Combinatoire des polynômes de Koornwinder à $q = t$

Sylvie Corteel (CNRS Paris 7)

travail en cours avec O. Mandelshtam, D. Stanton et L. Williams

Journées de Combinatoire de Bordeaux 2017

1. Mise en jambe : chemins

2. Polynômes orthogonaux à une variable

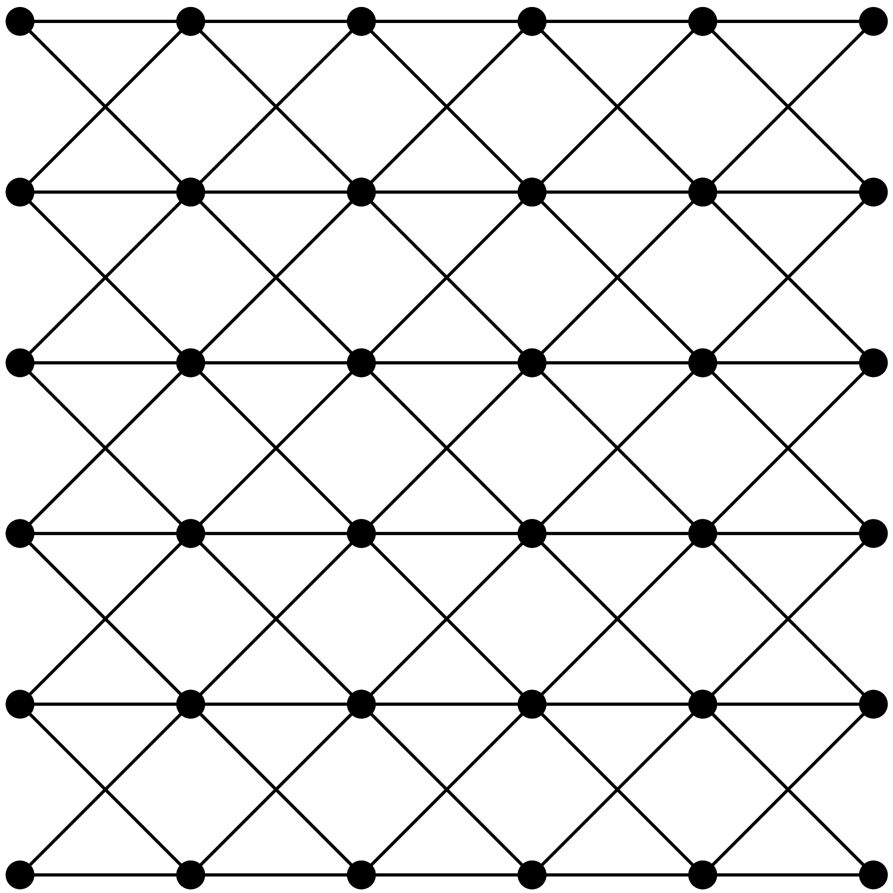
3. Moments des polynômes orthogonaux

4. Polynômes orthogonaux à plusieurs variables

5. Liens avec le 2-PASEP

6. Conjecture

1. Mise en jambe



Grille $n \times m$

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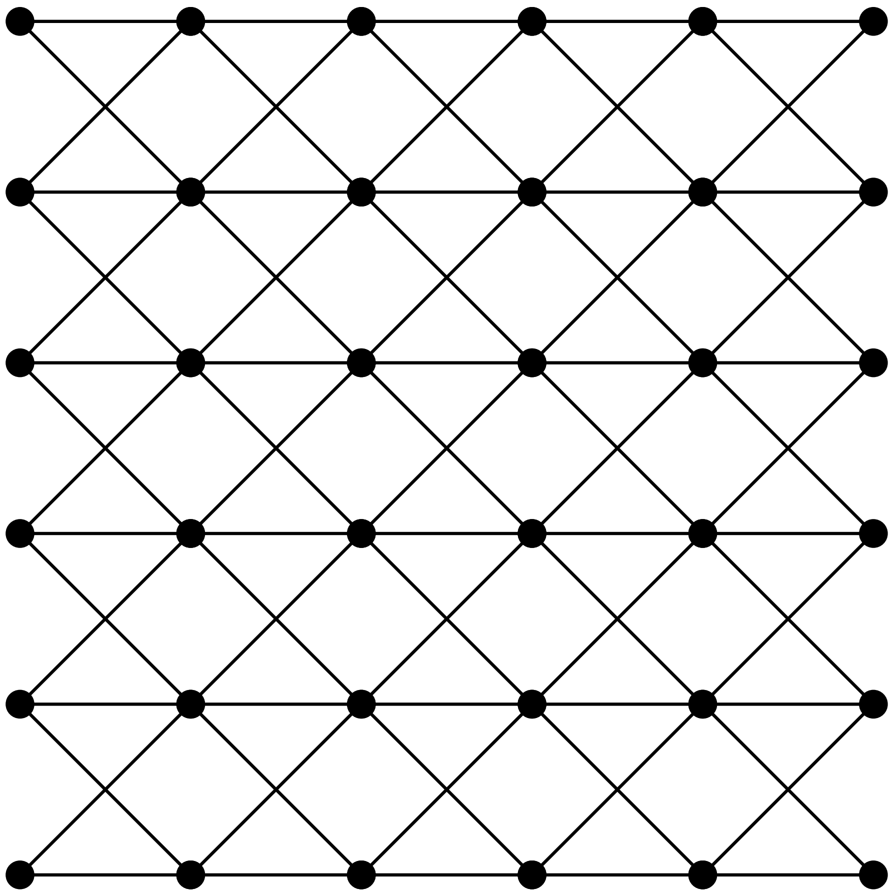
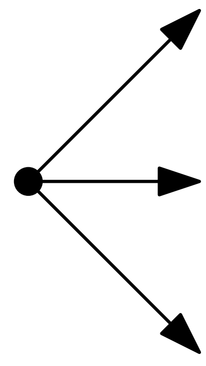


Grille $n \times m$

Sommets avec trois arêtes sortantes



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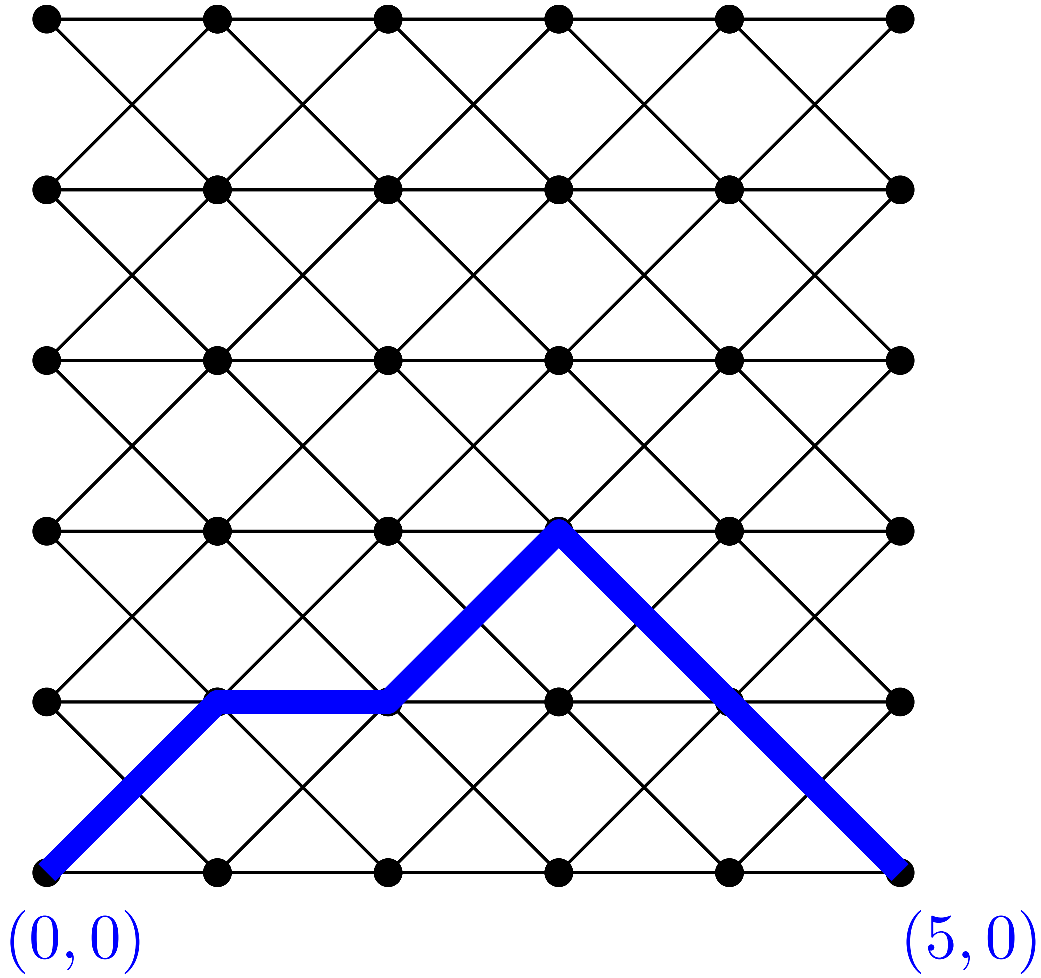

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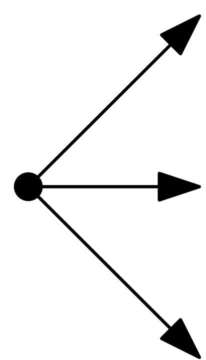

Compter le nombre de chemins de $(0,0)$ à $(n,0)$

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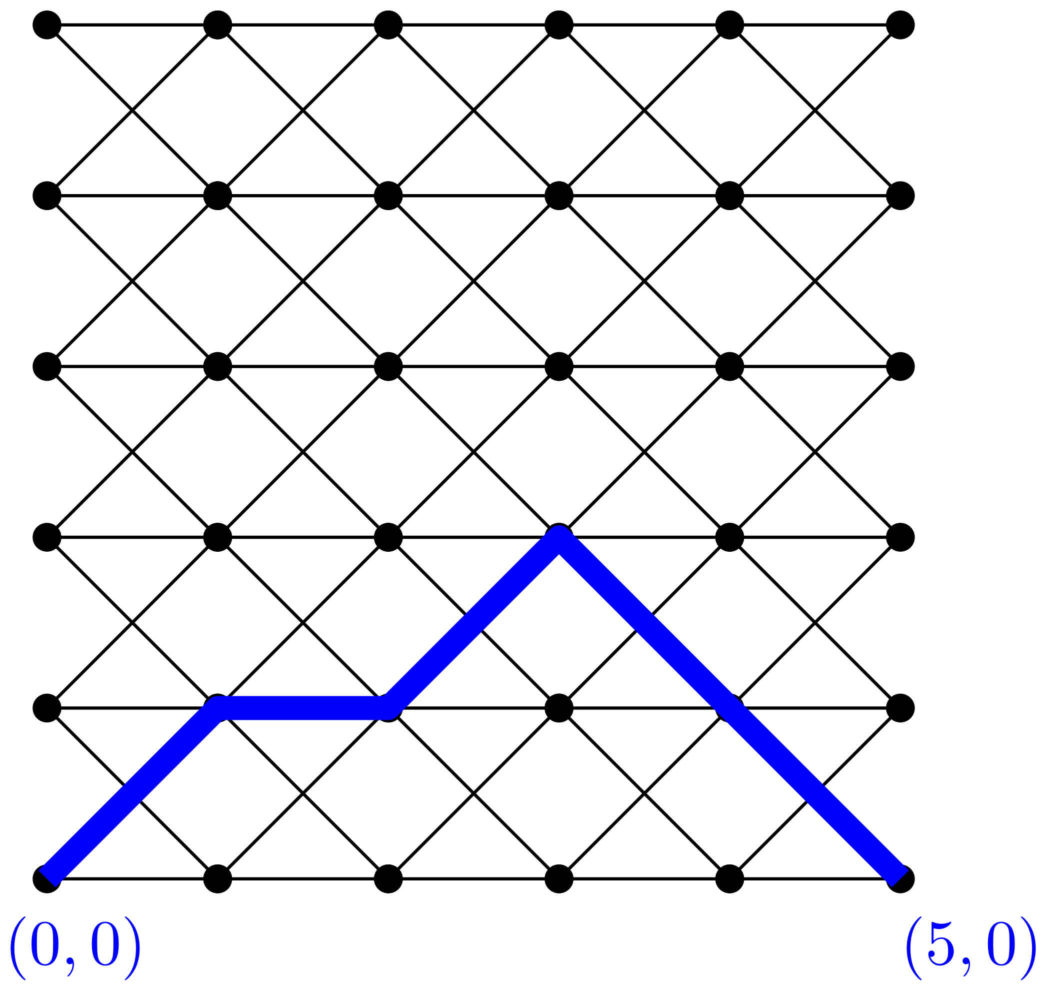

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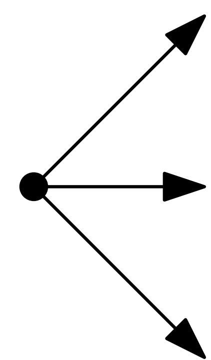
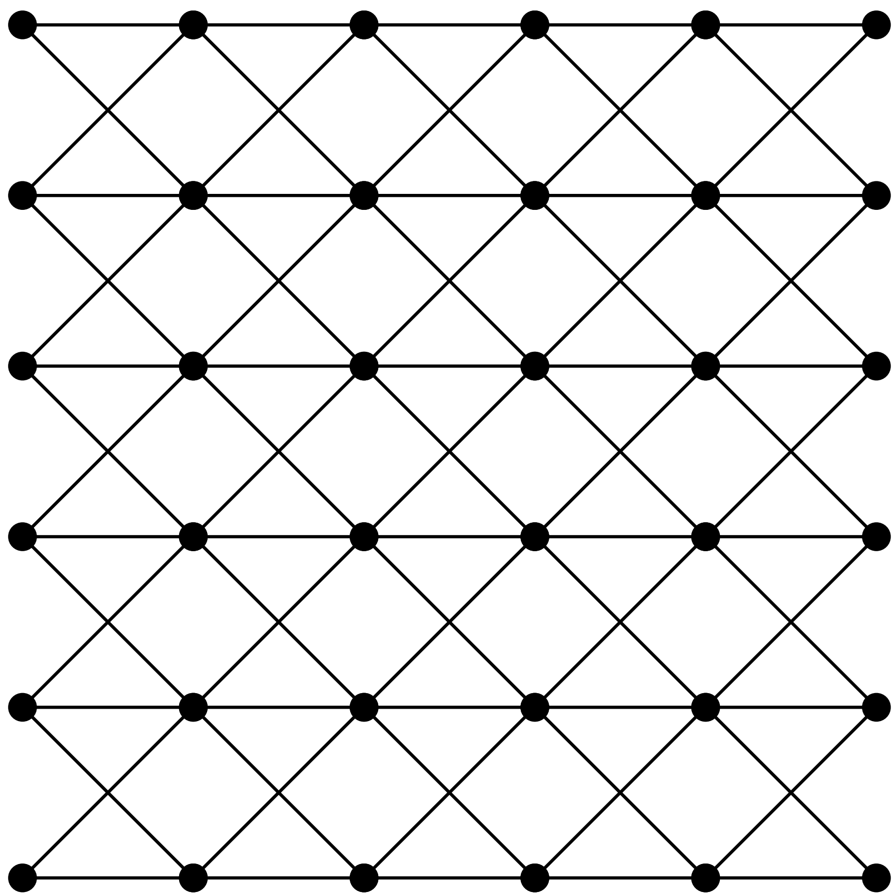


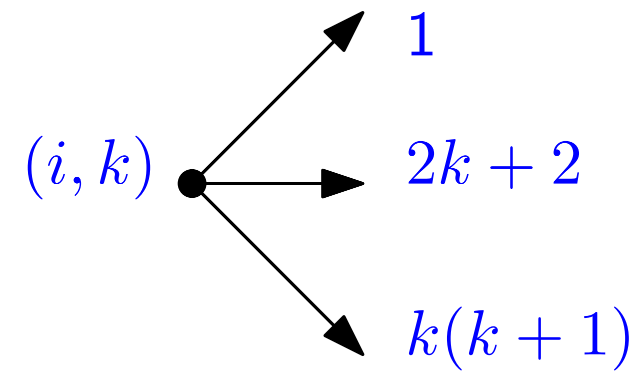
Compter le nombre de chemins de $(0,0)$ à $(n,0)$

$$M_n = \frac{1}{n+1} \sum_k \binom{n+1}{k} \binom{n+1-k}{k+1}$$

Mise en jambe pondérée



Arêtes sortantes pondérées



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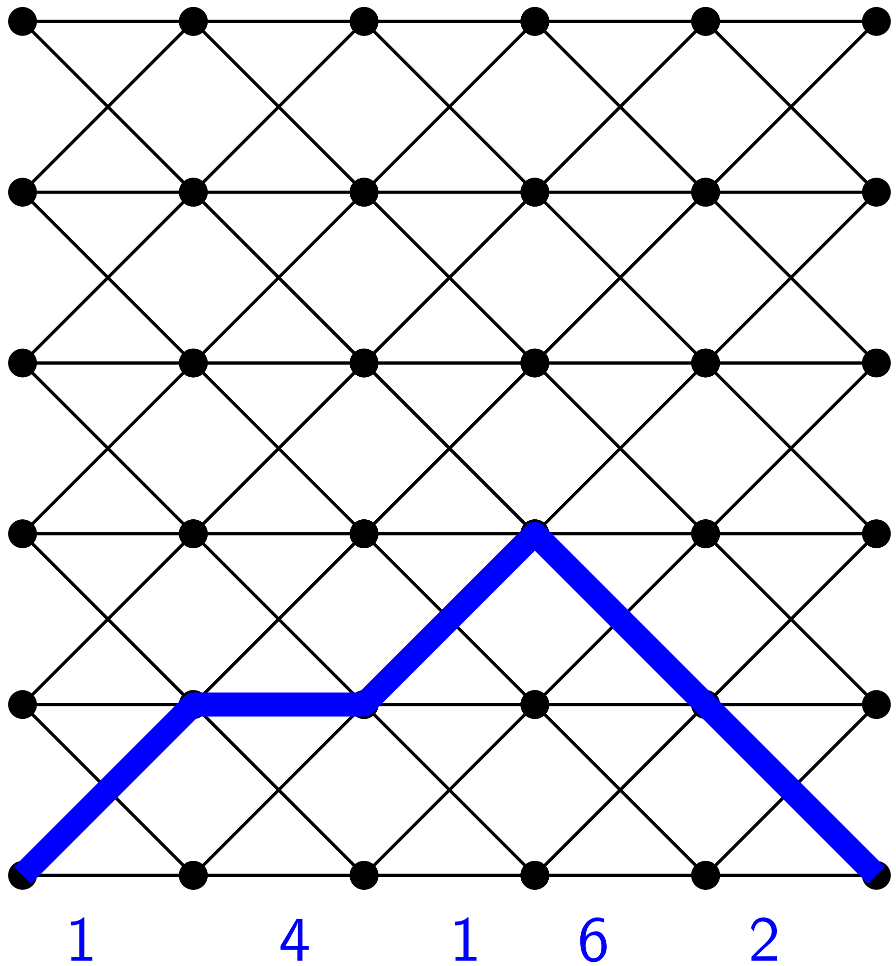
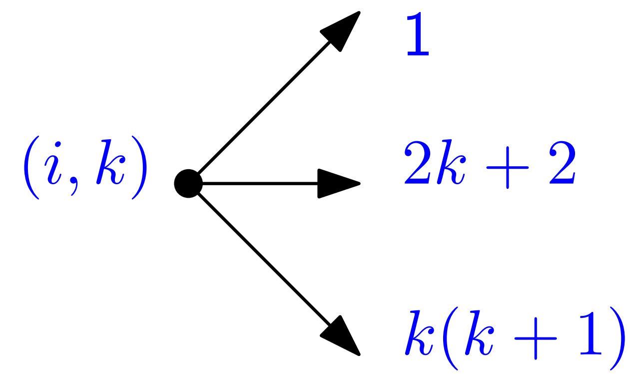


Poids du chemin = 1*4*1*6*2

Arêtes sortantes pondérées



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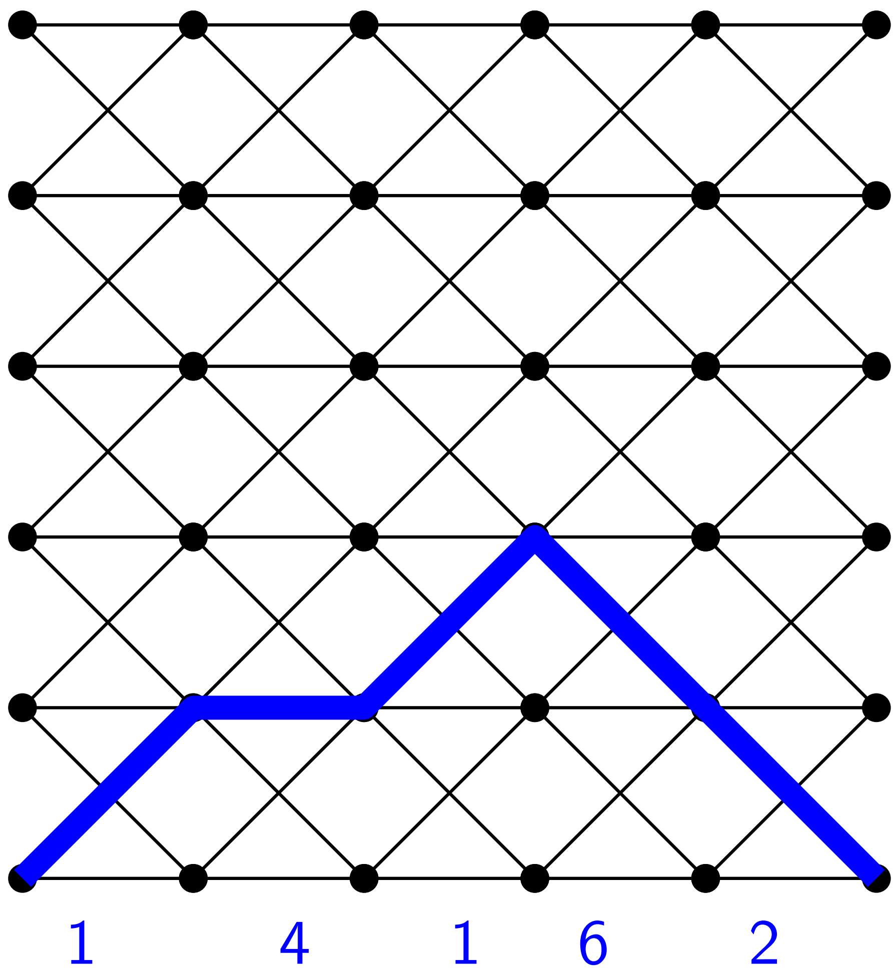

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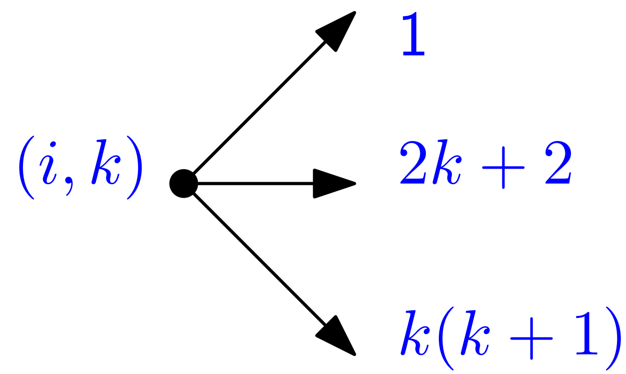
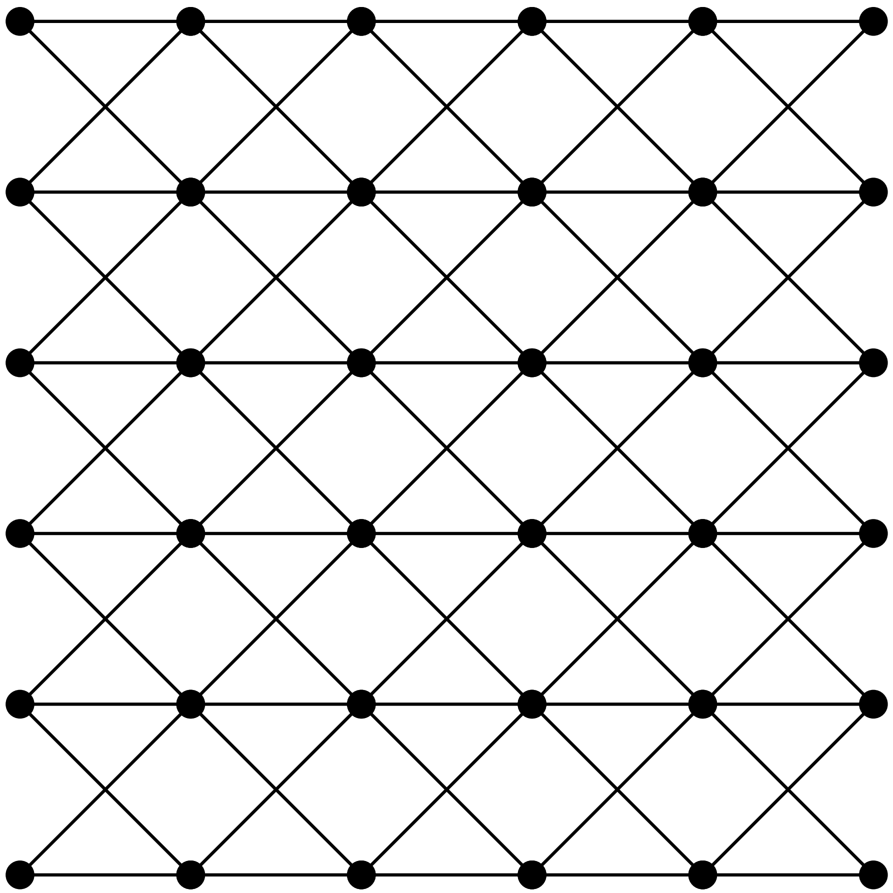


Chemins pondérés de $(0,0)$ à $(n,0)$

$$M_n = (n+1)!$$

Mise en jambe pondérée



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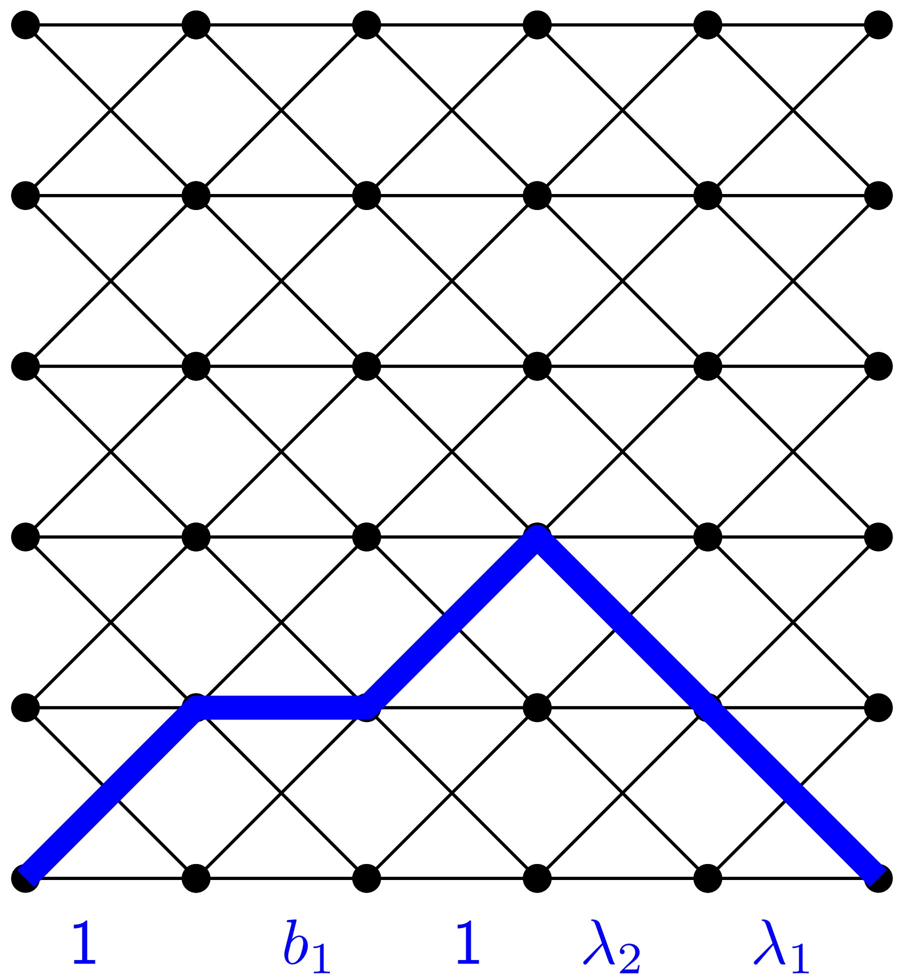

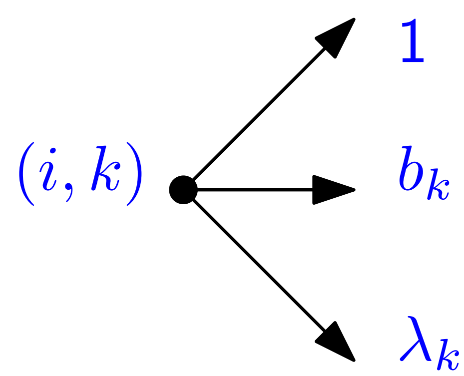

Poids du chemin $= b_1\lambda_2\lambda_1$

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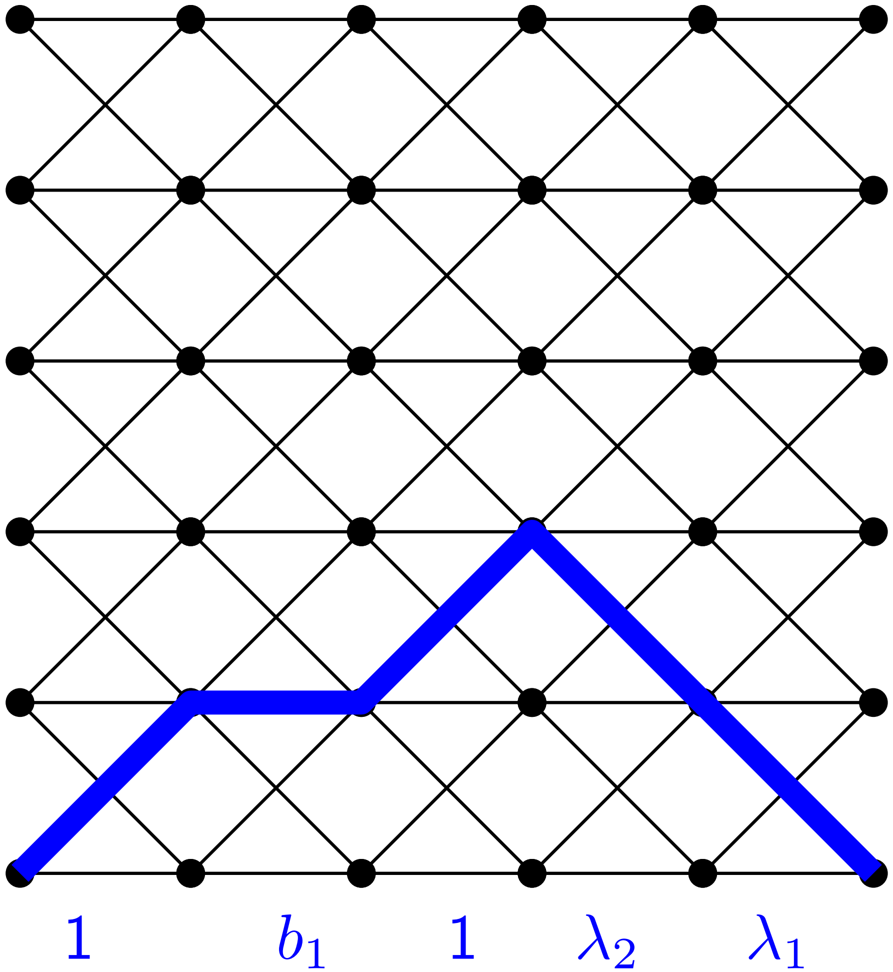
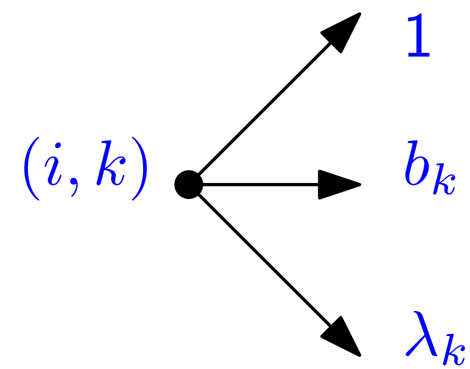
Arêtes sortantes pondérées



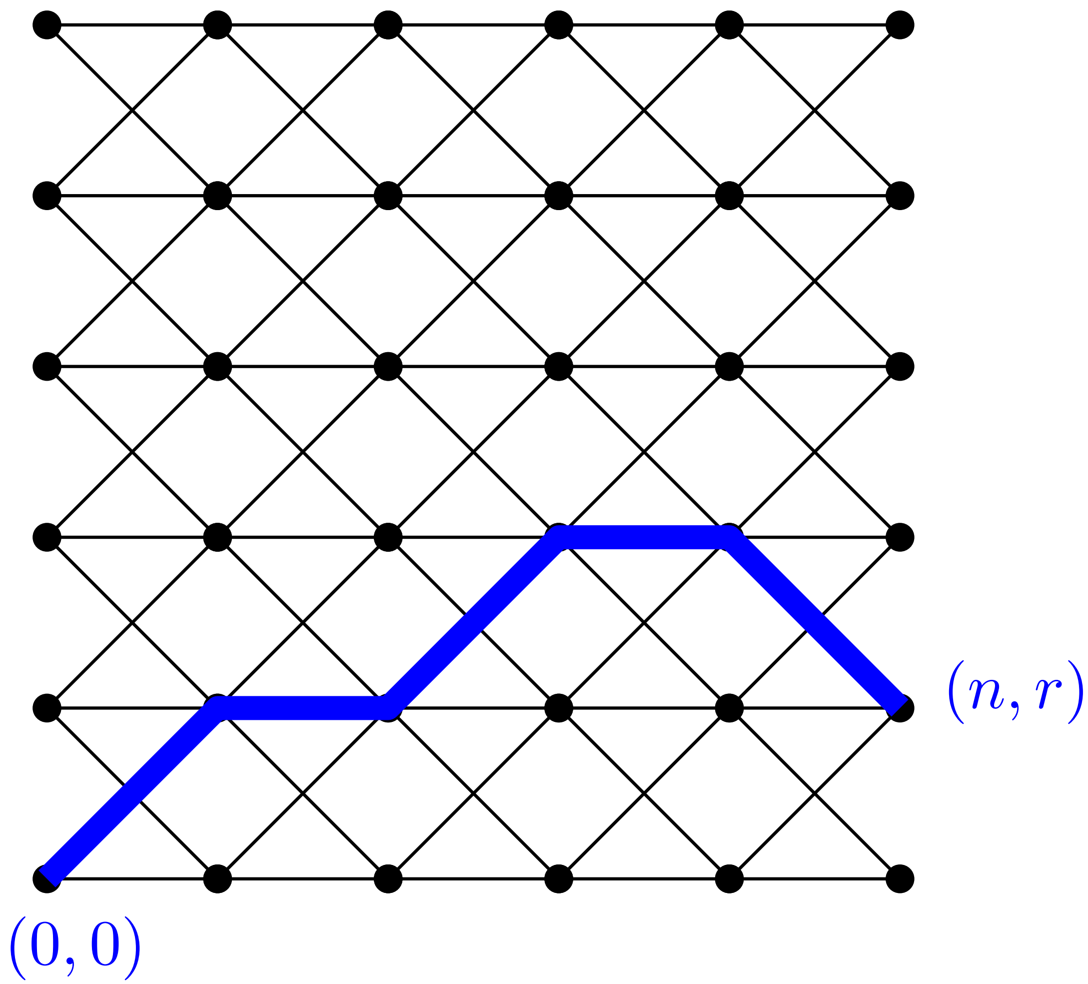


Poids du chemin $= b_1\lambda_2\lambda_1$

$$\sum_n M_n z^n =$$

$$\cfrac{1}{1-zb_0-\cfrac{z^2\lambda_1}{1-zb_1\cfrac{z^2\lambda_3}{1-zb_2-\cfrac{z^2\lambda_2}{\vdots}}}}$$

Mise en jambe pondérée



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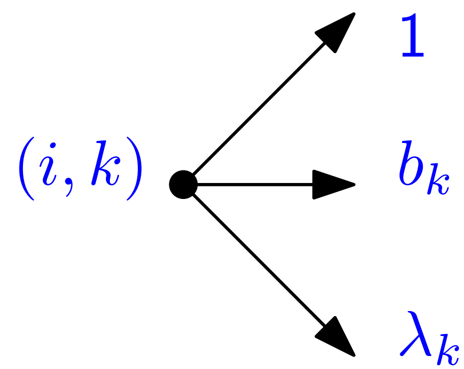
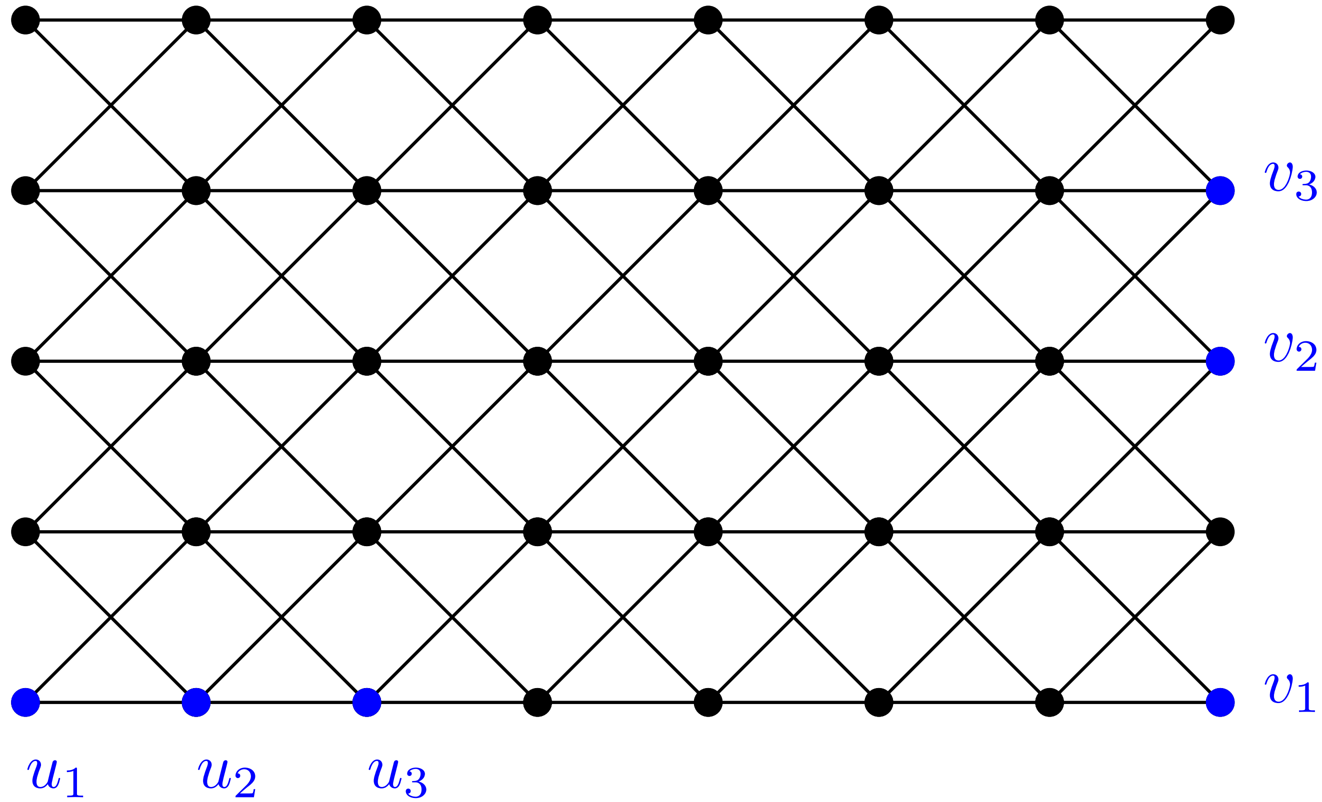


$M_{n,r}$: Chemins pondérés de $(0, 0)$ à (n, r)

Mise en jambe non-intersectante



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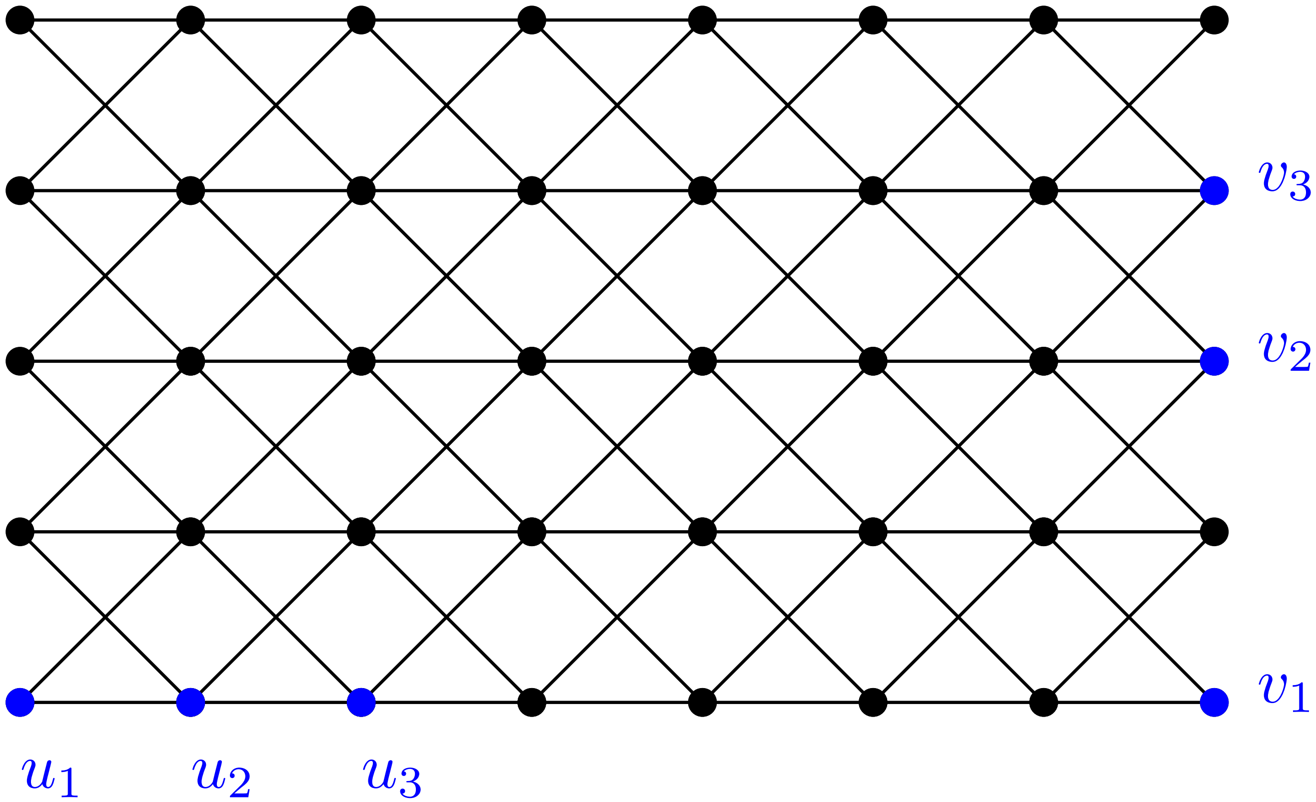
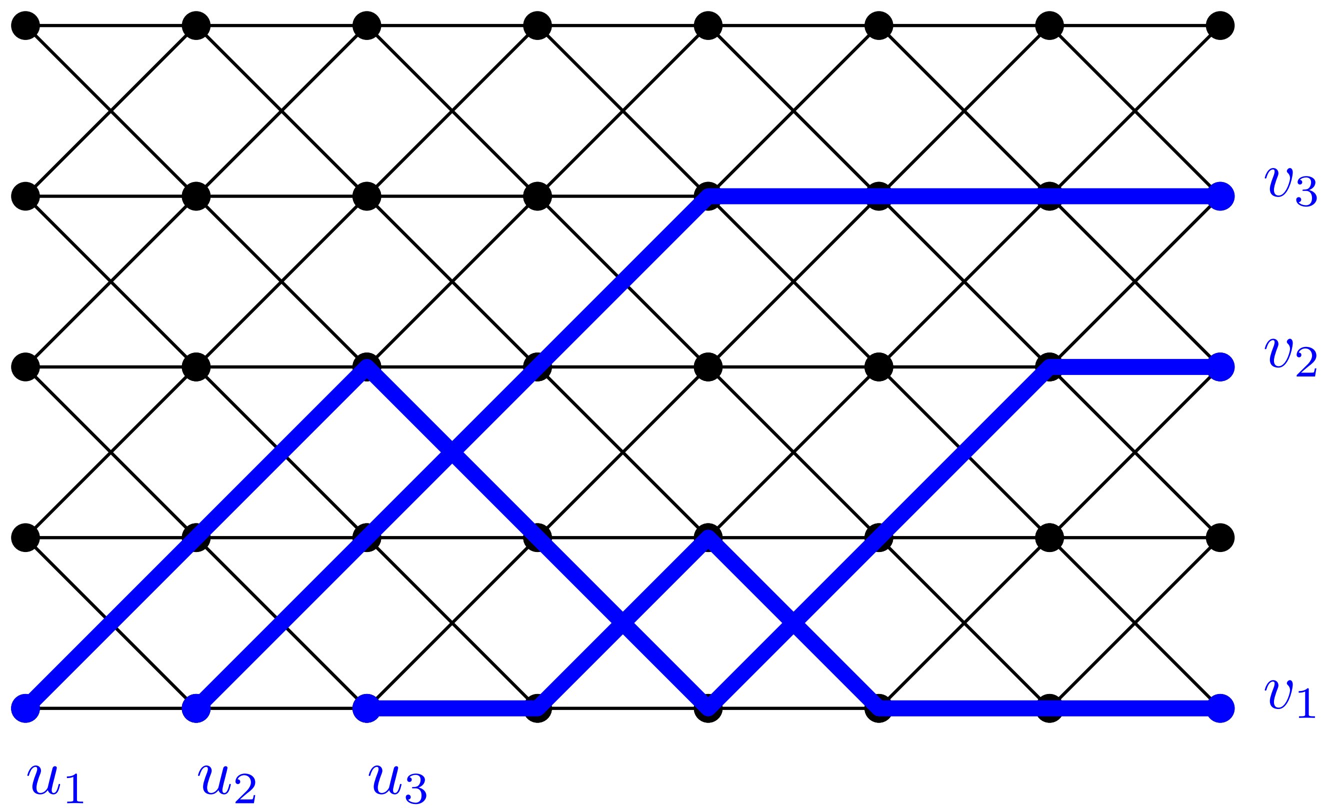


σ permutation

$M(\sigma)$ chemins pondérés
non intersectants de u_i à $v_{\sigma(i)}$

Mise en jambe non-intersectante



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Exemple: $\sigma = (2, 3, 1)$

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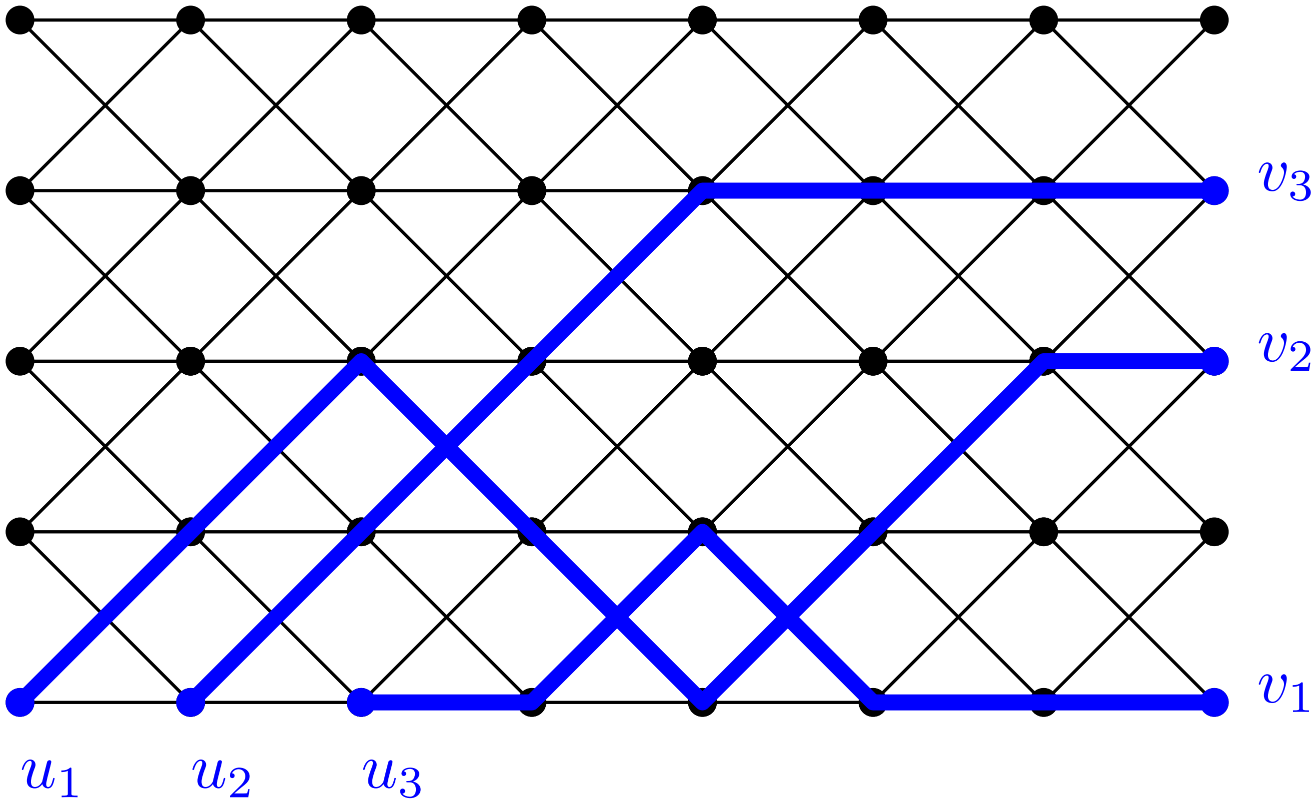

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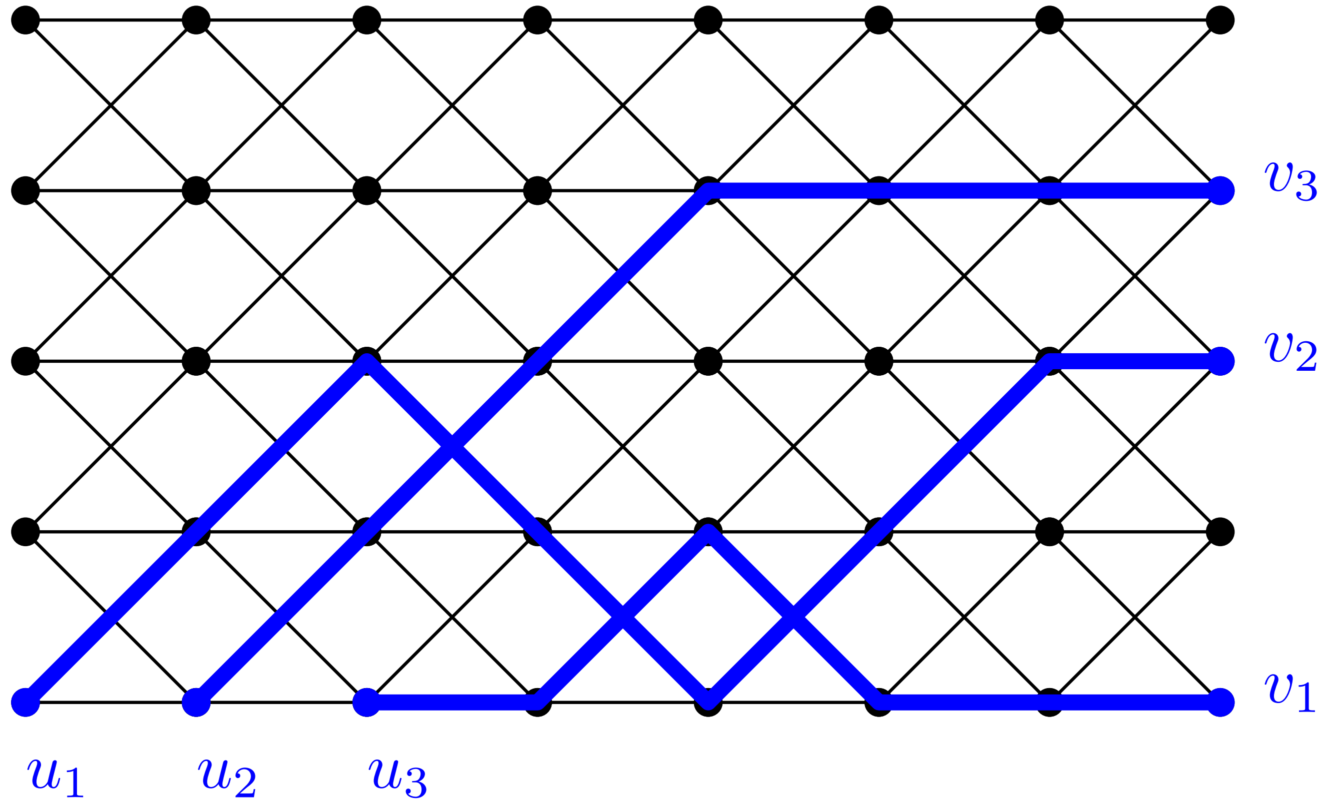
Chemins non intersectants: aucun sommet commun

Poids $=$ produit du poids des chemins

Exemple: poids $\lambda_1^2 b_0^3 \lambda_2 b_2 b_3^3$

Mise en jambe non-intersectante



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Exemple: $\sigma = (2, 3, 1)$

$u_i = (i-1, 0)$, $v_i = (n, y_i)$ Lindström-Gessel-Viennot Lemma

$$\sum_\sigma (-1)^{\epsilon(\sigma)} M(\sigma) = \det(M_{n+1-i, y_j})$$

2. Polynômes orthogonaux à une variable

$\{b_n\}, \quad \{\lambda_n\}$

$$p_{n+1}(x) = (x - b_n)p_n(x) - \lambda_n p_{n-1}(x)$$

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Askey Wilson polynomials

$$A_n = \frac{(1-abq^n)(1-acq^n)(1-adq^n)(1-abcdq^{n-1})}{a(1-abcdq^{2n})(1-abcdq^{2n-1})}$$

$$C_n = \frac{a(1-q^n)(1-cbq^{n-1})(1-bdq^{n-1})(1-cdq^{n-1})}{(1-abcdq^{2n-2})(1-abcdq^{2n-1})}$$

$$b_n = \frac{1}{2}(a + 1/a - A_n - C_n),\ \lambda_n = \frac{1}{4}A_{n-1}C_n$$

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Orthogonalité

Densité $w(x)$

$$\int p_n(x)p_m(x)w(x)dx = 0 \text{ si } m \neq n$$

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Askey Wilson $w(x) = \frac{(z^2,z^{-2};q)_\infty}{(az,az^{-1},bz,bz^{-1},cz,cz^{-1},dz,dz^{-1};q)_\infty}$, $x = (z + z^{-1})/2$

$$(a_1, \ldots, a_k; q)_\infty = \prod_{i-1}^{k} \prod_{j=1}^{\infty} (1 - a_i q^j)$$

Example: q-Laguerre polynomials

Askey Wilson with $a = b = -q$, $c = d = 0$

$$p_{n+1}(x) = (x - q^{n+1})p_n(x) - \tfrac{(1-q^{n+1})(1-q^n)}{4}p_{n-1}(x)$$

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$$\boxed{p_n(1+x) = \sum_{k=0}^{n} \frac{x^k(1-q)^{n-k}}{2^{n-k}} G_{n,k}}$$

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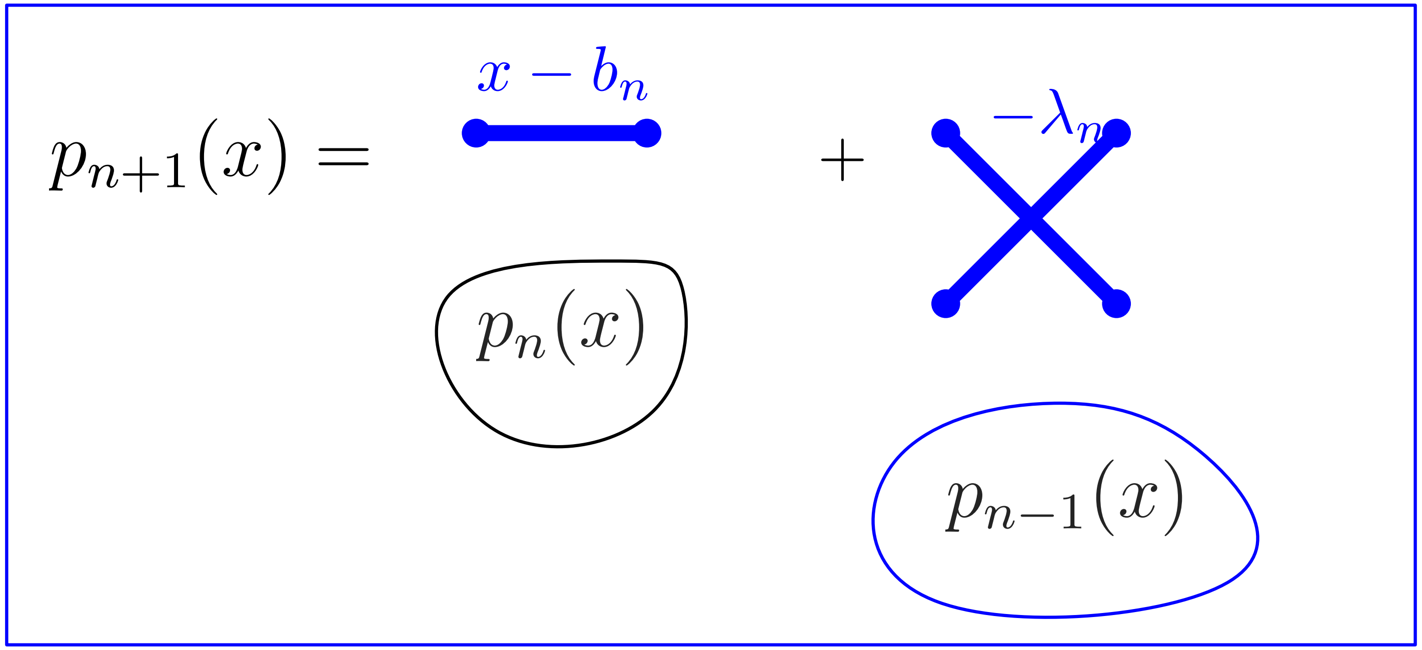
$$G_{n,0} = (n+1)[n]_q!$$

$$[n]_q = \tfrac{1-q^n}{1-q}$$

Combinatoire des polynômes orthogonaux



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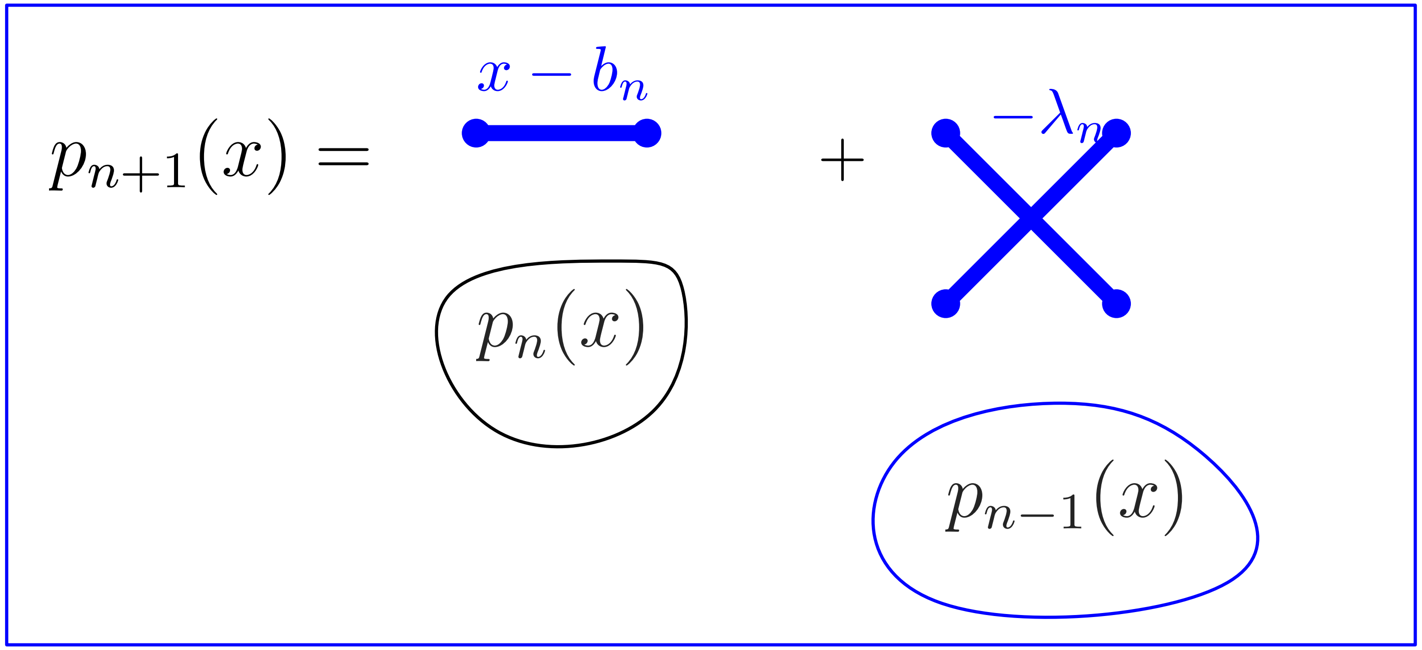

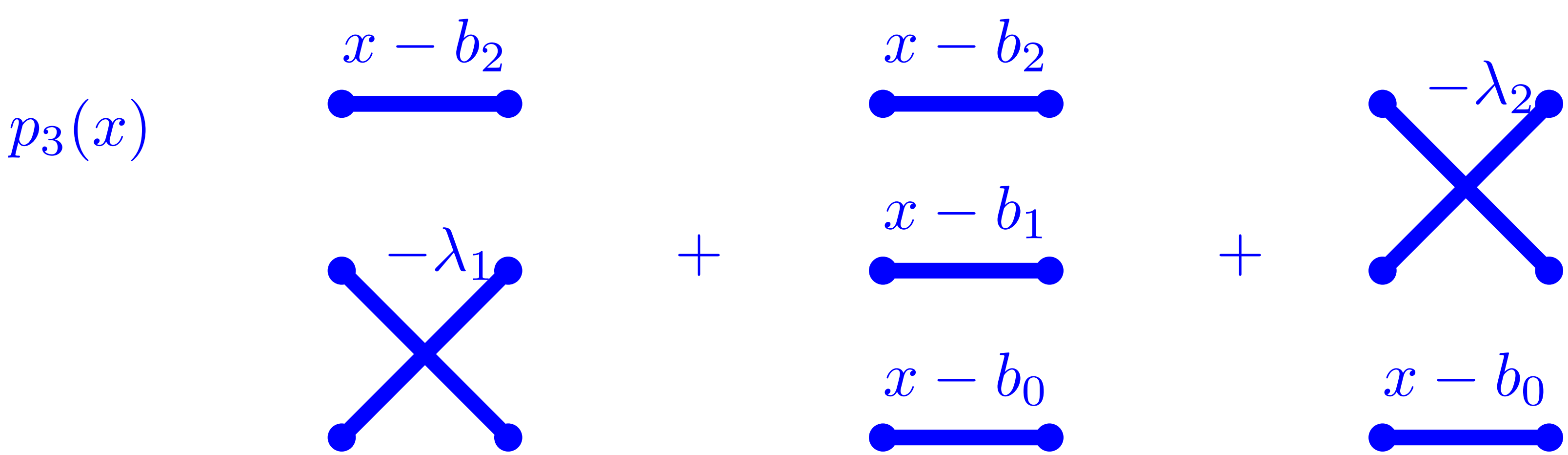

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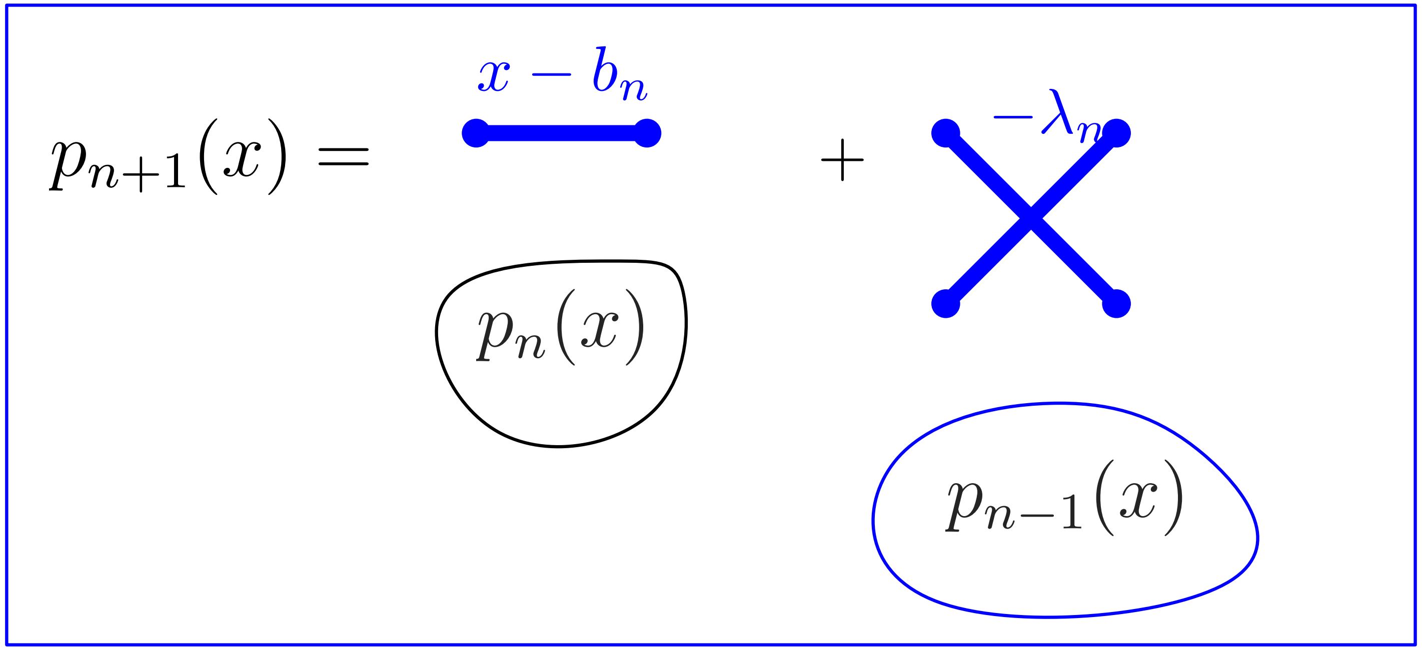
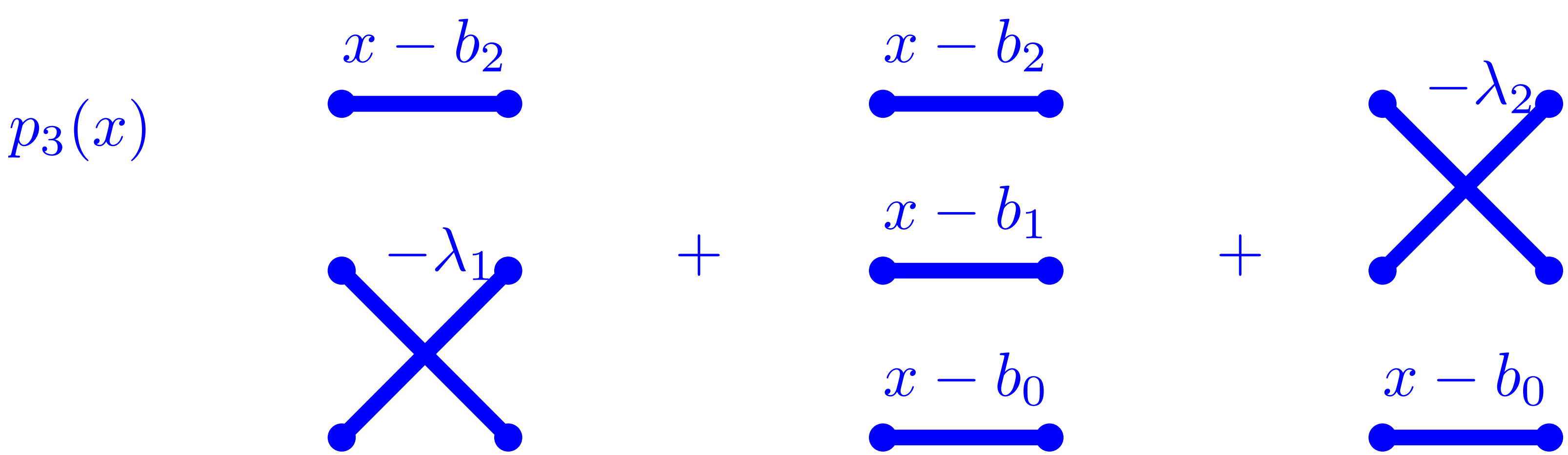
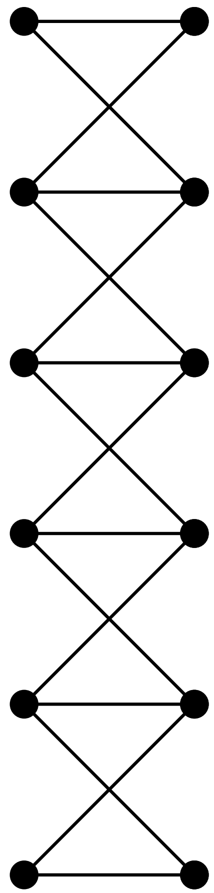




$$p_3(x) = (x - b_2) - \lambda_1 + (x - b_2)(x - b_1)(x - b_0) - \lambda_2(x - b_0)$$

Combinatoire des polynômes orthogonaux



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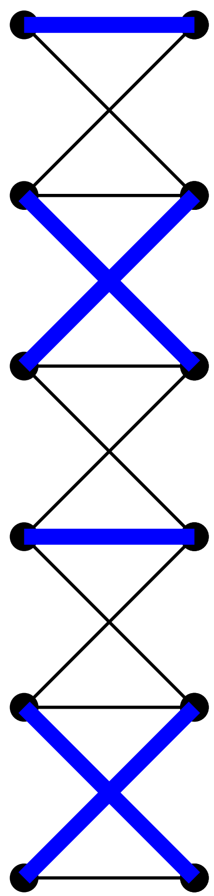
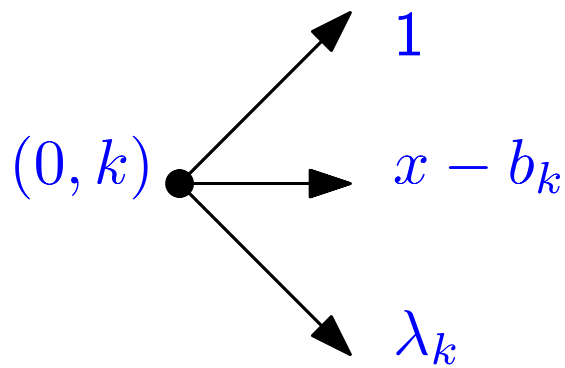


Chemins non-intersectants de $(0, i)$ à $(1, j)$



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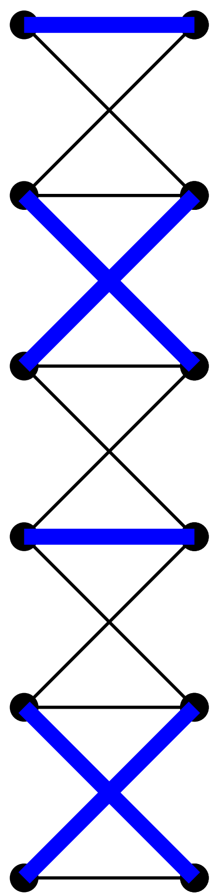

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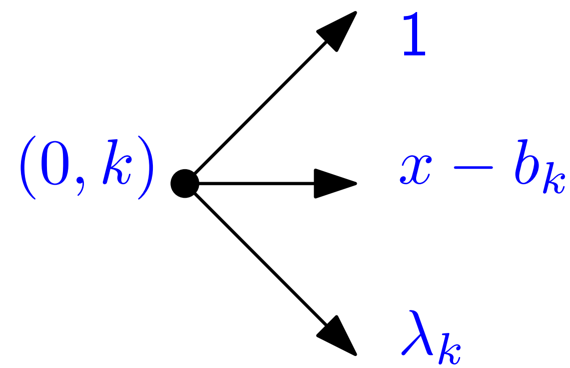
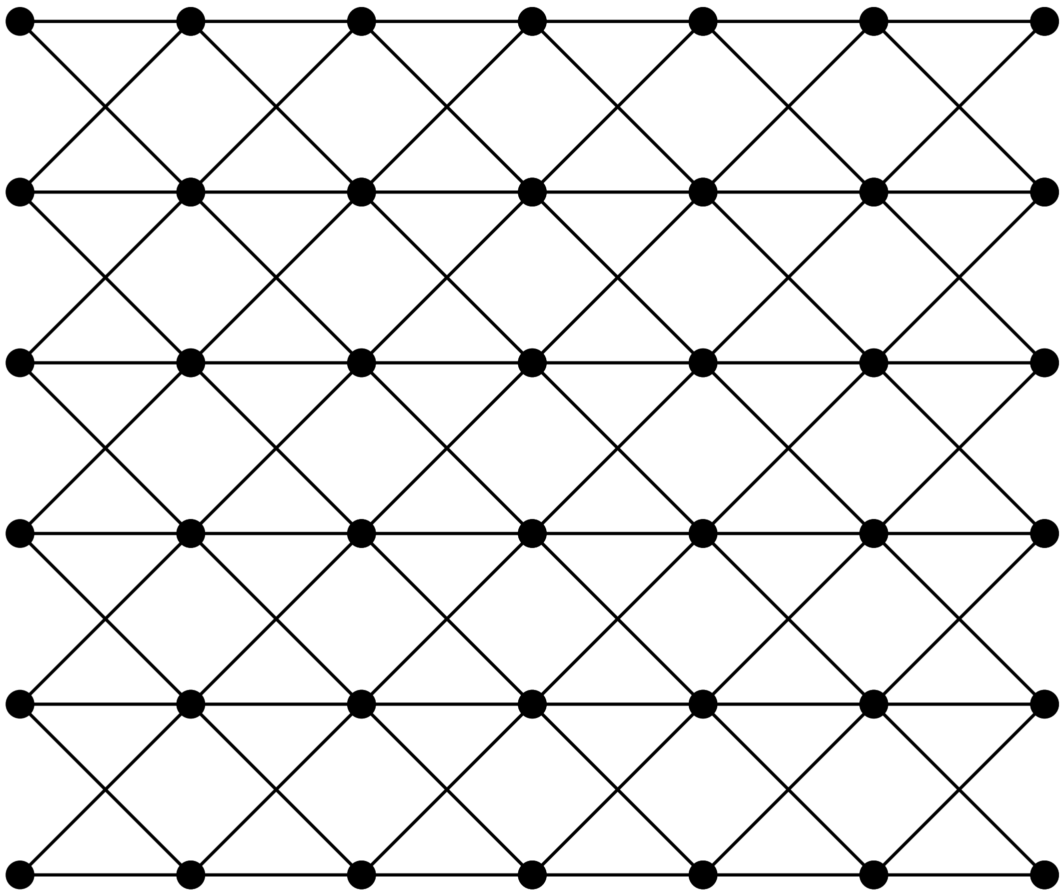


Lemme LGV

$$p_n(x) = \det \begin{pmatrix} x-b_0 & 1 & 0 & 0 & 0 \\ \lambda_1 & x-b_1 & 1 & 0 & 0 \\ 0 & \lambda_2 & x-b_2 & 1 & 0 \quad \cdots \\ 0 & 0 & \lambda_3 & x-b_3 & 1 \\ \vdots & & \vdots & & \end{pmatrix}$$

Combinatoire des polynômes orthogonaux



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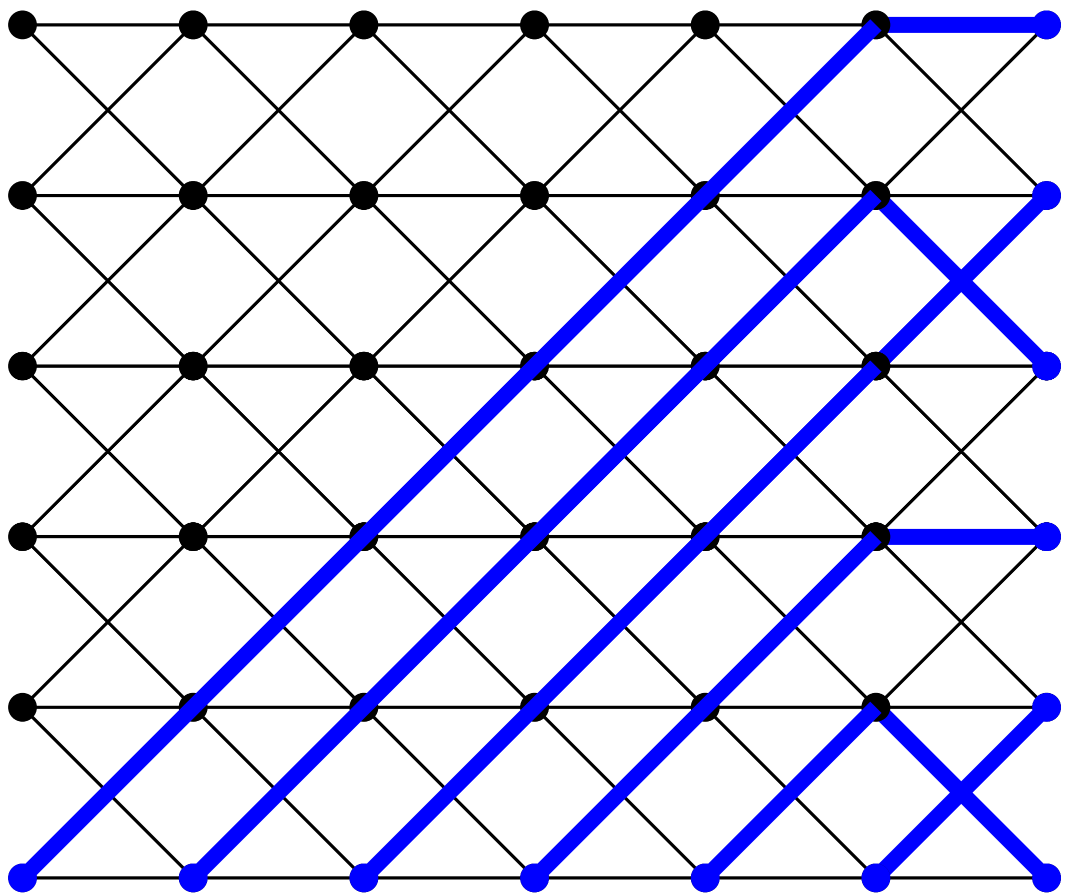
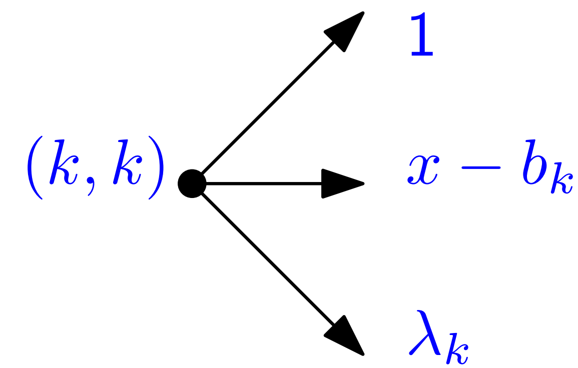


Chemins non-intersectants de $(i, 0)$ à (n, j)



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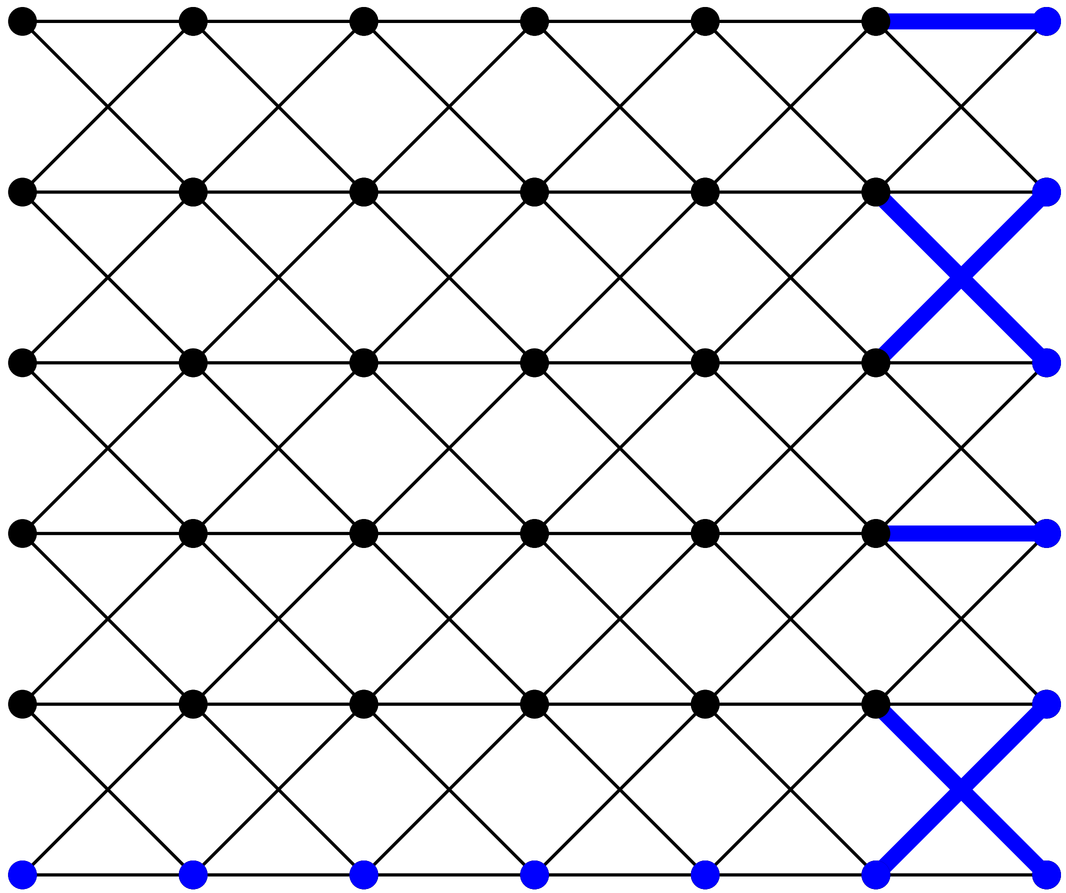

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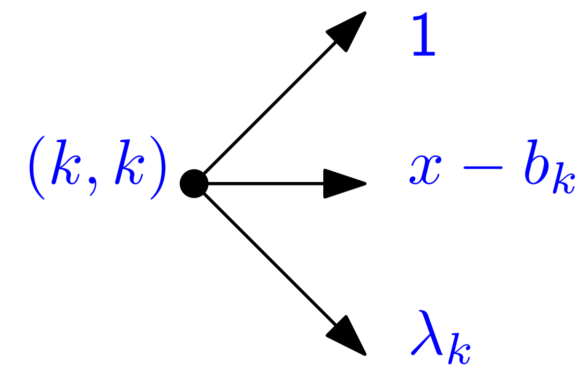

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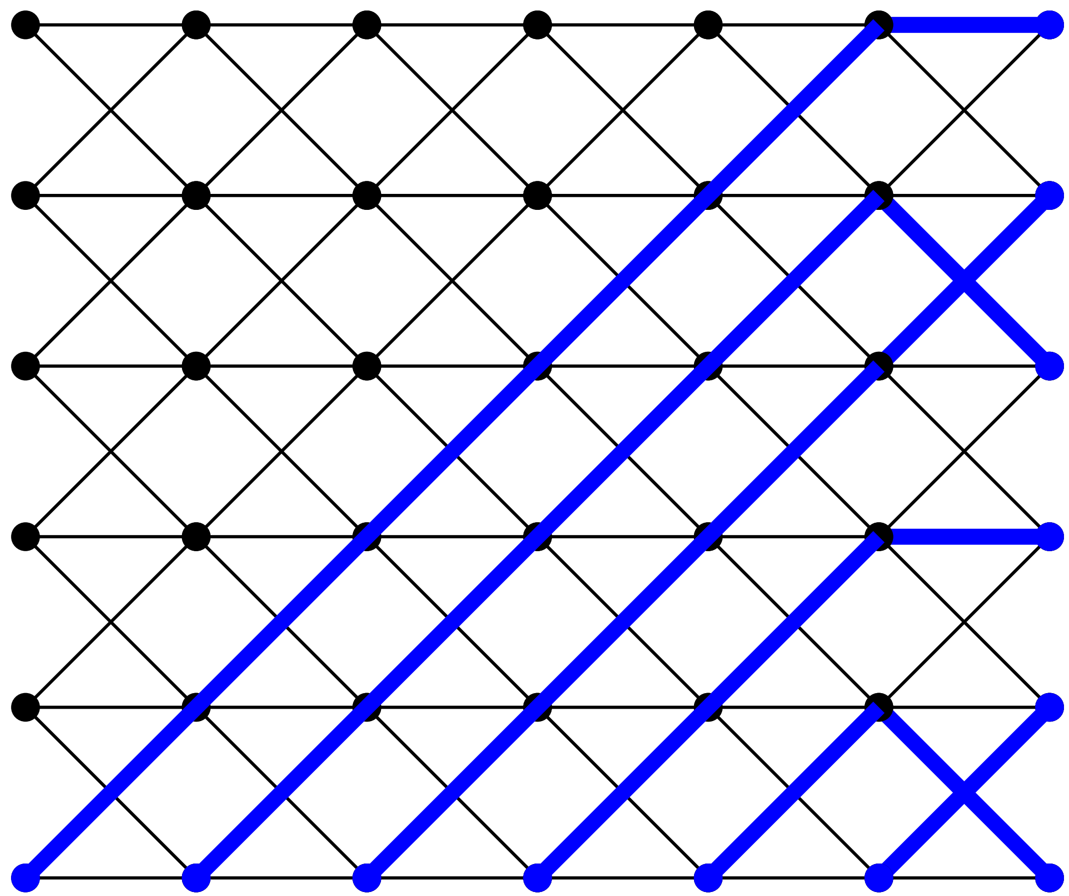

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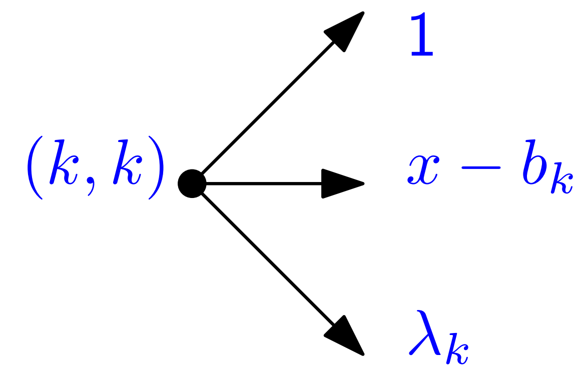

Lemme LGV

$$\boxed{p_n(x) = \det\Big(M_{n+1-i,j}\Big)}$$

3. Moments des polynômes orthogonaux

$$\boxed{\mu_n = \int x^n w(x) dx}$$

Flajolet (78) - Viennot (80s)

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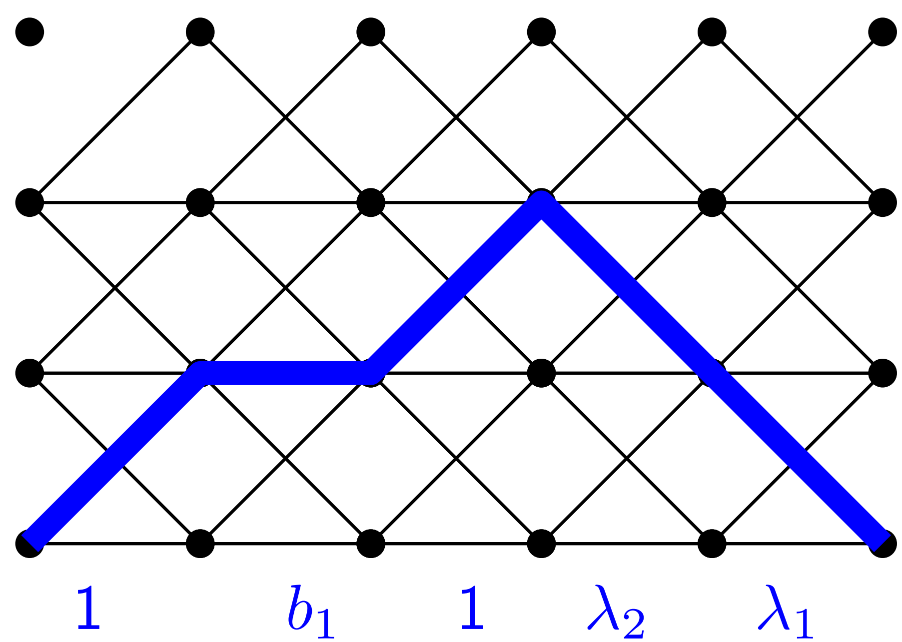
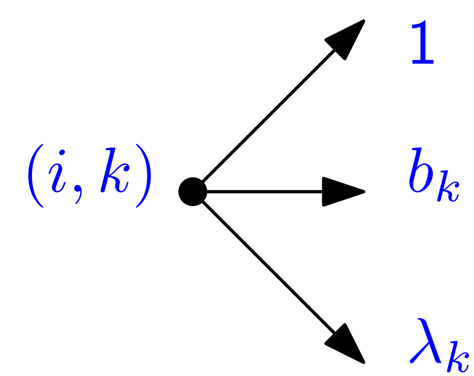


Arêtes sortantes pondérées



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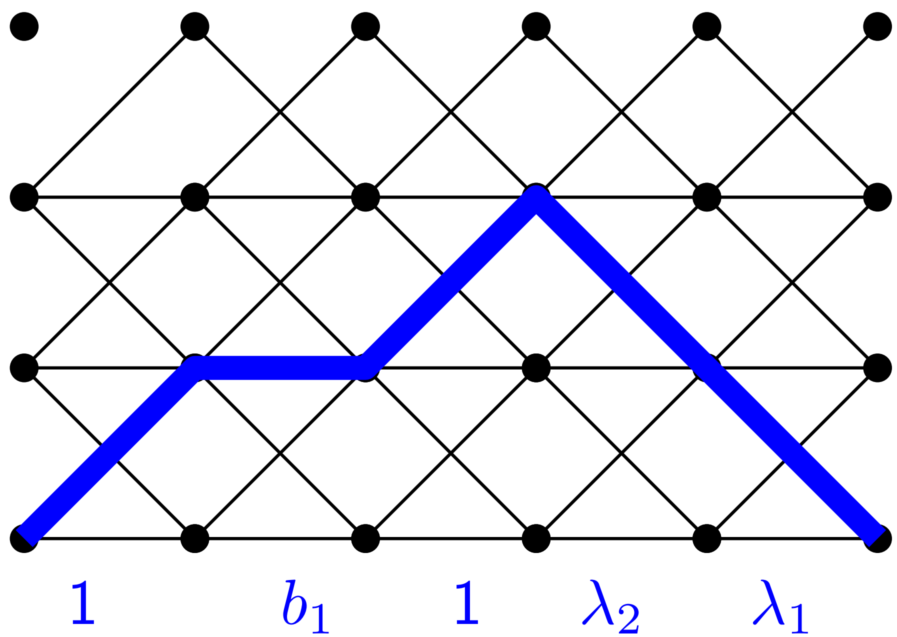

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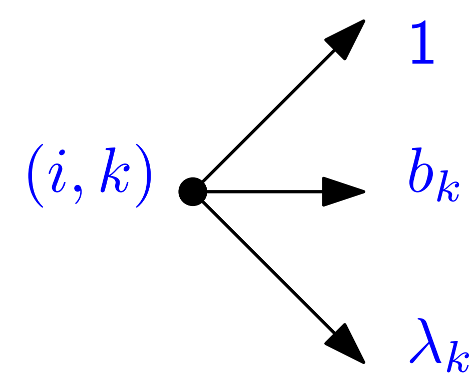

$$\boxed{\mu_n = M_n}$$

Moments des polynômes orthogonaux

$$\boxed{\mu_{n,k,r} = \int x^n p_k(x) p_r(x) w(x) dx}$$

Viennot (80s)

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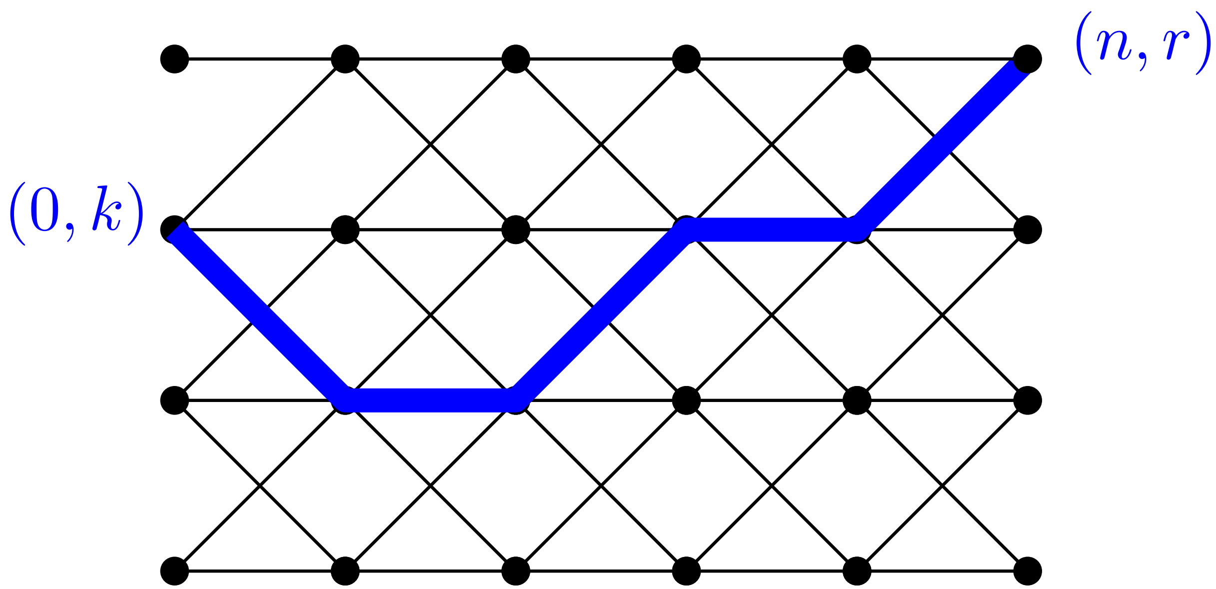

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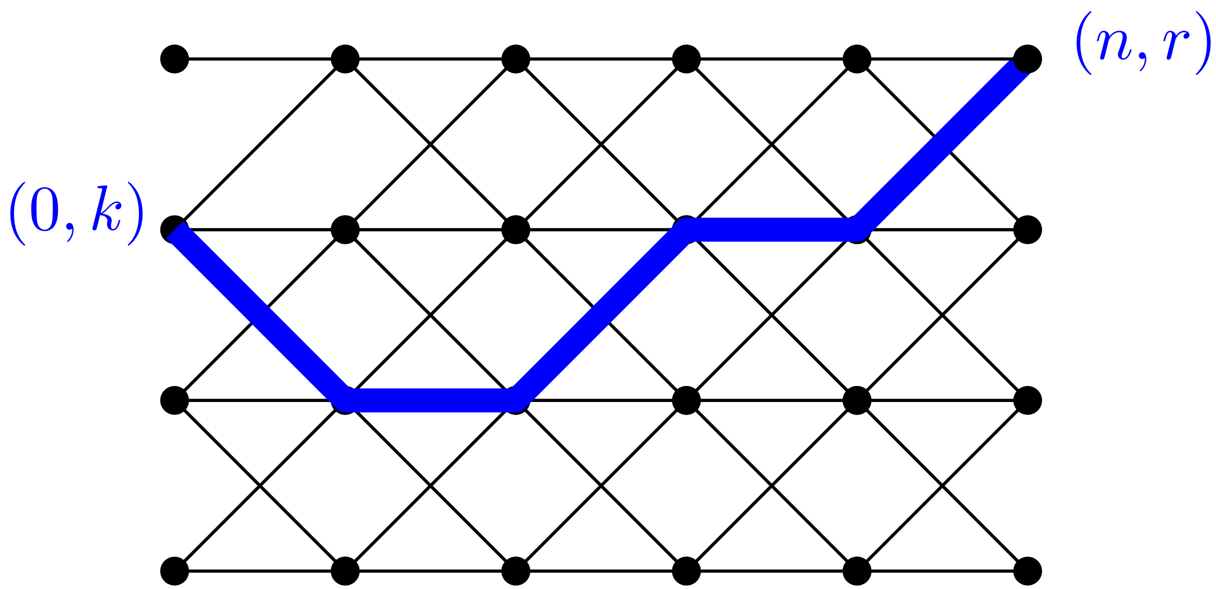
Viennot (80s)



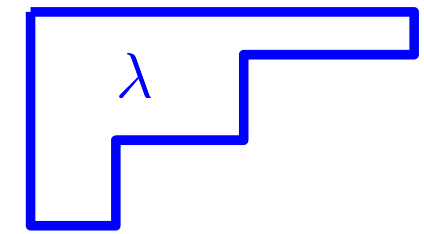
$$\boxed{\mu_{n,o,r} = M_{n,r}}$$

4. Polynômes orthogonaux à plusieurs variables

Polynômes de Schur

$$\lambda = (\lambda_1, \ldots, \lambda_n) \qquad \qquad \lambda_1 \geq \ldots \geq \lambda_n \geq 0$$

4. Polynômes orthogonaux à plusieurs variables



Polynômes de Schur

$$\lambda = (\lambda_1, \ldots, \lambda_n) \qquad \lambda_1 \geq \ldots \geq \lambda_n \geq 0$$

$$\boxed{S_\lambda(\mathbf{x}) = \frac{\det\left(x_i^{n-j+\lambda_j}\right)}{\det\left(x_i^{n-j}\right)}}$$

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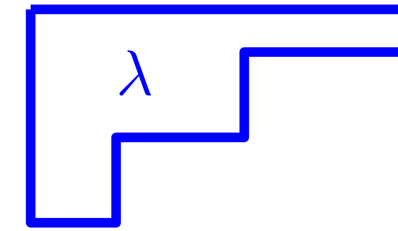

Polynômes de Schur $\mathbf{x} = (x_1, \ldots, x_n)$

$\lambda = (\lambda_1, \ldots, \lambda_n)$ $\lambda_1 \geq \ldots \geq \lambda_n \geq 0$

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Polynôme de Koornwinder à $q = t$

$$\boxed{P_\lambda(x_1, \ldots, x_n; a, b, c, d, q)}$$

Polynômes d'Askey Wilson

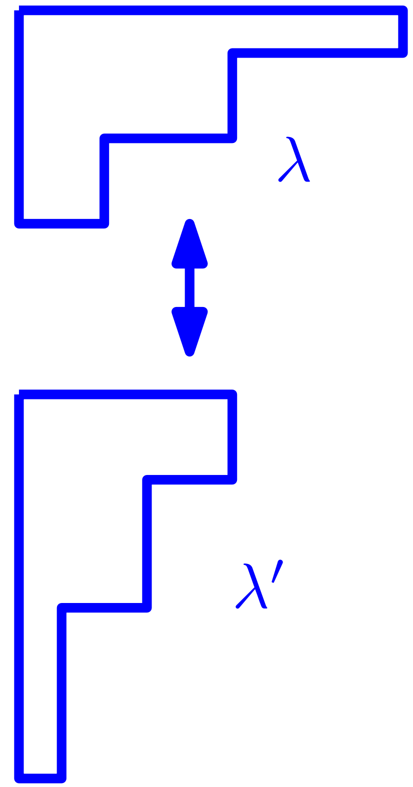
$$p_{n-j+\lambda_j}(x_i)$$

Polynômes orthogonaux à plusieurs variables

Polynômes de Schur: formule de Jacobi-Trudi $\lambda_1 \leq m$

$$S_\lambda(\mathbf{x}) = \det\left(e_{\lambda'_i + j - i}\right)_{1 \leq i,j \leq m}$$

$$e_k = S_{(1^k, 0^{n-k})}(\mathbf{x})$$



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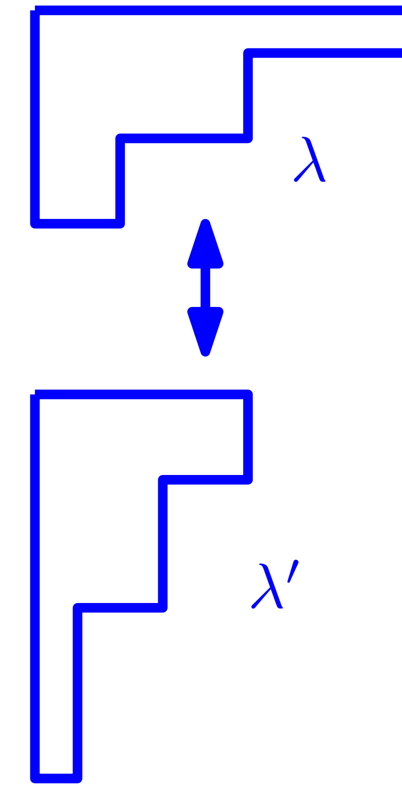

Polynômes de Koornwinder à $q = t$: formule de Jacobi-Trudi

J. Nakagawa, M. Noumi, M. Shirakawa, and Y. Yamada, 2000

$$P_\lambda(\mathbf{x}) = \det\left(e^{(n-1+j)}_{\lambda'_i+j-i}\right)_{1\leq i,j\leq m}$$

$$e^{(n)}_k = P_{(1^k,0^{n-k})}(\mathbf{x})$$

Determinant $\Longrightarrow$ Chemins non-intersectants

Determinant $\Longrightarrow$ Chemins non-intersectants

Peut on interpréter les e comme des chemins?

Determinant $\Rightarrow$ Chemins non-intersectants

Peut on interpréter les e comme des chemins?

Combinatoire des $e_k^{(n)}(\mathbf{x})$?

Formule Cauchy pour les Koornwinder généraux

Théorème (Mimachi)

$$\prod_{i=1}^{n} \prod_{j=1}^{m} (x_i - y_j) = \sum_{\lambda \in m^n} (-1)^{|\mu|} P_\lambda(\mathbf{x}; q, t) P_\mu(\mathbf{y}; t, q)$$

Formule Cauchy pour les Koornwinder généraux

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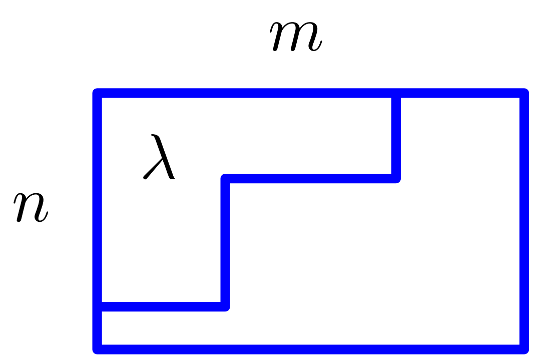
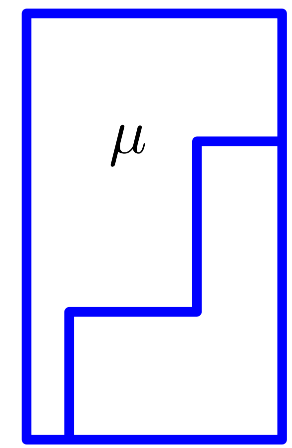


$$\mu = (n-\lambda'_m,\dots,n-\lambda'_1)$$



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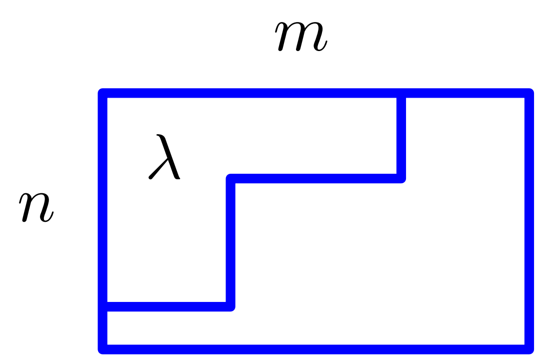

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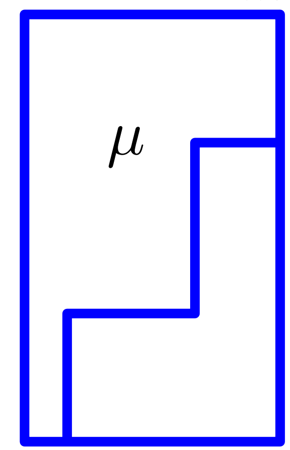

Corollaire (Mimachi) $m = 1$

$$P_{1^{n-r}}(\mathbf{x};q) = (-1)^{n-r}\int\prod_{i=1}^{n}(y-x_i)P_r(y;q)w(y)dy$$

$$e_{n-r}^{(n)}(\mathbf{x})$$

polynômes d'Askey-Wilson

Combinatoire des polynômes élémentaires

$$e_{n-r}^{(n)}(\mathbf{x};q) = (-1)^{n-r} \int \prod_{i=1}^{n} (y - x_i) p_r(y) w(y) dy$$

On applique la théorie de Viennot

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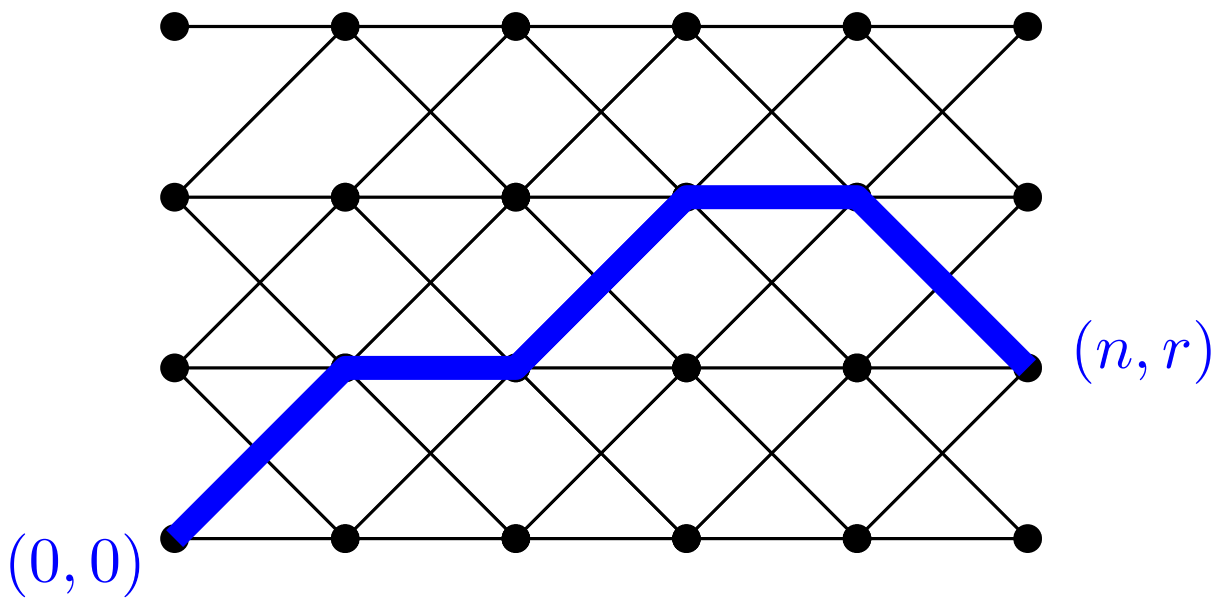

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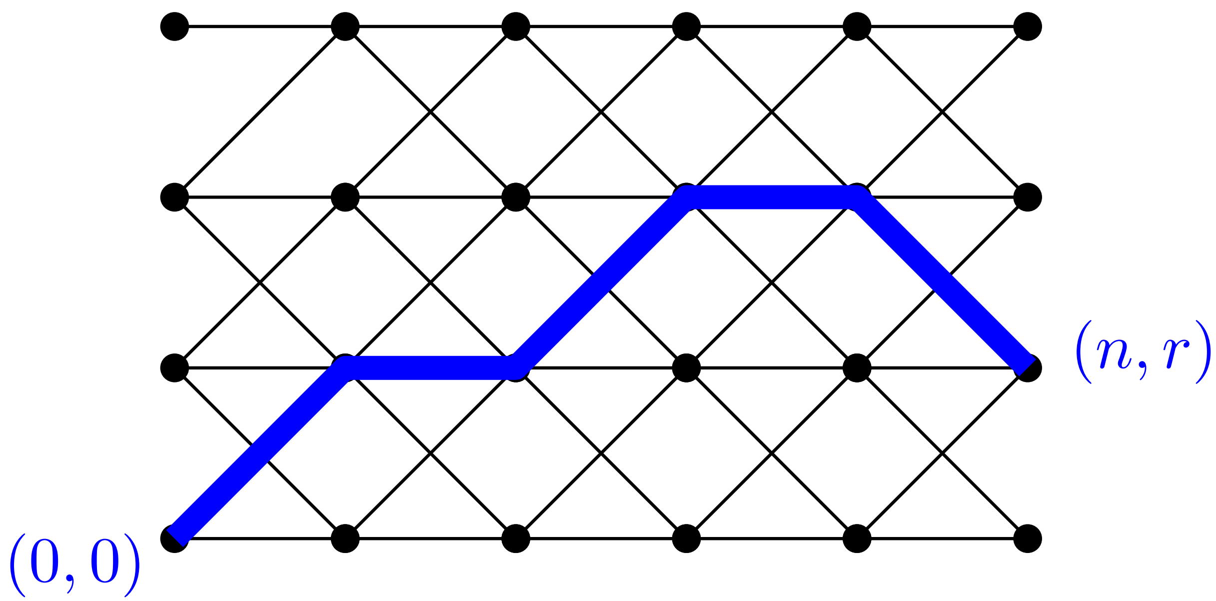
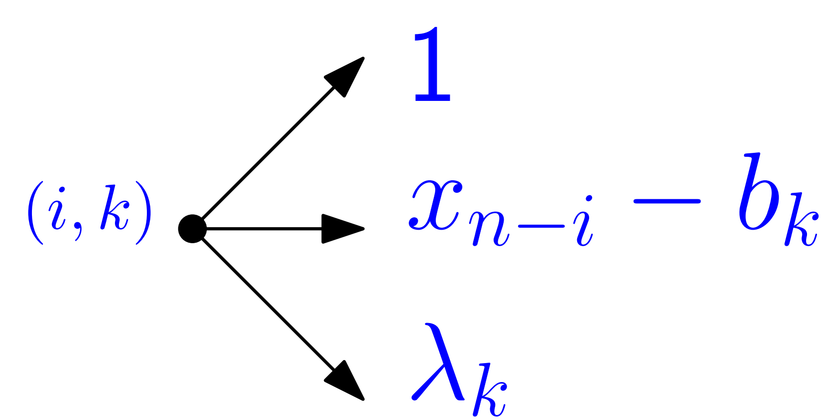


Arêtes sortantes pondérées



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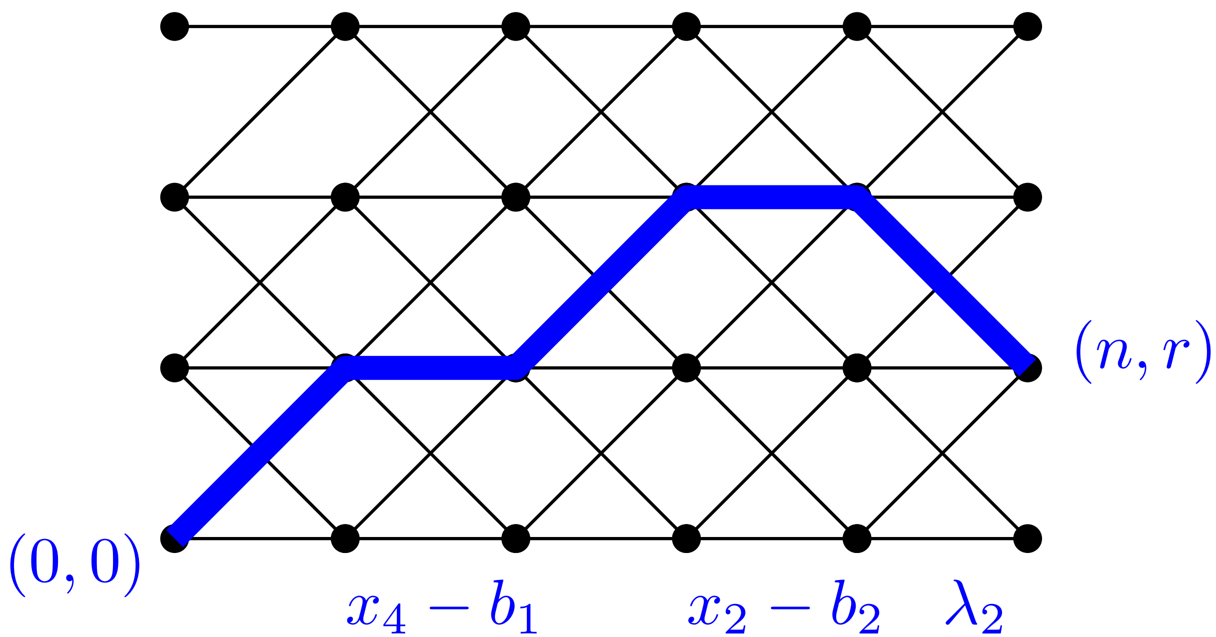

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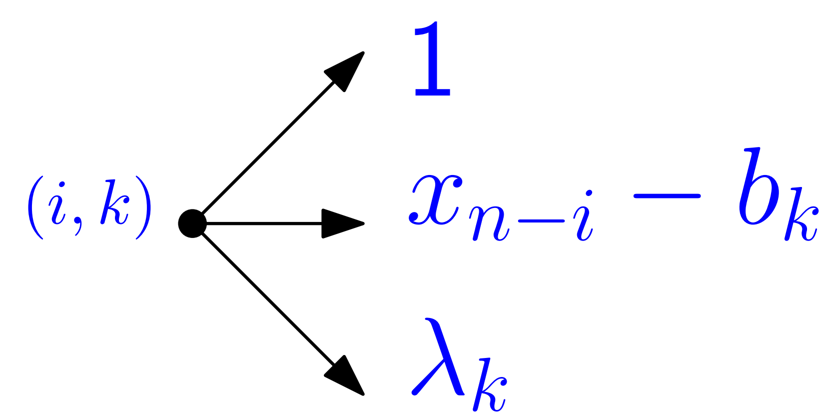

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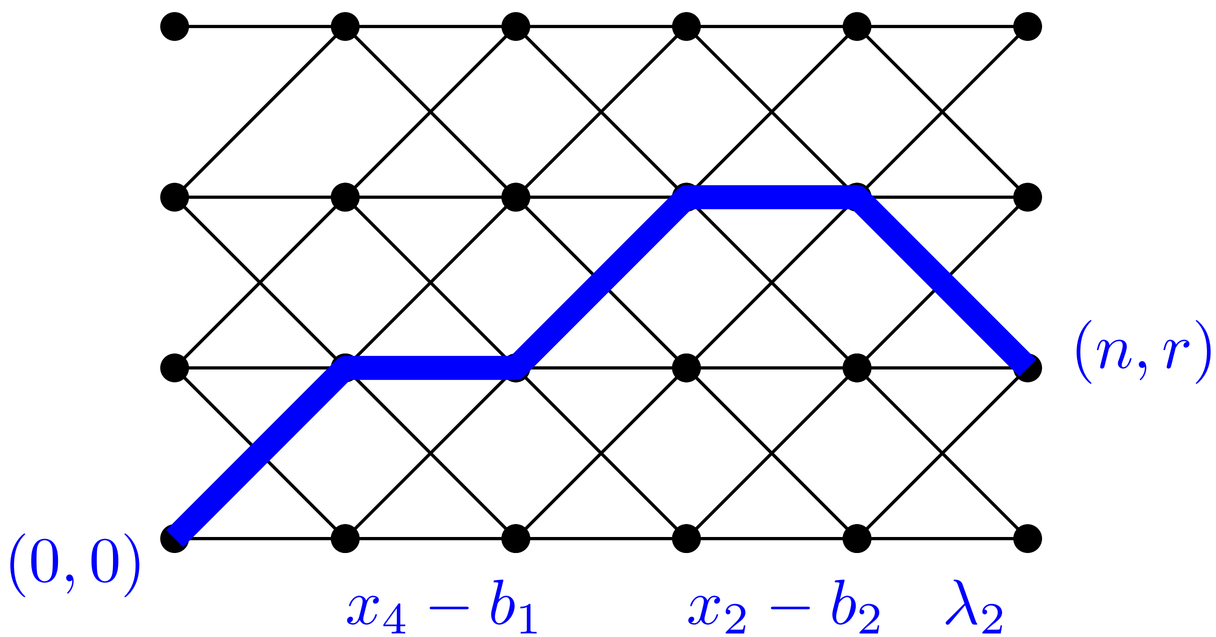

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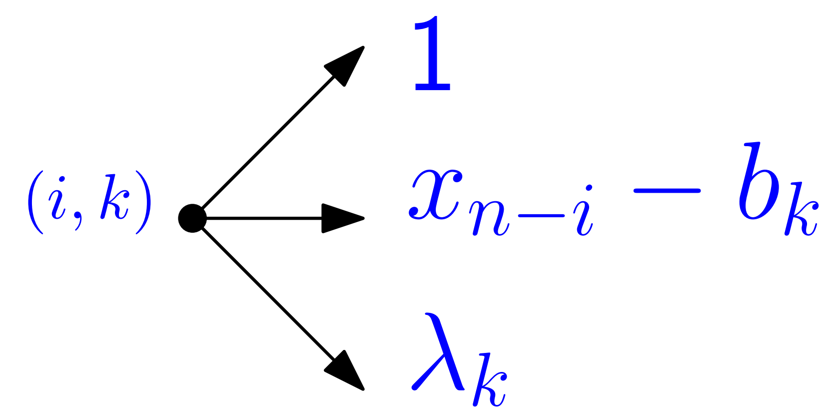
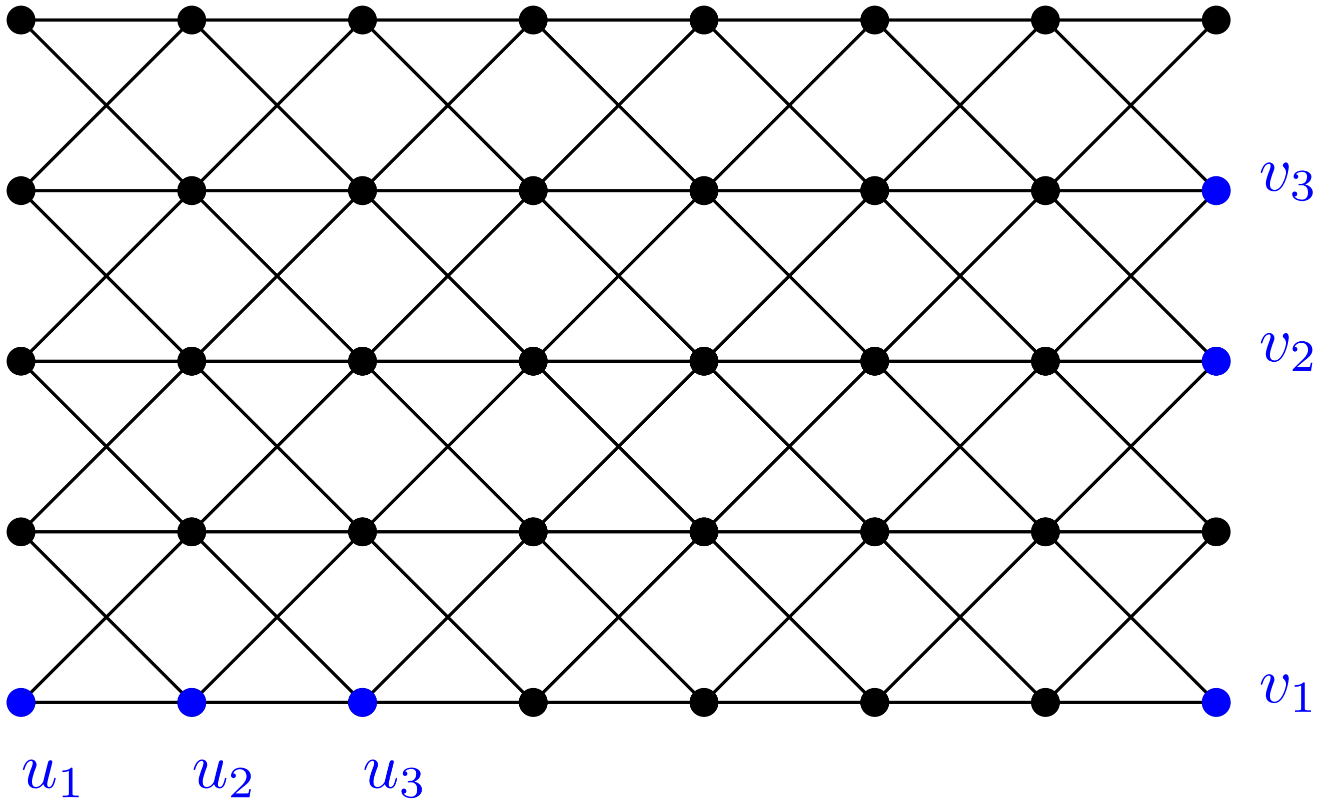


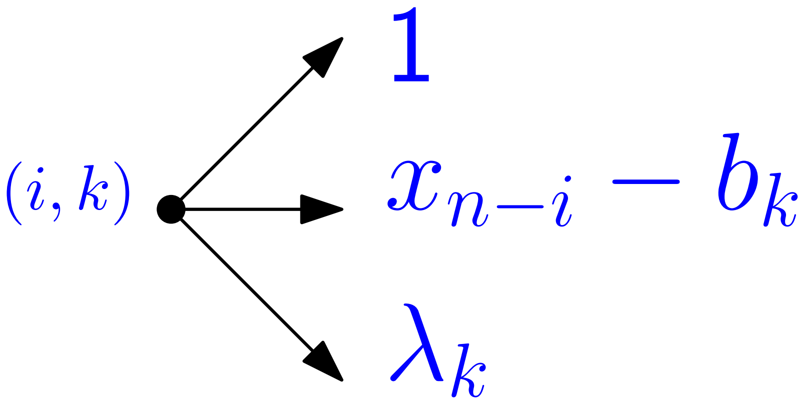
Remarque $b_k = \lambda_k = 0$ Cas classique

Combinatoire des polynômes de Koornwinder à $q = t$



$$u_i = (i-1, 0)$$

$$v_j = (n+m-1, \lambda'_{m+1-j} + j - 1)$$



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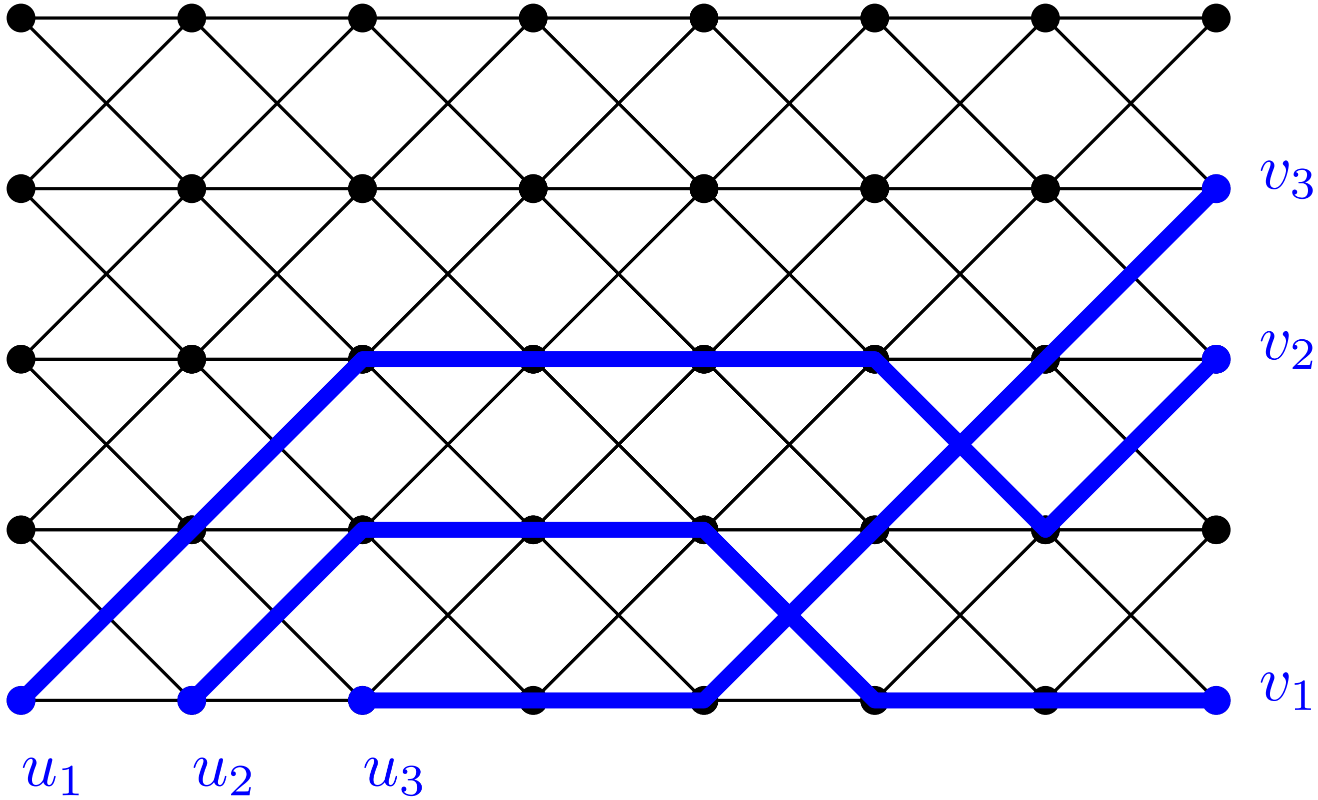

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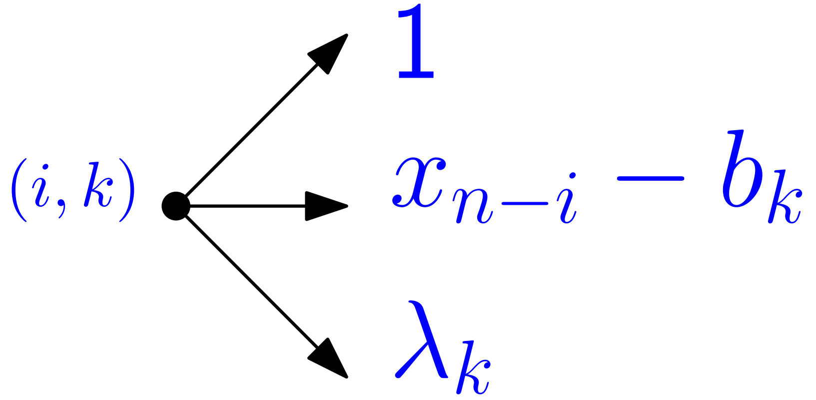
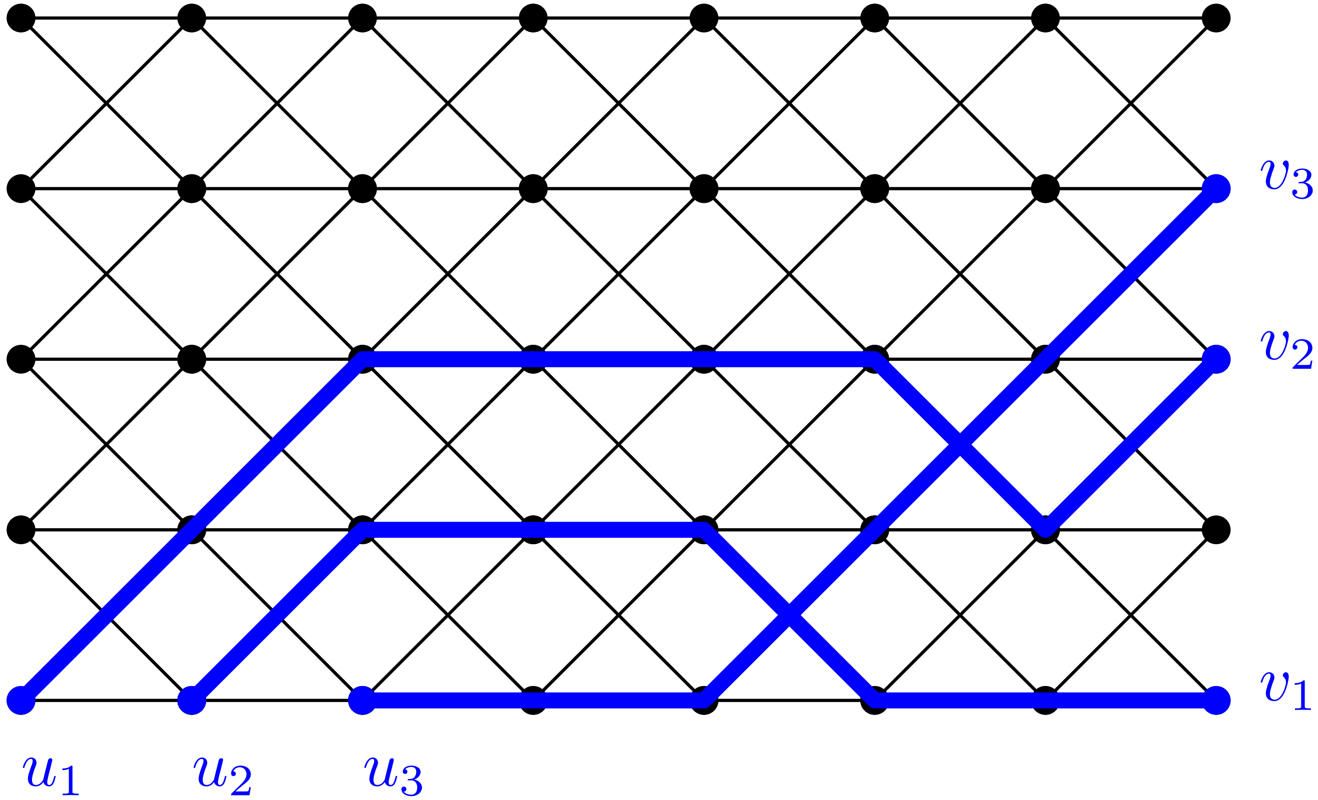


Théorème

P_λ: chemins non-intersectants de u_i à v_j

Combinatoire des polynômes de Koornwinder à $q = t$



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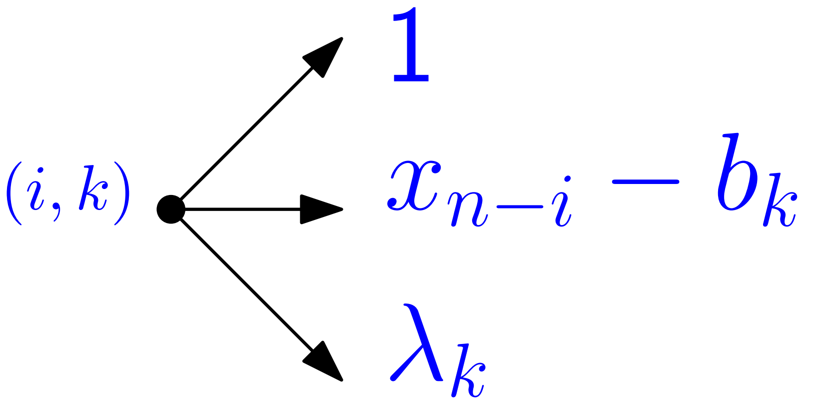


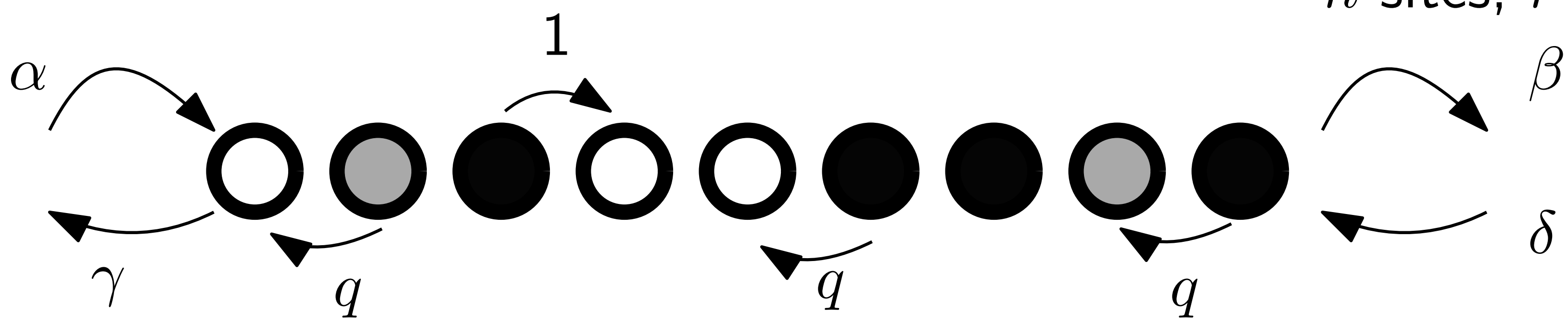
Théorème

P_λ: chemins non-intersectants de u_i à v_j

Si $b_k = \lambda_k = 0$ polynômes de Schur, tableaux semi-standards

5. Lien avec le 2-PASEP

n sites, r particules grises



- Les particules grises ne rentrent pas et ne sortent pas
- Les particules noires entrent (resp. sortent) par la gauche avec probabilité α (resp. γ)
- Les particules noires entrent (resp. sortent) par la droite avec probabilité β (resp. δ)

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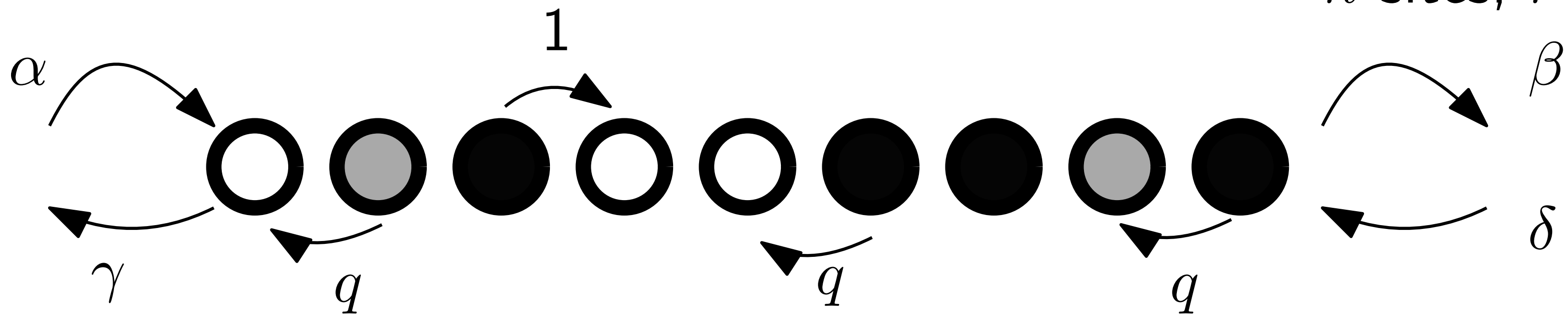

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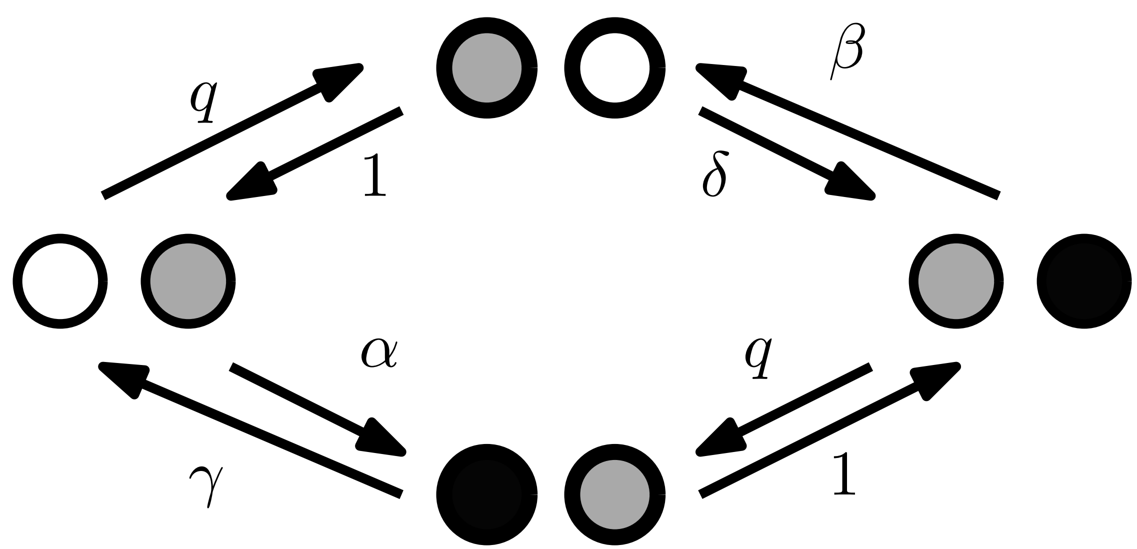

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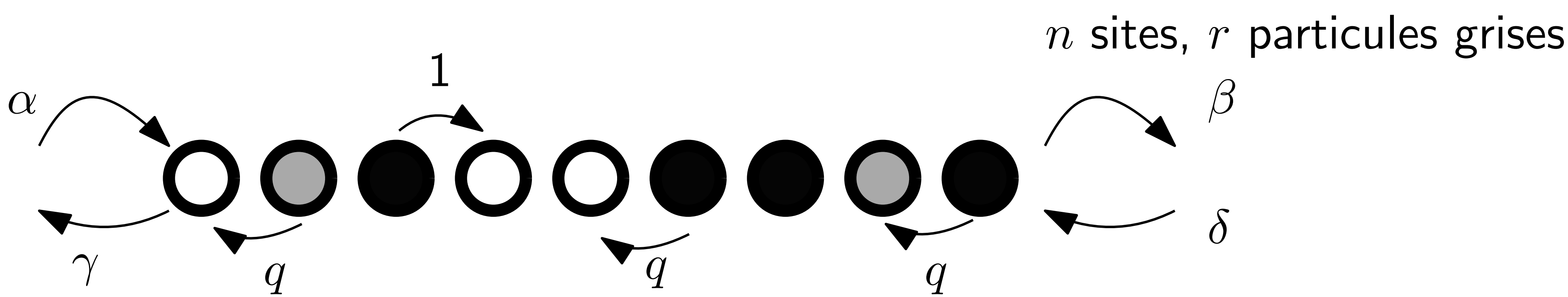

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Théorème (Cantini, C. Williams 2015)

La fonction de partition du 2-PASEP est $e_{n-r}^{(n)}(1,\ldots,1)$

Combinatoire des $e^{(n)}_{n-r}$

Proposition (C, Mandelshtam, Williams 15)

$$e^{(n)}_{n-r}(1,\ldots,1) = \frac{(1-q)^{n-r}}{2^{n-r}\prod_{i=0}^{n-1}(\alpha\beta-\gamma\delta q^i)} Z_{n,r}$$

avec $Z_{n,r}$ polynôme en $\alpha, \beta, \gamma, \delta, q$ à coefficients positifs

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Quand $q=1$

$$Z_{n,r}(\alpha,\beta,\gamma,\delta,1) = \binom{n}{r}\prod_{i=r}^{n-1}((\alpha+\gamma)(\beta+\delta)i+\alpha+\beta+\gamma+\delta)$$

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Rhombic staircase tableaux

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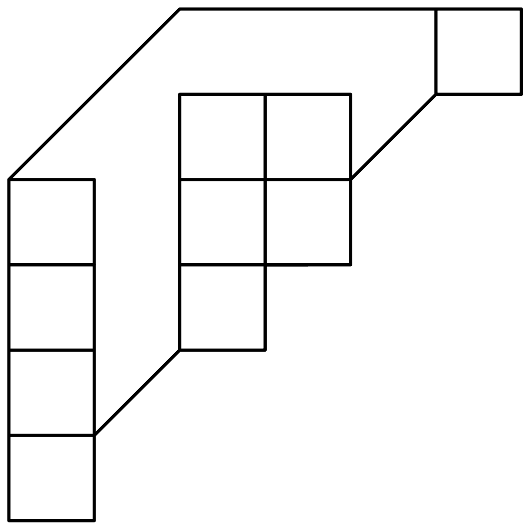
Rhombic staircase tableaux (C., Mandelshtam, Williams 15)



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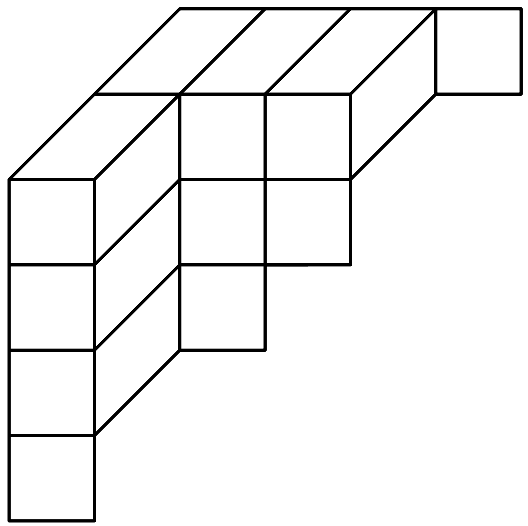

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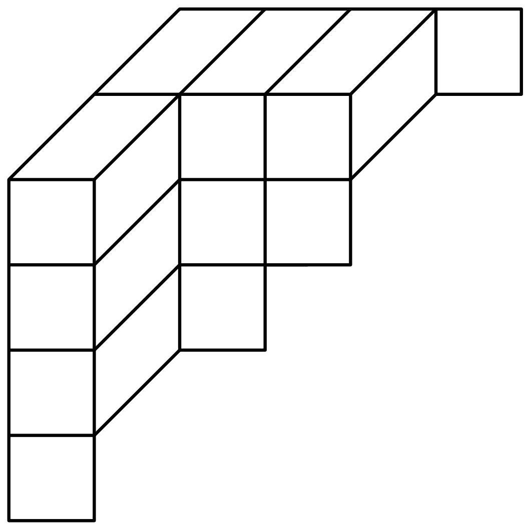

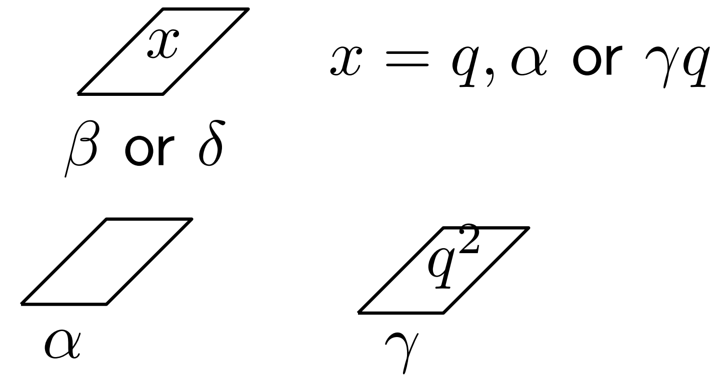

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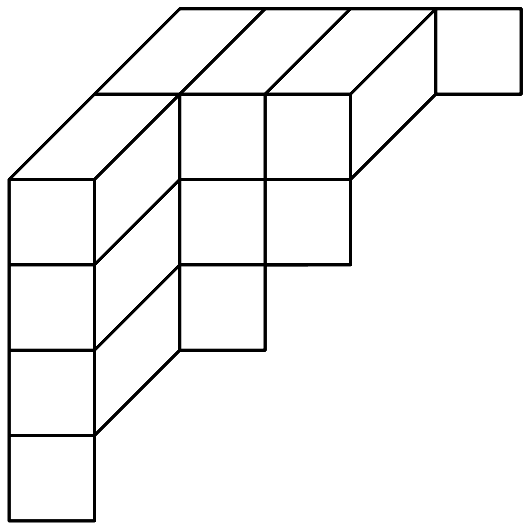

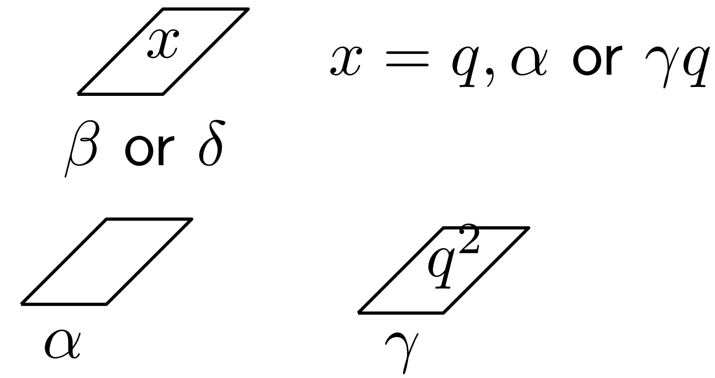

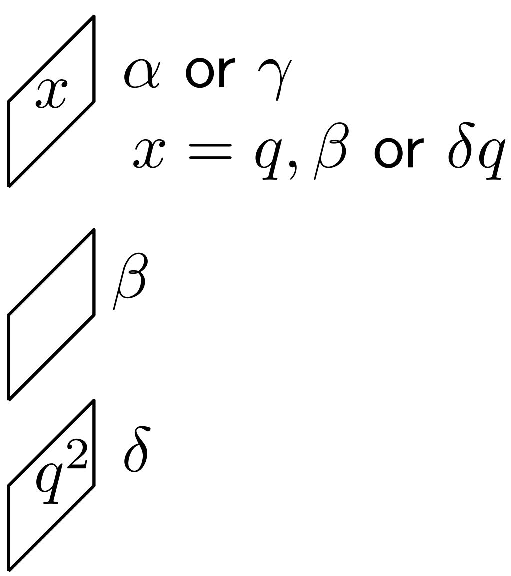

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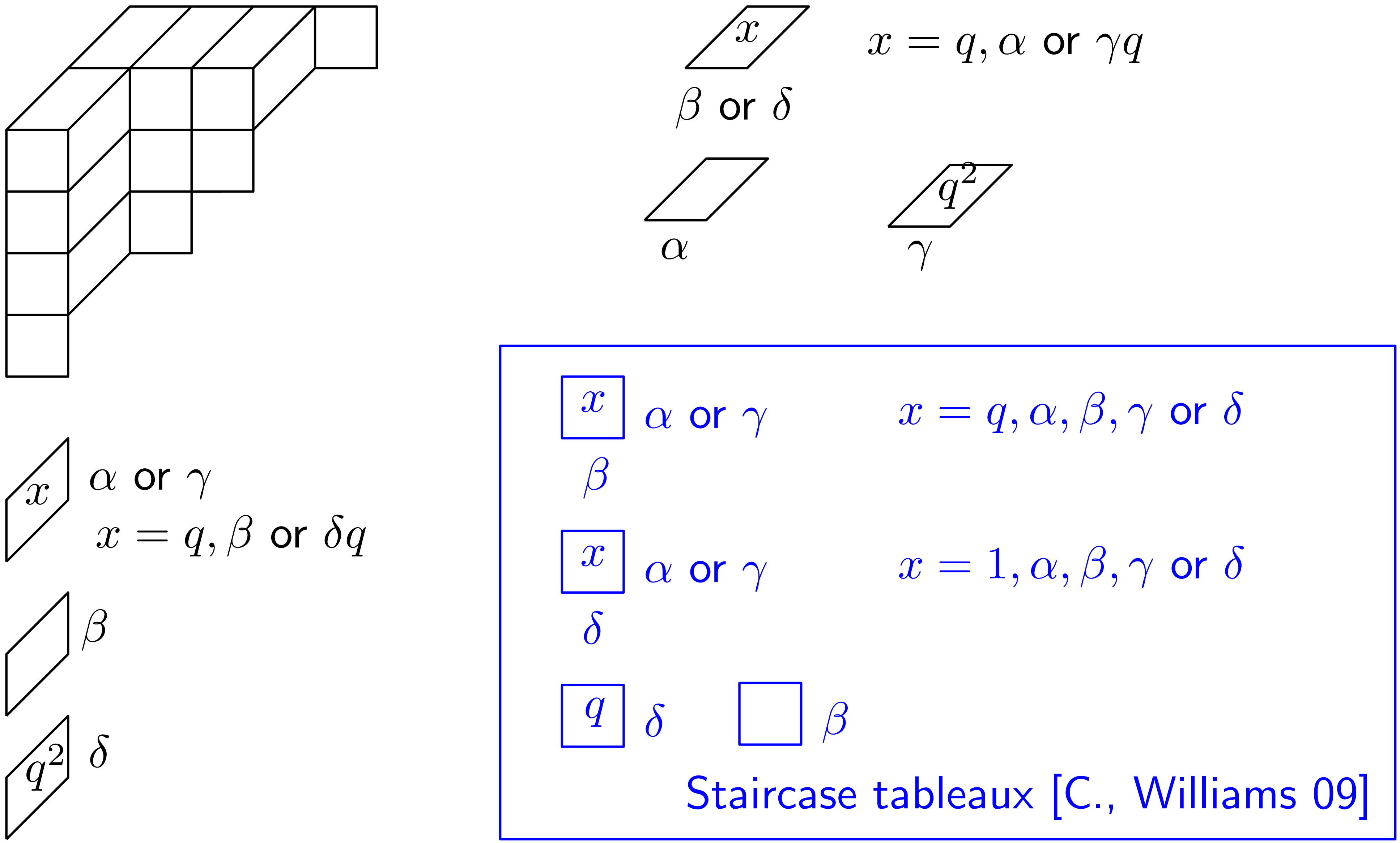

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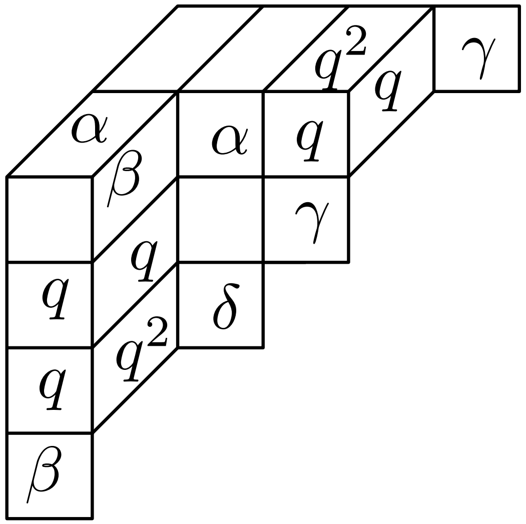
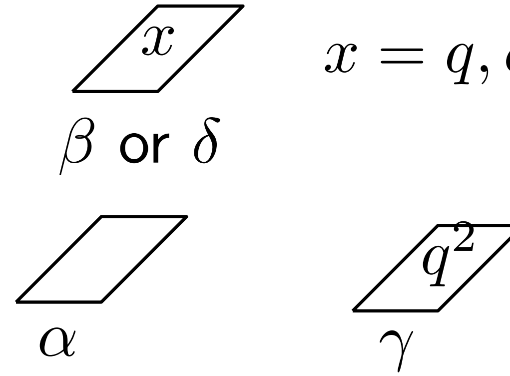
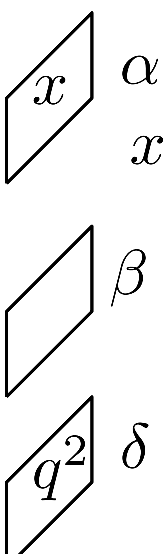




$x = q, \alpha$ or γq

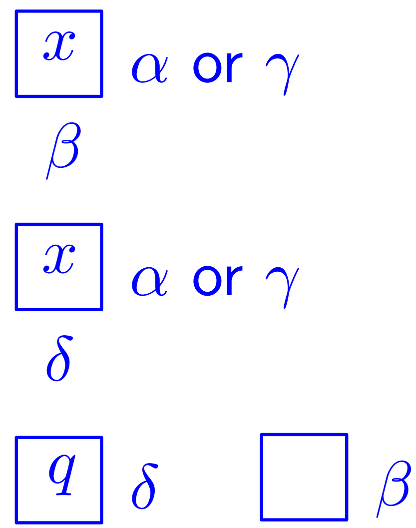


α or γ

$x = q, \beta$ or δq

β

δ



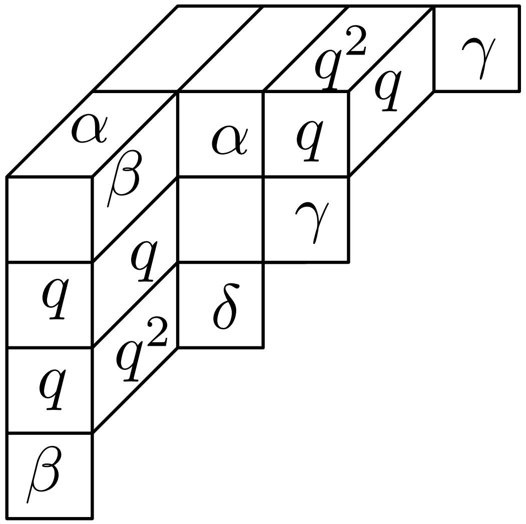
$x = q, \alpha, \beta, \gamma$ or δ

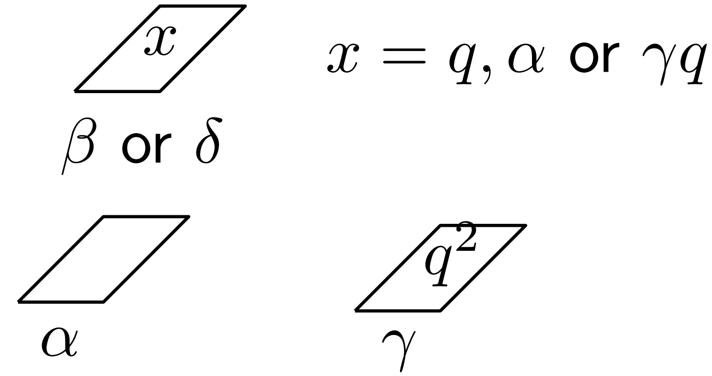
$x = 1, \alpha, \beta, \gamma$ or δ

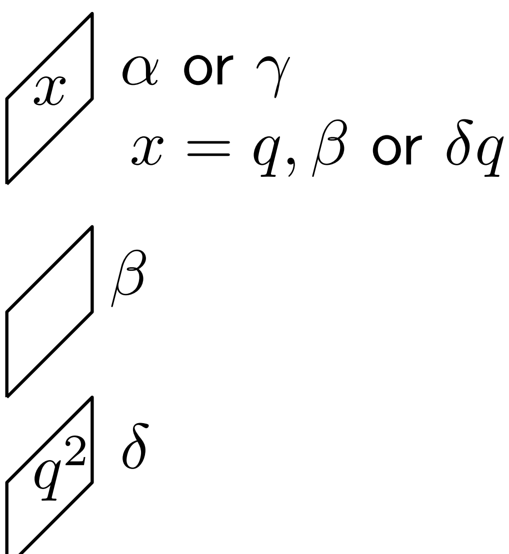
Staircase tableaux [C., Williams 09]

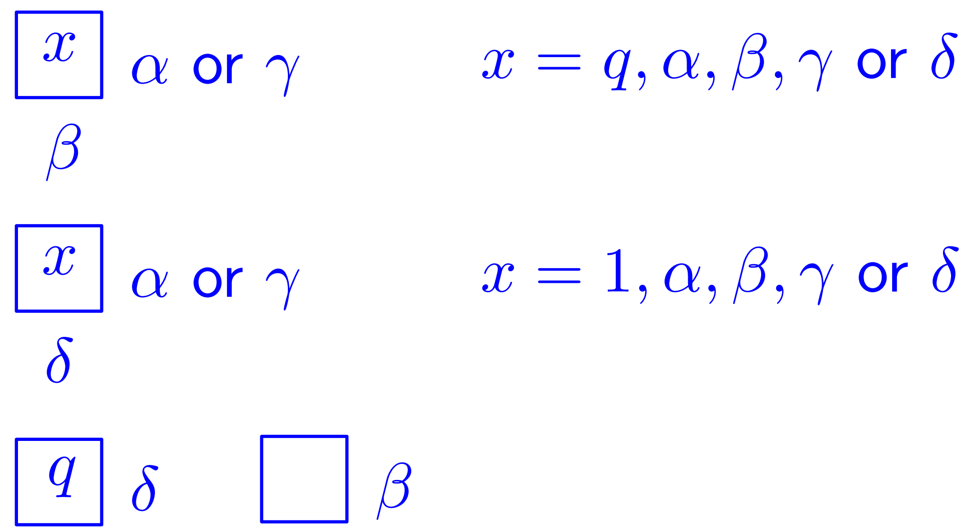
$Z_{n,r}$ tableaux avec $n-r$ coins et r pas diagonaux

Rhombic staircase tableaux (C., Mandelshtam, Williams 15)









Staircase tableaux [C., Williams 09]

$Z_{n,r}$ tableaux avec $n-r$ coins et r pas diagonaux

$\gamma = \delta = 0$: RATS (Mandelshtam, Viennot 2015)

6. Conjecture

$$\alpha = \frac{(1-1/q)ac}{(1-a)(1-c)} \qquad \beta = \frac{(1-1/q)bd}{(1-b)(1-d)} \qquad \gamma = \frac{1/q-1}{(1-a)(1-c)} \qquad \delta = \frac{1/q-1}{(1-b)(1-d)}$$

$$P_\lambda(1,\ldots,1) = \frac{(1-q)^{??}}{2^{??}\prod_i(\alpha\beta-\gamma\delta q^i)} Z_\lambda$$

avec Z_λ polynôme en $\alpha, \beta, \gamma, \delta, q$ à coefficients positifs

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Conjecture vraie pour $\lambda = (1^{n-r}, 0^r)$
Conjecture vraie pour $q = 1$
Conjecture vraie pour $\alpha = \beta = 1$, $\gamma = \delta = 0 \Rightarrow$ polynômes de q-Laguerre

Autre combinatoire du 2-PASEP

- Tableaux alternatifs rhombiques (Mandelshtam, Viennot 15)

  $$\gamma = \delta = 0$$

  Combinatoire des assemblées de permutation

  Forest like tableaux? (Aval, Boussicault, Nadeau 12)

- Histoires de Laguerre marquées (C., Nunge 16)

  $$\alpha = \beta = 1, \ \gamma = \delta = 0$$

  Combinatoire des permutations partiellement signées

Liens avec les algèbres combinatoires indicées par les compositions segmentées

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- Généralisation des polynômes de MacDonald
- Si $q = 1$ lien avec un modèle type PASEP (Cantini, de Gier et al 16)

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