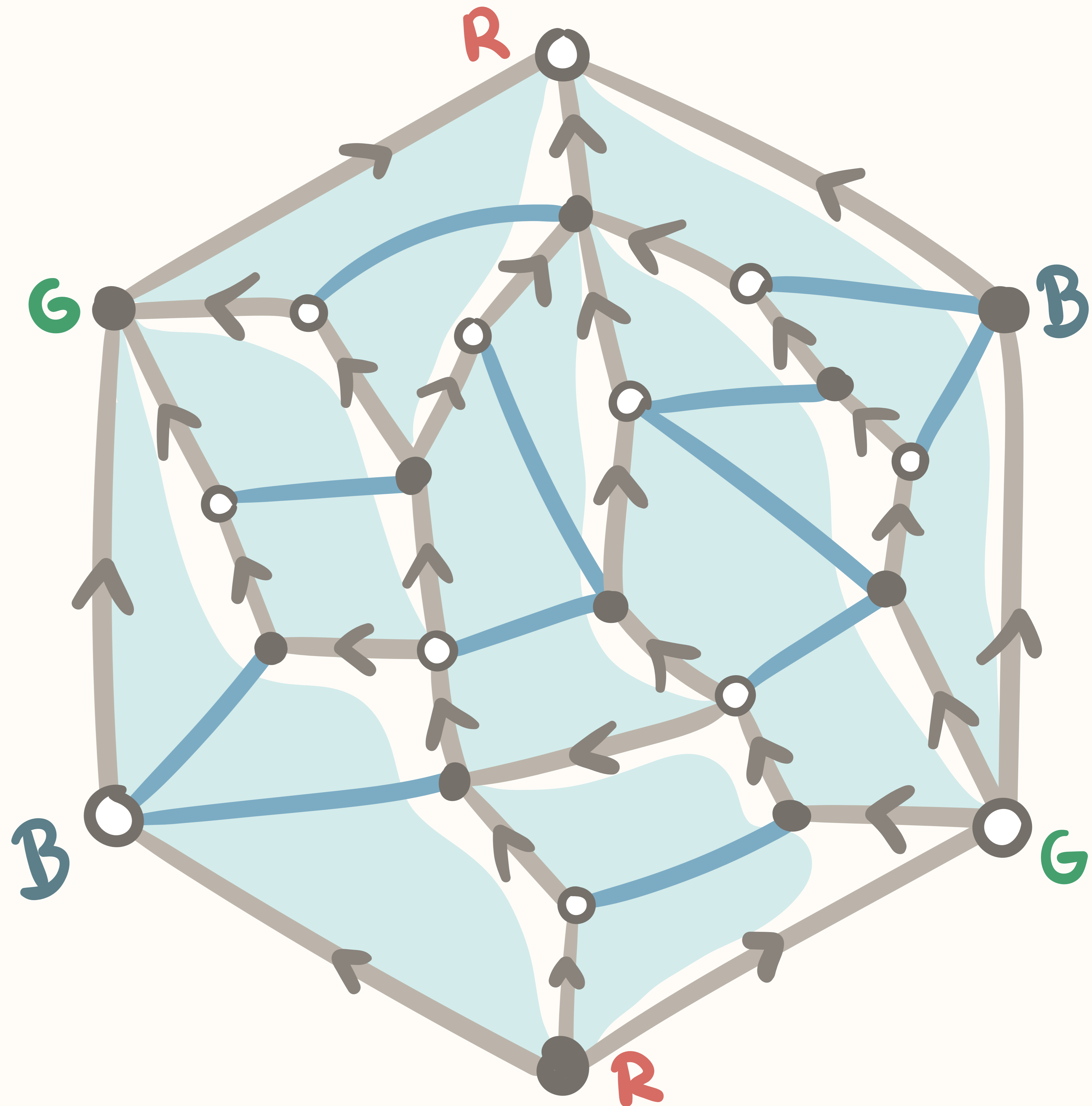
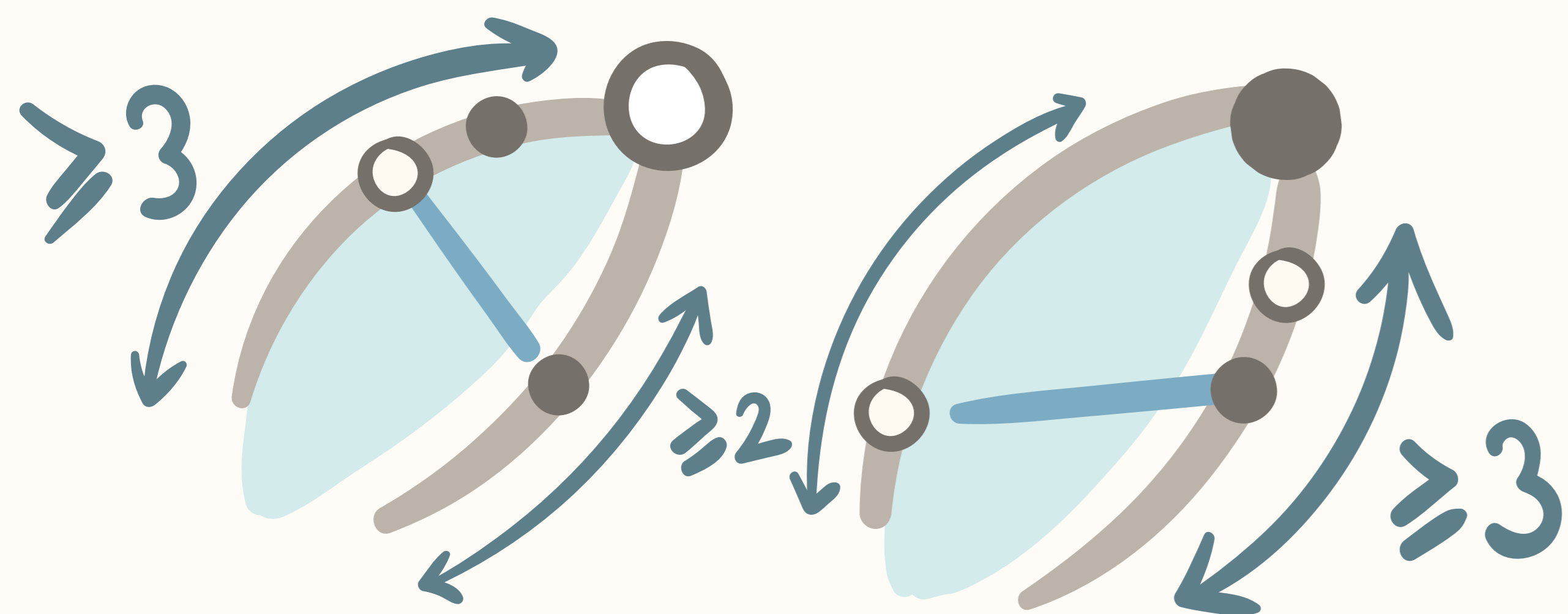
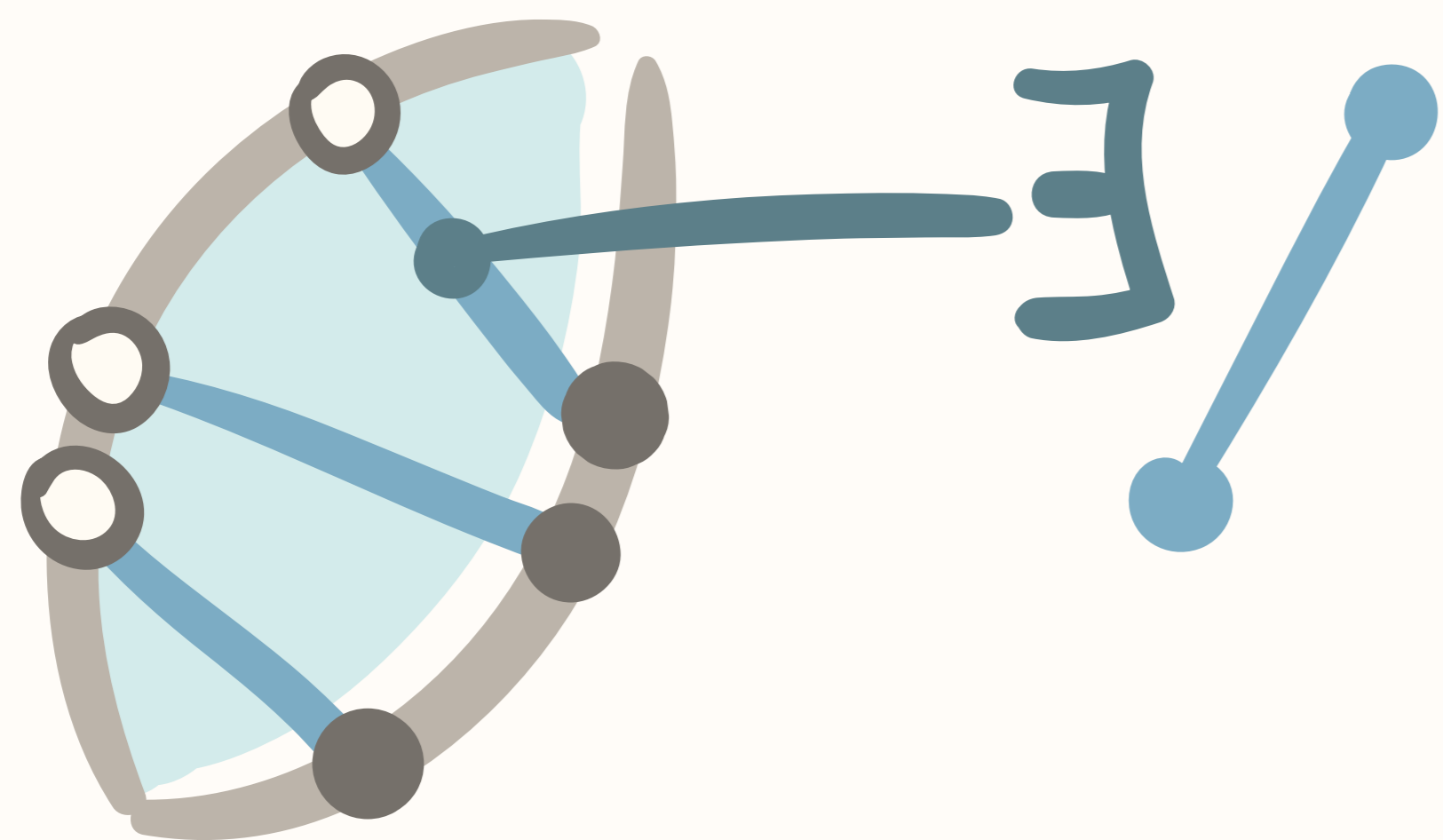
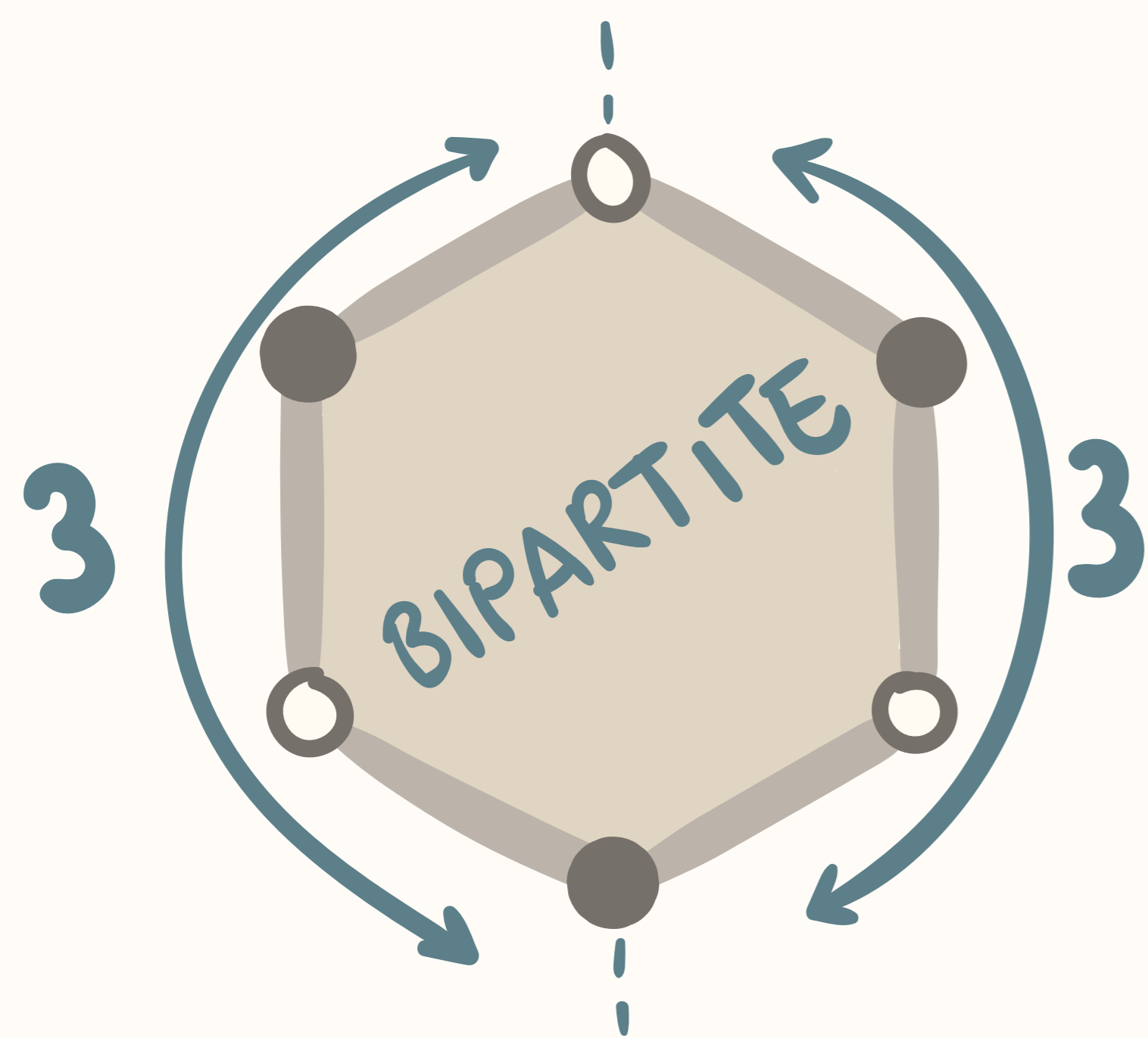
Schnyder labellings



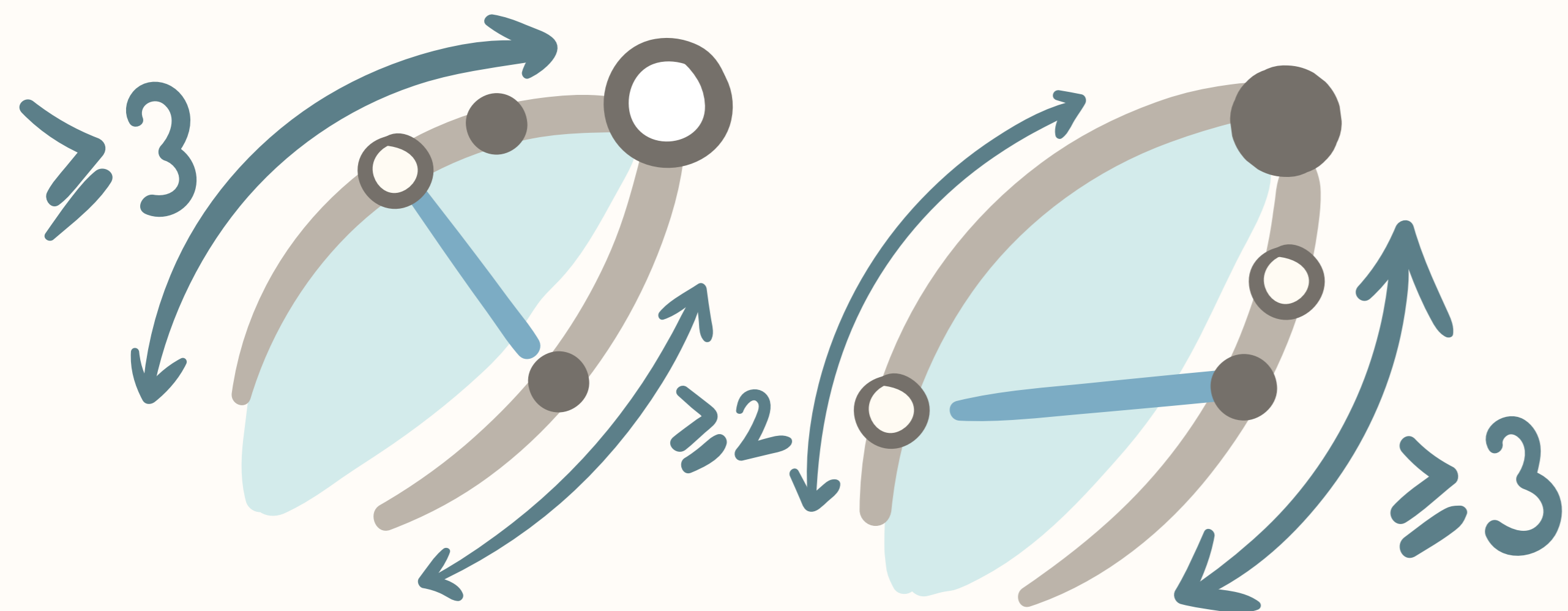
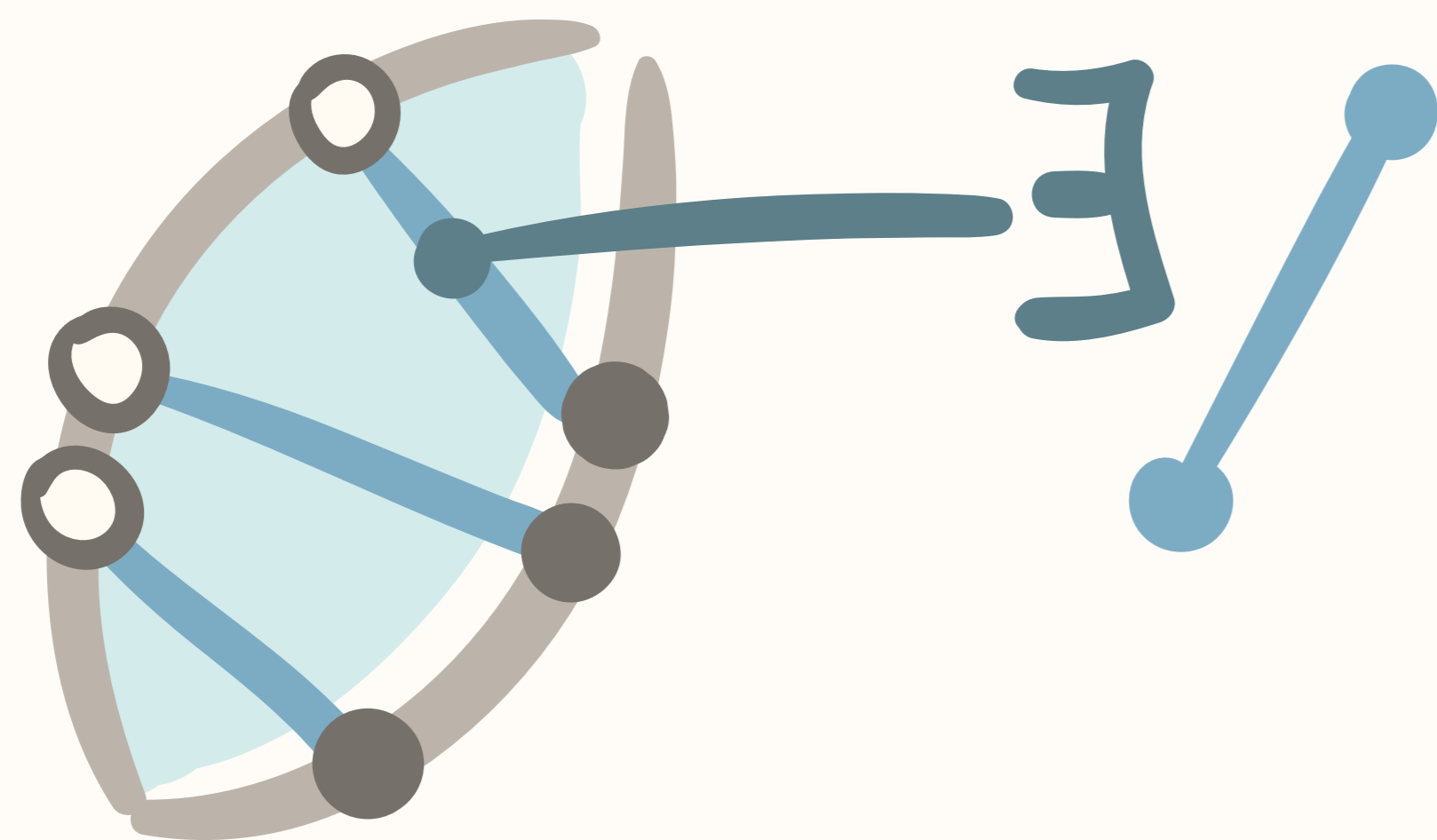
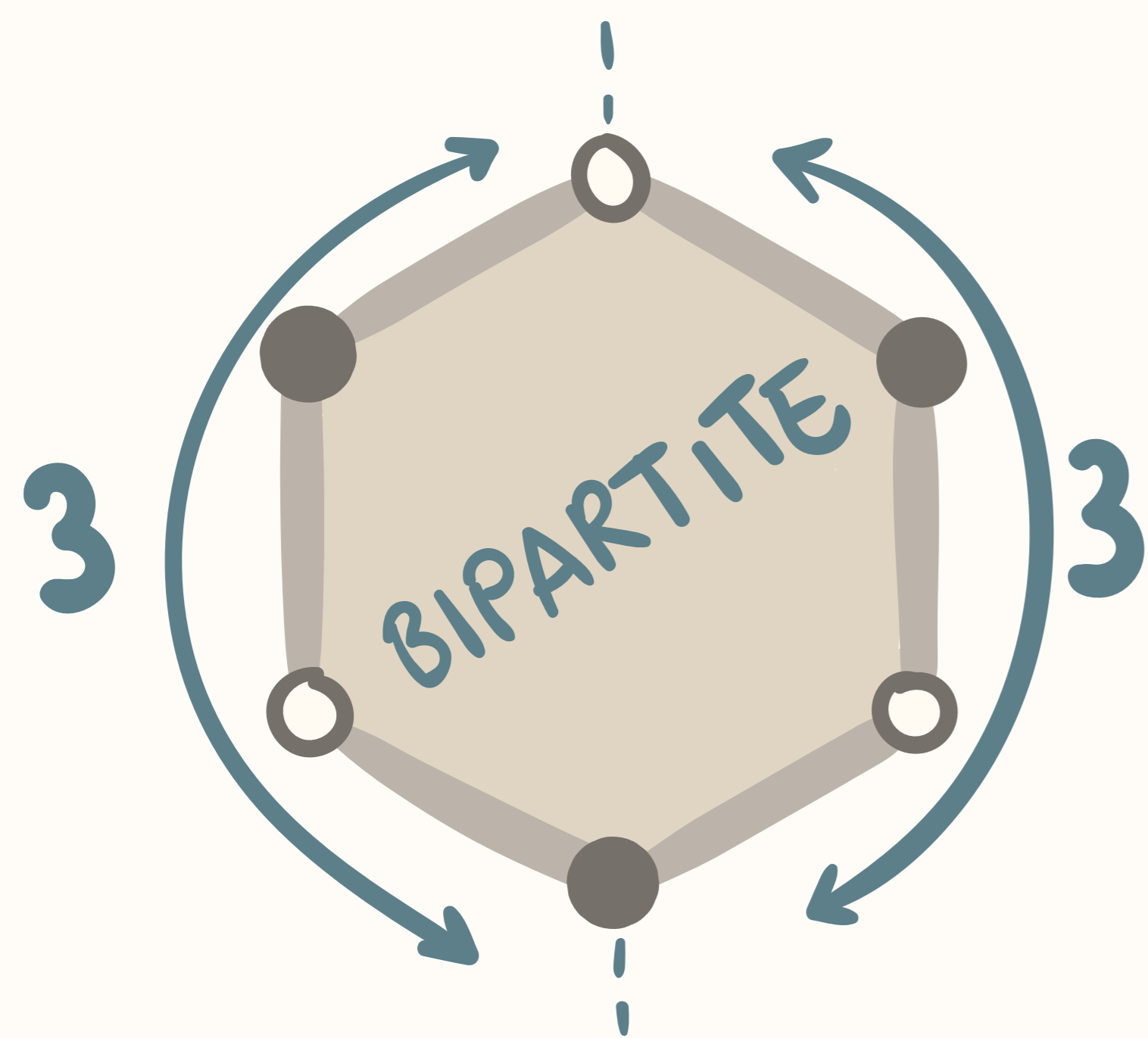
Schnyder labellings



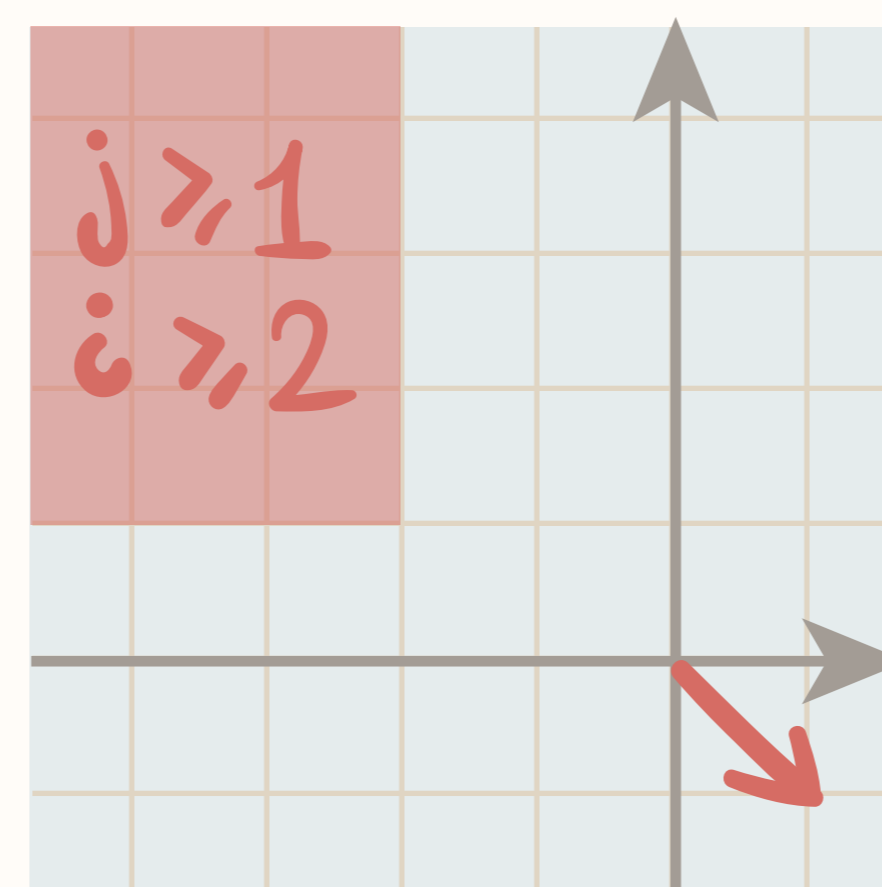
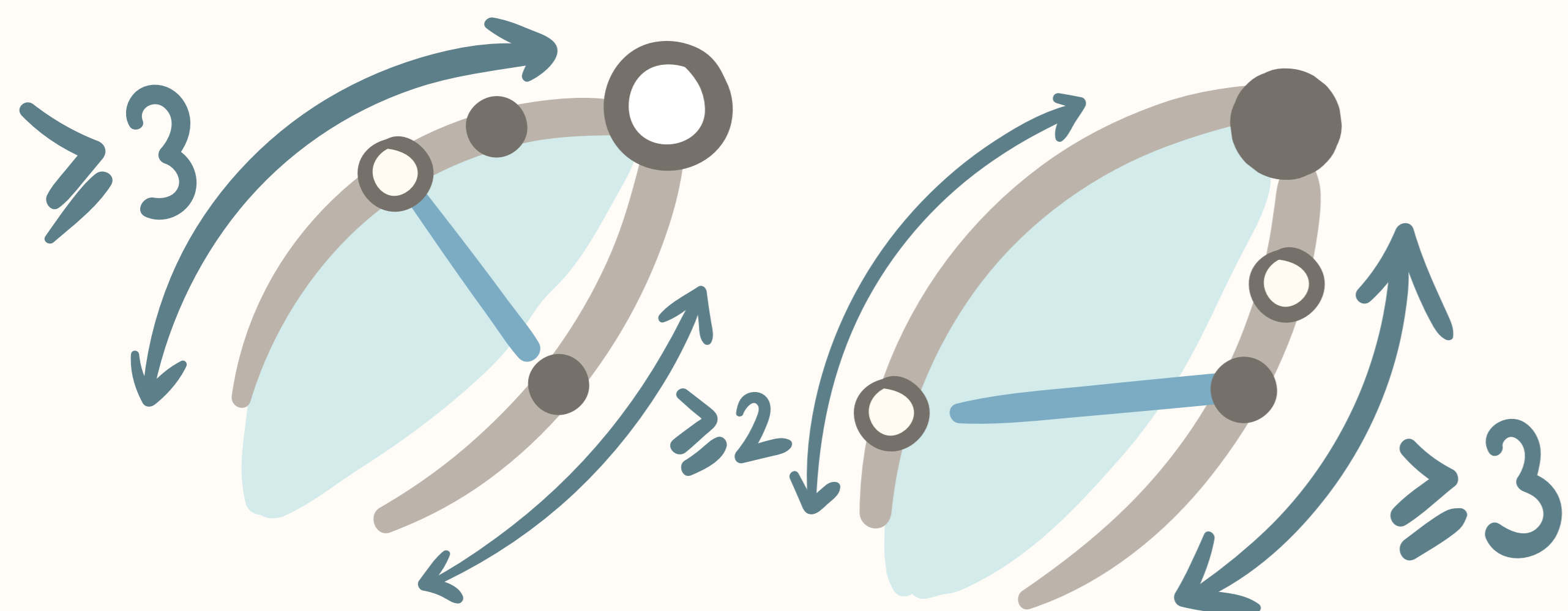
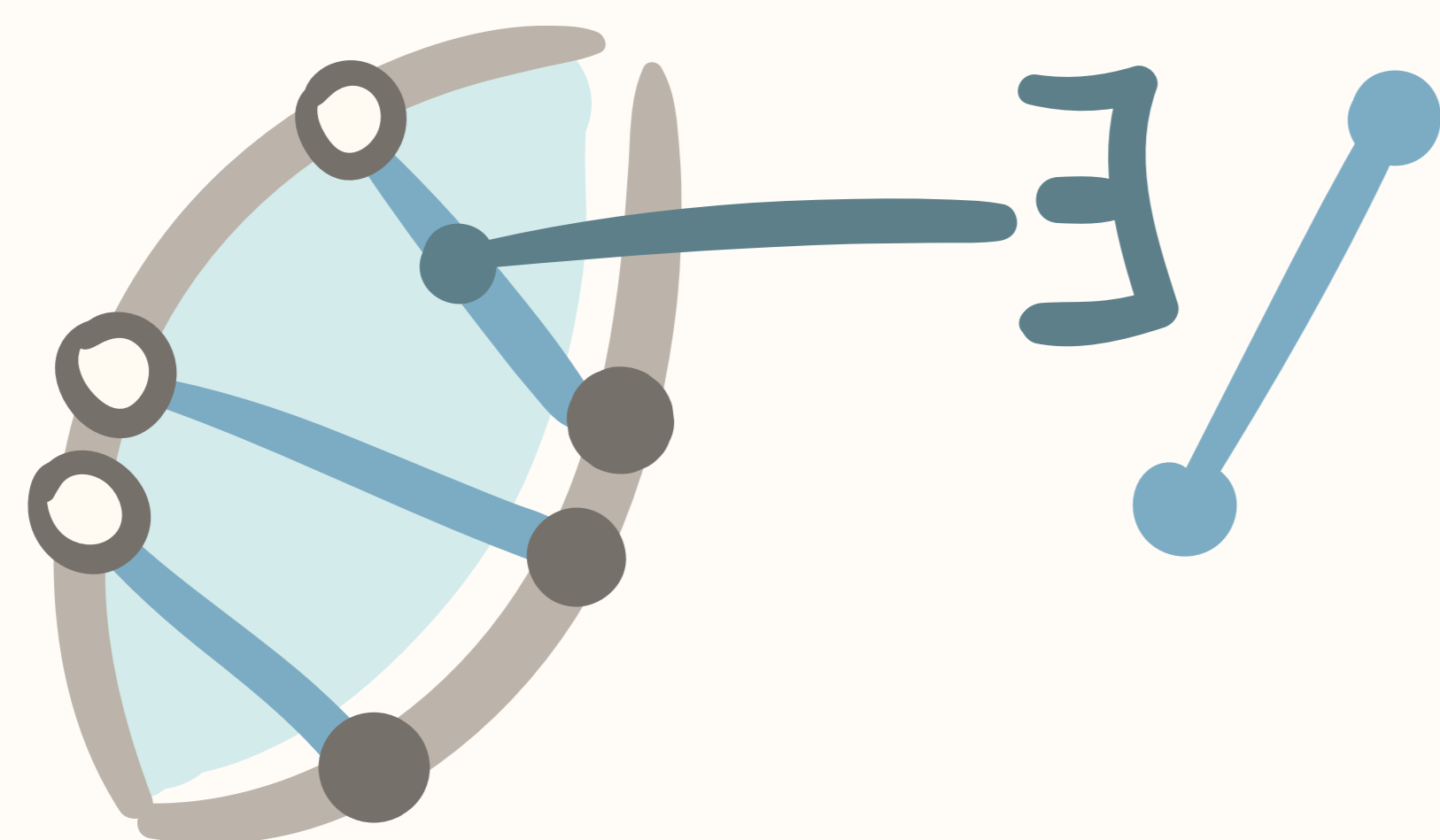
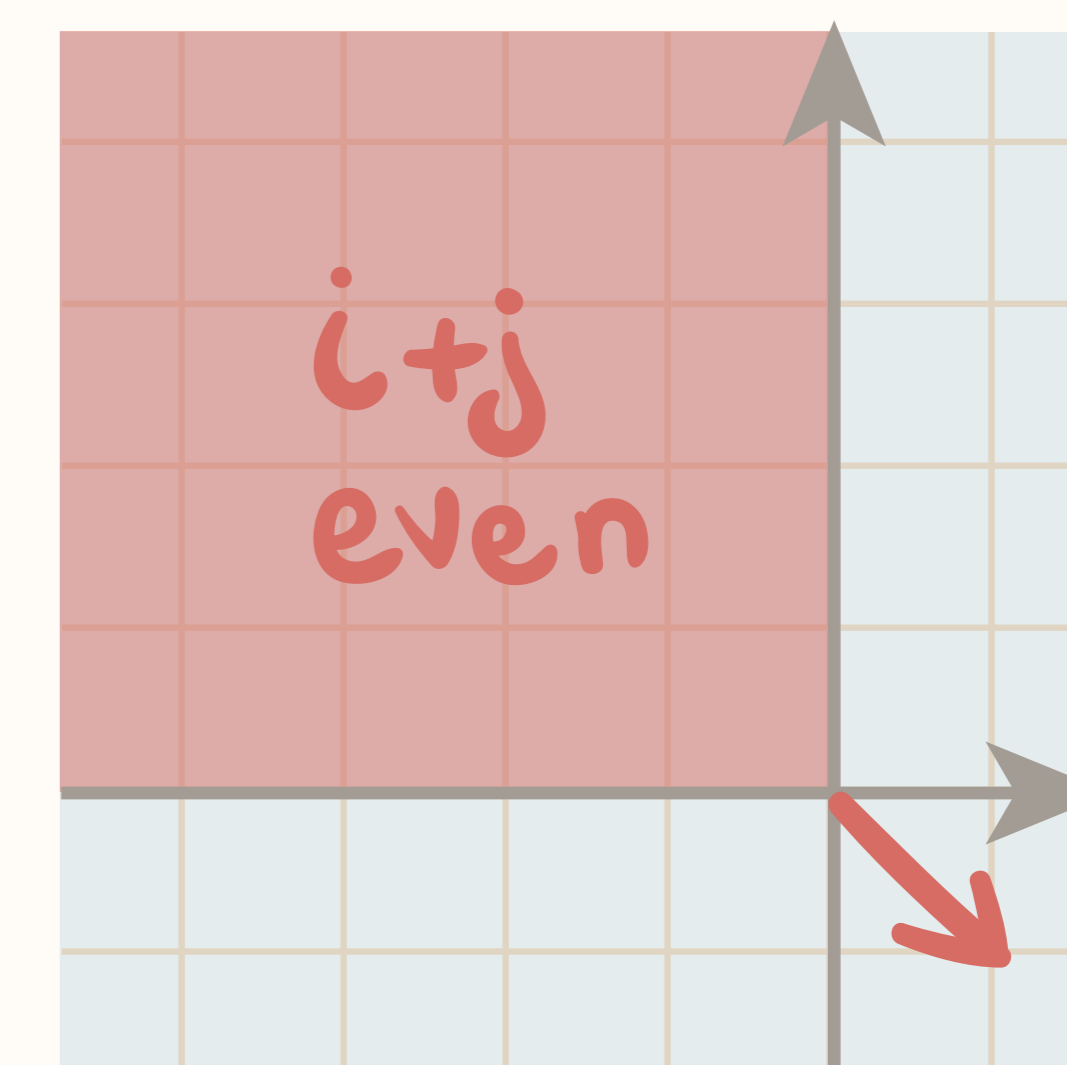
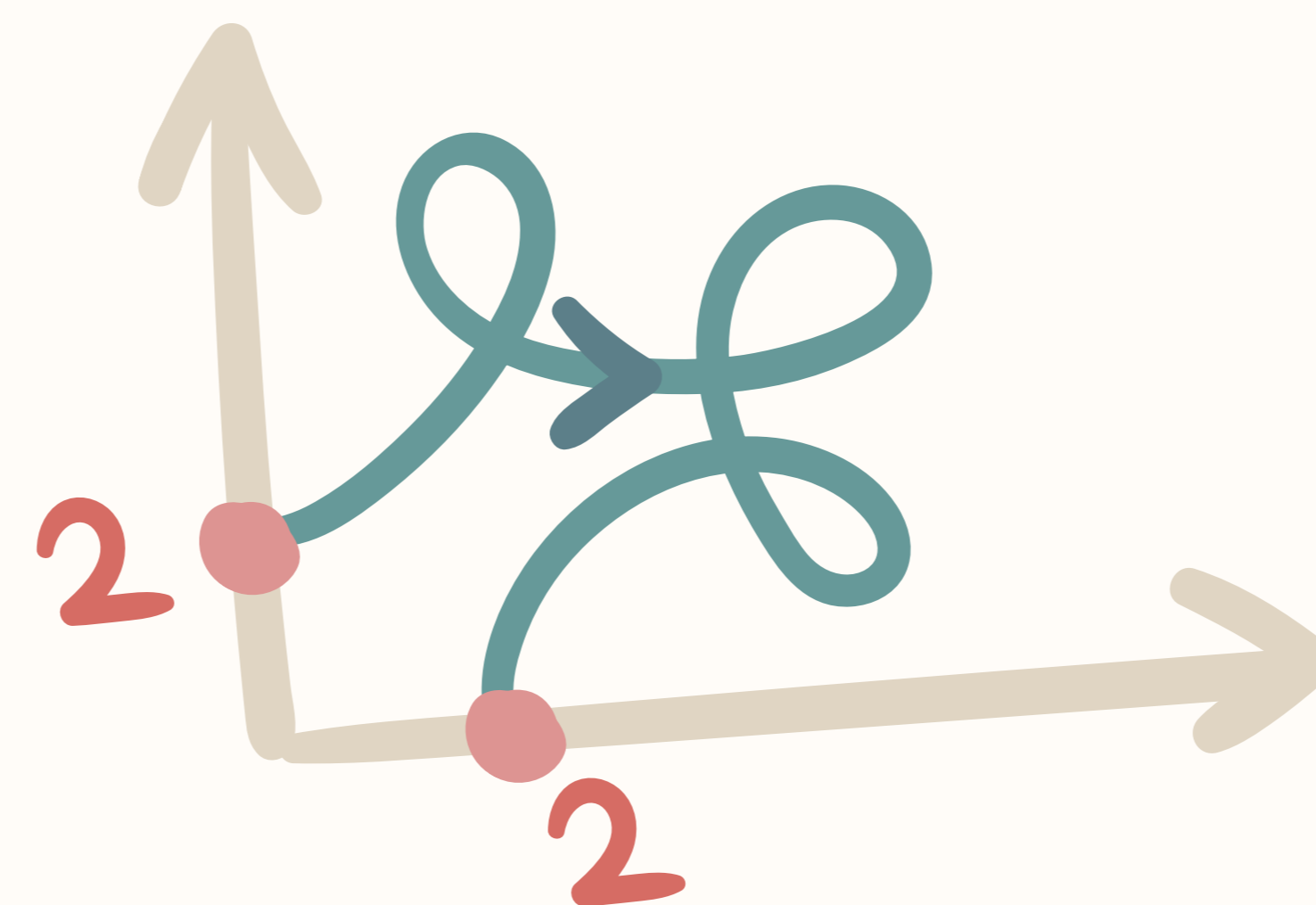
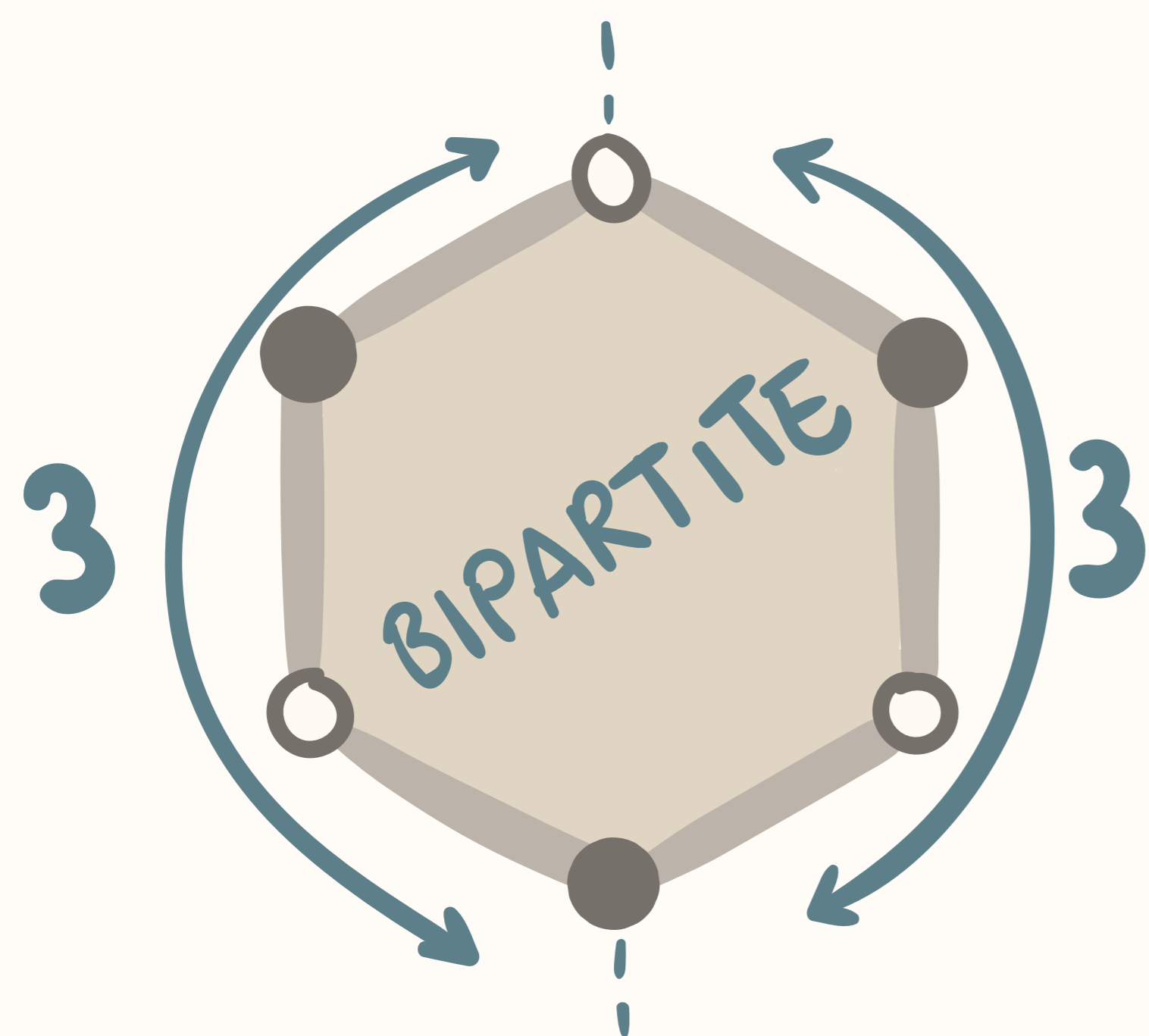
Schnyder labellings



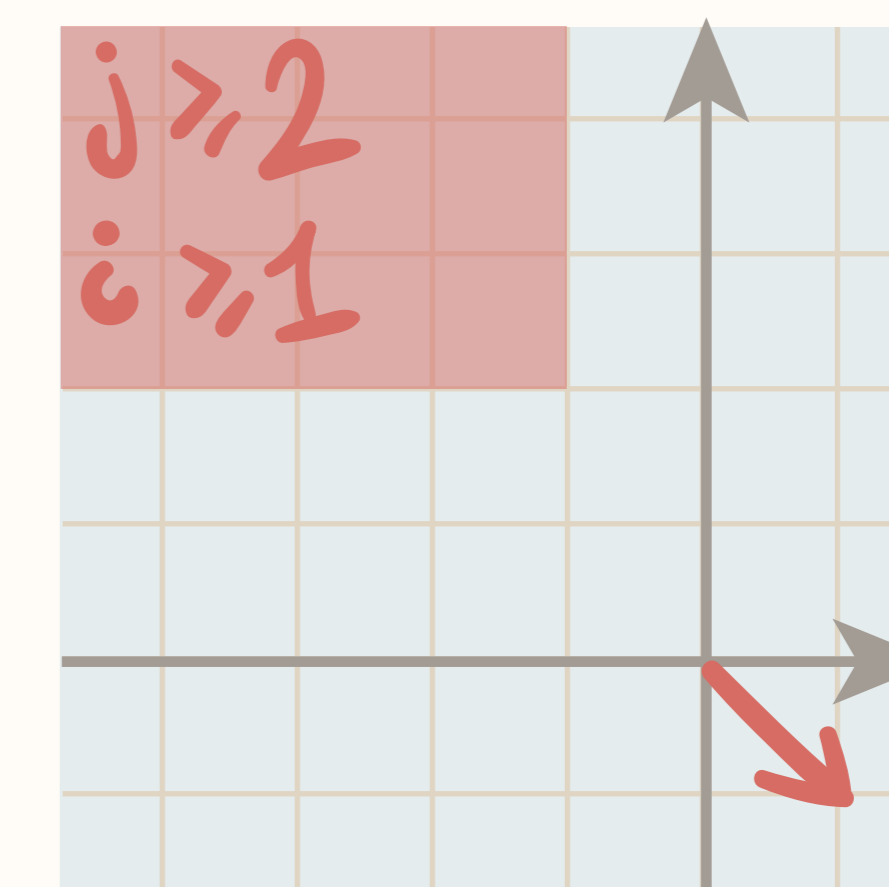
Schnyder labellings KMSW



Schnyder labellings KMSW

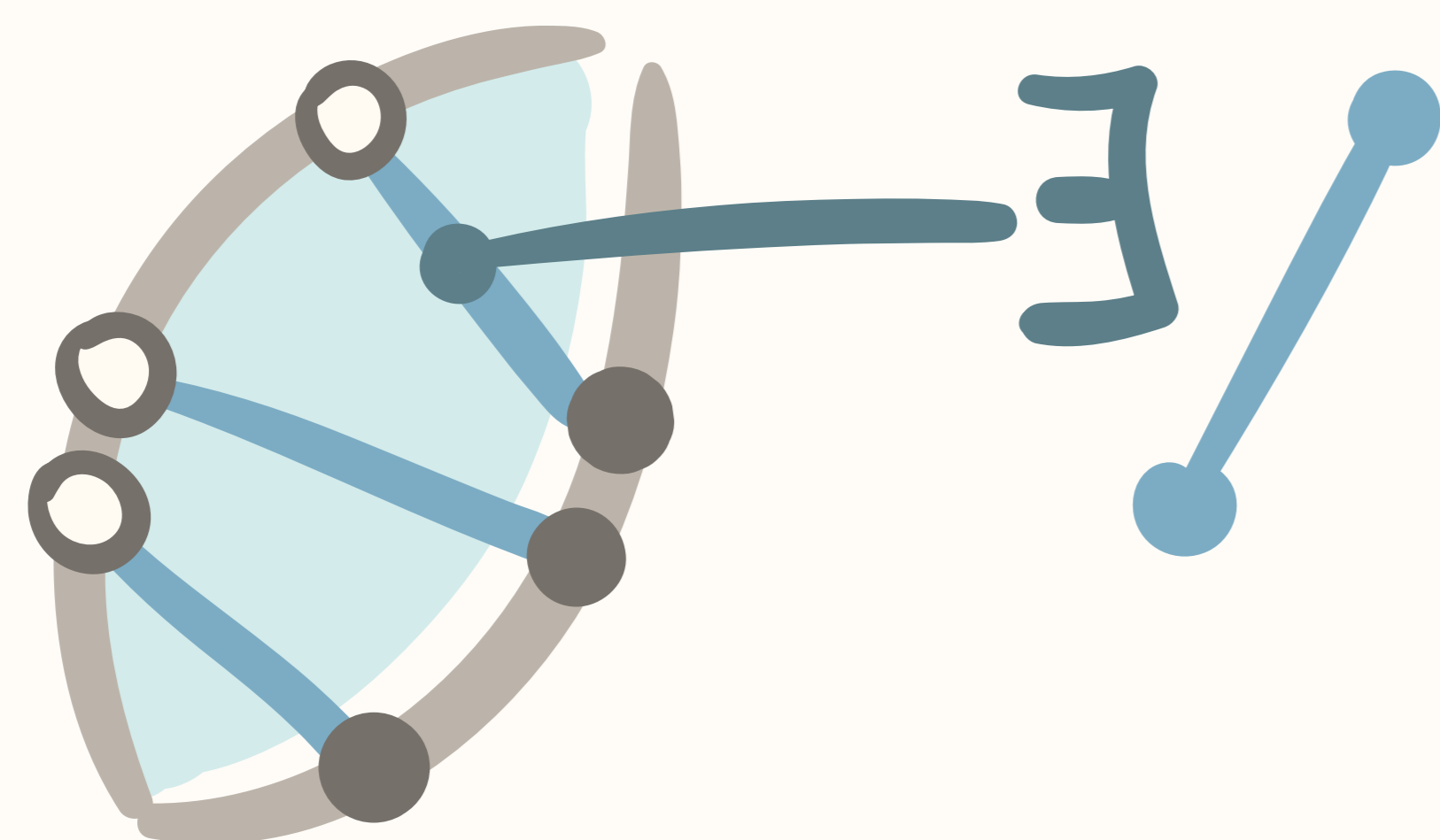
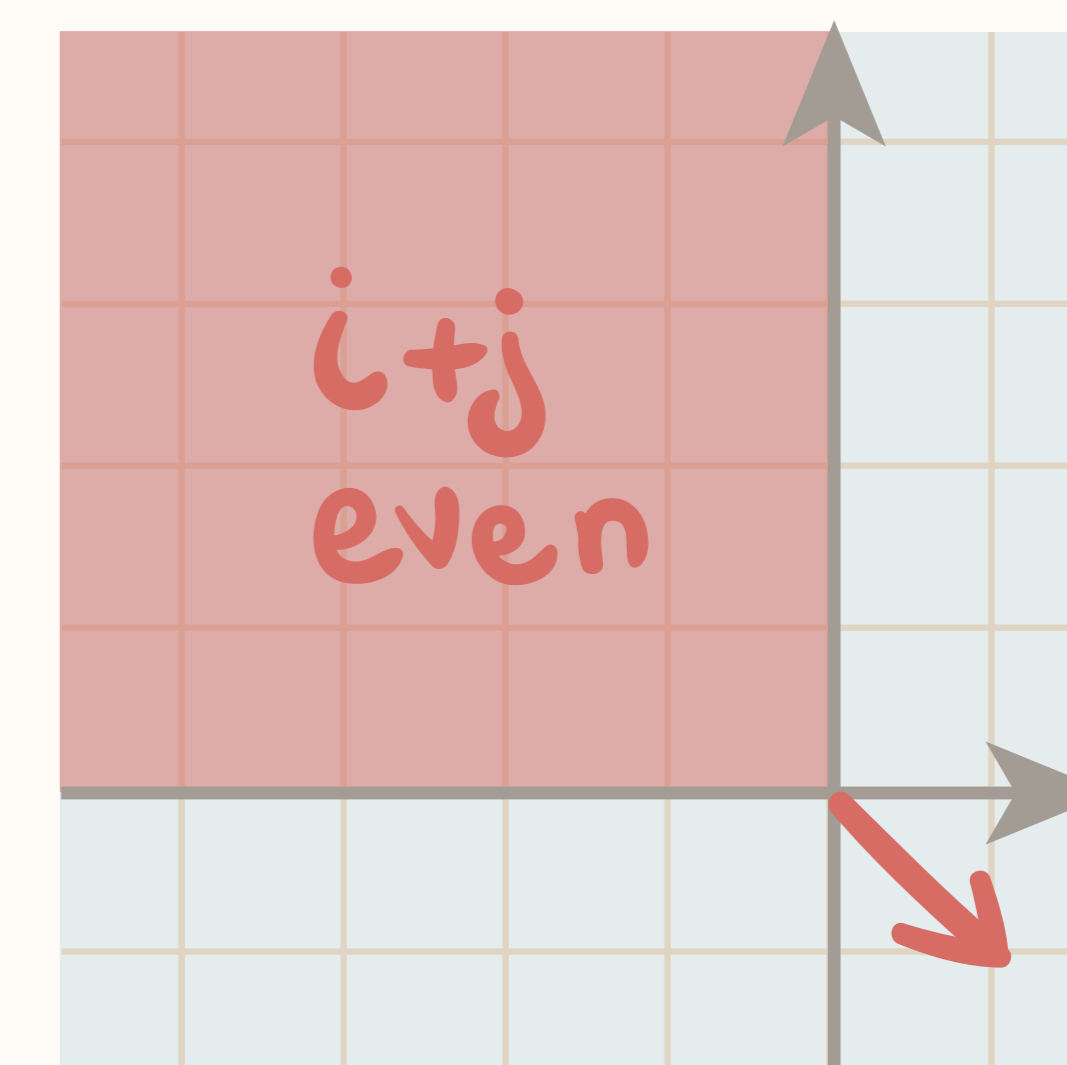
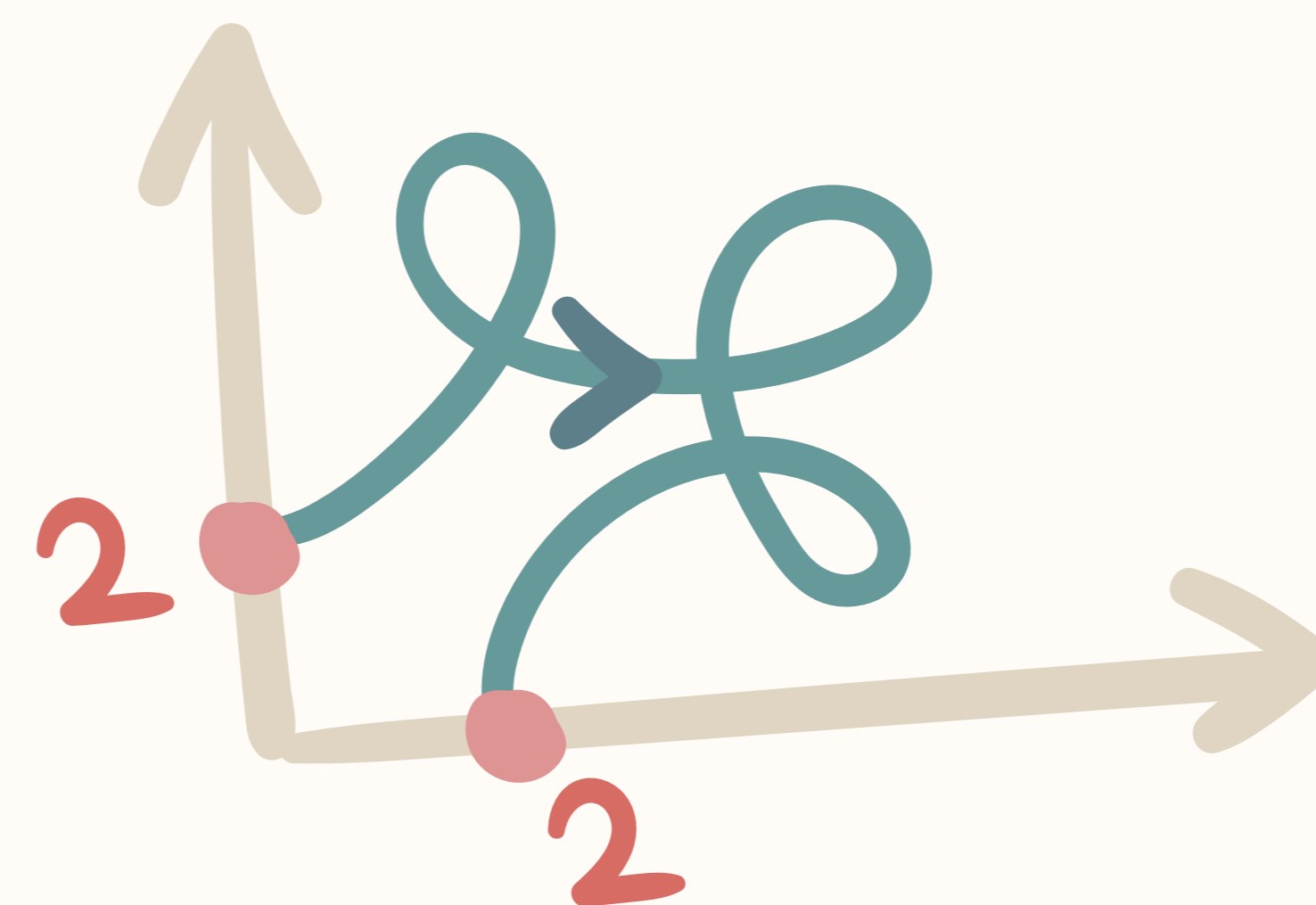
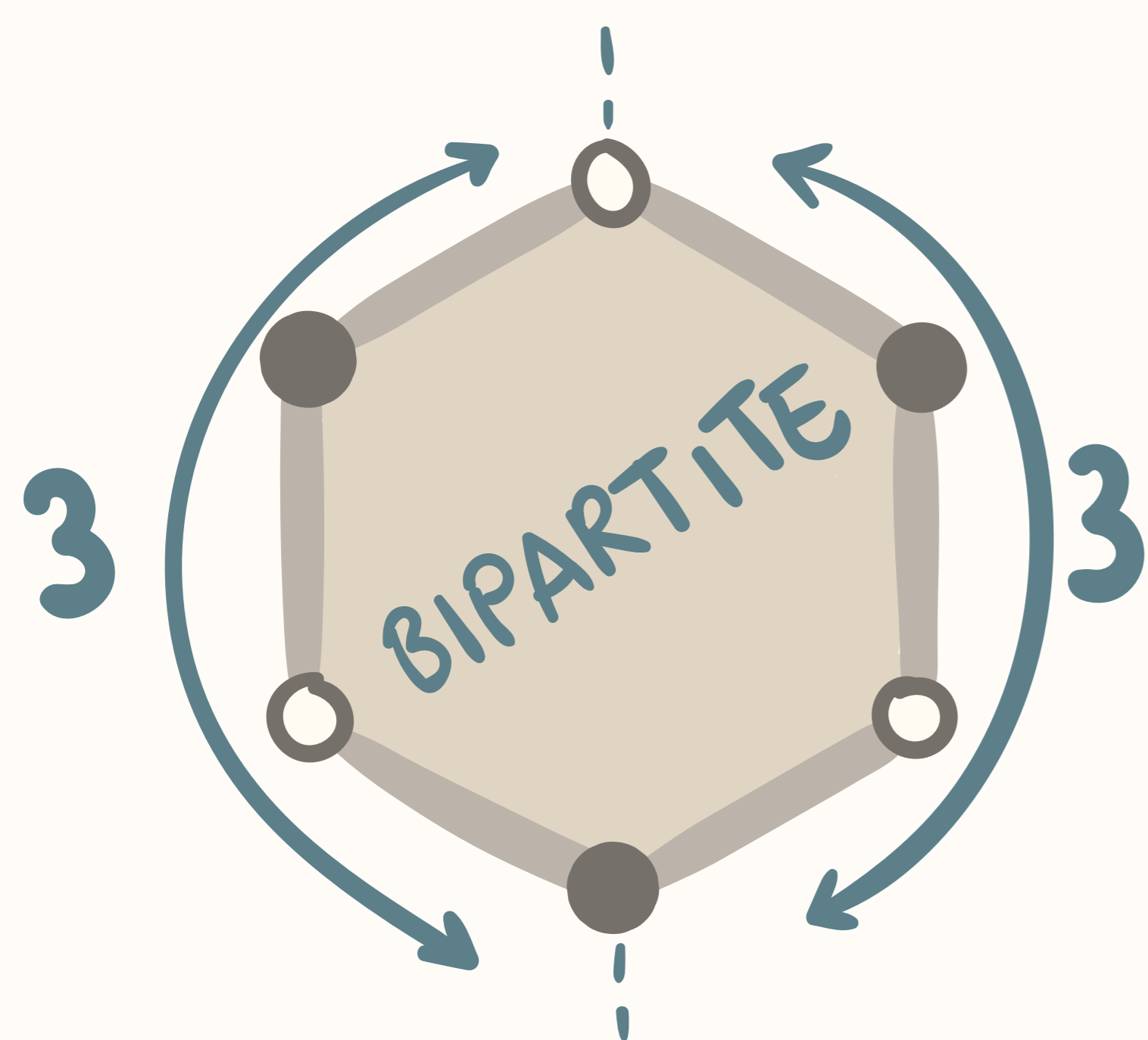


when x, y
are even

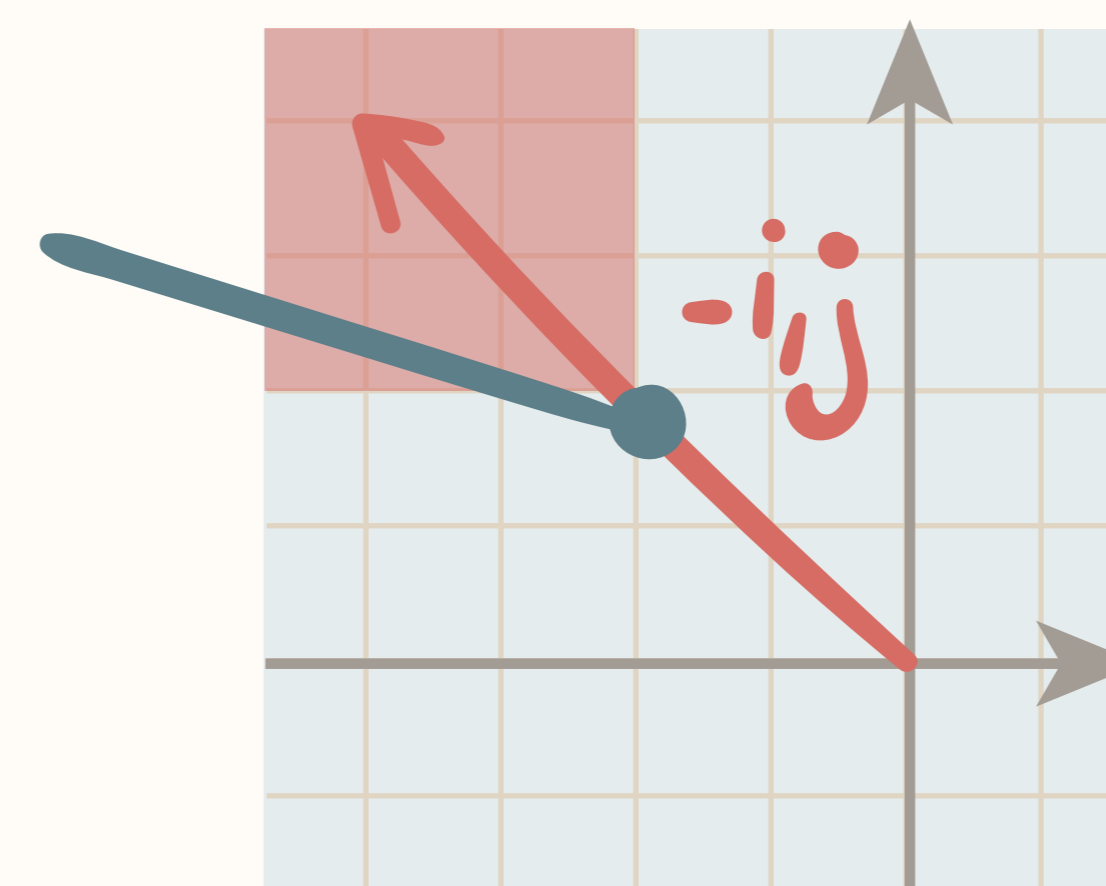


when x, y
are odd

Schnyder labellings KMSW

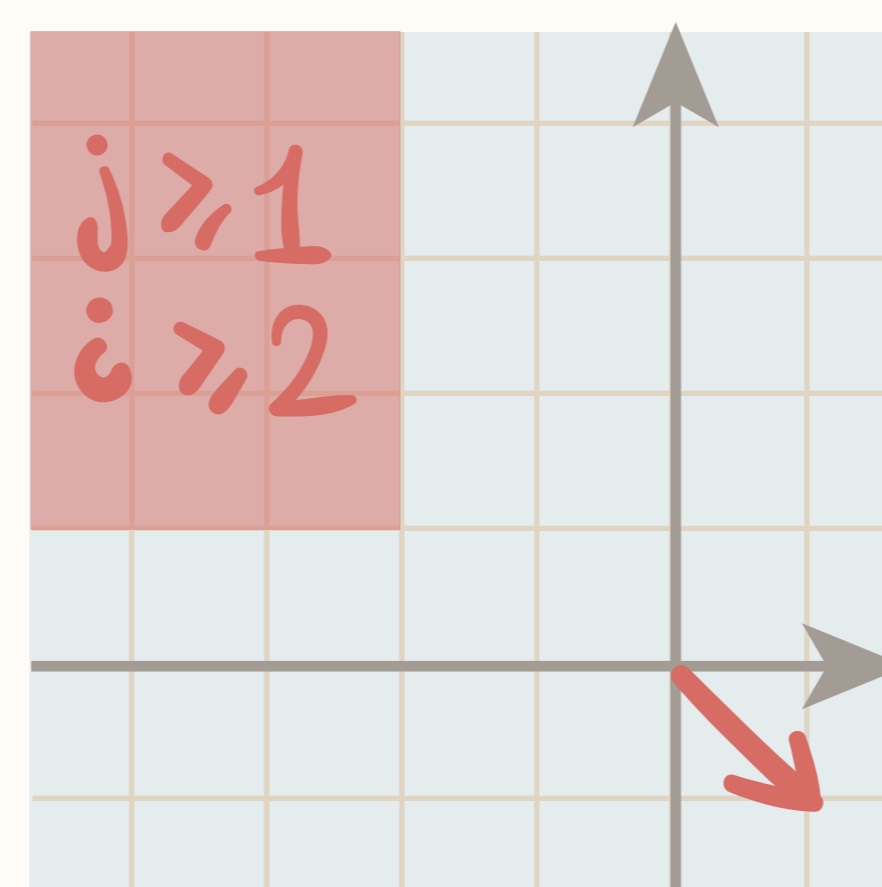
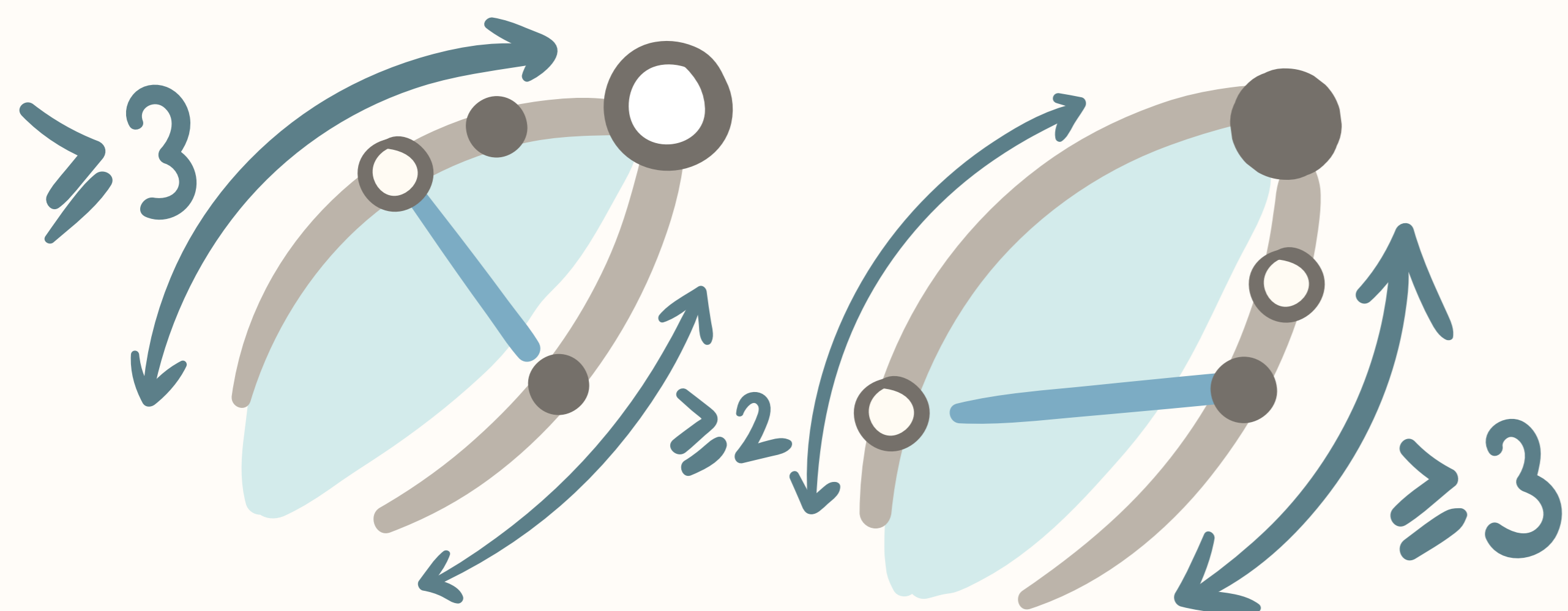


$$\binom{k+l}{k}$$

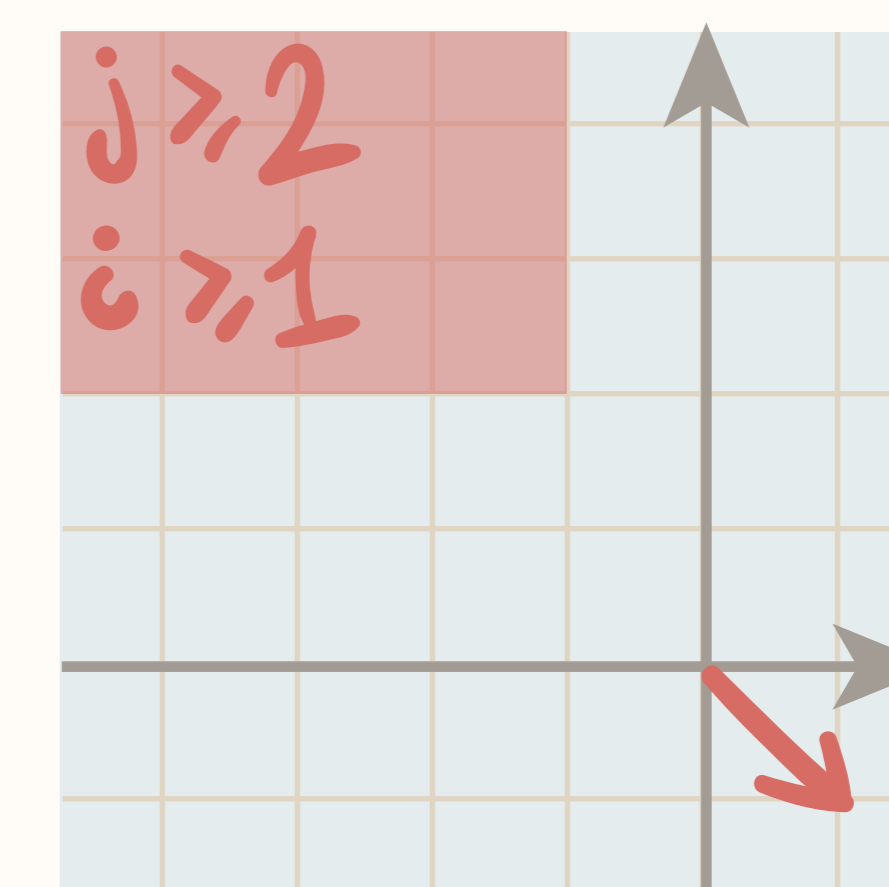


regarding
 x, y parity

with
 $i = 2k+2$
 $j = 2l+2$



when x, y
are even



when x, y
are odd

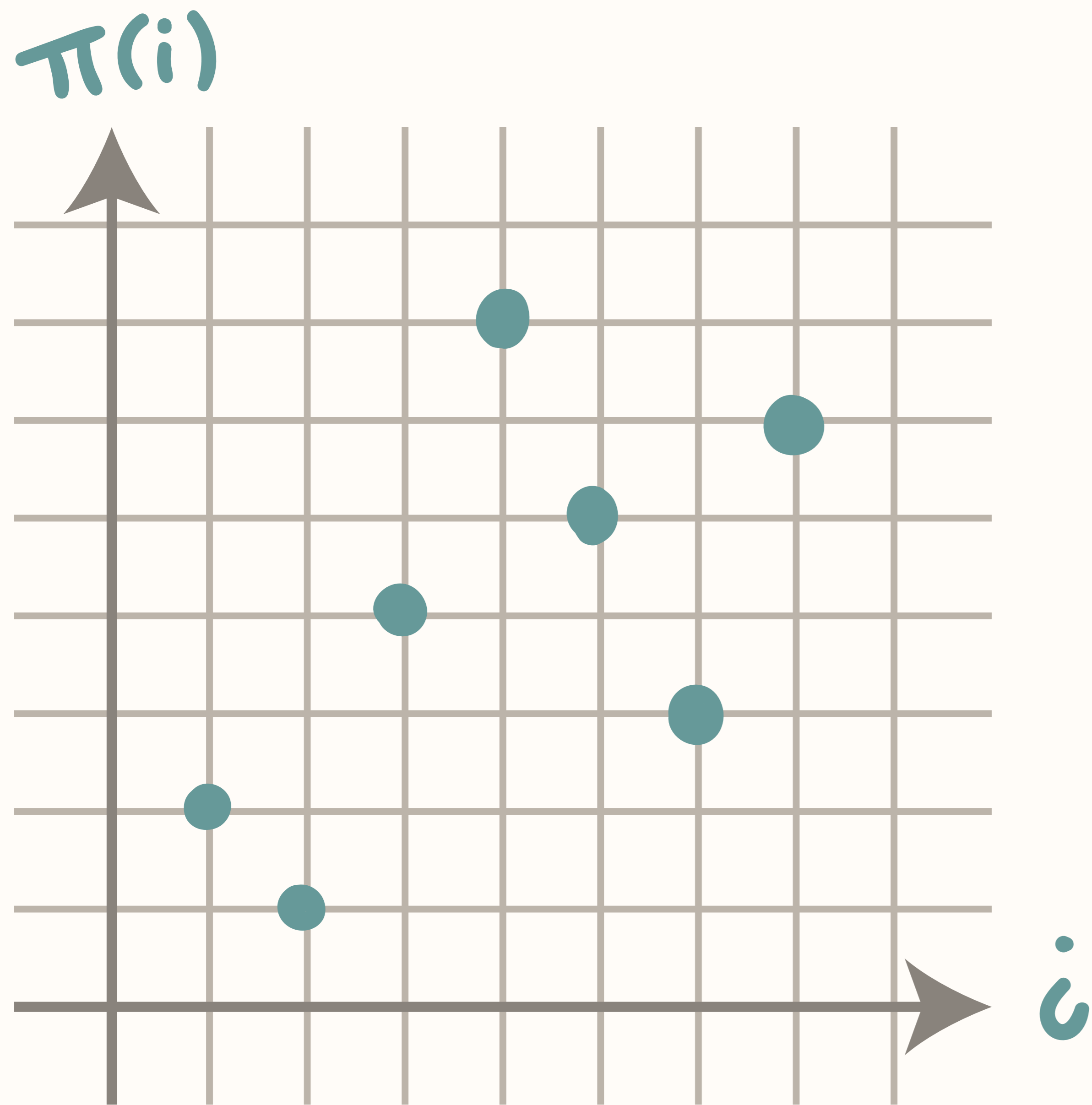
ANNEXES

A. Generalization for transversal structures

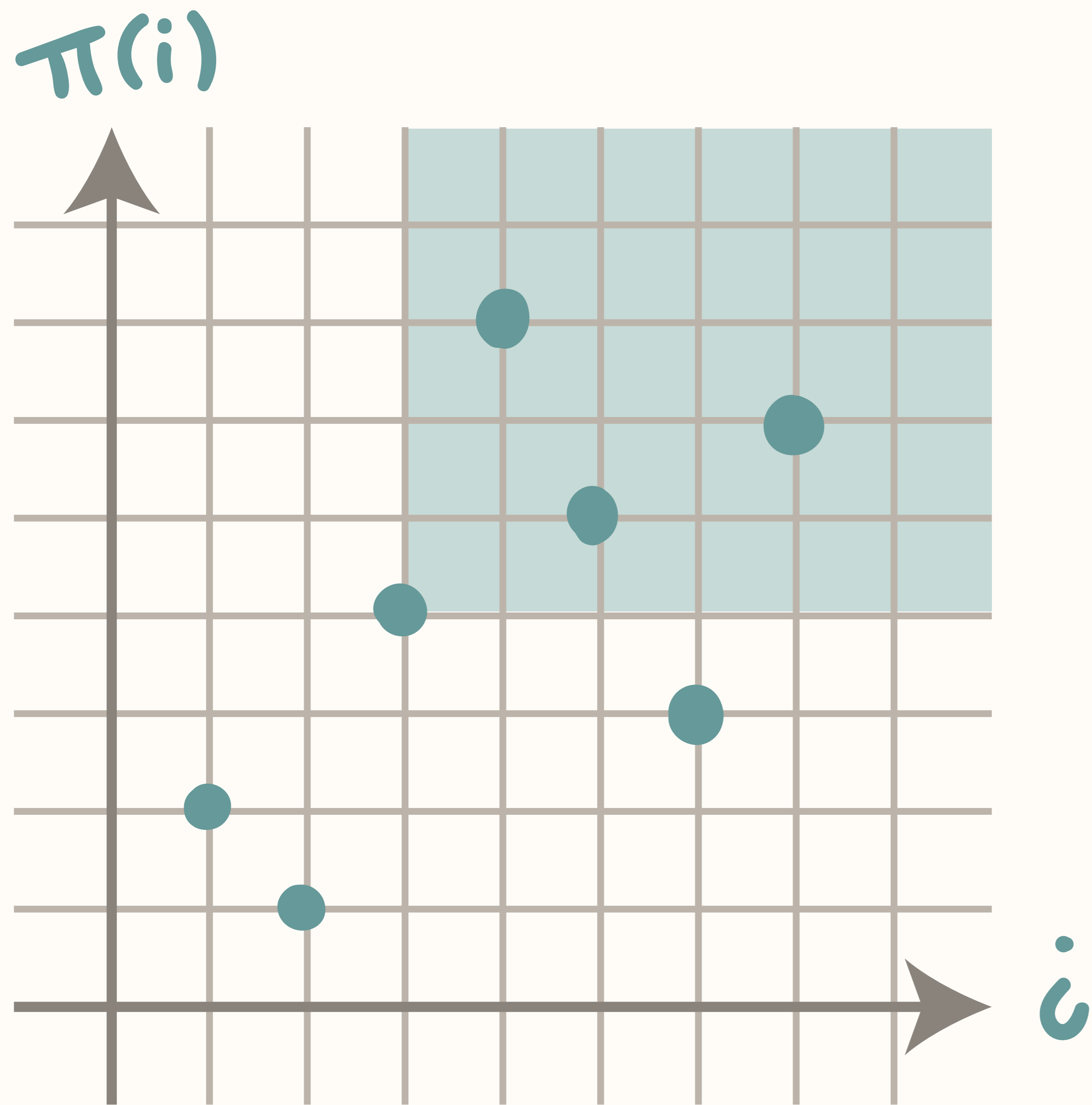
B. Schnyder labelings

C. Bijection for plane permutations

Plane permutations

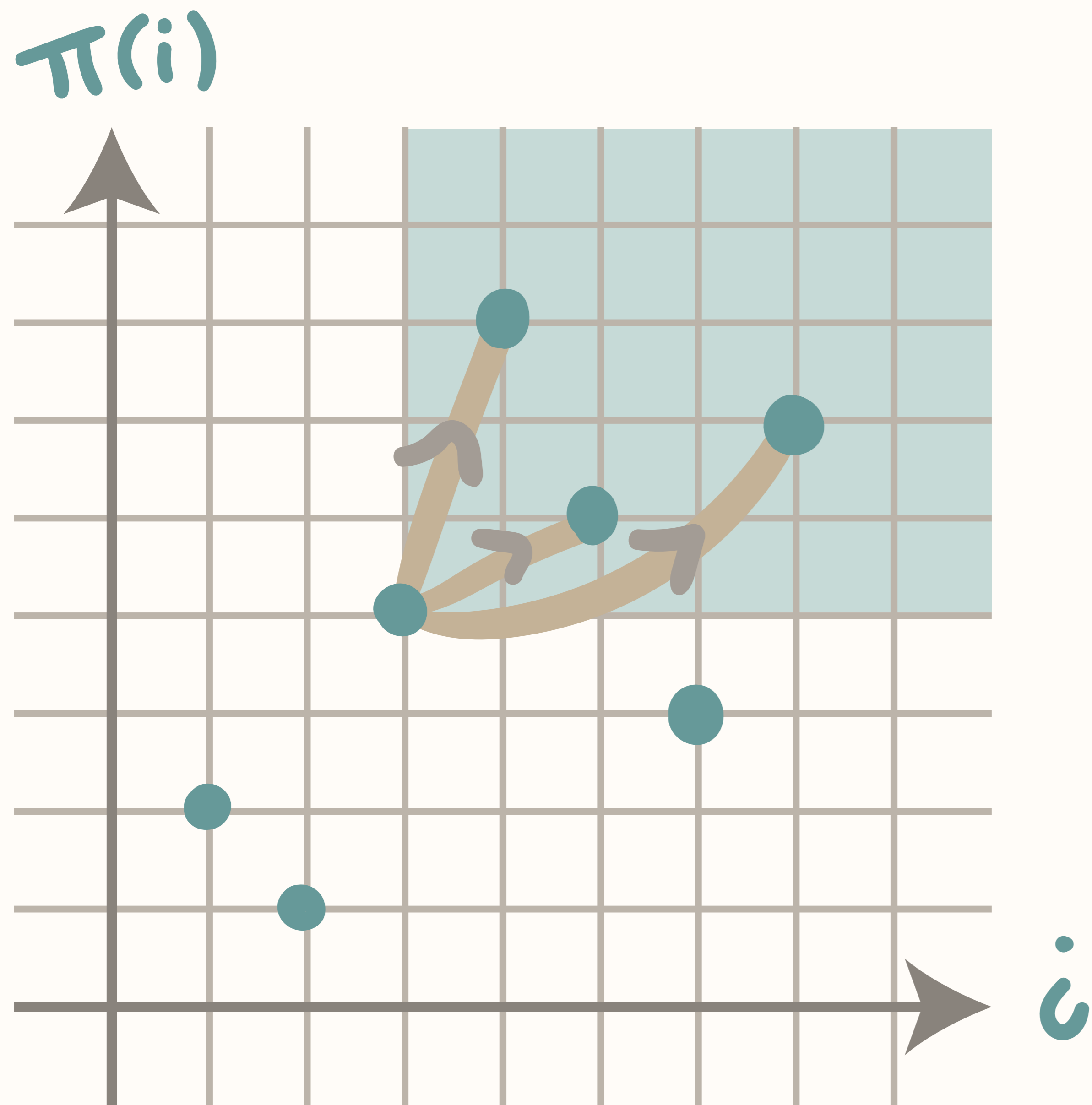


Plane permutations



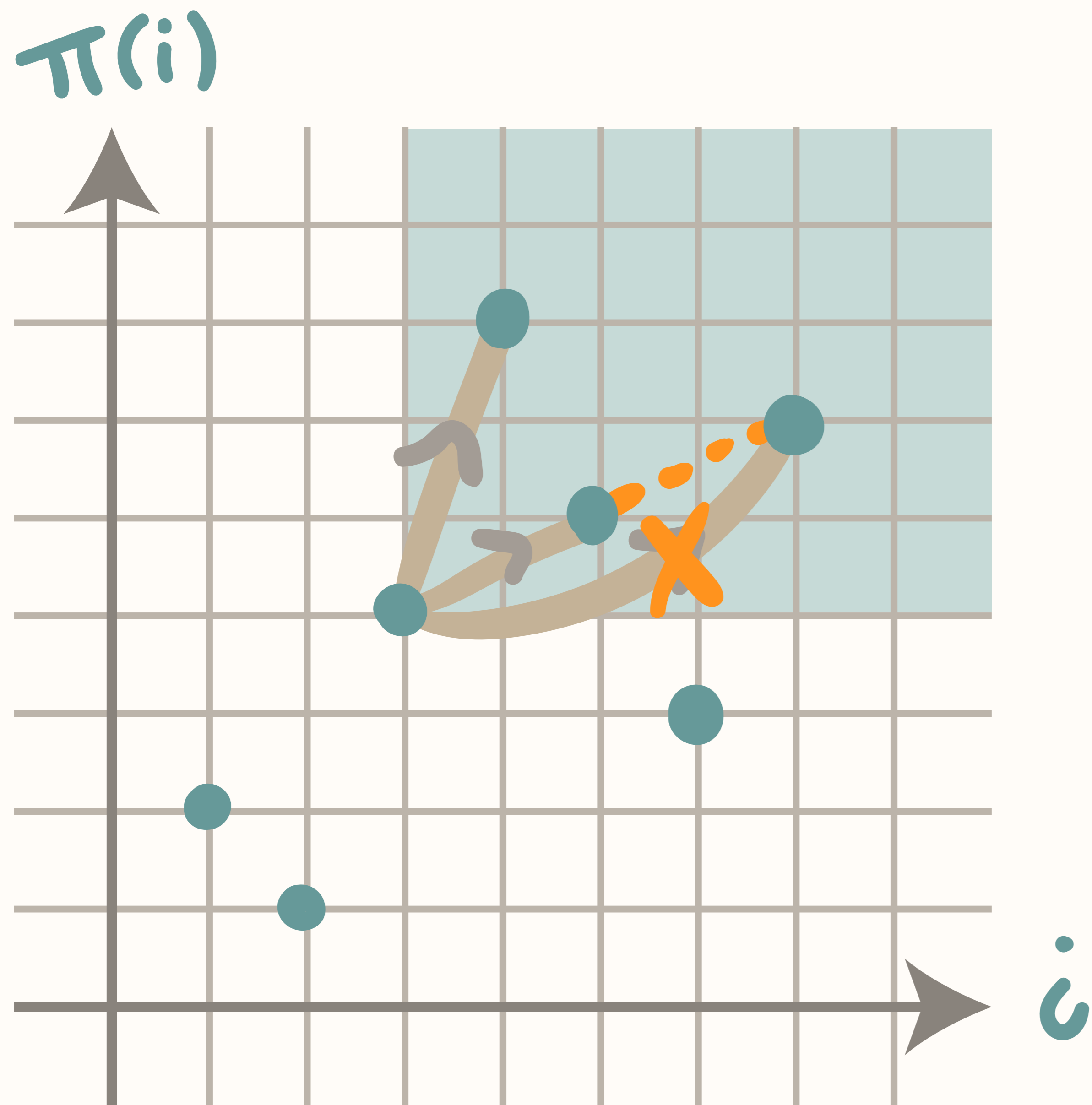
Dominance relation

Plane permutations



Dominance relation

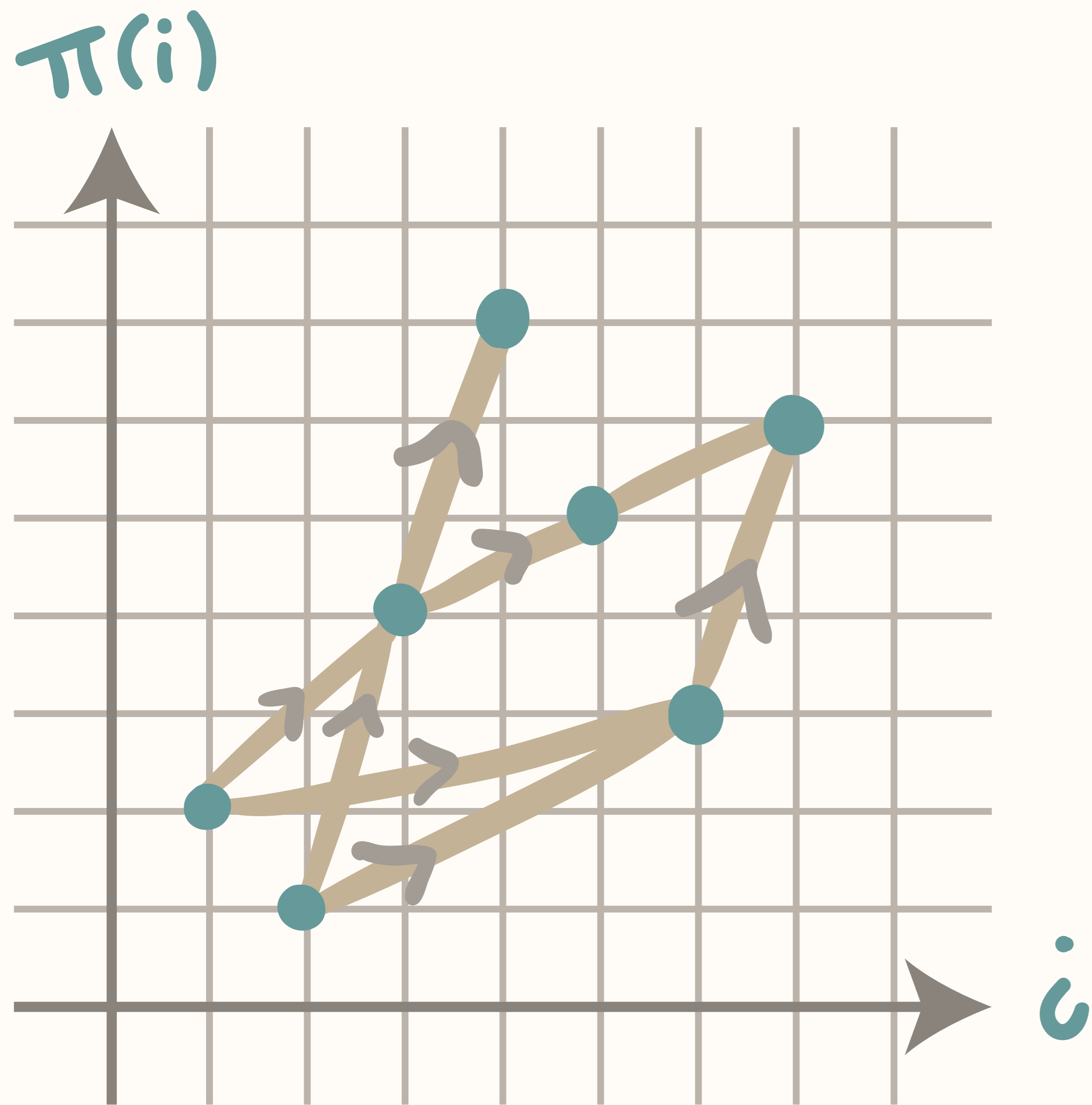
Plane permutations



***Dominance
diagram***

***= Dominance relation
with no transitive edges***

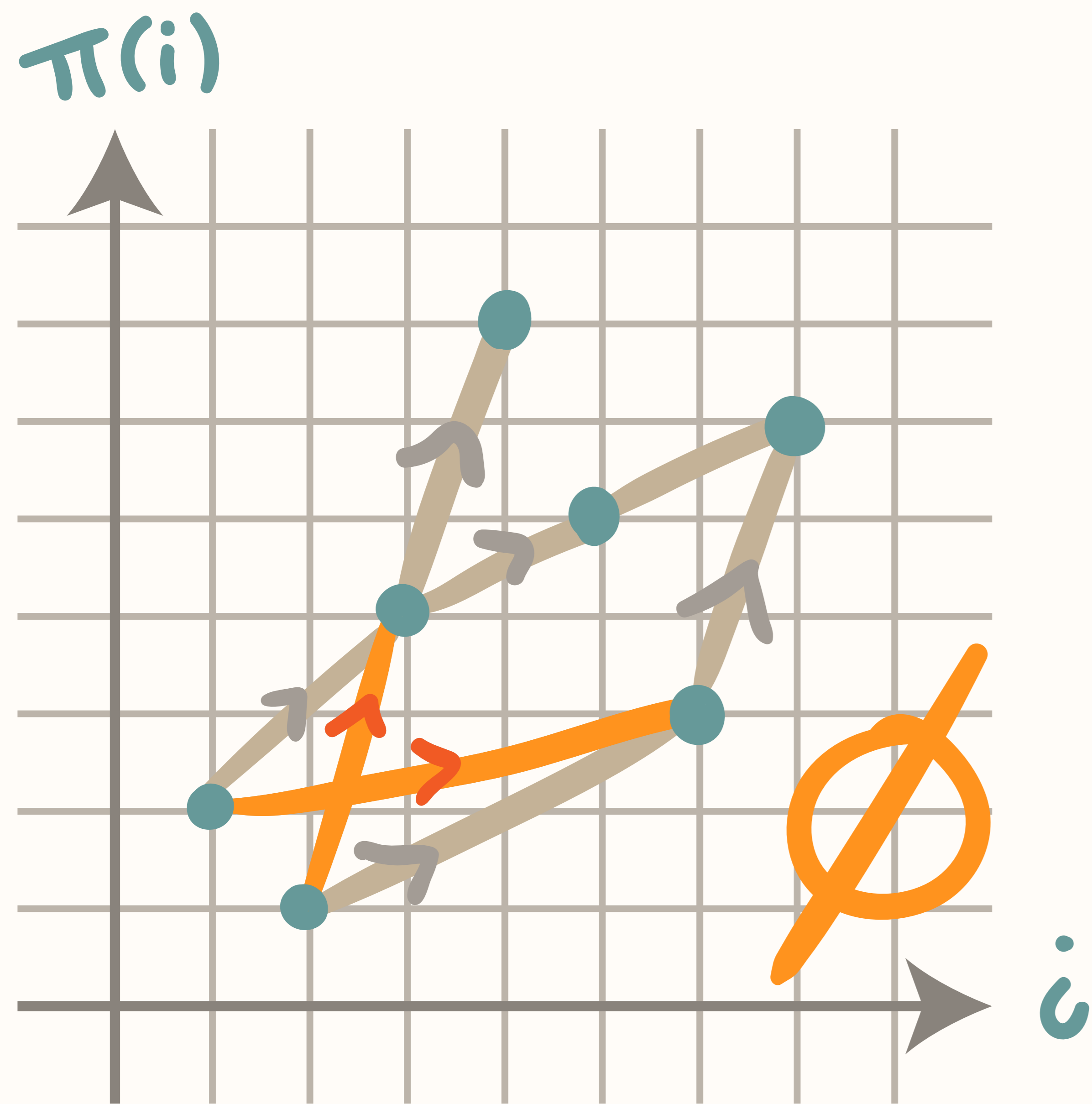
Plane permutations



***Dominance
diagram***

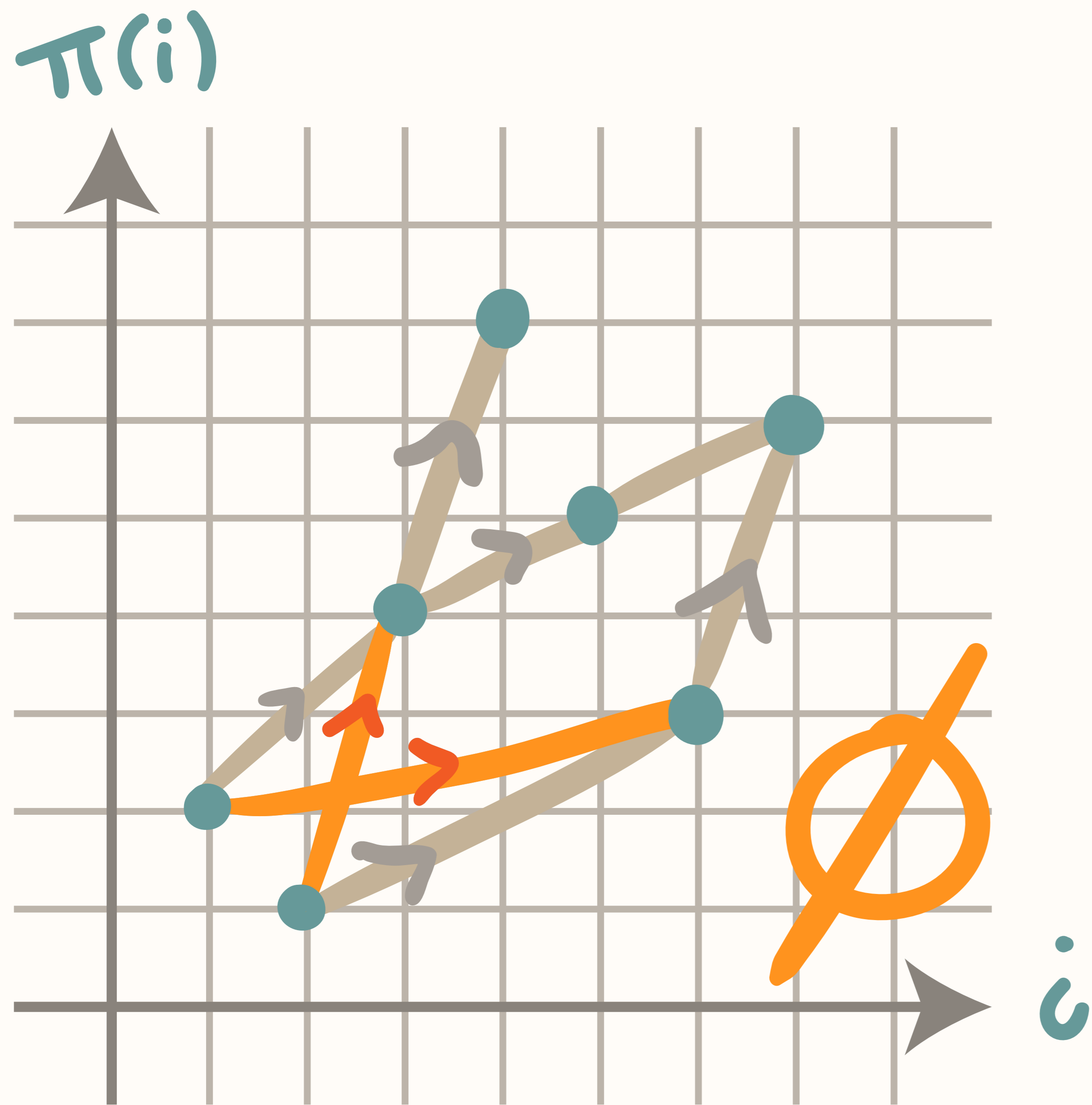
***= Dominance relation
with no transitive edges***

Plane permutations



Plane permutation
= No edge crossing in the dominance diagram

Plane permutations

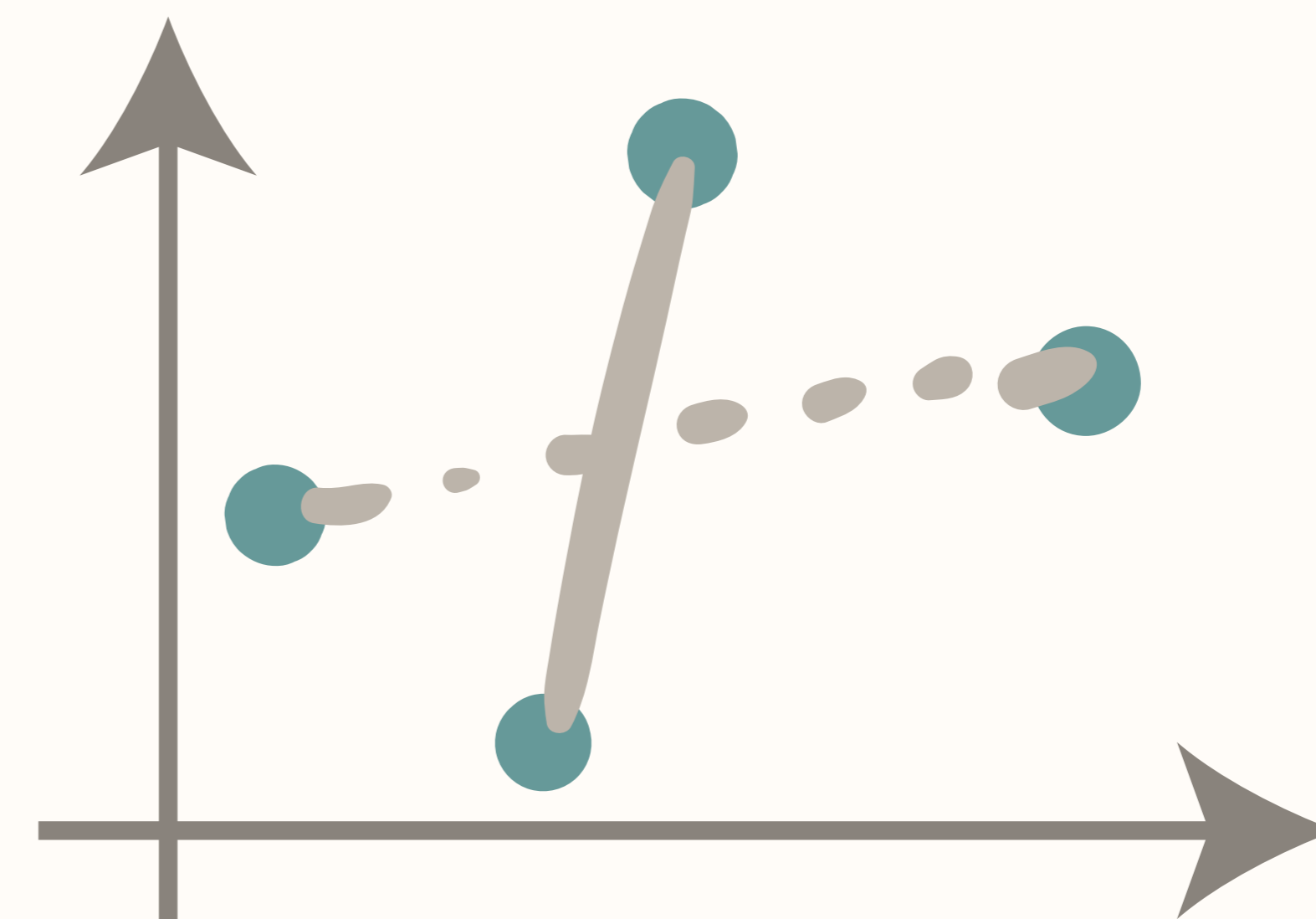


Plane permutation

= No edge crossing in the dominance diagram

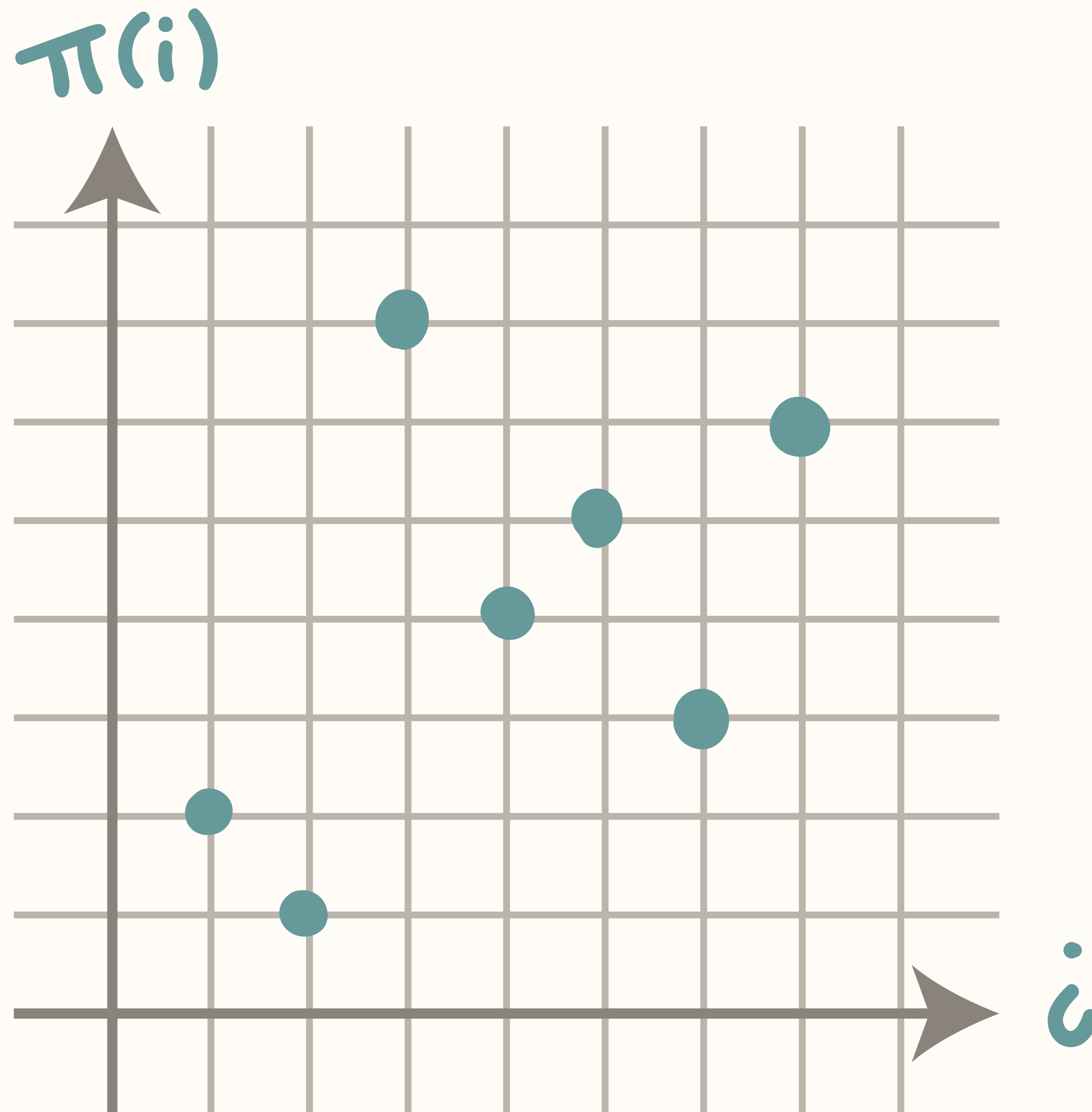
= Avoid the vincular pattern :

$2 \underline{14} 3$



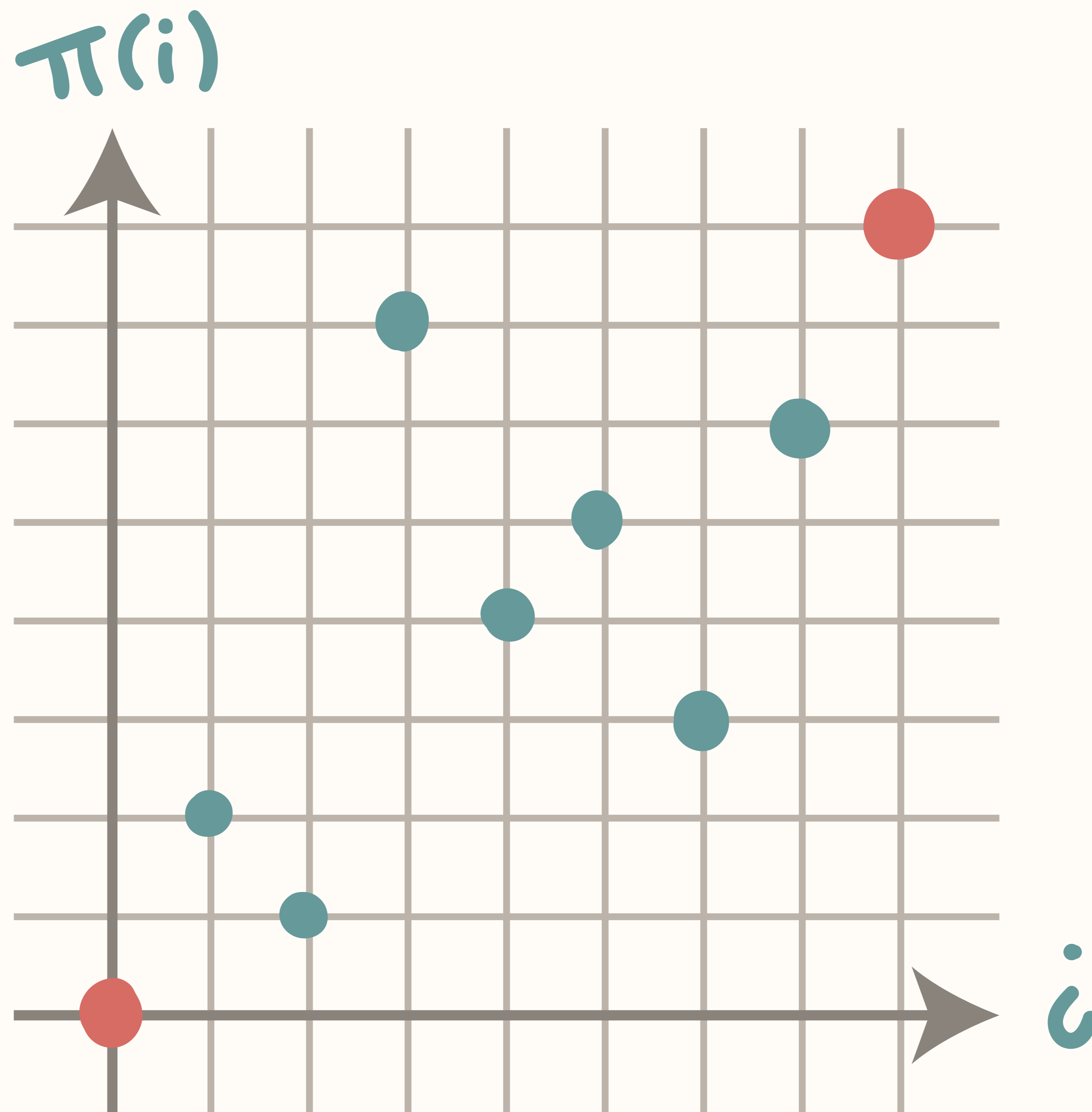
Link with plane permutations

Plane permutation \longrightarrow Poset



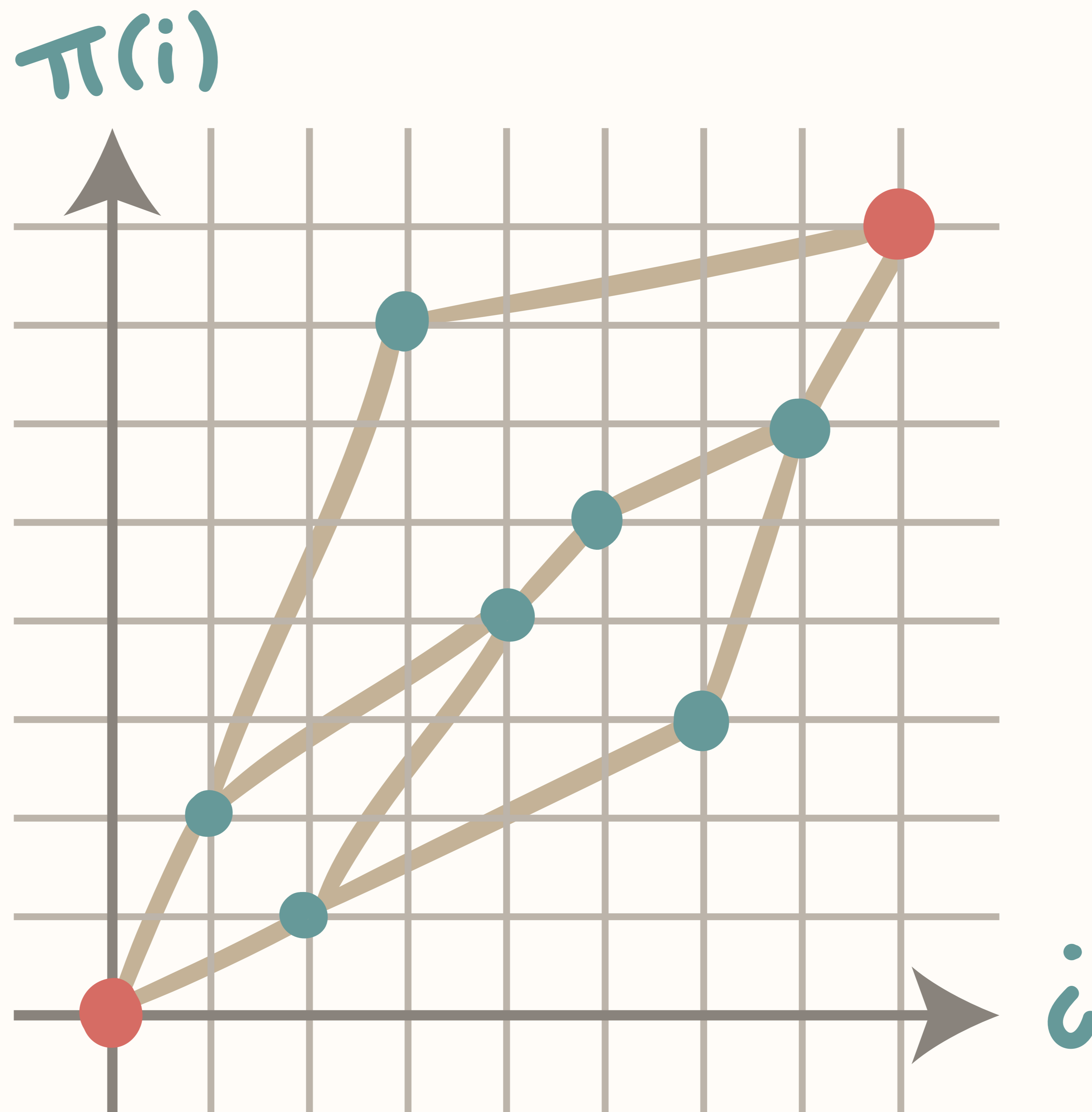
Link with plane permutations

Plane permutation \longrightarrow Poset



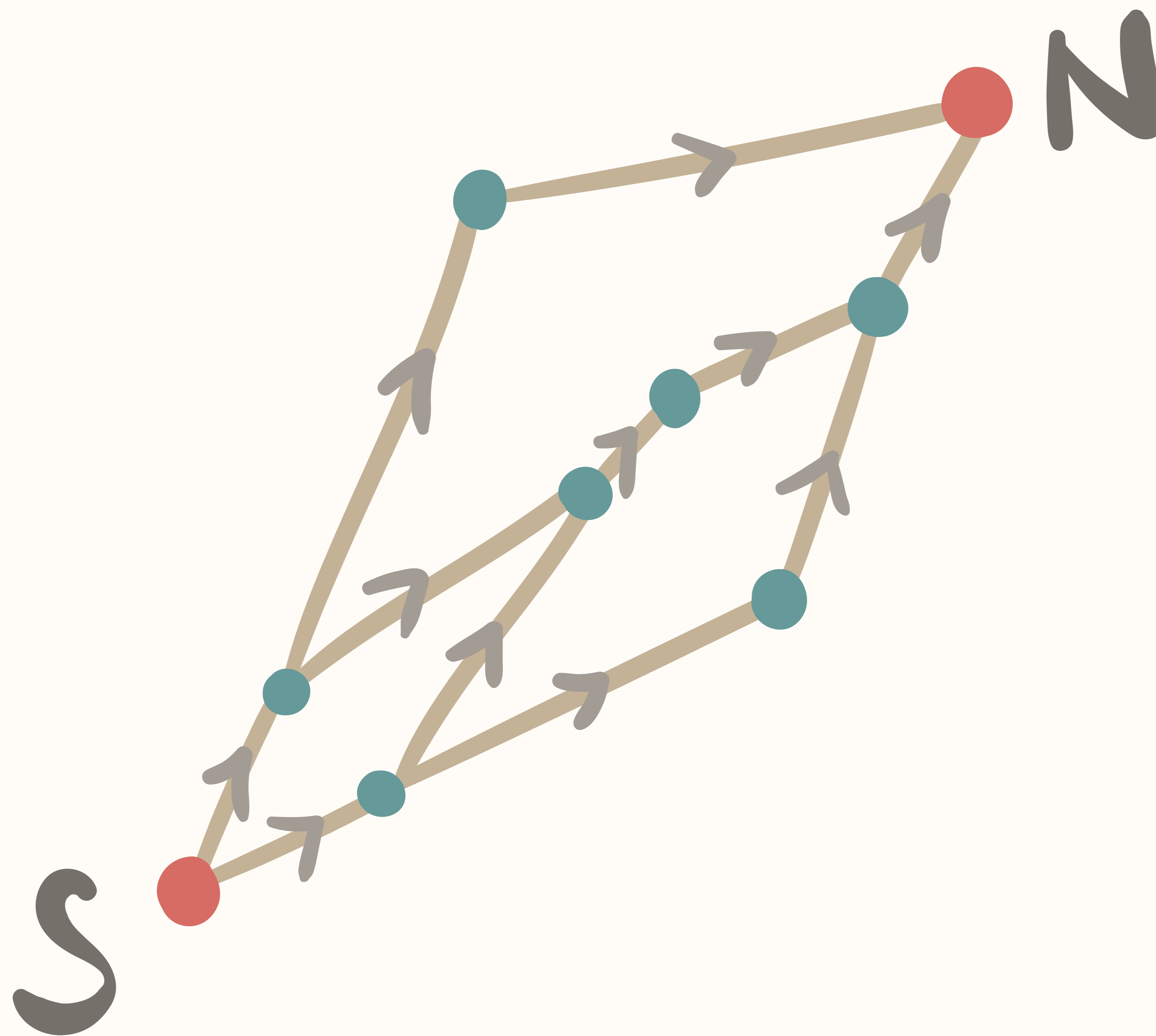
Link with plane permutations

Plane permutation \longrightarrow Poset



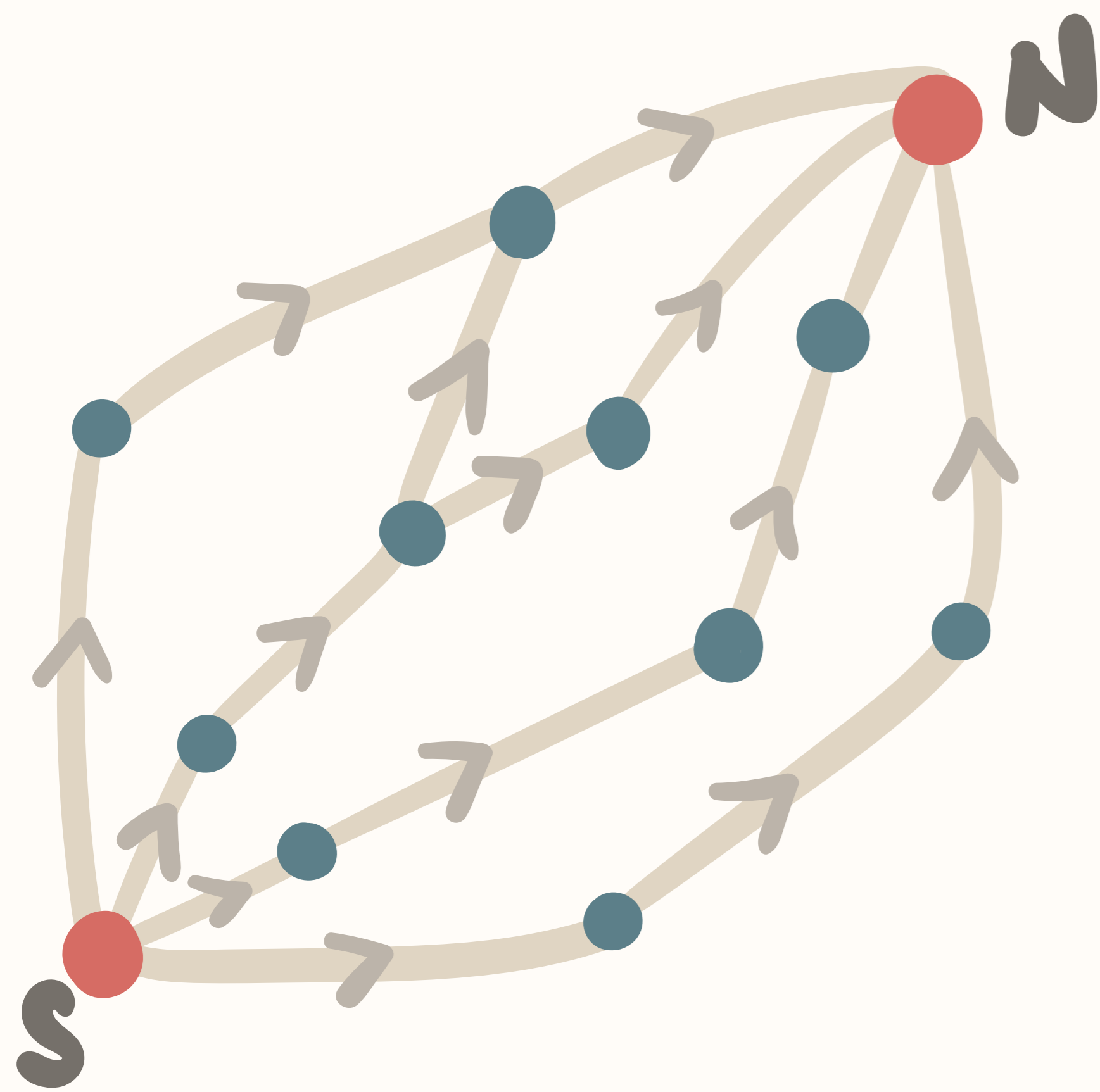
Link with plane permutations

Plane permutation \longrightarrow Poset



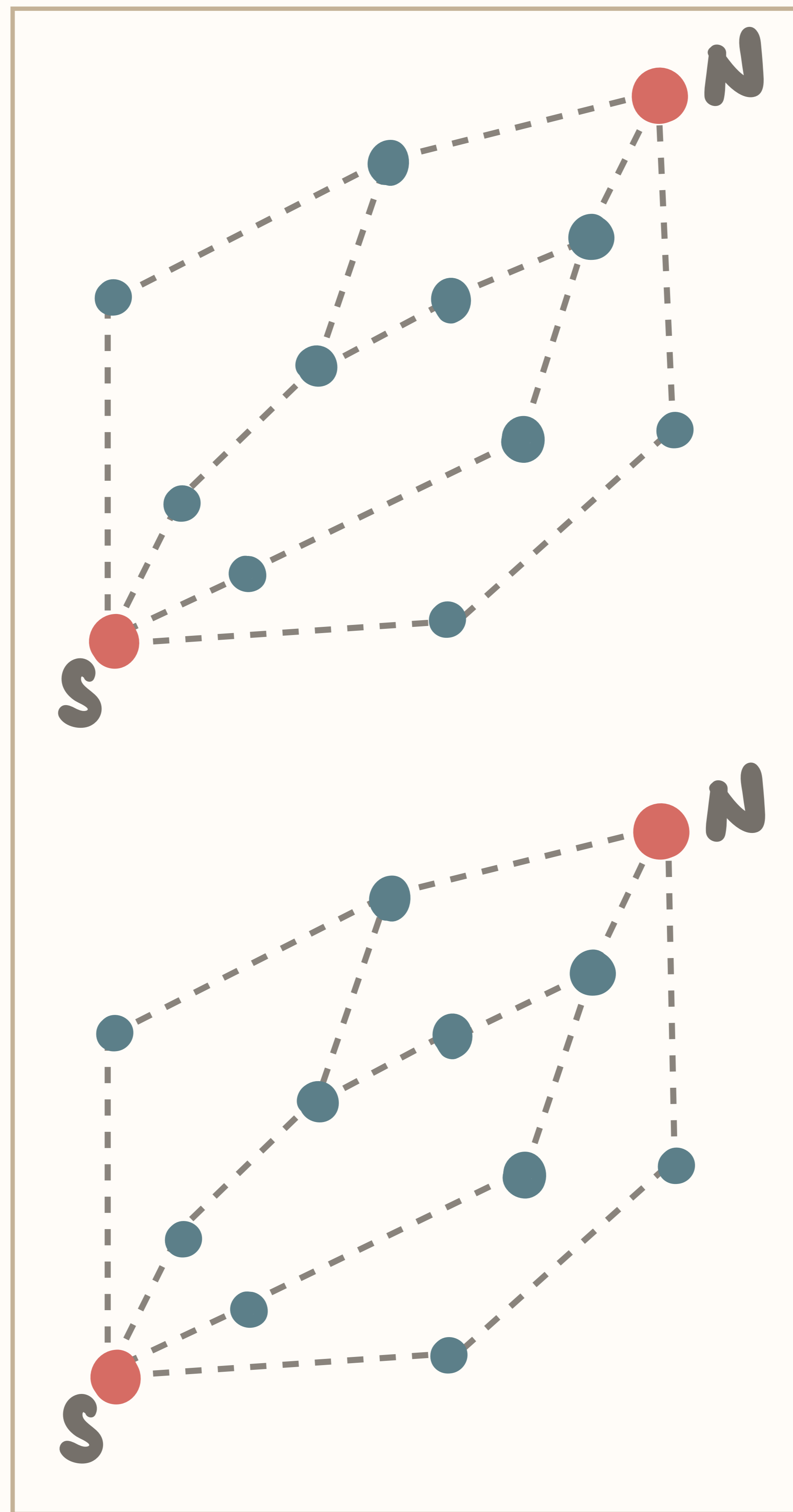
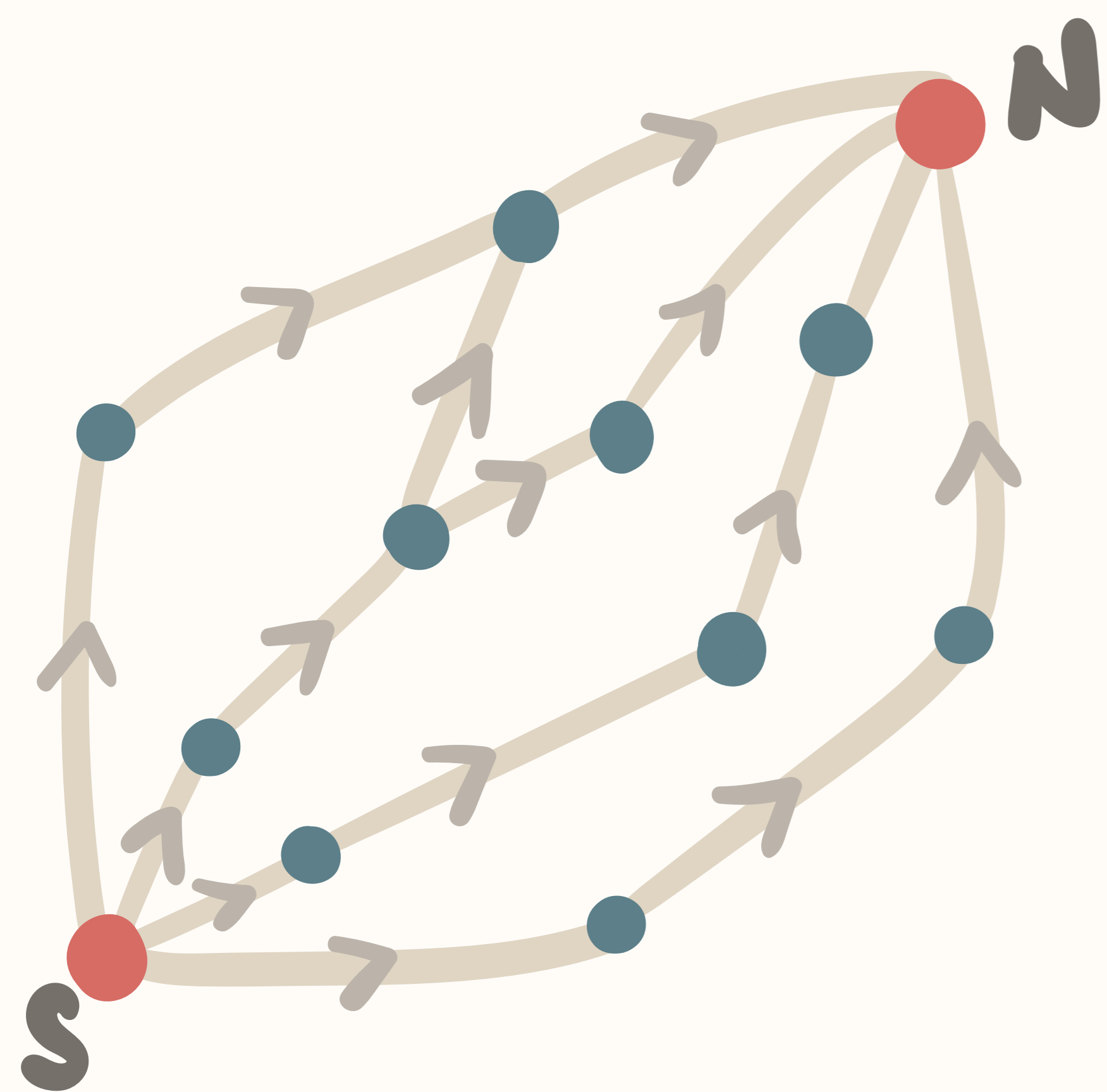
Link with plane permutations

Poset \longrightarrow *Plane permutation*



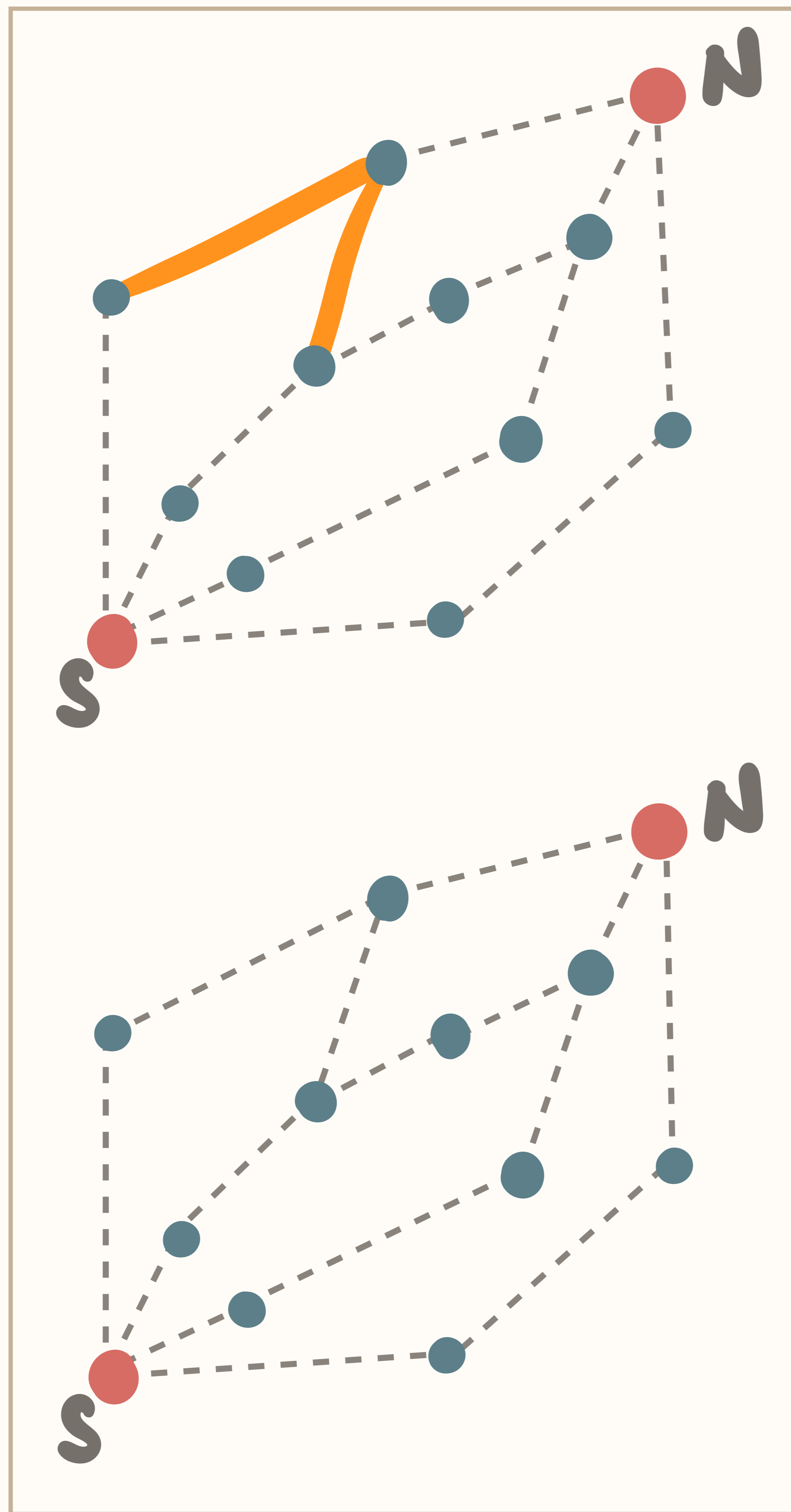
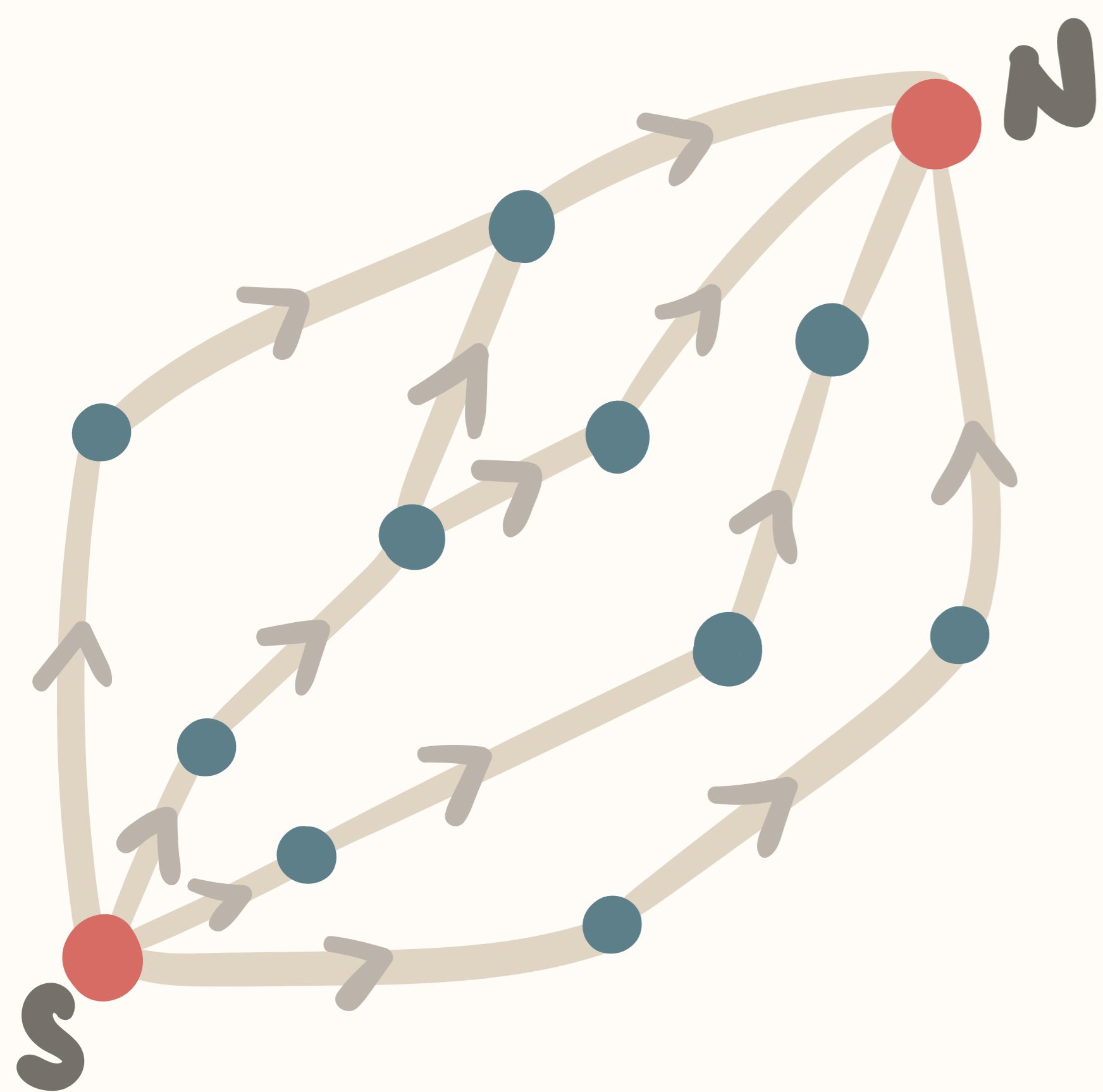
Link with plane permutations

Poset \longrightarrow *Plane permutation*



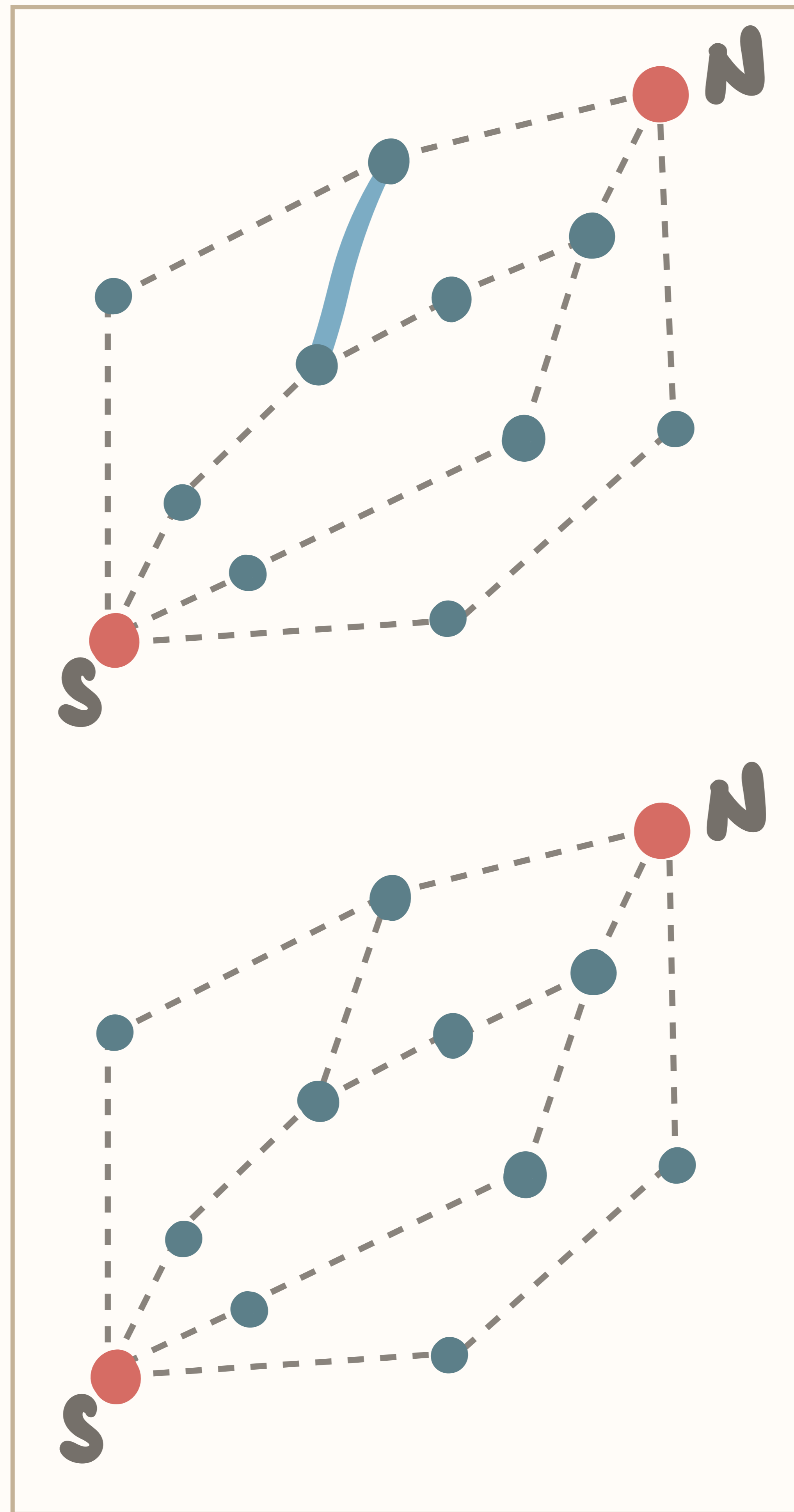
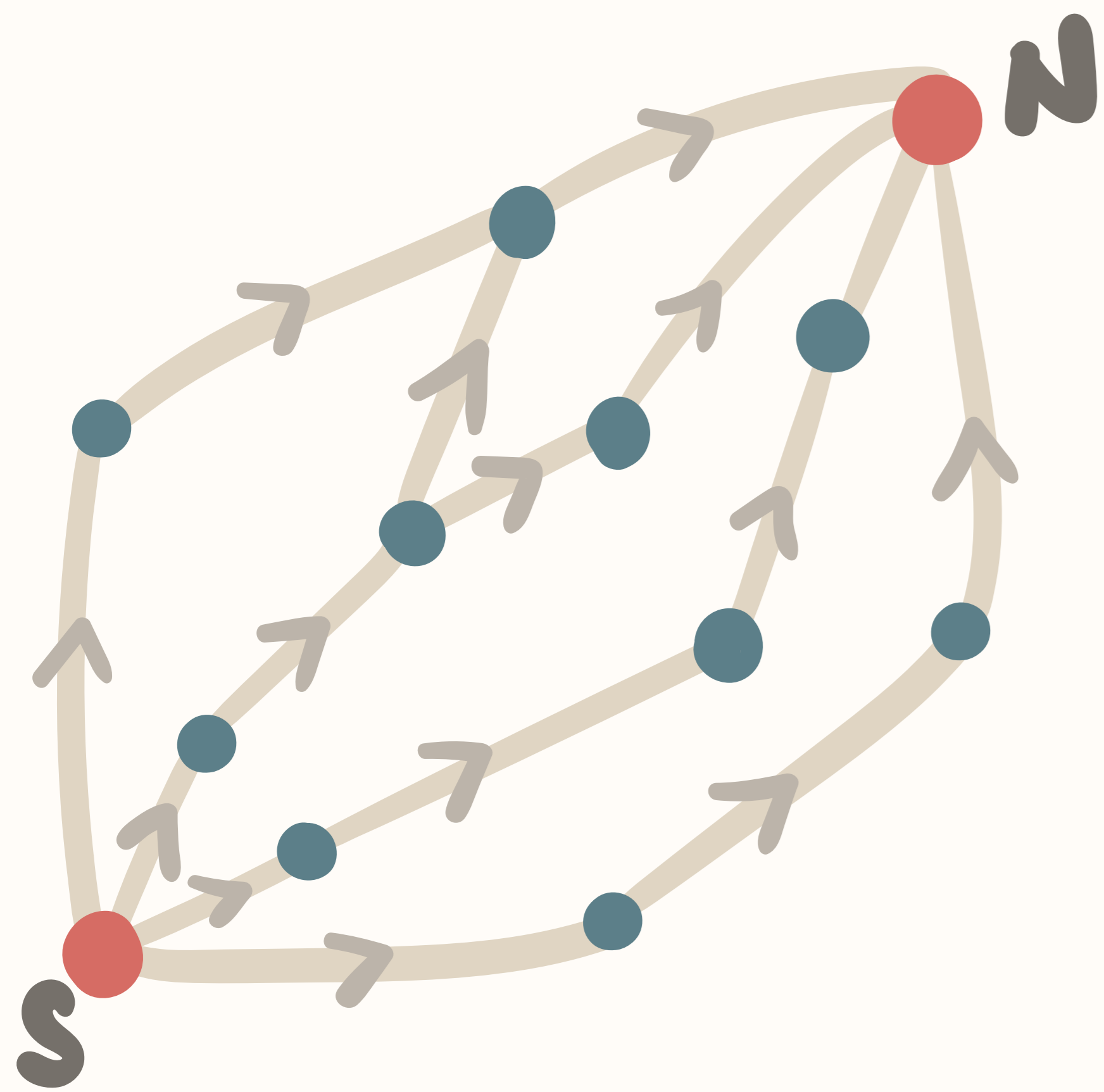
Link with plane permutations

Poset \longrightarrow Plane permutation



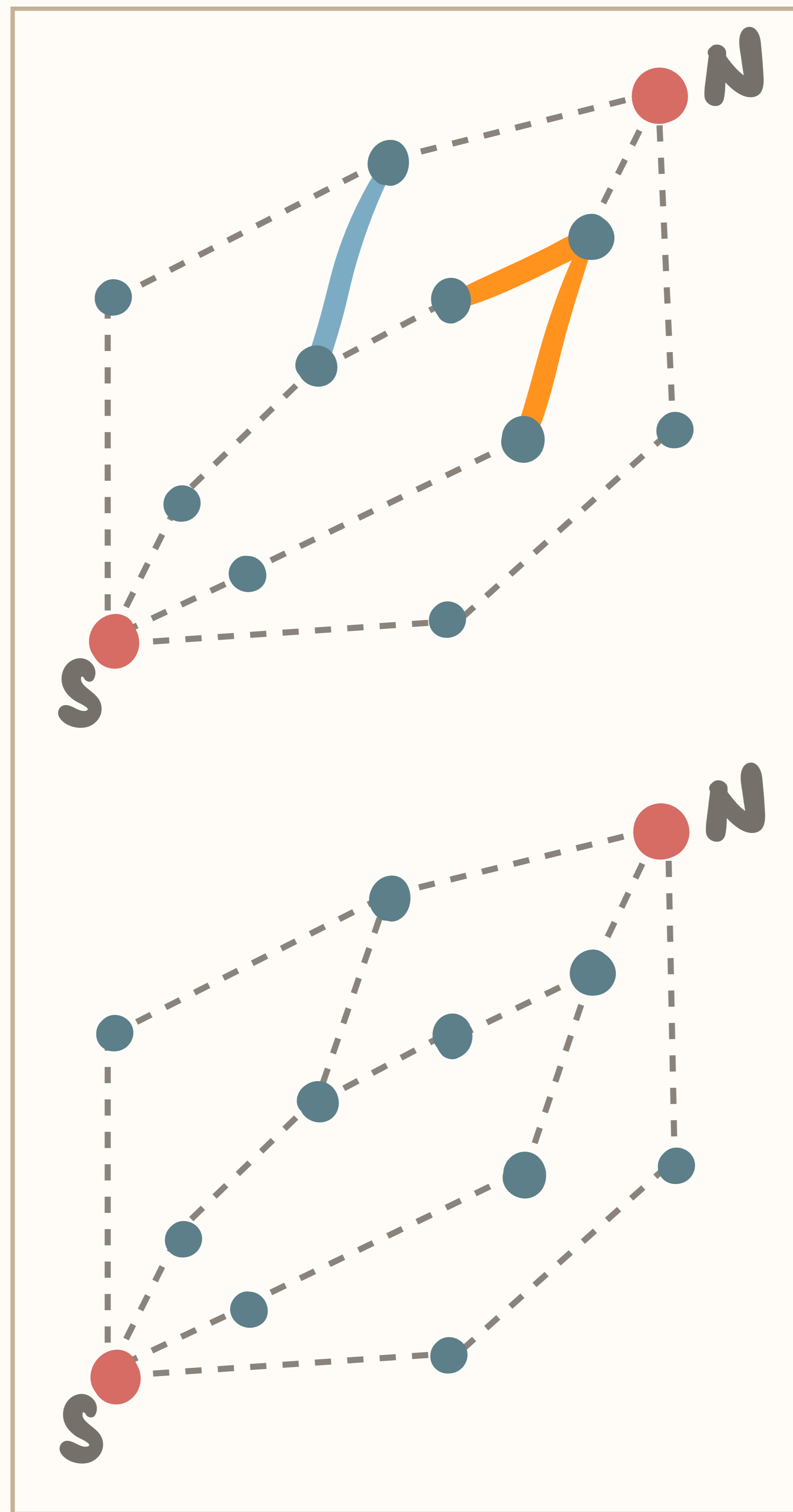
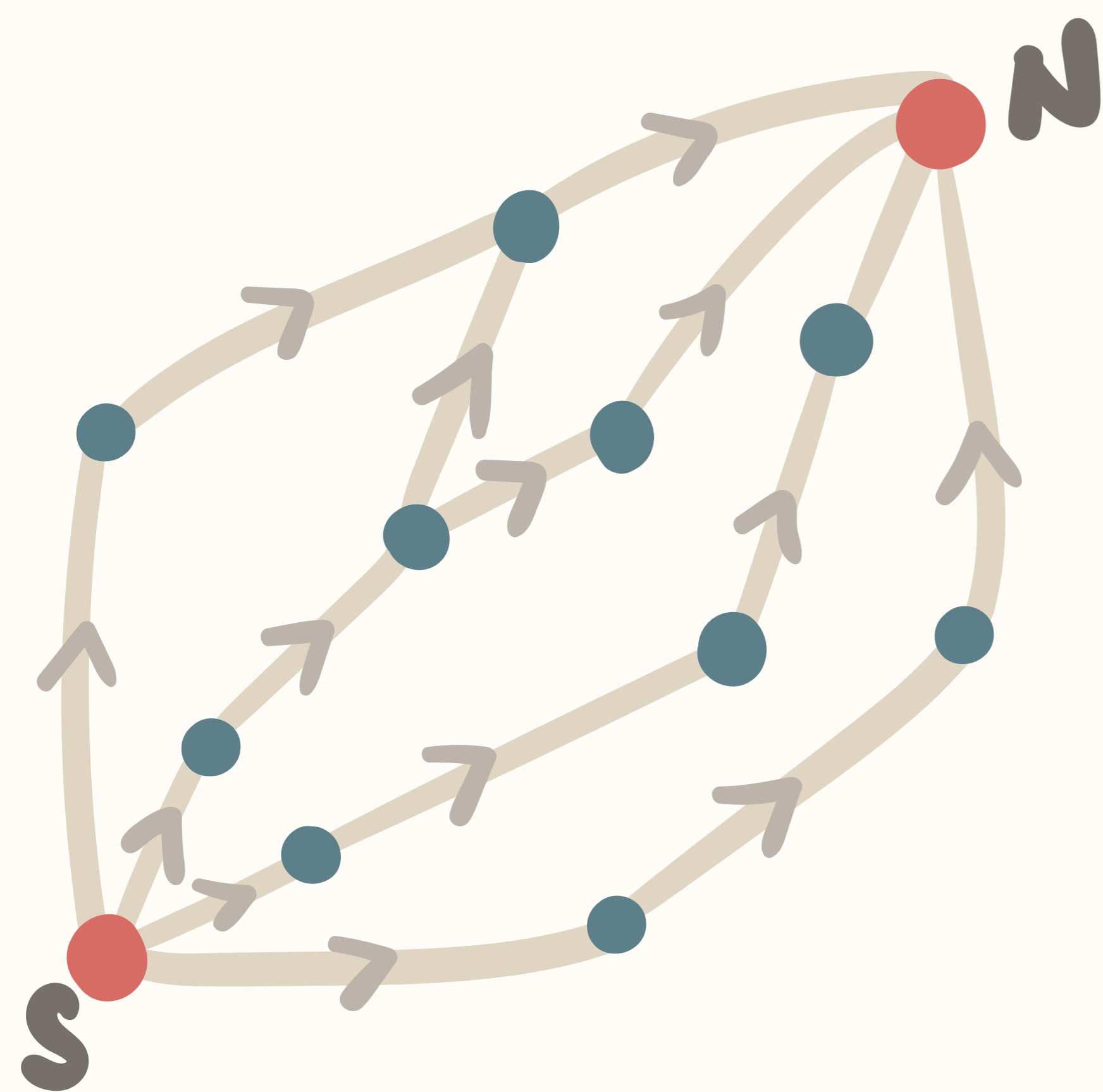
Link with plane permutations

Poset \longrightarrow Plane permutation



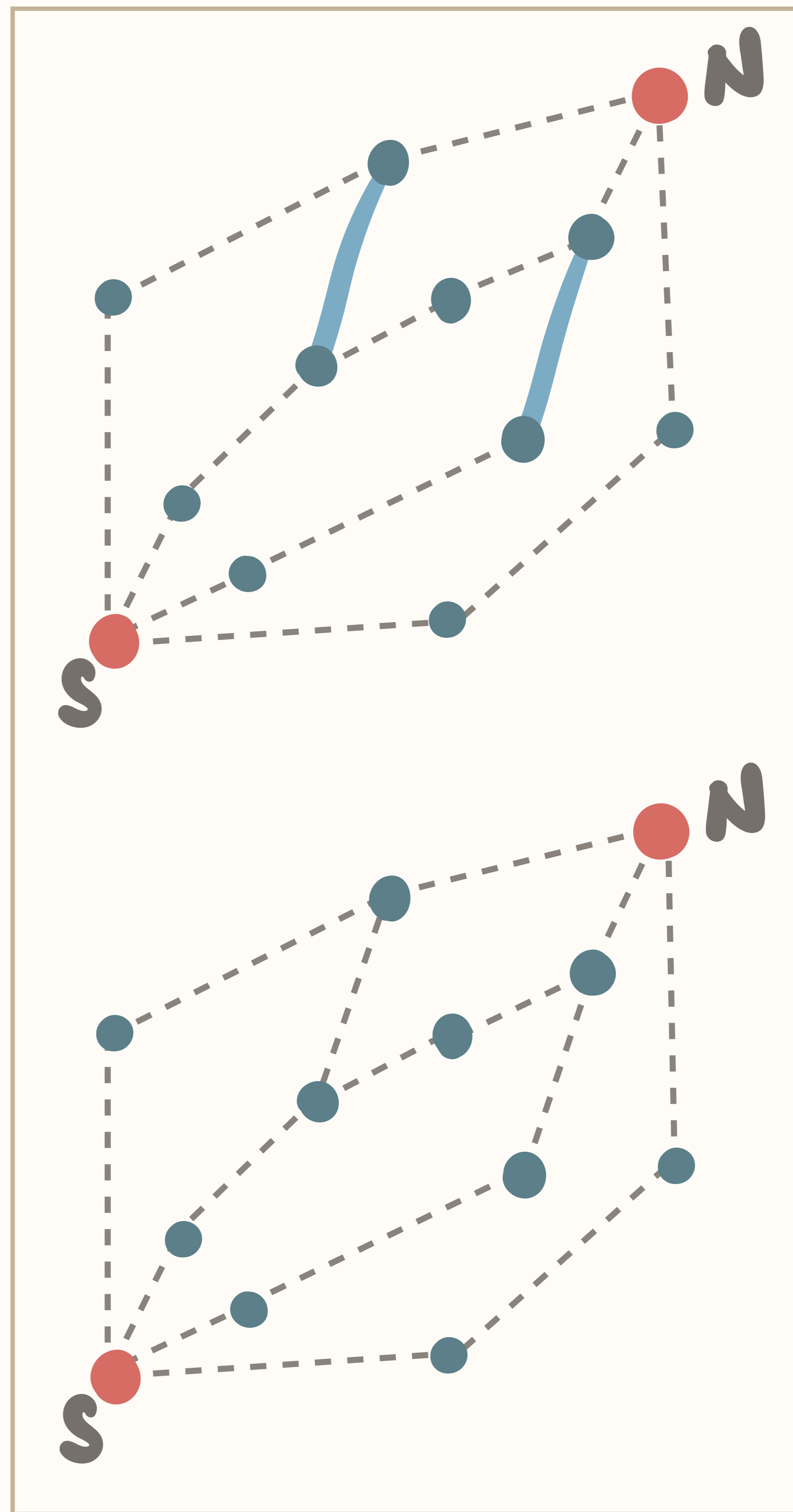
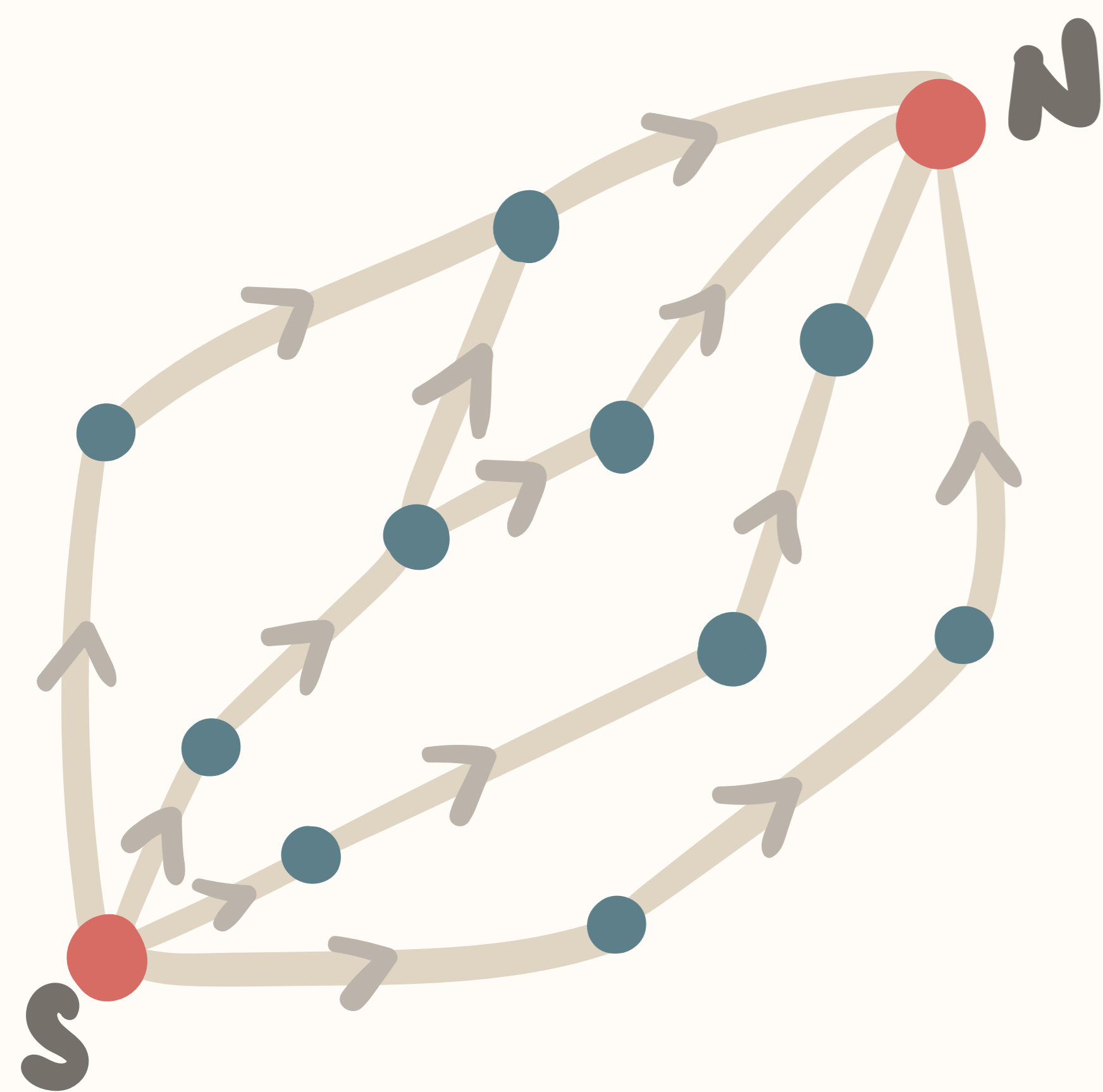
Link with plane permutations

Poset \longrightarrow Plane permutation



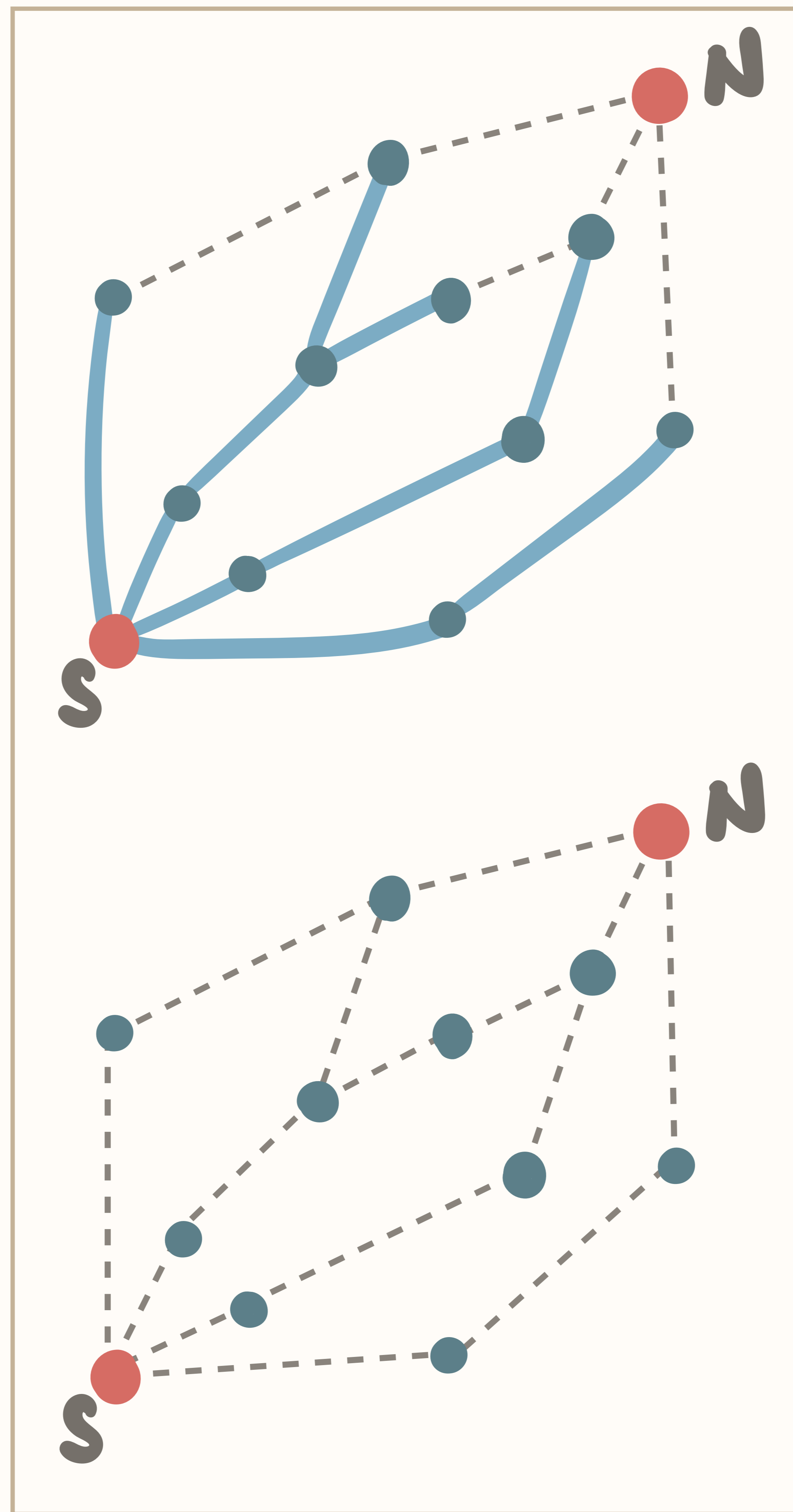
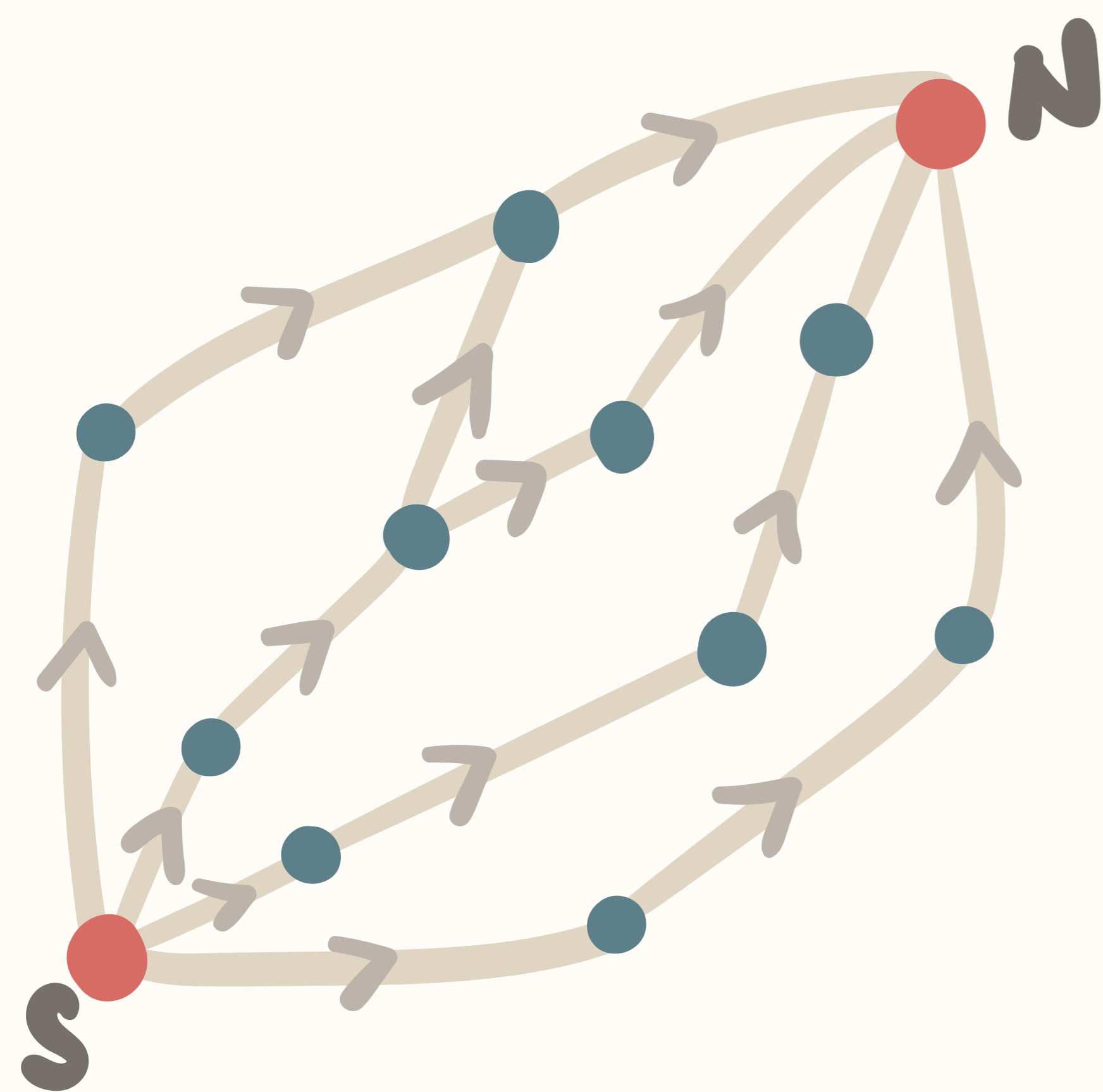
Link with plane permutations

Poset \longrightarrow Plane permutation



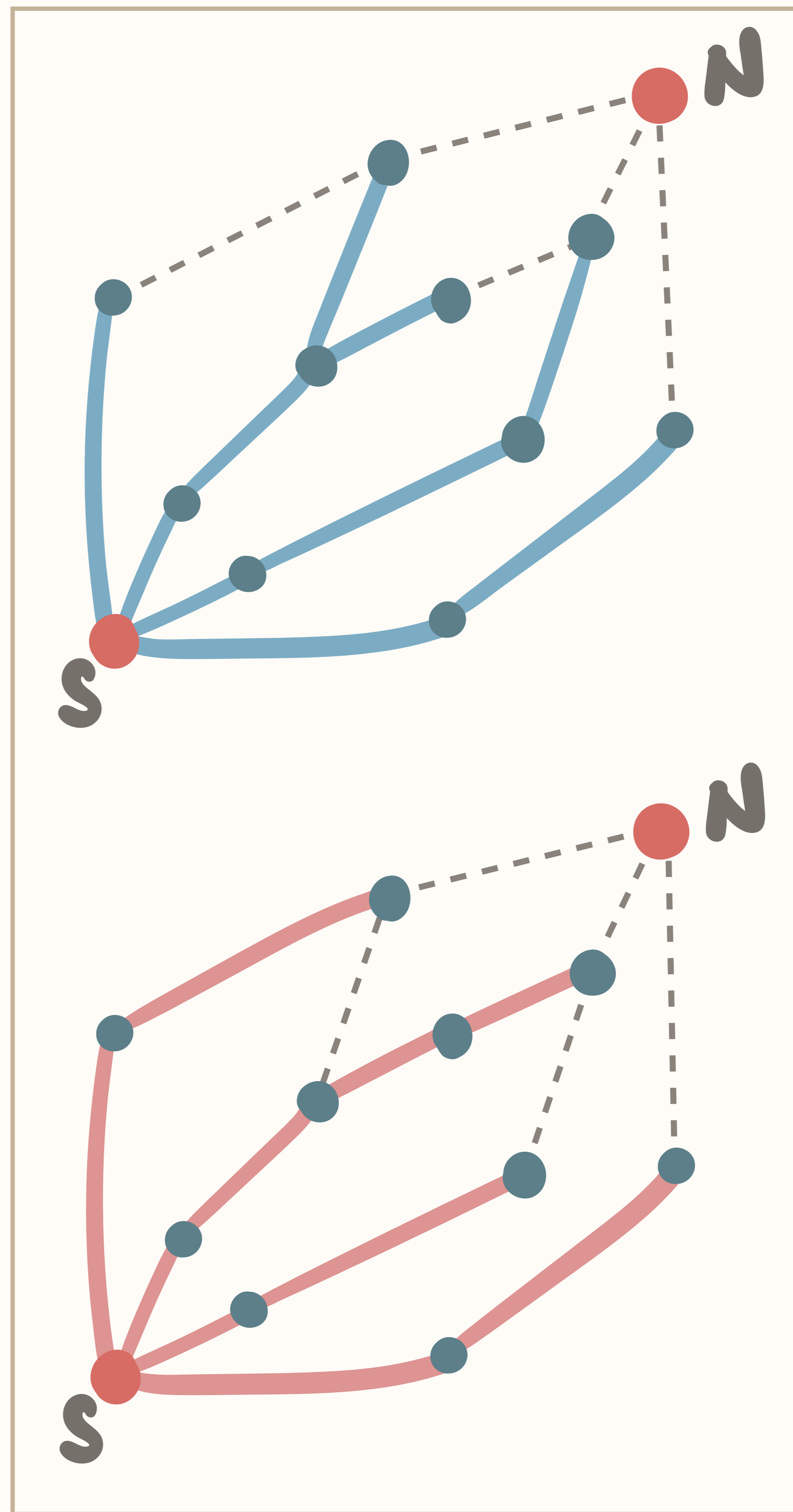
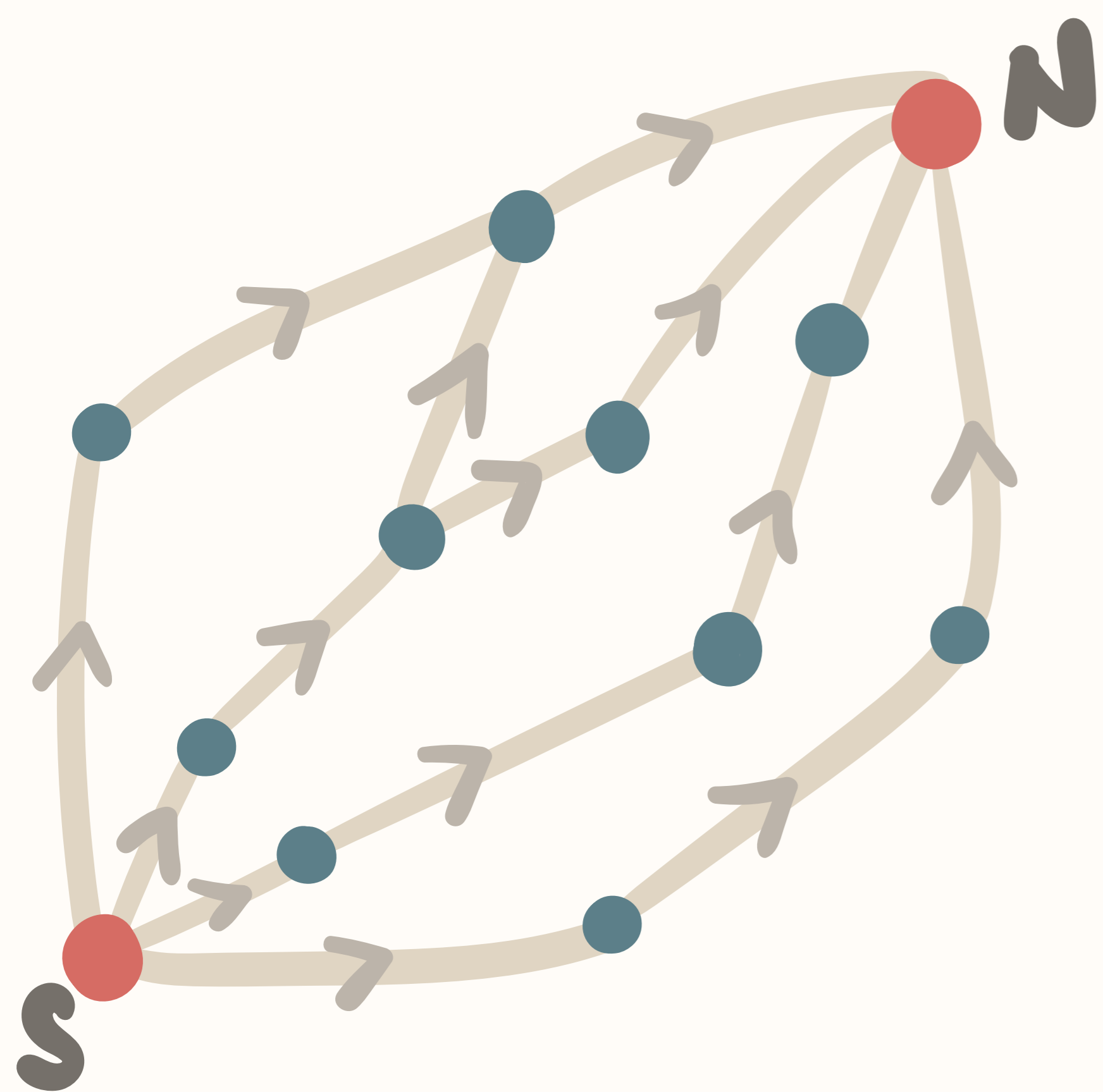
Link with plane permutations

Poset \longrightarrow Plane permutation



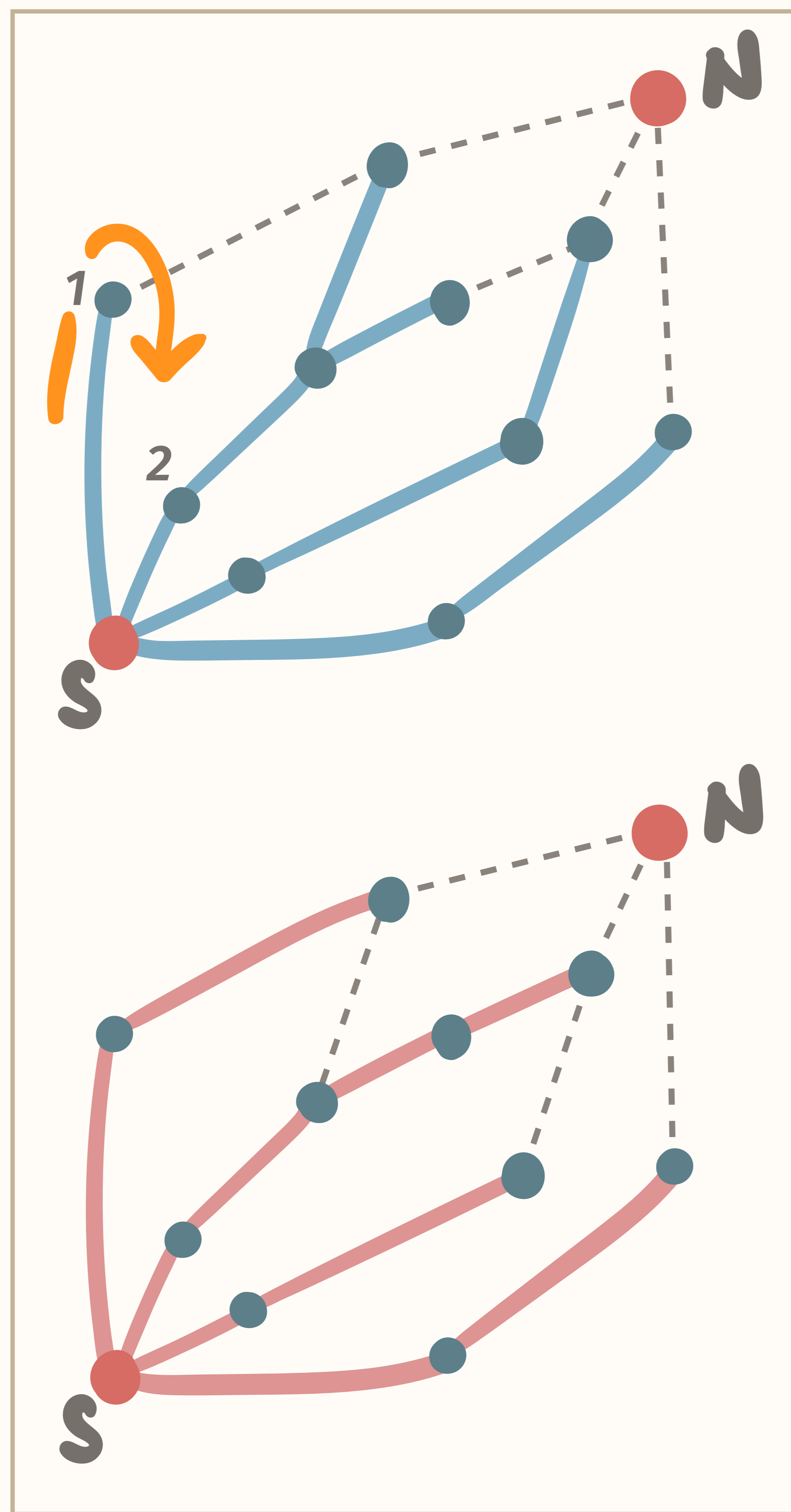
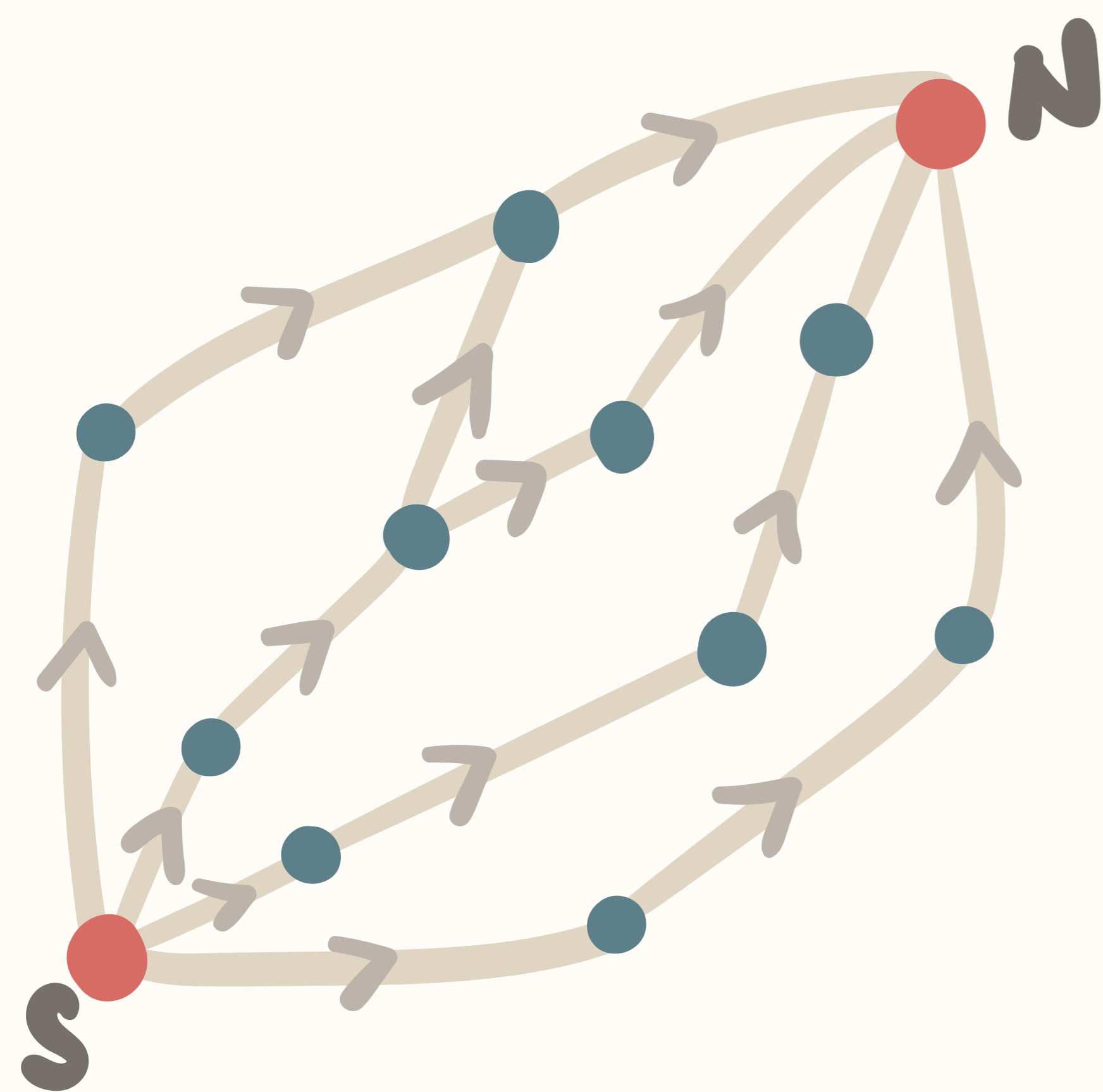
Link with plane permutations

Poset \longrightarrow Plane permutation



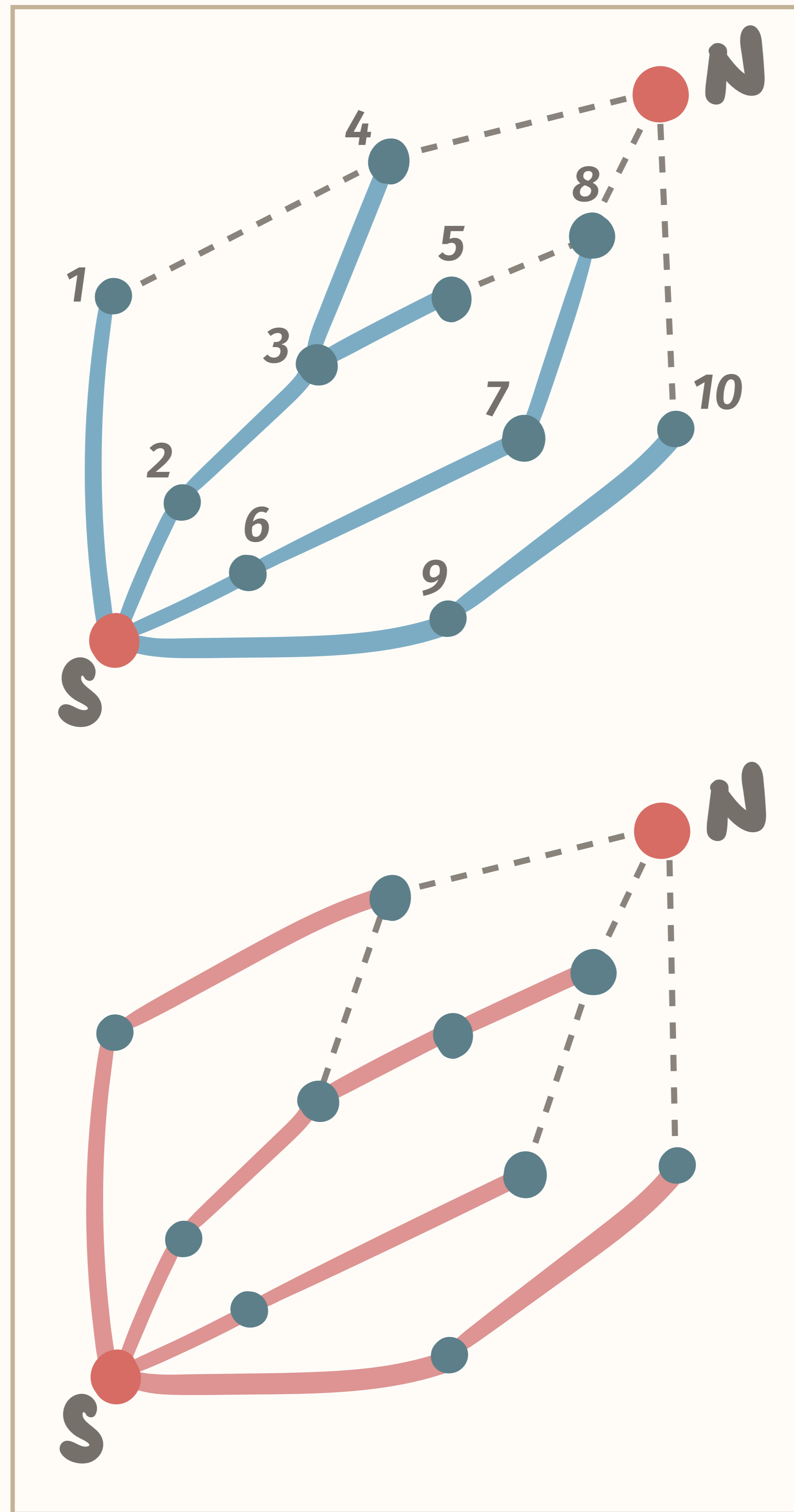
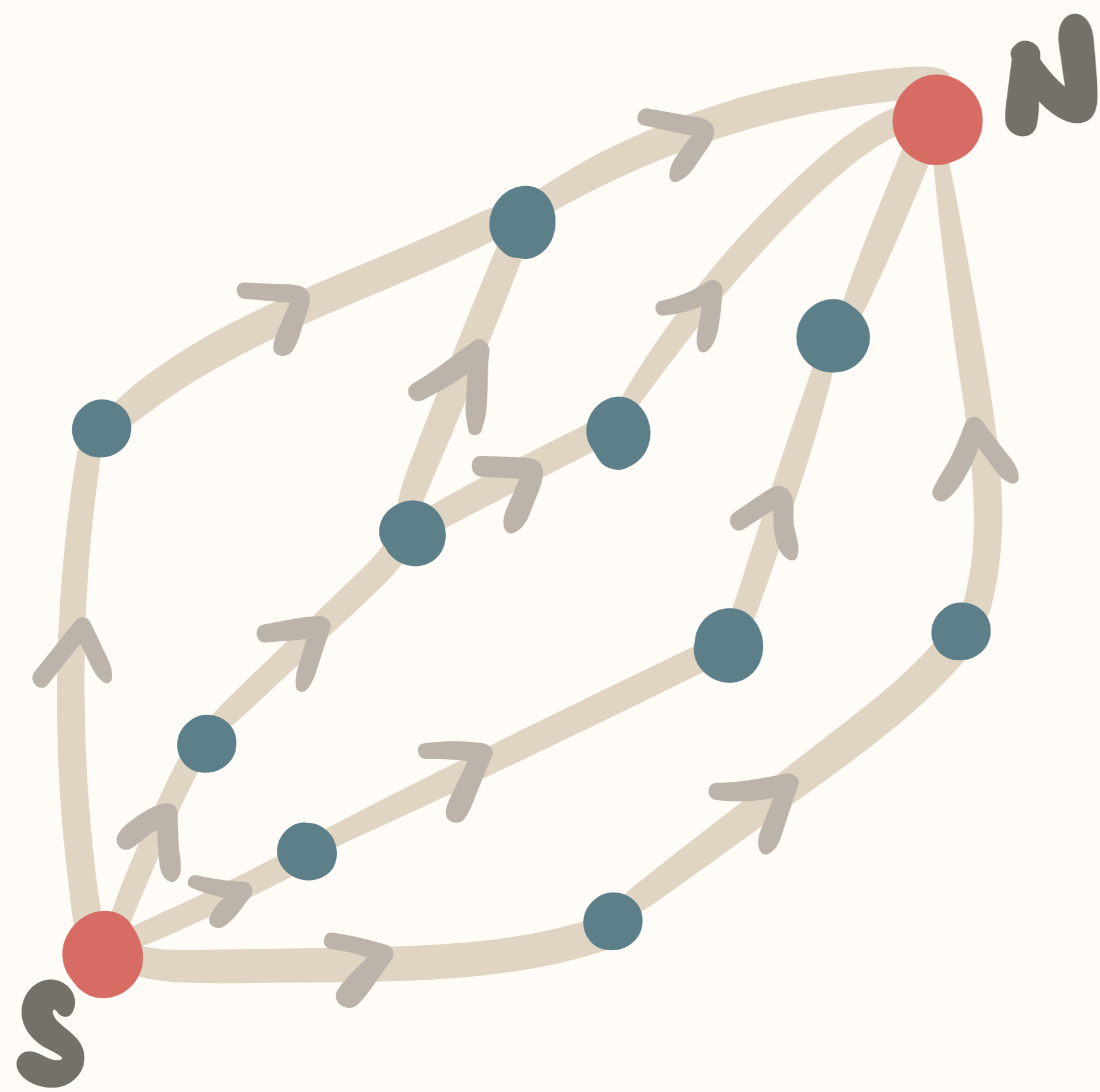
Link with plane permutations

Poset \longrightarrow Plane permutation



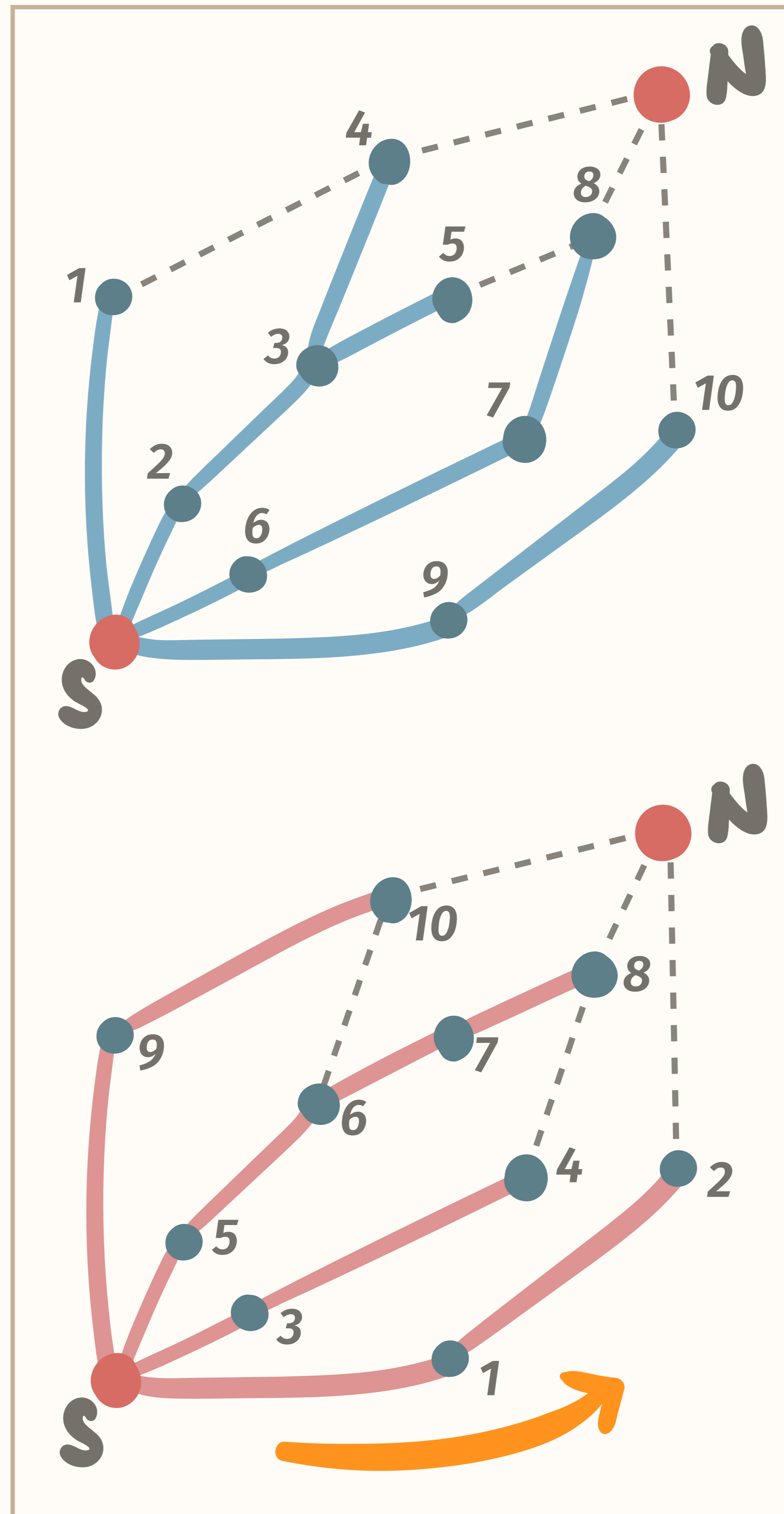
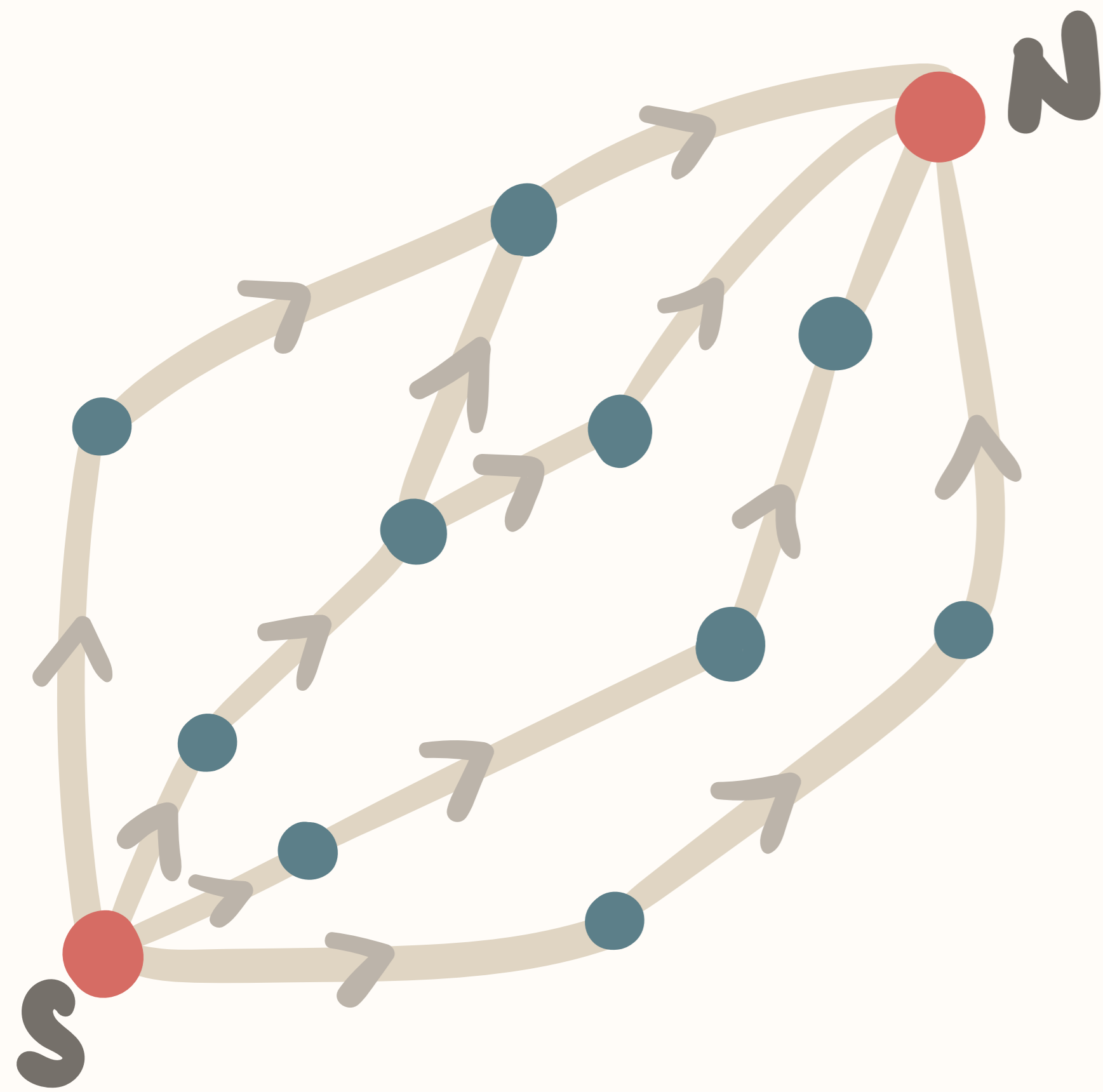
Link with plane permutations

Poset \longrightarrow Plane permutation



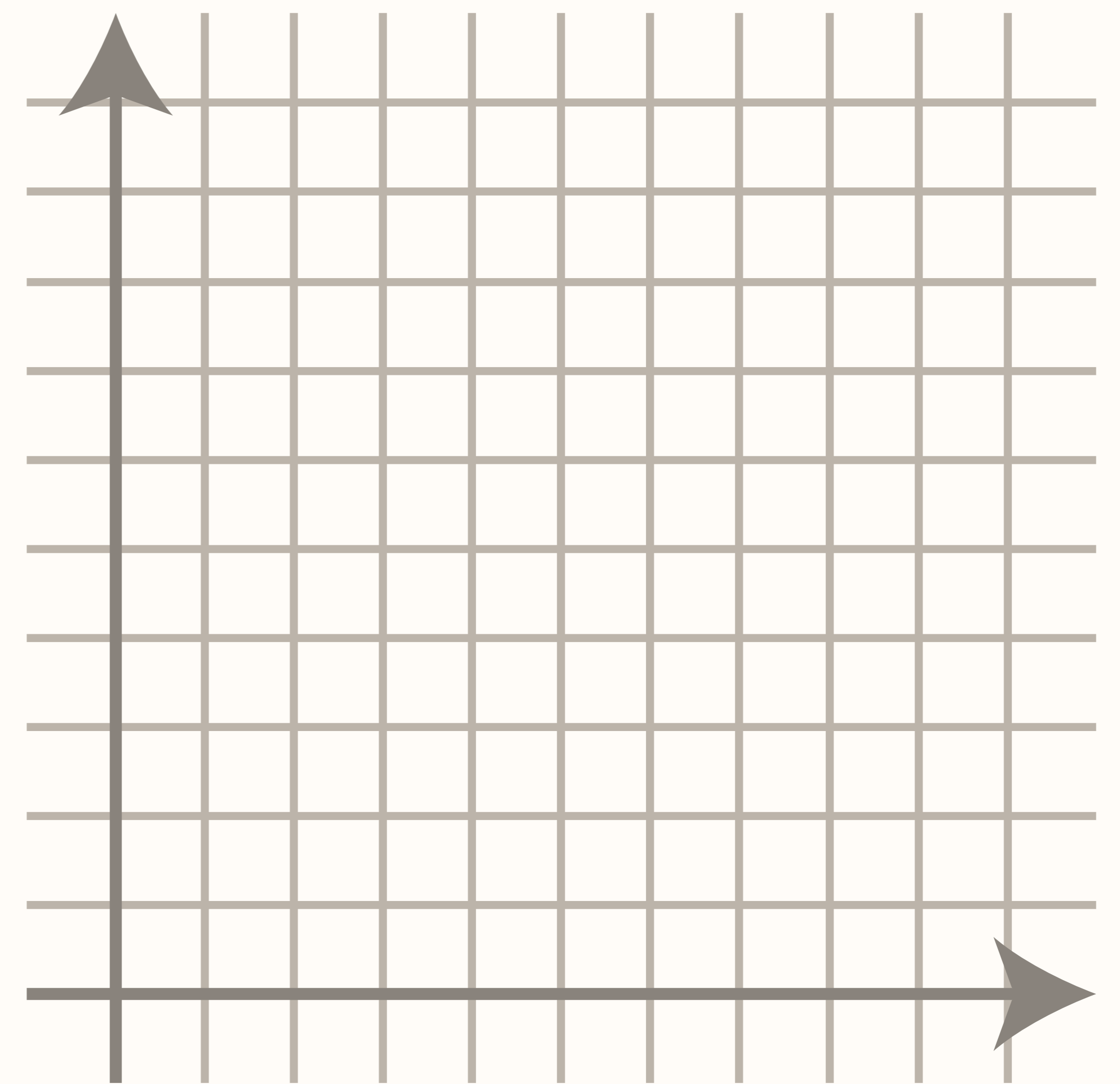
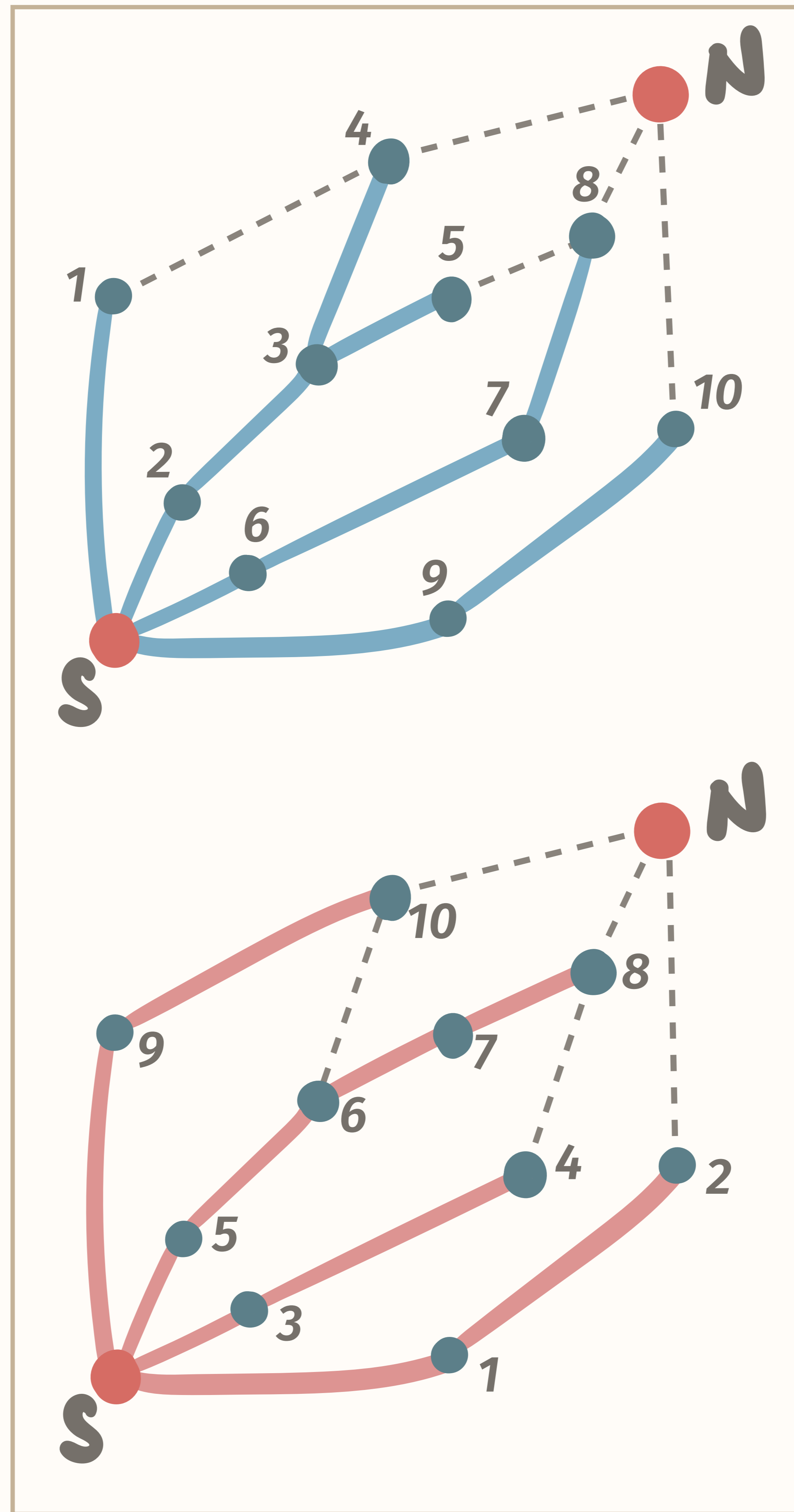
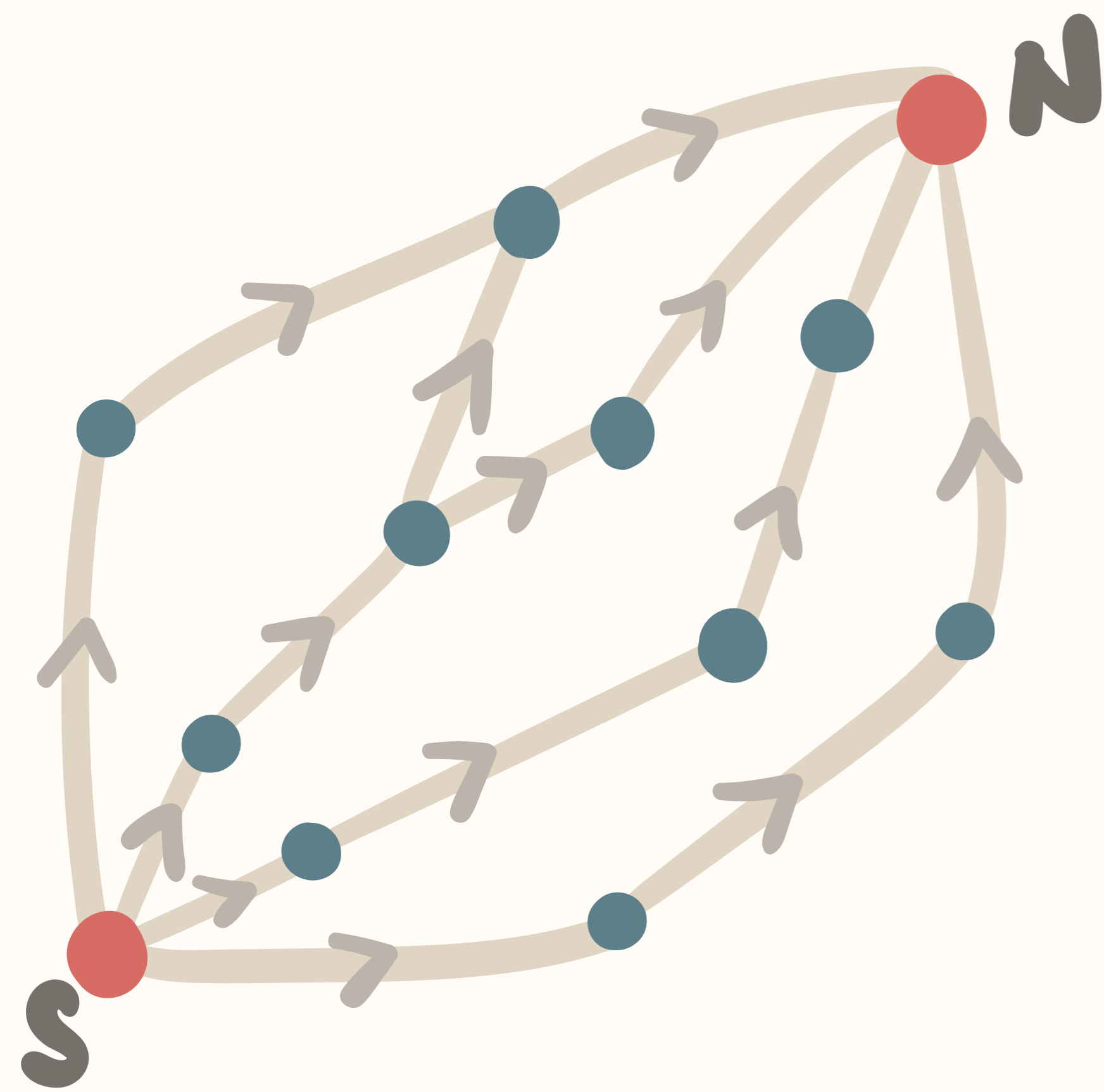
Link with plane permutations

Poset \longrightarrow Plane permutation



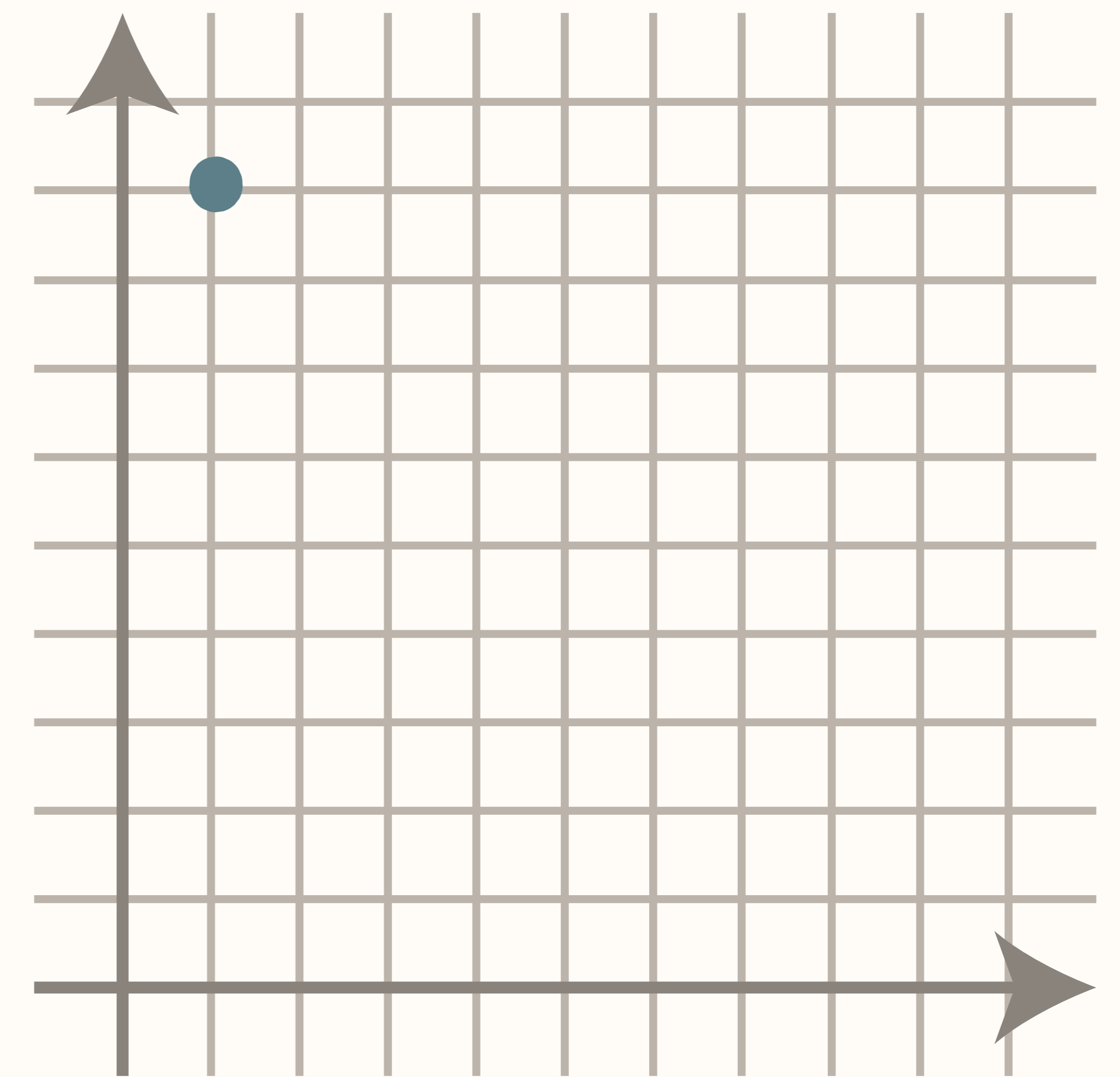
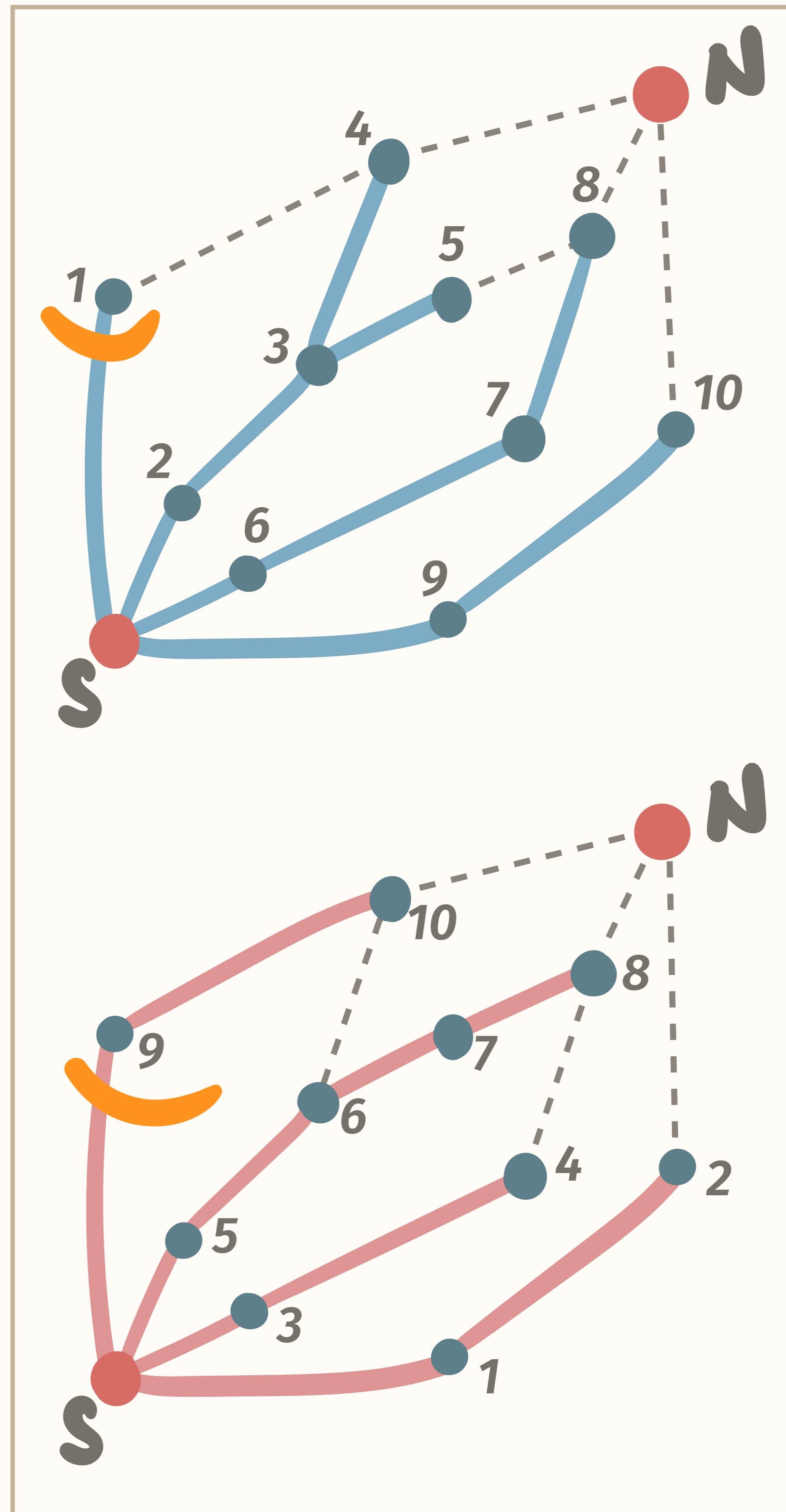
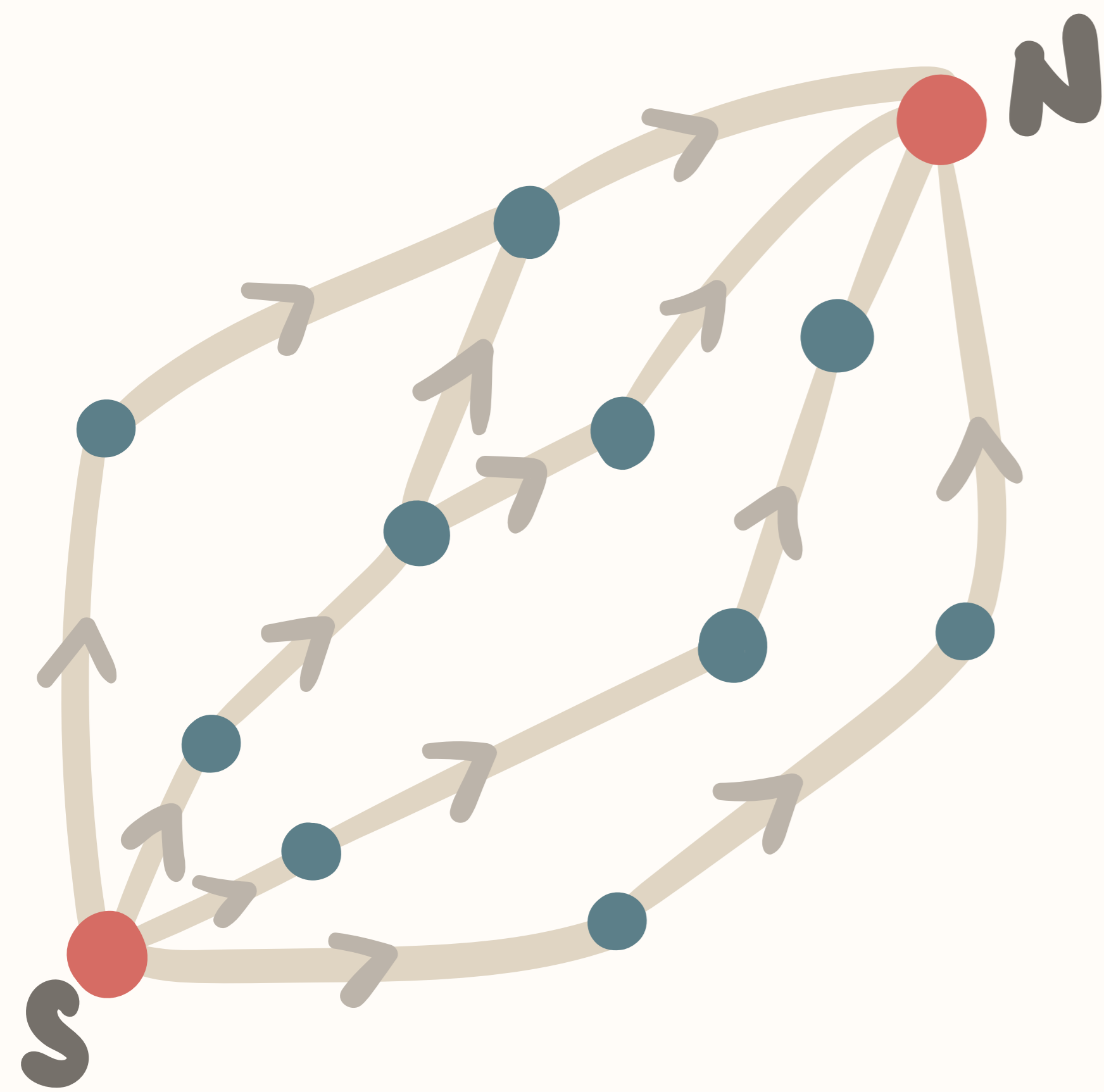
Link with plane permutations

Poset \longrightarrow Plane permutation



Link with plane permutations

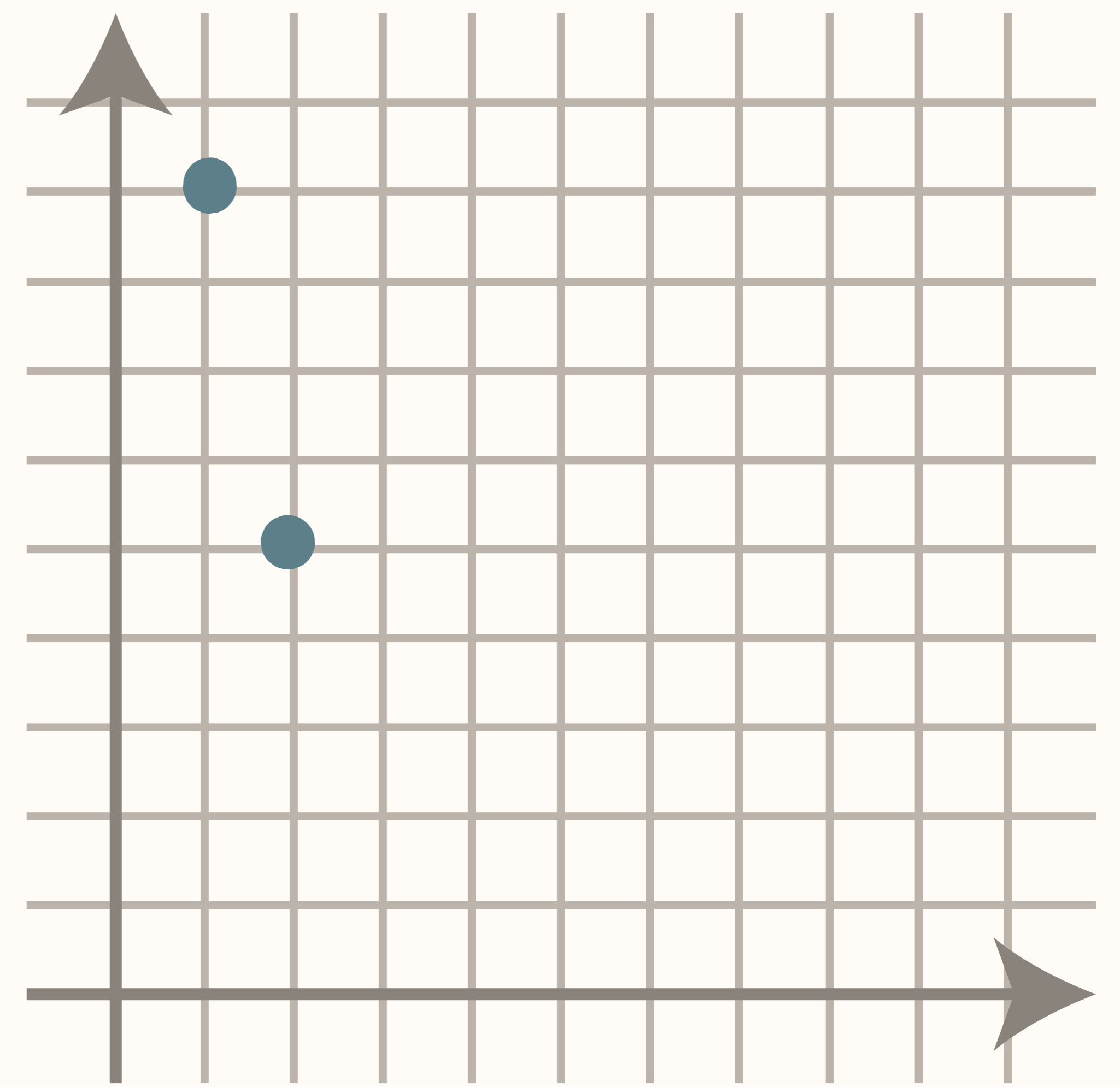
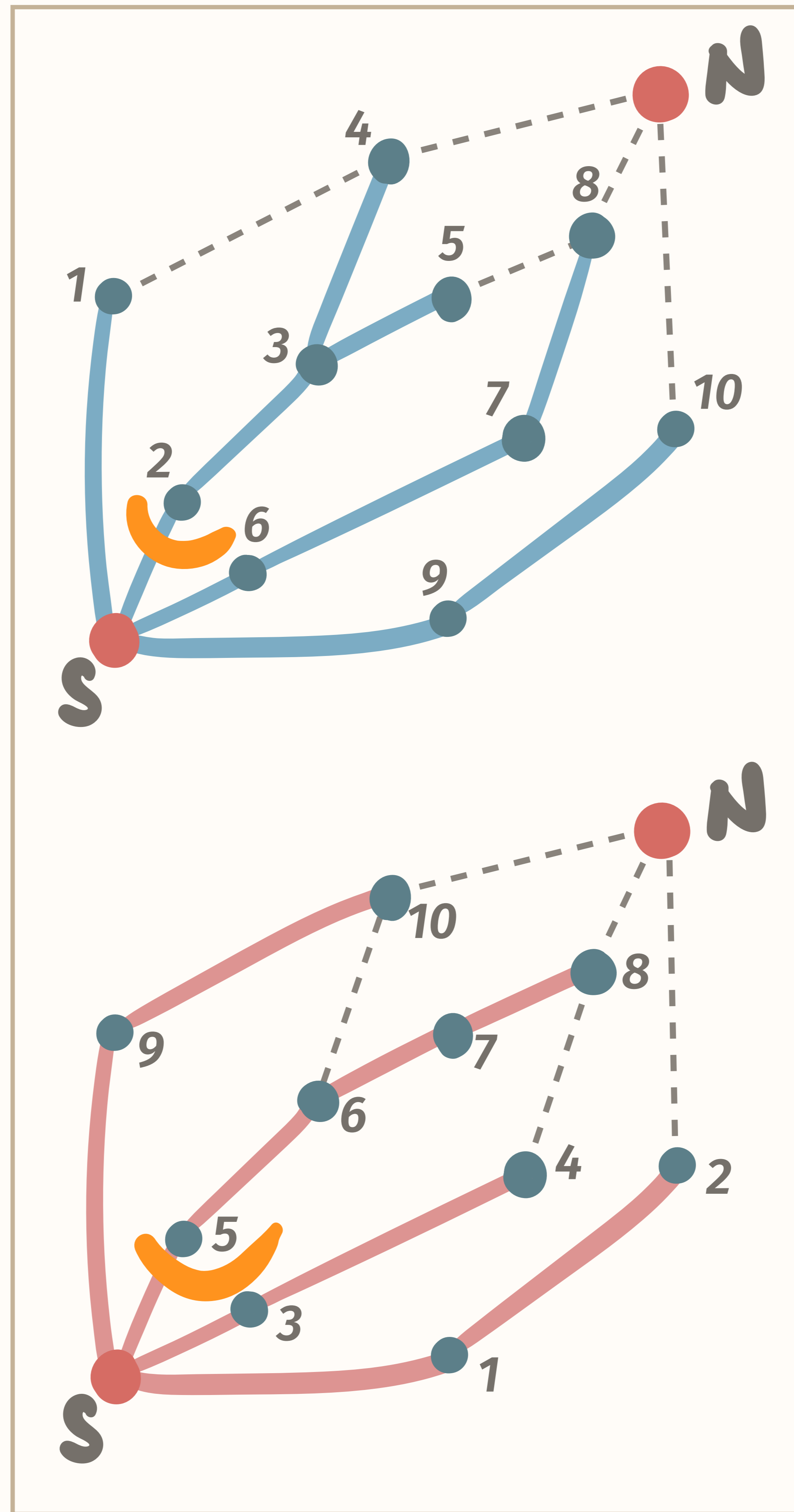
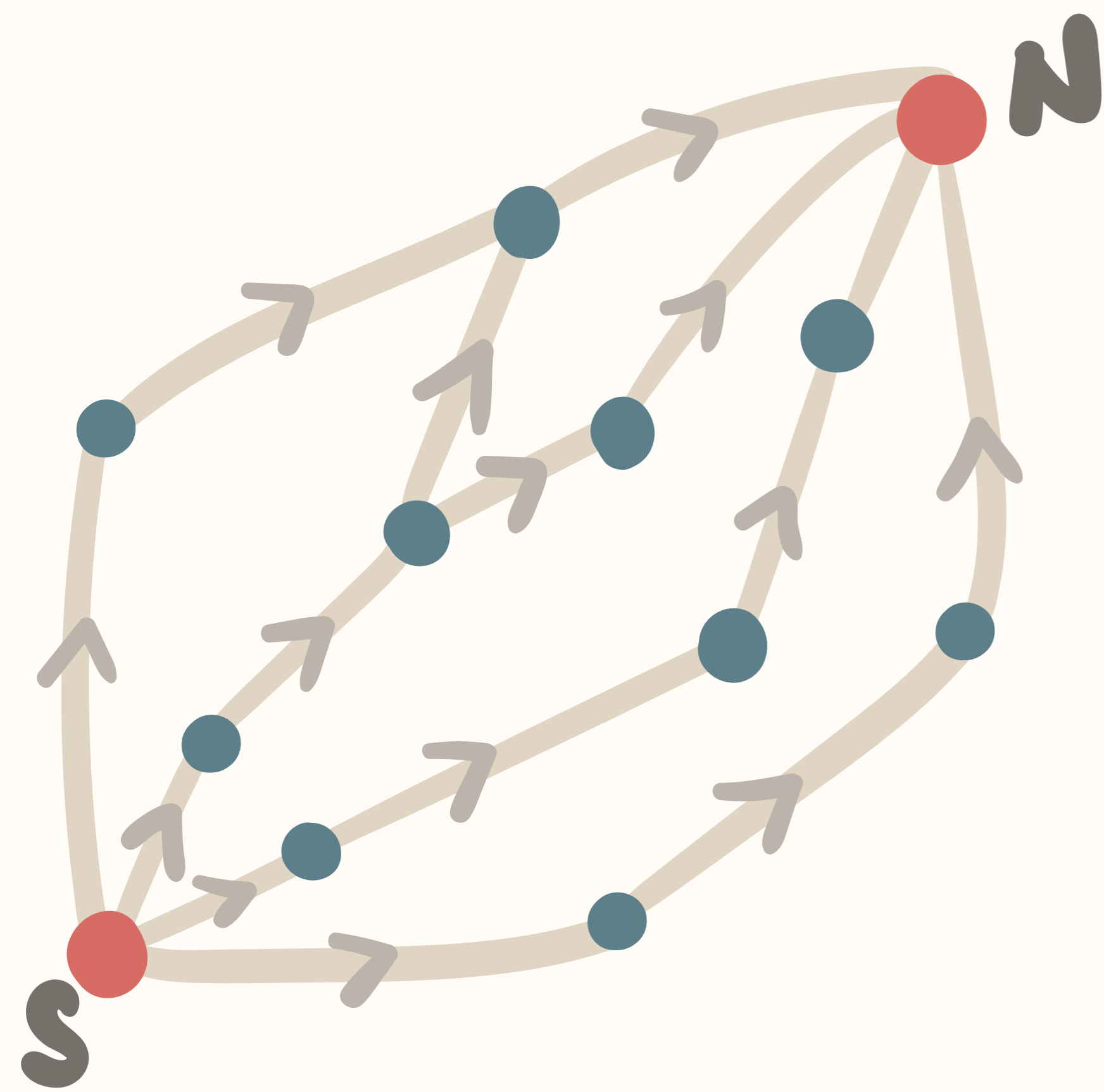
Poset \longrightarrow Plane permutation



$$\pi : 1 \rightarrow 9$$

Link with plane permutations

Poset \longrightarrow Plane permutation

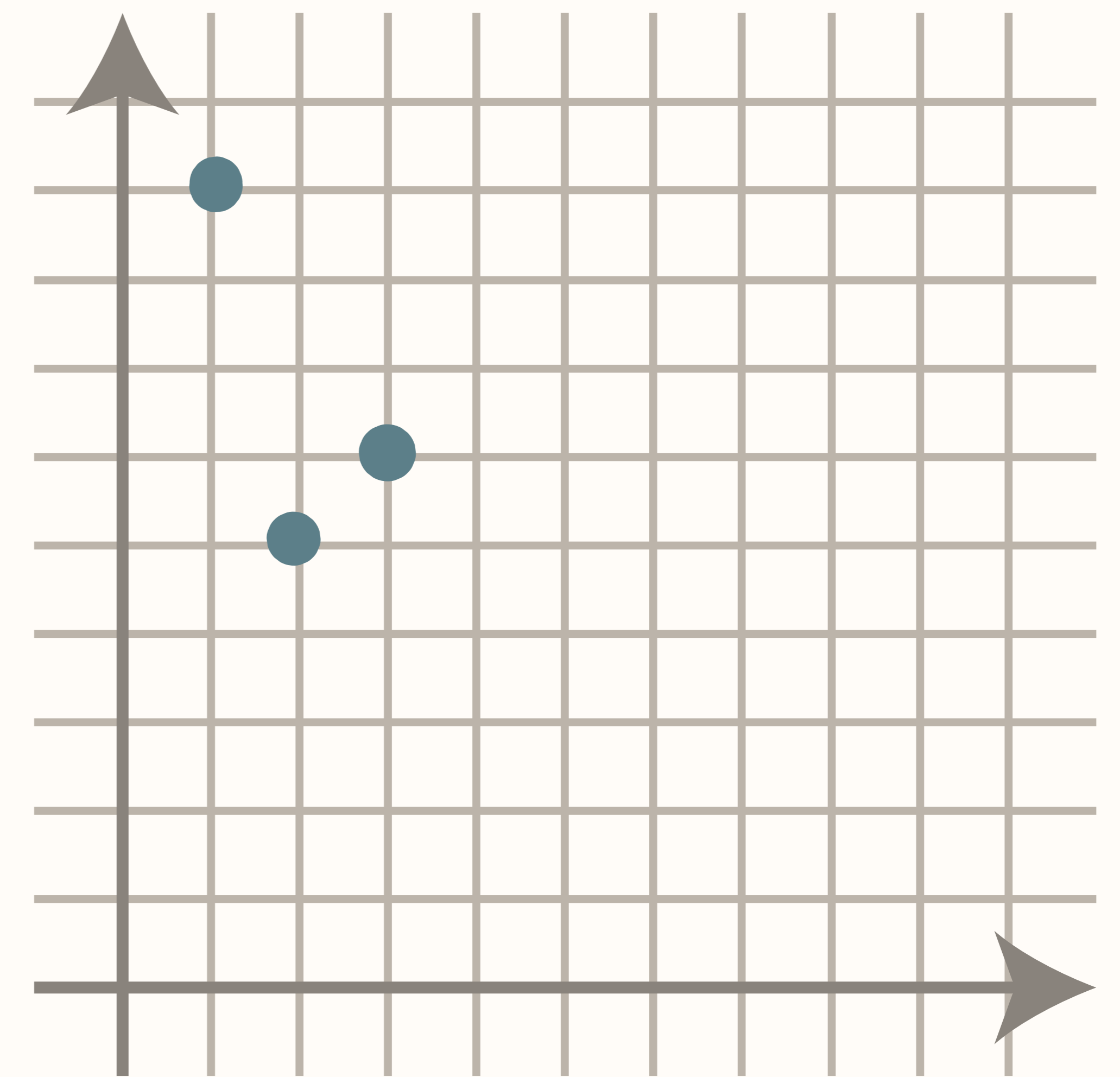
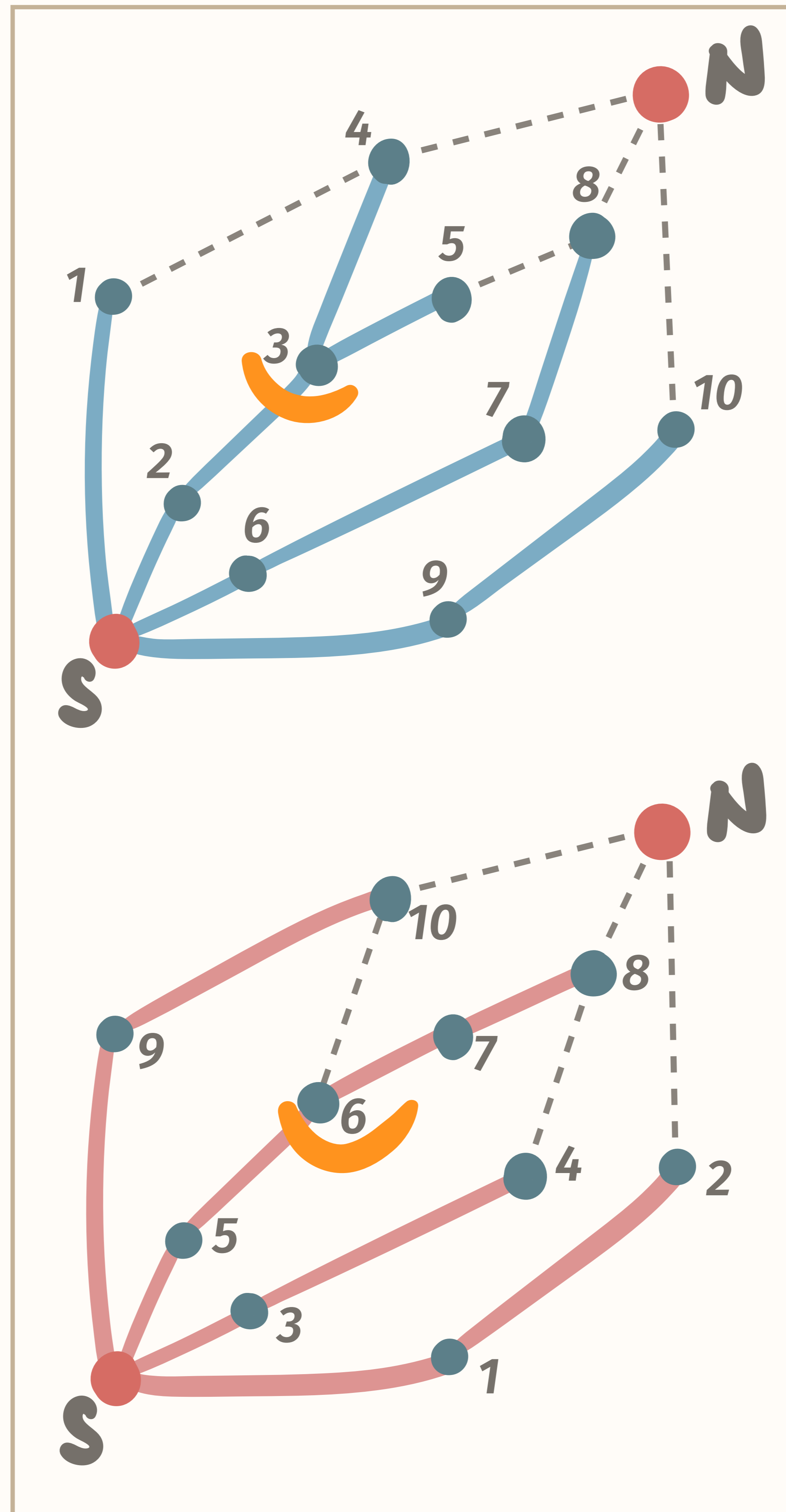
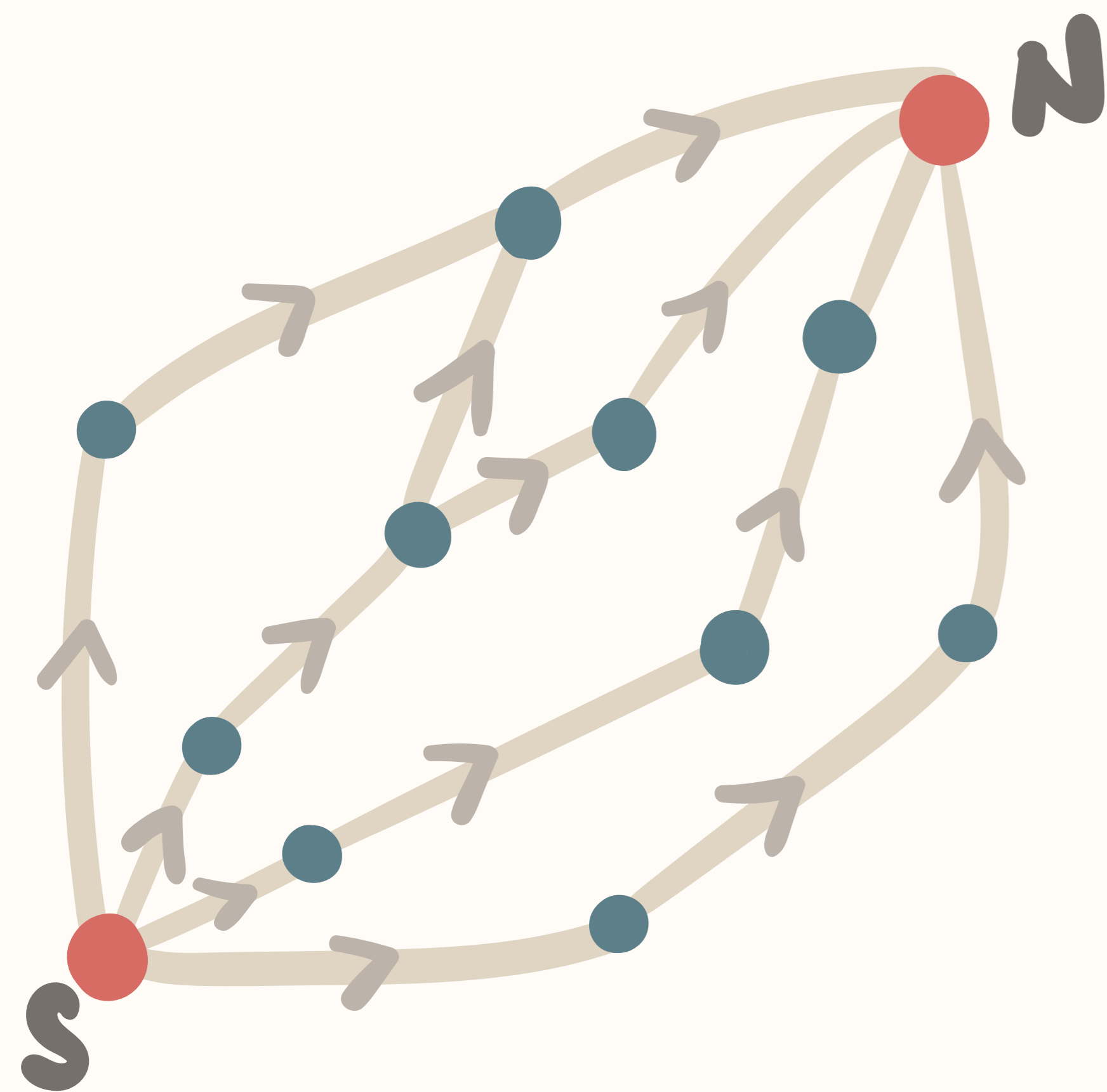


$$\pi: 1 \rightarrow 9$$

$$2 \rightarrow 5$$

Link with plane permutations

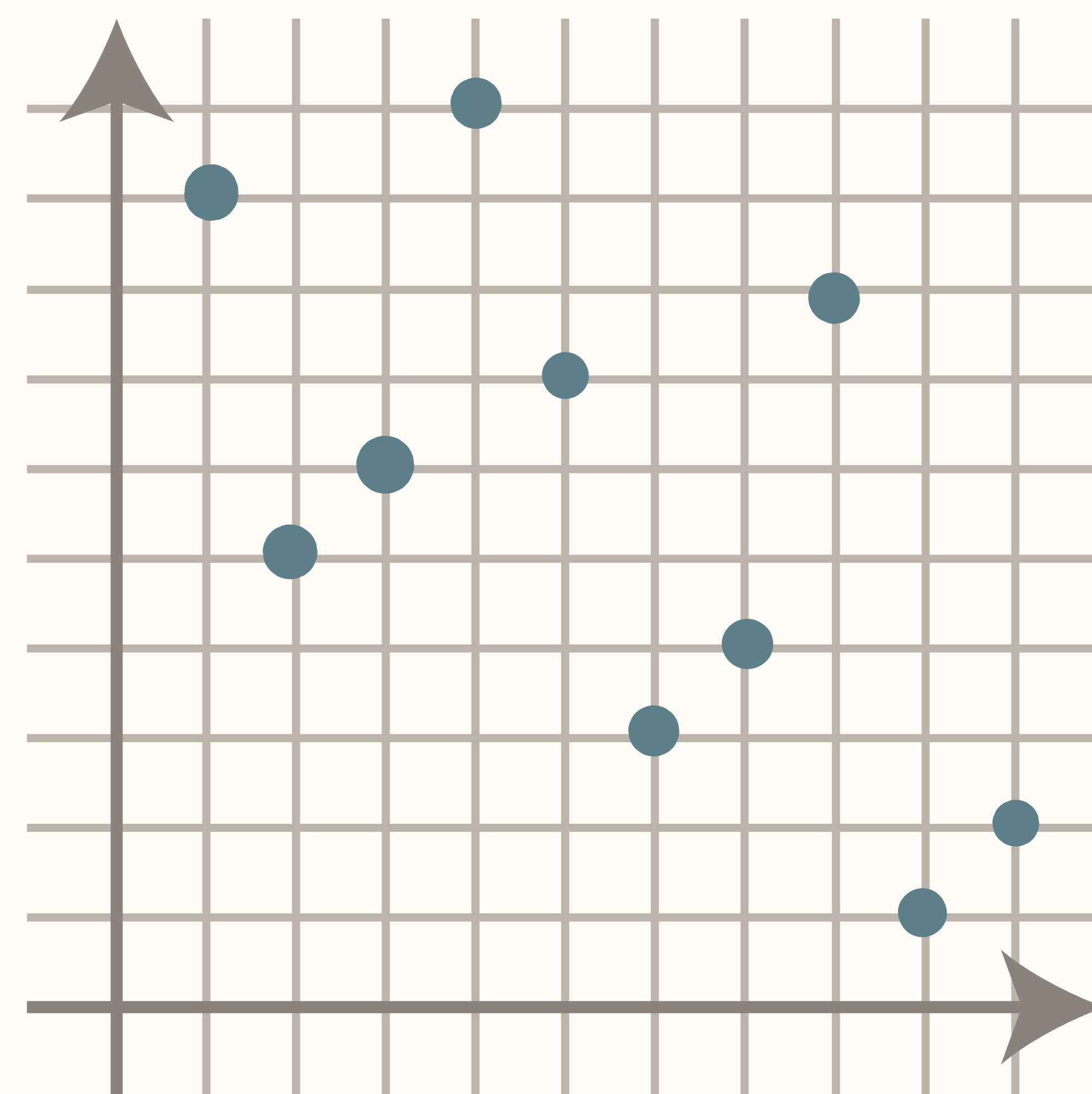
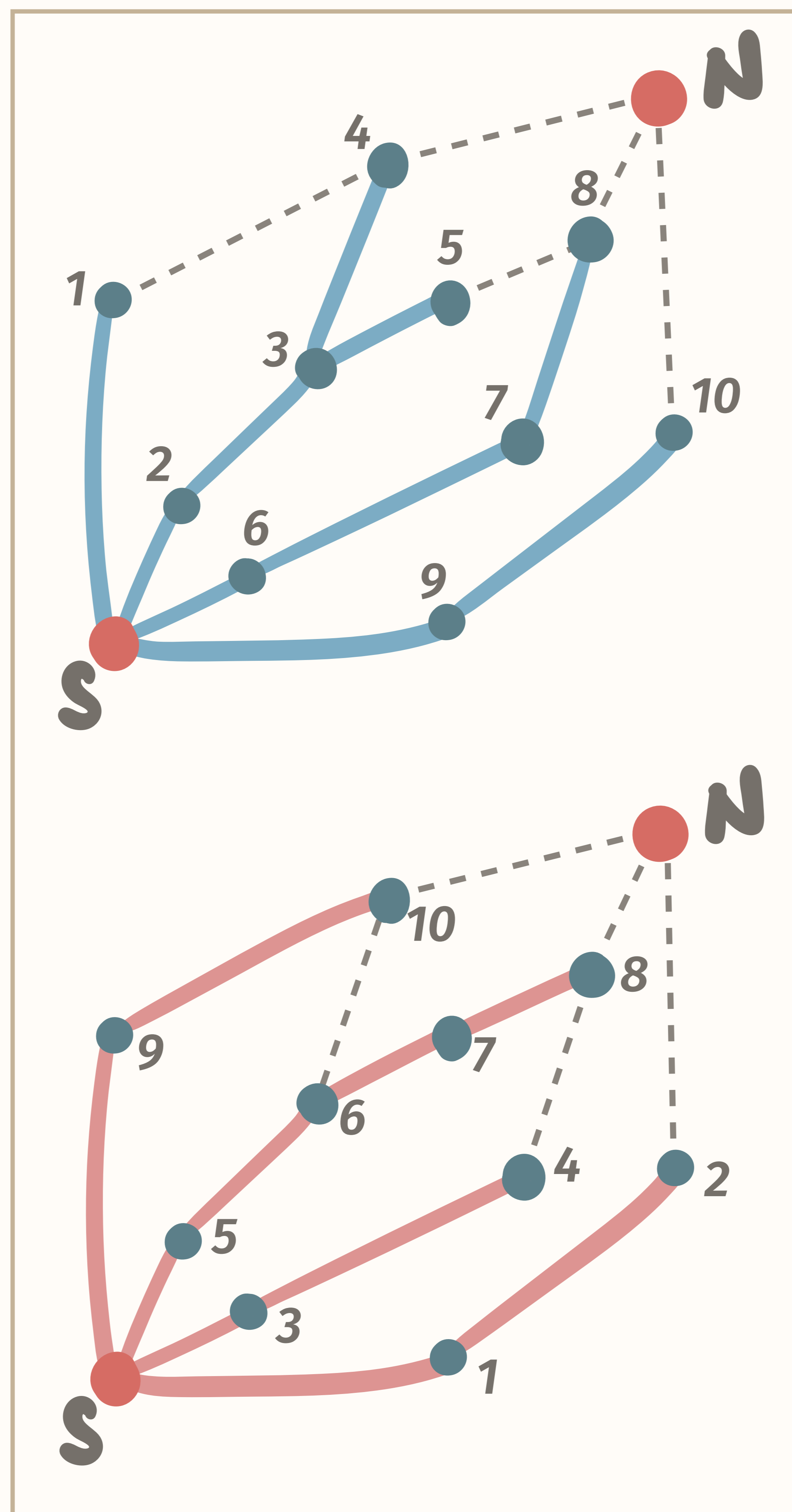
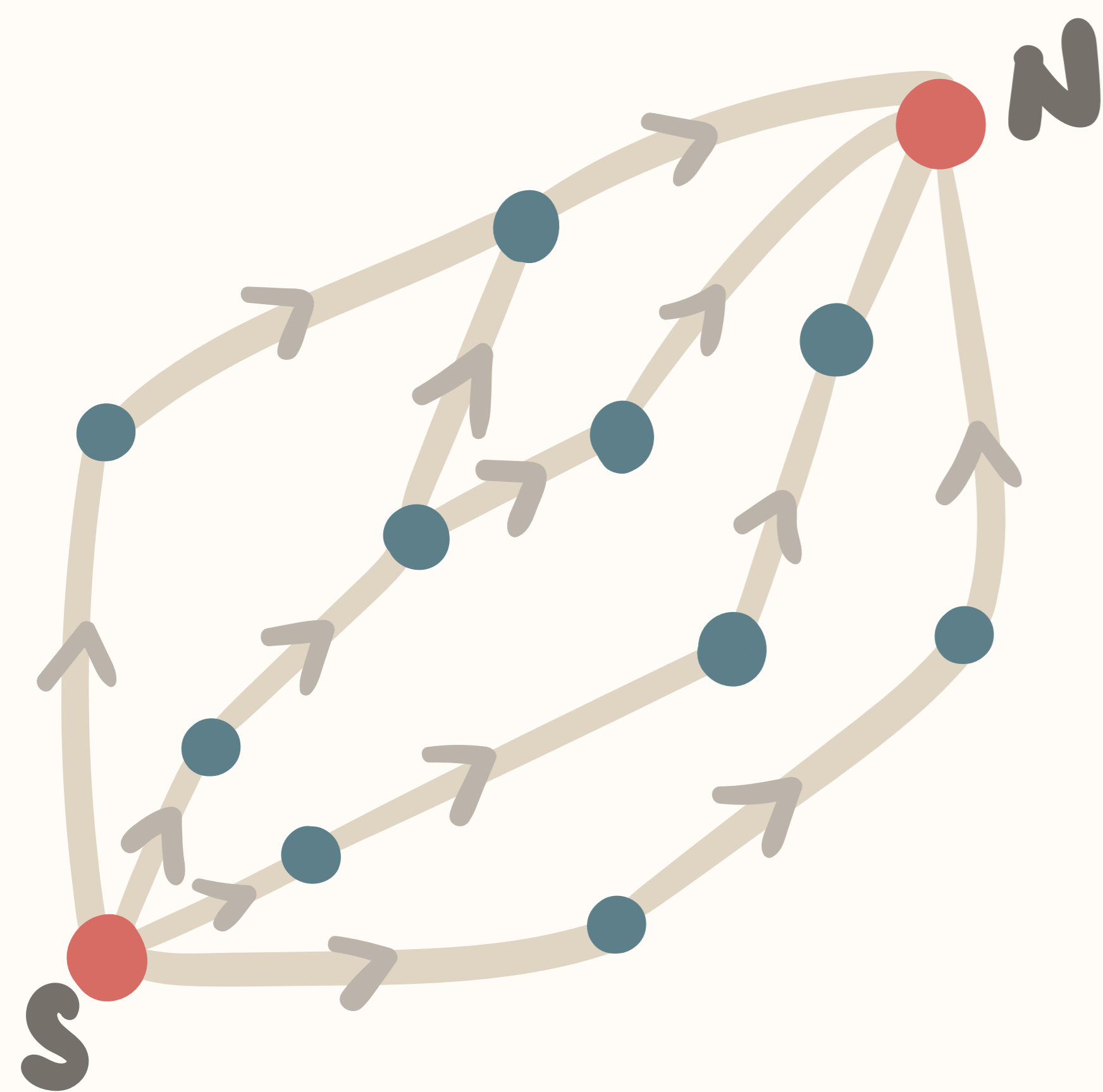
Poset \longrightarrow Plane permutation



$$\pi: \begin{array}{l} 1 \rightarrow 9 \\ 2 \rightarrow 5 \\ 3 \rightarrow 6 \end{array}$$

Link with plane permutations

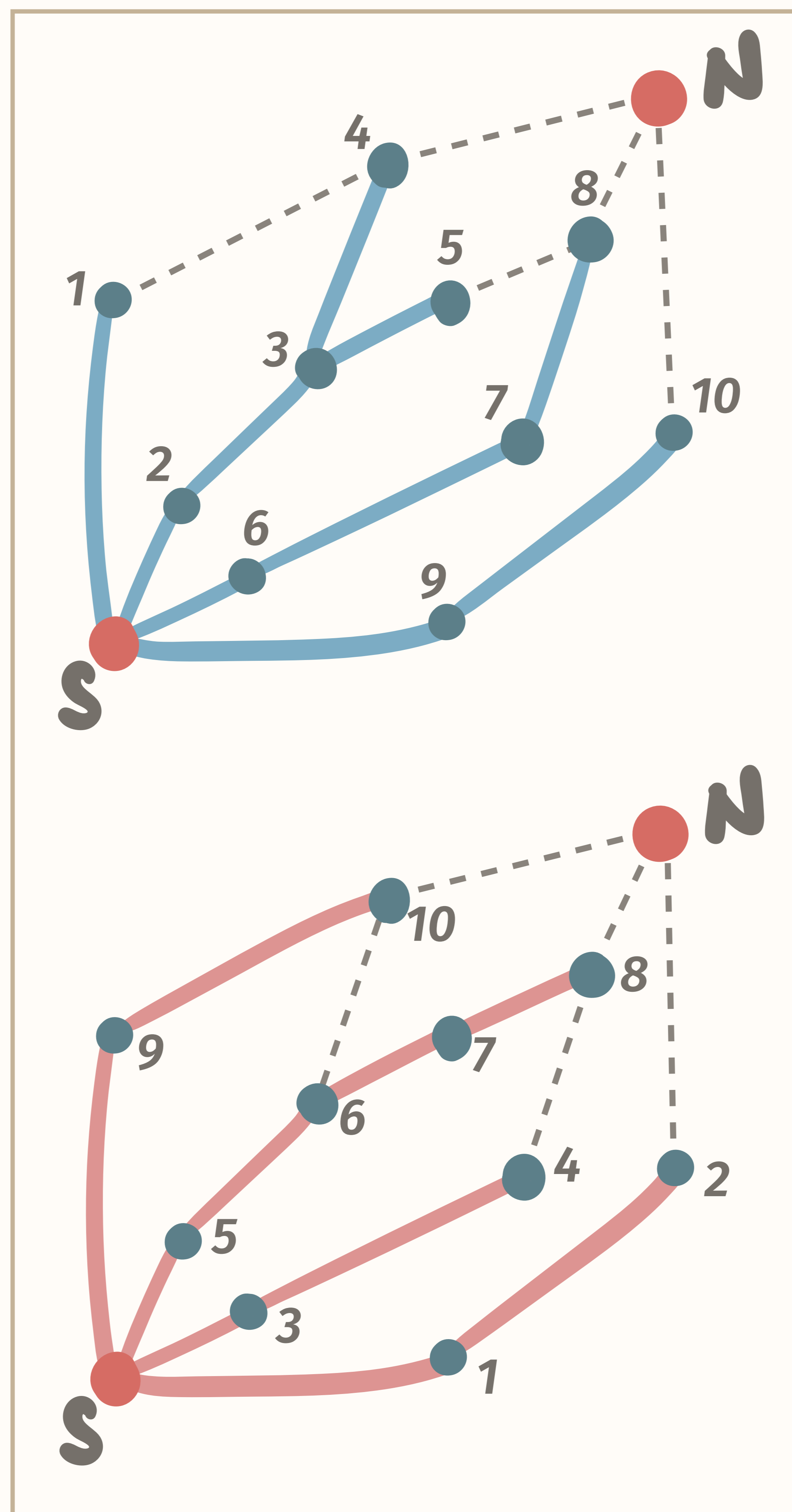
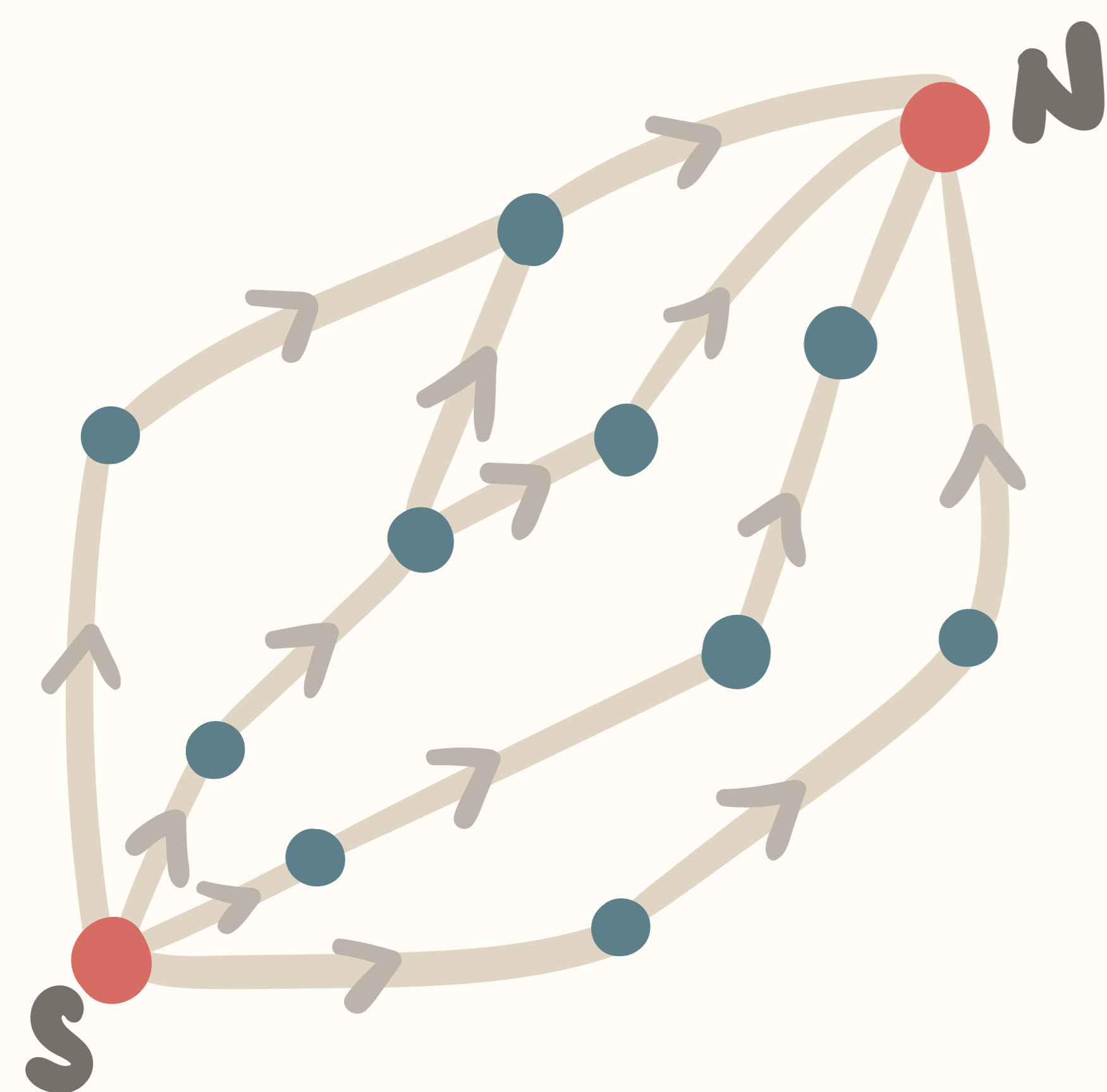
Poset \longrightarrow Plane permutation



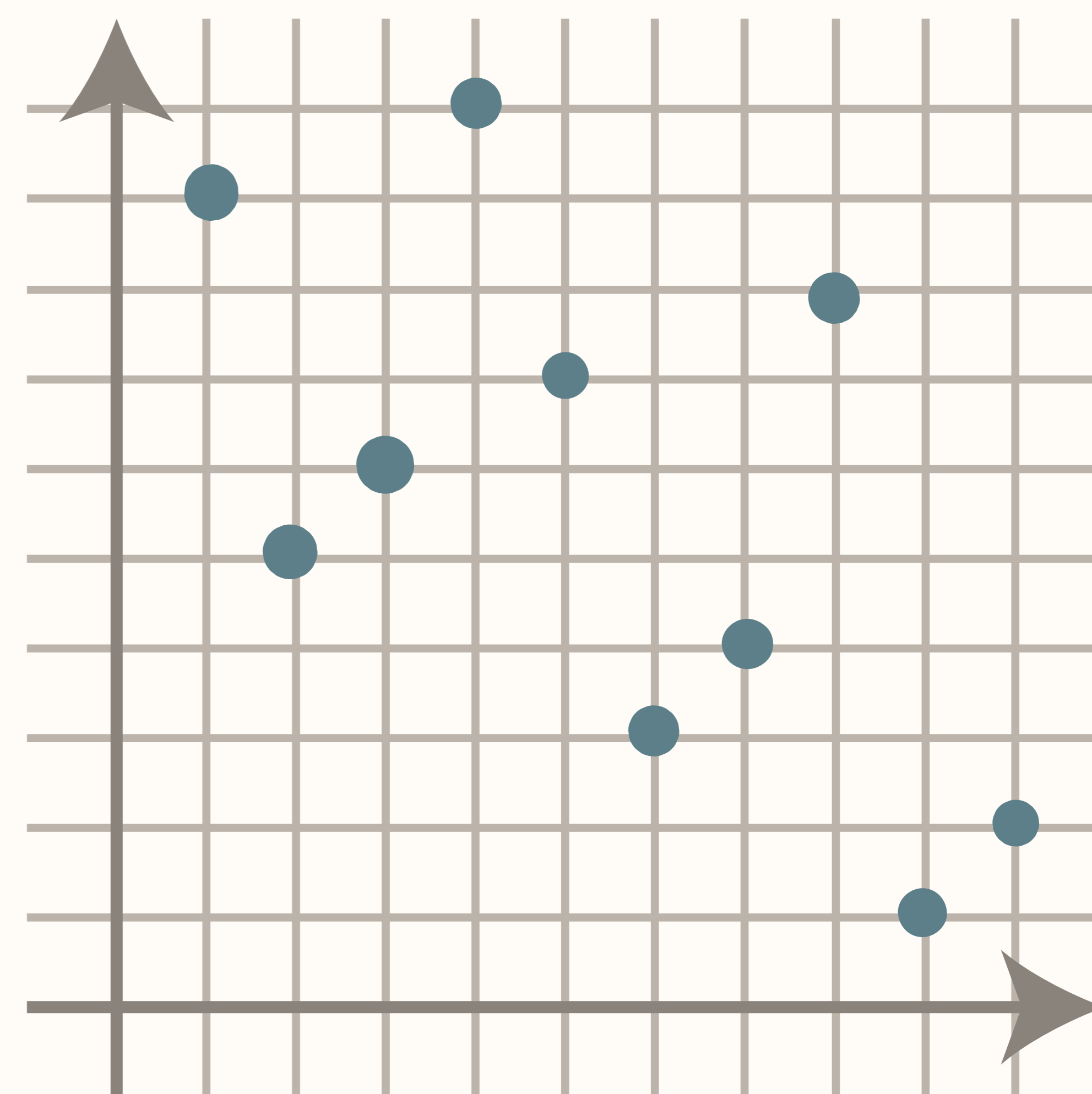
$$\pi: \begin{array}{ll} 1 \rightarrow 9 & 6 \rightarrow 3 \\ 2 \rightarrow 5 & 7 \rightarrow 4 \\ 3 \rightarrow 6 & 8 \rightarrow 8 \\ 4 \rightarrow 10 & 9 \rightarrow 1 \\ 5 \rightarrow 7 & 10 \rightarrow 2 \end{array}$$

Link with plane permutations

Poset \longrightarrow Plane permutation



\Rightarrow Area requirement and symmetry display of planar upward drawings, G. Di Battista, R. Tamassia, and I. G. Tollis (1992)



$$\pi: \begin{array}{ll} 1 \rightarrow 9 & 6 \rightarrow 3 \\ 2 \rightarrow 5 & 7 \rightarrow 4 \\ 3 \rightarrow 6 & 8 \rightarrow 8 \\ 4 \rightarrow 10 & 9 \rightarrow 1 \\ 5 \rightarrow 7 & 10 \rightarrow 2 \end{array}$$