Power domination in triangulations

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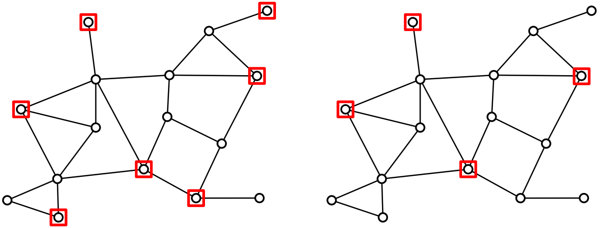
Domination

- Some vertices in a set S of sensors
- $N[S] = M$ = monitored vertices ($\overline{M} = V(G) \setminus M$)

The **domination number** $\gamma(G)$ of G is the minimum size of S s.t. $M = V(G)$.



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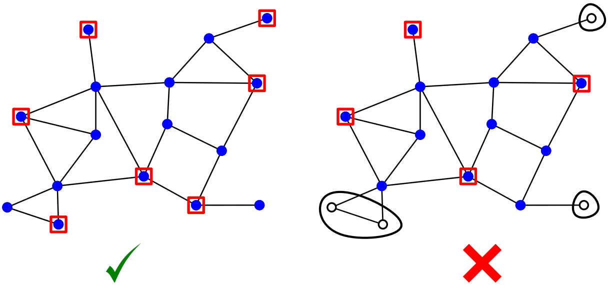

Some results on domination

Dominating set

Input: $G, k \geq 0$.
Question: Is there a dominating set $S \subseteq V(G)$ with $|S| \leq k$?

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For sufficiently large order n, every triangulation G has $\gamma(G) \leq \varepsilon.n$, with $\frac{n}{4} \leq \varepsilon \leq \frac{n}{3}$. (Conjecture: $\varepsilon = \frac{n}{4}$)

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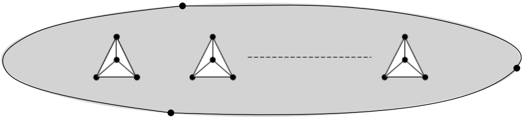
Lower bound: each facial K_4 needs a vertex in S



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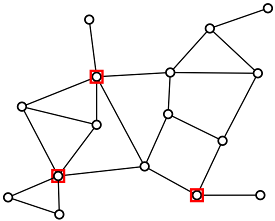

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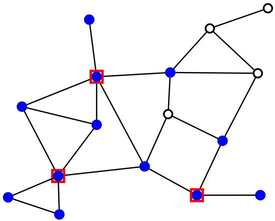

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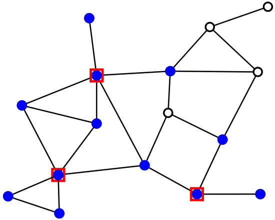

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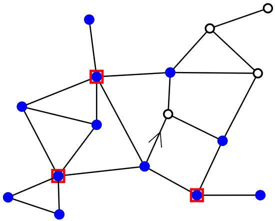

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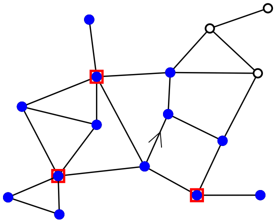

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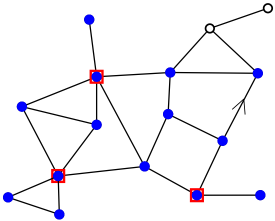

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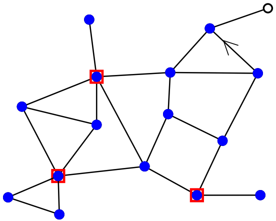

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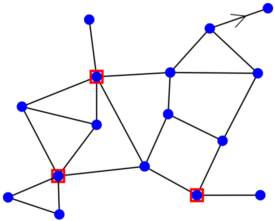

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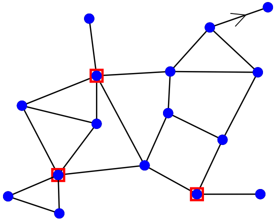

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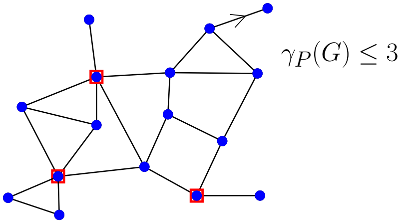

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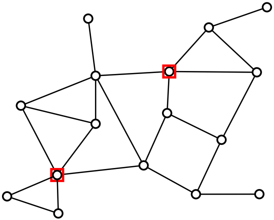

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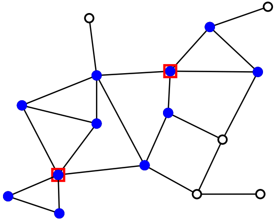

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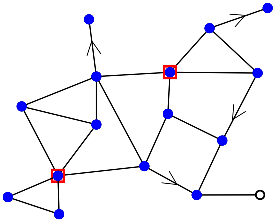

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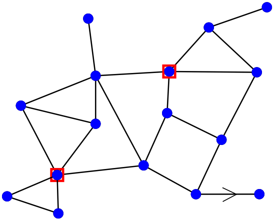

$\gamma_P(G) \leq 3$
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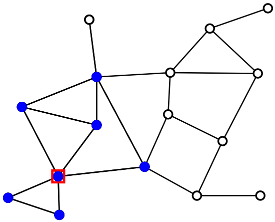

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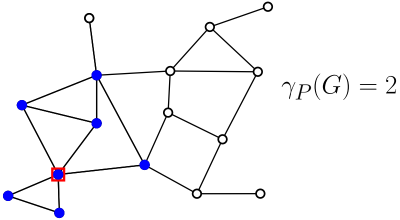

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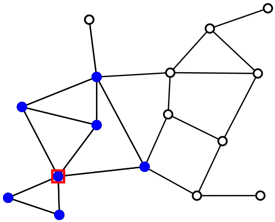

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Background story: Placing a minimal number of "sensors" to monitor an electrical system

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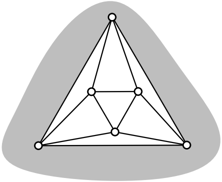

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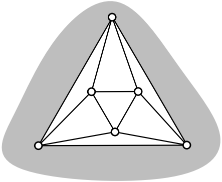

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Other results: hexagonal grids [Ferrero et al. '11], some products of paths [Dorfling and Henning '06, Dorbec et al. '08]...

Main result

G: triangulation (= maximal planar graph) with n vertices.

Theorem

$$\gamma_P(G) \leq \frac{n-2}{4} \text{ if } G \text{ is a triangulation with } n \geq 6 \text{ vertices.}$$

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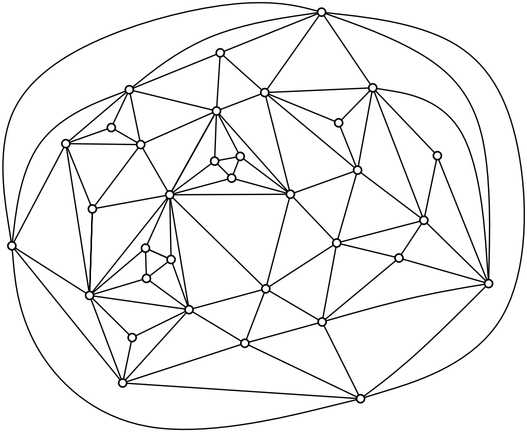
- (Step 3) "Holes" → add some (few) other vertices to S.

Steps 1 and 2

- **Step 1.** Monitor the octahedra with $l \leq n'/6$ sensors



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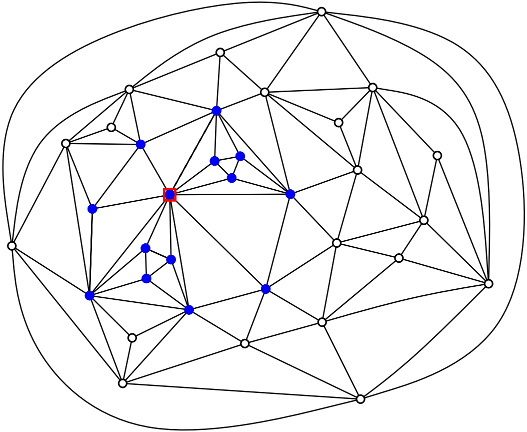

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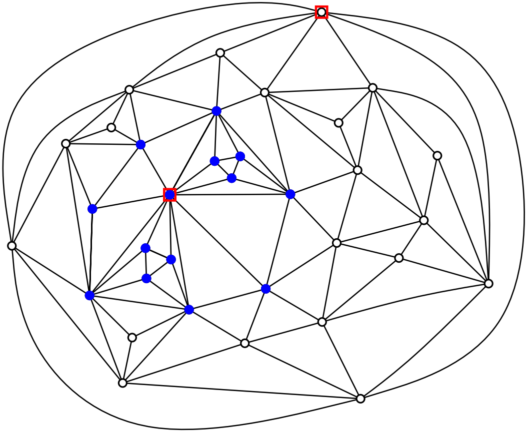

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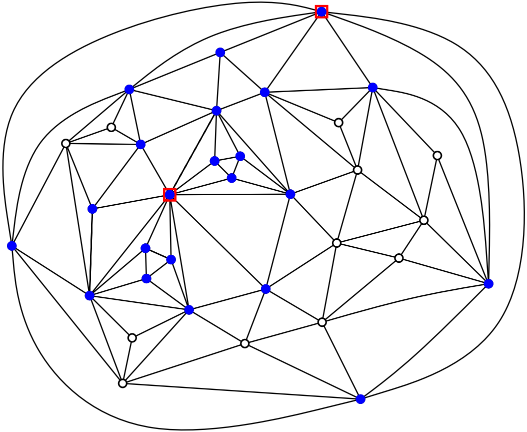

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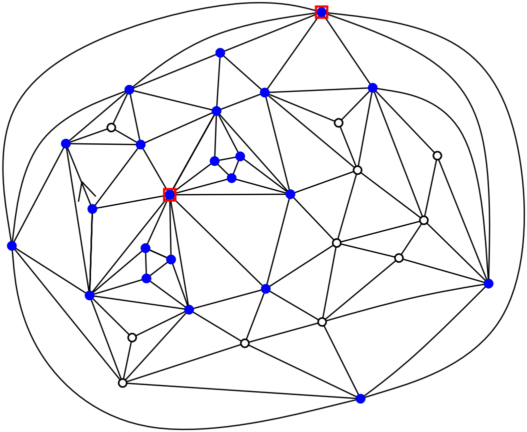

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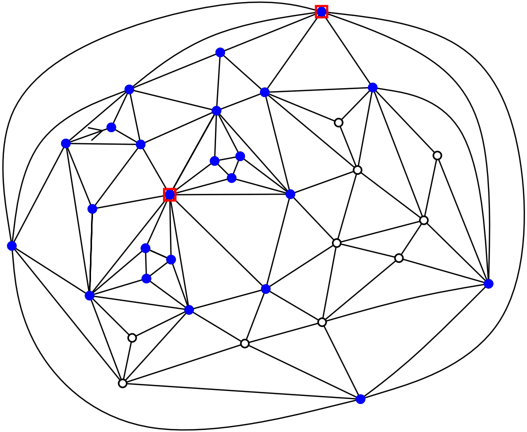

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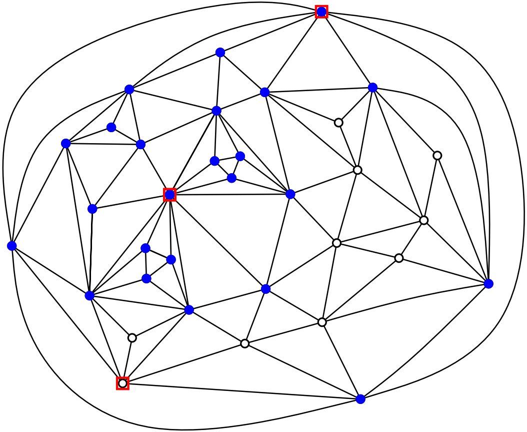

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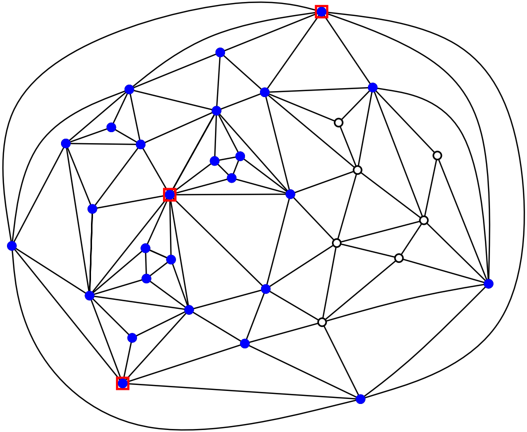

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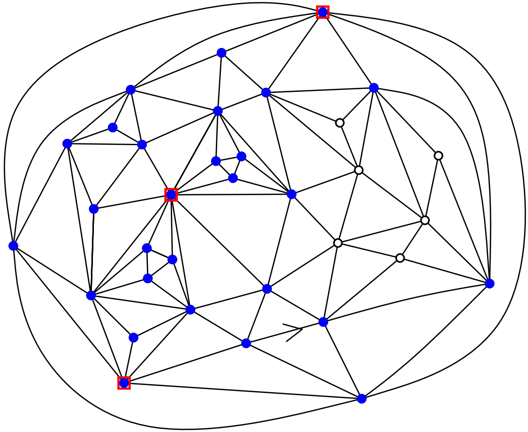

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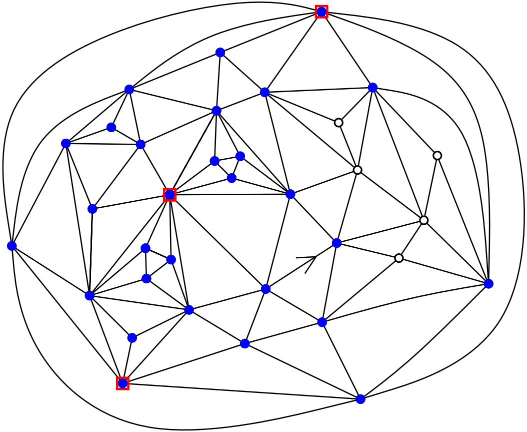

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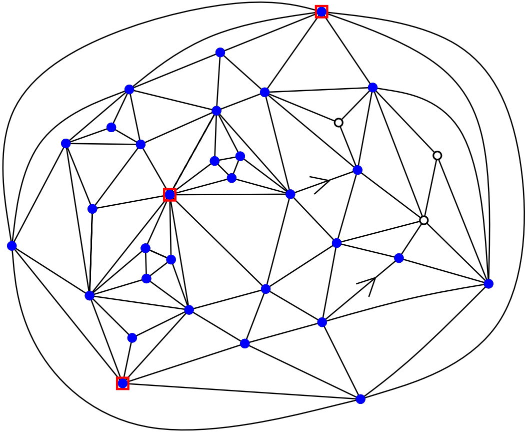

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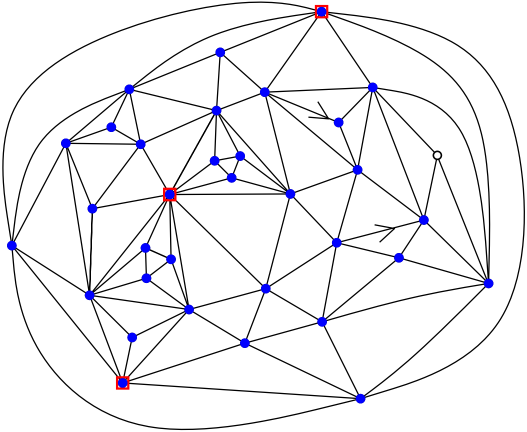

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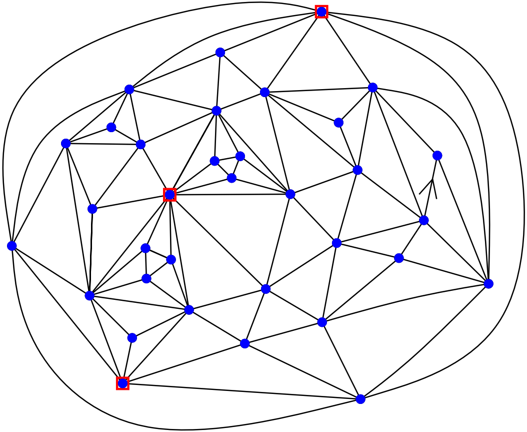

After Step 2

(a) $G[\overline{M}]$ has maximum degree 2 (it is a cycle or a path).

(b) Each connected component of $G[\overline{M}]$ has at most three vertices.

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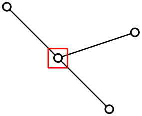

(b) Each connected component of $G[\overline{M}]$ has at most three vertices.

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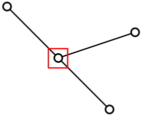
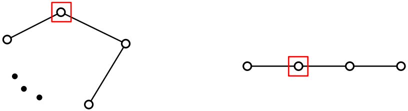


(b) Each connected component of $G[\overline{M}]$ has at most three vertices.



After Step 2

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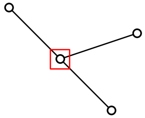
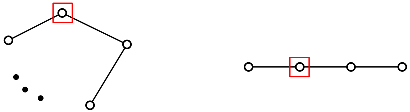
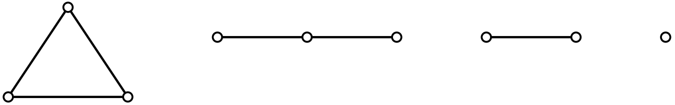


(b) Each connected component of $G[\overline{M}]$ has at most three vertices.

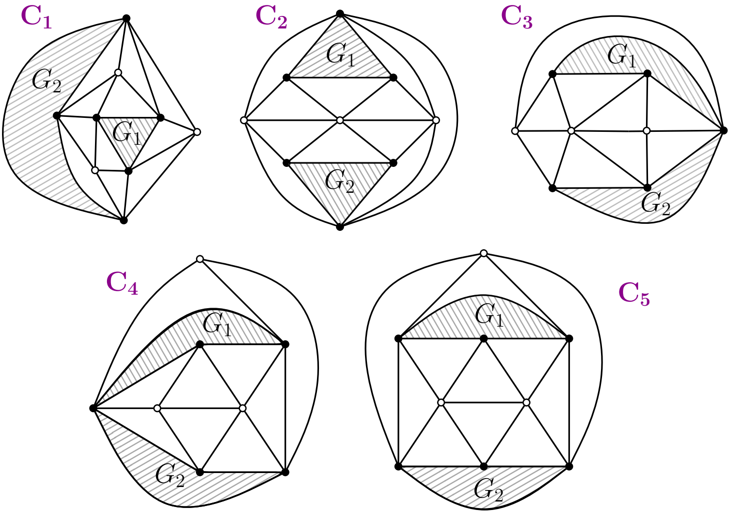


→ A connected component of $G[\overline{M}]$ is isomorphic to K_3, P_3, P_2 or K_1.



After Step 2

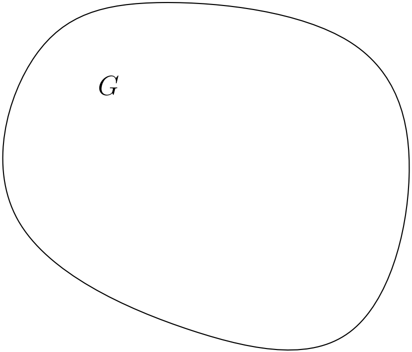
After Step 2, each non-monitored induced triangulation G' of G is isomorphic to one configuration of $\{C_1, \cdots, C_5\}$:



Step 3

Tree-like decomposition of G via the configurations
(leaves: there are no non-monitored vertices)

(in G)



Step 3

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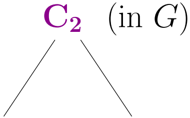

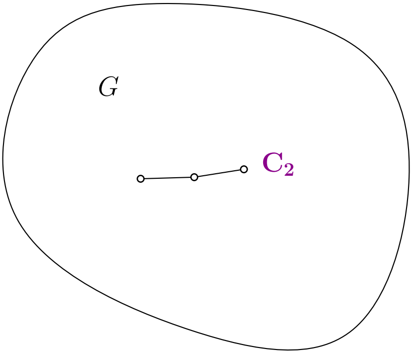

Step 3

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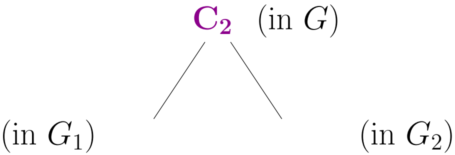

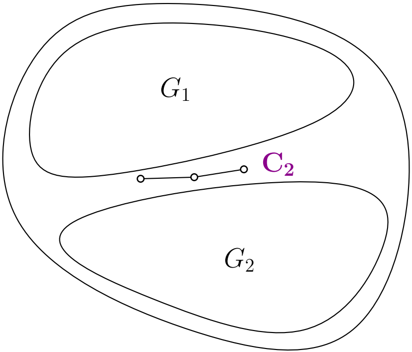

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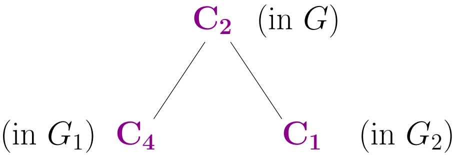

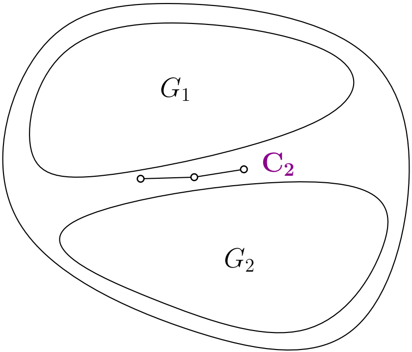

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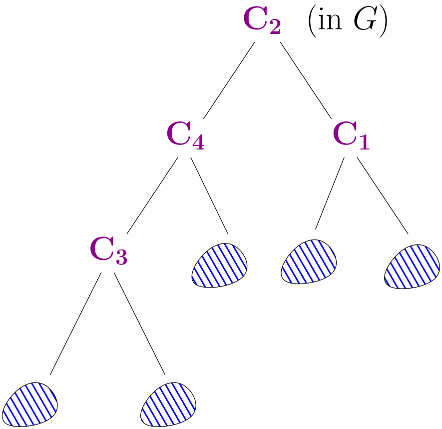

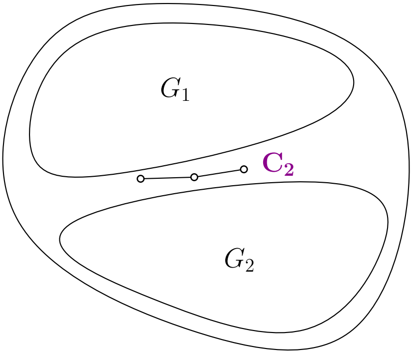

Step 3

Bottom-up addition of vertices to S:

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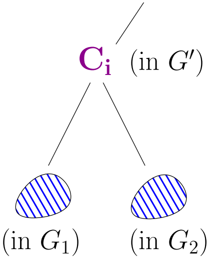
Induced triangulation G' with $C_i = \{u, v, w\}$

G_1, G_2 monitored with sets S_1, S_2



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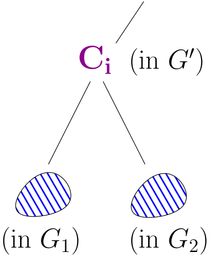

Induced triangulation G' with $C_i = \{u, v, w\}$

G_1, G_2 monitored with sets S_1, S_2

$$|S_1| \leq \frac{n_1 - 2}{4}, \; |S_2| \leq \frac{n_2 - 2}{4}$$

Step 3

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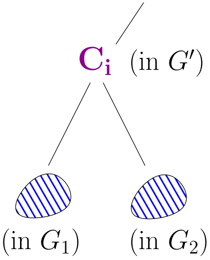

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Step 3

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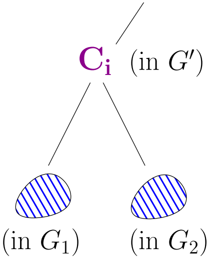

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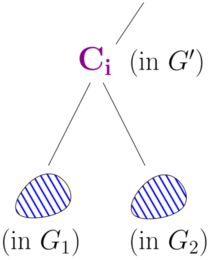

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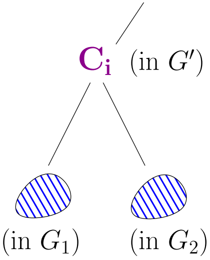

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$|S'| \leq \dfrac{n_1 + n_2}{4}$ and $|G'| \geq n_1 + n_2 + 2$ $\rightarrow |S'| \leq \dfrac{|G'| - 2}{4}$

S has the good number of nodes after of Step 3.

And now?

Theorem

$$\gamma_P(G) \leq \frac{n-2}{4} \text{ if } G \text{ is a triangulation with } n \geq 6 \text{ vertices.}$$

Some open questions:

- Reduce the gap between $\frac{n}{6}$ and $\frac{n-2}{4}$?
- Is the decision problem NP-Complete for triangulations?
- What if a vertex can propagate to up to k vertices? E.g. $k=2$?
- What if we allow only propagation at distance at most d? E.g. $d=1$?

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Thank you!